Inhomogeneous Turbulent Dispersion across the Nearshore Induced by Surfzone Eddies

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ABSTRACT

In various oceanic regions, drifter-derived diffusivities reach a temporal maximum and subsequently decrease. Often, these are regions of inhomogeneous eddies, however, the effect of inhomogeneous turbulence on dispersion is poorly understood. The nearshore region (spanning from the surfzone to the inner shelf) also has strong cross-shore inhomogeneous turbulence. Nearshore Lagrangian statistics are estimated from drifter trajectories simulated with a wave-resolving two-dimensional Boussinesq model with random, normally incident, and directionally spread waves. The simulation is idealized and does not include other (wind, tidal, Coriolis) processes. The eddy field cross-shore inhomogeneity affects both the mean cross-shore drift and cross- and alongshore diffusivities. Short-time diffusivities are locally ballistic, and the mean drift is toward the eddy velocity variance maximum. The diffusivities reach a maximum and subsequently decrease, that is, are subdiffusive. The diffusivity maximum and time to maximum are parameterized taking into account the eddy field inhomogeneity. At long times, the cross- and alongshore diffusivities scale as $t^{-1/2}$ and $t^{-1/4}$, respectively, which is related to the offshore decay of the eddy intensity. A diffusion equation, with a space-dependent Fickian diffusivity related to the eddy velocity variance, reproduced the short-, intermediate-, and long-time behavior of the mean drift and cross-shore diffusivity. The small Middleton parameter, indicating fixed float dispersion, suggests the Eulerian time scale can parameterize the Lagrangian time scale in this region. Although this idealized simulation had no mean currents, and thus no shear dispersion or mixing suppression, inhomogeneous turbulence effects may be relevant in other regions such as the Antarctic Circumpolar Current (ACC) and western boundary current extensions.

1. Introduction

The effect of turbulence on scalar mixing is a fundamental problem across geophysical fluid dynamics. For example, the meridional overturning circulation depends on both small scale [$O(10^{-2})$ m] vertical mixing that influences stratification (e.g., Wunsch and Ferrari 2004), and large-scale (10–100 km) horizontal mixing, induced by mesoscale eddies, that is critical to the meridional heat flux (e.g., Marshall et al. 2006). Turbulent mixing also affects ocean anthropogenic contaminant evolution on both large (basin) length scales and multiyear time scales, such as radiation levels from the

Fukushima nuclear disaster (Nakano and Povinec 2012), and on small scales such as coastal wastewater plumes on $O(1)$ km length scales and hour time scales (e.g., Grant et al. 2005). Close to shore within the surfzone (region of depth-limited wave breaking), horizontal turbulence rapidly mixes tracer on $O(10)$ m length and $O(100)$ s time scales (Spydell and Feddersen 2009) and is critical to the exchange of heat, material, and passive biology between the surfzone and the inner shelf (e.g., Hally-Rosendahl et al. 2014; Morgan et al. 2018), the region denoted here as the nearshore.

Our conceptual understanding of turbulent scalar mixing is based mainly on idealized turbulence that is isotropic, homogeneous, and stationary. In this idealized setting, the eddy diffusivity K quantifies turbulent scalar mixing and is defined as the ensemble mean spreading rate

$$K(t) = \frac{1}{2} \frac{d}{dt} D^2(t), \quad (1)$$

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where $D^2(t)$ is the variance of drifter positions, or the second moment of tracer concentration, at time t after release. The eddy diffusivity has two important limits (Taylor 1922)

$$K(t) = \begin{cases} \mathcal{U}^2 t, & \text{for } t \ll T_L, \\ \mathcal{U}^2 T_L, & \text{for } t \gg T_L, \end{cases} \quad (2a)$$

$$(2b)$$

denoted ballistic and Brownian dispersion, respectively. Here, $\mathcal{U}^2$ is the eddy velocity variance, and T_L is the Lagrangian time scale: the decorrelation time of Lagrangian velocities. For long-time Brownian dispersion, the asymptotic diffusivity is $K^\infty \equiv \mathcal{U}^2 T_L$. Although K^∞ is fundamentally a Lagrangian flow property due to T_L , for isotropic and homogeneous turbulence, it can be related to Eulerian eddy time and length scales (Middleton 1985).

In many instances, particularly near boundaries, turbulent inhomogeneities must be accounted for when estimating turbulent transport and dispersion. Two examples where boundary-induced inhomogeneity affects dispersion are the atmospheric boundary layer (ABL) and turbulent channel flow. For the ABL, the mean flow, eddy velocity variance, and turbulent length scales depend on the vertical z , such that vertical variations in the eddy velocity and Lagrangian time scale must be accounted for to accurately model scalar dispersion (Wilson et al. 1981). Due to inhomogeneous turbulence and boundary effects, rather than approaching a constant K^∞ , the ABL vertical eddy diffusivity $K_z(t)$ decreases in time after reaching a maximum (Dosio et al. 2005) indicative of subdiffusive dispersion. A similar time dependence is observed for the cross-channel diffusivity for turbulent channel flow (Choi et al. 2004). Horizontal eddy diffusivity maxima with subsequent decrease often has been observed or modeled in a broad array of surfzone, coastal, to global oceanographic contexts (e.g., Dever et al. 1998; Lumpkin et al. 2002; Spydel and Feddersen 2012b; Klocker et al. 2012; Zhurbas et al. 2014). Recently, the diffusivity maximum and subsequent subdiffusion has been attributed to mixing suppression (e.g., Spydel 2016; Klocker et al. 2012) induced by the mean current and eddy propagation velocity difference (e.g., Ferrari and Nikurashin 2010). However, the difference between the maximum (in time) horizontal eddy diffusivity and asymptotic (in time) eddy diffusivity is often highest in regions of highest eddy kinetic energy and the role of inhomogeneous turbulence on the eddy diffusivity has not been explored.

The nearshore region, spanning the surfzone and inner shelf, has dramatically different dynamical regimes resulting in cross-shore inhomogeneous turbulence. The surfzone horizontal eddy field (vertical vorticity)

is stochastically forced by the depth-limited wave-breaking of finite crested waves (Peregrine 1998; Clark et al. 2012) resulting in a range of eddy time and length scales (Spydel and Feddersen 2009; Feddersen 2014). The eddy forcing is strongly controlled by the wave directional spread (Suanda and Feddersen 2015). In contrast, the inner shelf is, by definition, not breaking-wave eddy forced. Also inducing inhomogeneity (and anisotropy), the surfzone has a shoreline boundary, and the depth varies in the cross-shore inducing vortex stretching (e.g., Arthur 1962). These effects lead to a complex surfzone and inner-shelf eddy field (e.g., Suanda and Feddersen 2015) where horizontal eddy magnitudes can vary $O(1)$ over 2–3 times the surfzone width L_{sz} (Feddersen et al. 2011) spanning from the surfzone to inner shelf. As the surfzone width L_{sz} is similar to the length scale of the energy containing eddies, the assumption of homogeneous turbulence is strongly violated.

Despite the inhomogeneities, surfzone diffusivities have generally been estimated for the entire surfzone—mostly due to the limitations of the observations—tacitly assuming cross-shore homogeneity. Dye tracer has been used to estimate surfzone cross-shore diffusivity from dye moments integrated across the surfzone (Clark et al. 2010). Drifter-derived time-dependent diffusivities $K_x(t)$ have been calculated for $\leq 10^3$ s with estimated Lagrangian time scales T_L of 100–200 s (Spydel et al. 2009; Brown et al. 2009; Spydel and Feddersen 2012b). Drifter-derived alongshore (y) diffusivities $K_y(t)$ increase with the strength of the mean alongshore current (Spydel and Feddersen 2012b) due to shear dispersion that takes into account T_L (Spydel and Feddersen 2012a). In Spydel et al. (2007) and Brown et al. (2009), separate diffusivities were calculated in the regions within and seaward of the surfzone. However, the binning was too coarse with insufficient statistical reliability to illuminate inhomogeneous turbulence effects. In Spydel and Feddersen (2012b), explicit shoreline effects were considered, but otherwise, the turbulence was assumed homogeneous. The relationship between surfzone-averaged Lagrangian and Eulerian time scales was explored in Spydel et al. (2014). In these previous studies, cross-shore homogeneity is assumed because the number (between 10 and 30), and trajectory-length ($\leq 10^3$ s), of drifter observations were insufficient to estimate the cross-shore dependence of $K(t)$ or other Lagrangian statistics with any accuracy (Spydel et al. 2007; Brown et al. 2009; Spydel et al. 2009). Furthermore, because the diffusivity estimate error grows in time (e.g., Davis 1991), whether the surfzone cross-shore diffusivity approaches a constant Brownian regime is unclear (e.g., Spydel and Feddersen 2012b).

Thus, two important and inter-related questions remain regarding diffusion in the inhomogeneous turbulence spanning from the surfzone to inner shelf. One, why is there a diffusivity maxima with subsequent decrease and how can it be scaled? Two, how does inhomogeneous turbulence affect long-time dispersion spanning the surfzone and the inner shelf? Here, dispersion spanning the inhomogeneous turbulence of the surfzone to inner shelf is studied using idealized modeled trajectories 10 times longer than previously considered and many more trajectories than observationally possible. This allows the cross-shore structure of surfzone to inner-shelf particle transport and dispersion to be quantified in detail.

Drifters are tracked in an idealized but realistic nearshore simulation using the wave-resolving two-dimensional (2D) funwaveC Boussinesq model with random normally incident but directionally spread waves. Modeled drifter trajectories are used to calculate time- and release-location-dependent Lagrangian statistics (section 2). Wind, tidal, stratification, Coriolis, and other processes are neglected. The simulation had no mean flows but strongly cross-shore inhomogeneous turbulence. The time- and cross-shore release location dependence of the modeled cross-shore mean drift and cross- and alongshore diffusivities are presented in section 3. The short-time, intermediate-time diffusivity maxima, and long-time behavior of the mean drift and diffusivities are scaled in section 4. The ability of a Fickian diffusion equation, with a time- and cross-shore-dependent Fickian diffusivity, to represent the modeled mean drift and cross-shore diffusivity is examined in section 5. In the discussion (section 6), Lagrangian and Eulerian statistics are compared and the applicability of the results to other regions with inhomogeneous eddy fields is discussed. Results are summarized in section 7.

2. Model setup and definitions

a. The model

The nearshore Eulerian and Lagrangian statistics analyzed here are based on a single idealized simulation of the 2D Boussinesq model funwaveC (Feddersen et al. 2011). Time-dependent Boussinesq model equations are similar to the nonlinear shallow water equations, but include higher-order dispersive terms (and in some derivations higher-order nonlinear terms). The funwaveC model solves the finite difference approximations of the Boussinesq mass and momentum equations with nonlinear and dispersive effects (Nwogu 1993). Boussinesq models have been validated for laboratory waves (Shi et al. 2012), nearshore field observations

from surfzone to 200 m offshore of the surfzone (Feddersen et al. 2011; Feddersen 2014), observed nearshore drifters (Spydell and Feddersen 2009), observed surfzone dye plumes (Clark et al. 2011), and inner-shelf dye plumes from rip current ejections (Hally-Rosendahl and Feddersen 2016). Depth-limited wave breaking is parameterized with an eddy viscosity (Kennedy et al. 2000), which together with quadratic bottom friction and a surfzone-appropriate drag coefficient $c_d = 0.0023$ (Feddersen et al. 2011), provides energy dissipation. Wave run up is implemented with the “thin-layer” method (Salmon 2002). An offshore sponge layer ensures outgoing wave energy is not reflected.

For this idealized simulation, the alongshore uniform planar beach has bathymetry $h(x) = -sx$, where x is the cross-shore coordinate increasing onshore with shoreline at $x = 0$, and slope $s = 0.03$ typical of many beaches (Fig. 1a). At a depth of $h = 9$ m (at $x = -300$ m), the bathymetry is constant farther offshore ($x < -300$ m). The cross- and alongshore model domain size is $(L_x, L_y) = (627, 1600)$ m, with grid resolution $(\Delta x, \Delta y) = (1, 1.33)$ m, and is periodic in the alongshore direction. In $h = 9$ m water depth (at $x = -444$ m), the source function method (Wei et al. 1999; Suanda et al. 2016) generates random directionally spread waves with a Pierson–Moskowitz spectrum, significant wave height $H_s = 0.8$ m, peak period $T_p = 8$ s, and mean-normal incidence $\bar{\theta} = 0$. The $H_s(x)$ increases as waves shoal and then decreases through the surfzone due to breaking wave dissipation (Fig. 1b). The cross-shore location of maximum $H_s(x)$, defines an offshore limit of the surfzone $x_{sz} = -90$ m. At x_{sz} , the directional spread (Kuik et al. 1988) is $\sigma_\theta = 10^\circ$. This idealized simulation is forced only by waves and does not include wind, tidal, or other forcing. The model is run for 20 000 s and model data is output at 1 Hz.

In the nearshore, irrotational orbital velocities of surface gravity waves are usually larger than the rotational (nonzero vertical vorticity) motions associated with horizontal eddies, particularly offshore of the surfzone where irrotational velocities can be an order of magnitude larger than rotational velocities. However, nearshore horizontal eddies dominate dispersion (Spydell et al. 2007; Spydell and Feddersen 2009). These rotational motions are surfzone generated by depth limited wave breaking of finite crested waves (Peregrine 1998; Clark et al. 2012). Because depth-limited breaking occurs in the surfzone, eddy variance is concentrated there and weakens offshore. Because eddies and not surface gravity waves are responsible for dispersion, scalings will use eddy (rotational) velocity statistics. Model velocities $\mathbf{u}_m = (u_m, v_m)$ are separated into rotational, the velocities associated with vertical vorticity, and irrotational components,

FIG. 1. (a) Depth h , (b) significant wave height H_s , (c) cross-shore eddy velocity variance U^2 (black curve), and (d) alongshore eddy velocity variance V^2 (black curve) versus cross-shore distance x . In (c) and (d), analytic forms for the eddy velocity variance (U_{an}^2 and V_{an}^2) are shown as dashed red lines. The shoreline is at $x = 0$, and the cross-shore location of the maximum cross-shore $x_{U_{mx}}$ and alongshore eddy velocity $x_{V_{mx}}$ is indicated (gray vertical lines). The gray dash-dotted vertical line denotes the offshore limit of the surfzone x_{sz} , defined as the x location of maximum H_s .

$$\mathbf{u}_m = \nabla \times \psi \mathbf{k} + \nabla \phi. \quad (3)$$

The rotational, or eddy velocities, $\mathbf{u}$ are found from the stream function ψ

$$\mathbf{u} = \nabla \times \psi \mathbf{k}, \quad (4)$$

which is obtained from the vorticity $\nabla^2 \psi = \zeta$, where $\zeta = (\nabla \times \mathbf{u}_m) \cdot \mathbf{k}$. Note, rotational velocities are not necessarily nondivergent in the shallow-water continuity equation. For >100 s, irrotational velocities $\nabla \phi$ do not contribute to surfzone particle dispersion (Spydell et al. 2007; Spydell and Feddersen 2009). For the normally incident waves simulated here, the time-mean cross- and alongshore eddy velocity $(\langle u \rangle, \langle v \rangle)$ are statistically indistinguishable from zero. At mid-surfzone, the alongshore variation in the time mean of u and v is

weak ($1\text{--}3 \text{ cm s}^{-1}$) and also statistically indistinguishable from zero (i.e., alongshore uniform). The cross-shore-dependent Eulerian cross- and alongshore eddy velocity variances $[U^2(x), V^2(x)]$ are

$$(U^2, V^2) = (\langle u^2 \rangle, \langle v^2 \rangle)$$

where $\langle \rangle$ denotes time- and alongshore averaging.

From mid-surfzone to offshore of the surfzone, cross-shore $U^2(x)$ and alongshore $V^2(x)$ eddy velocity variances are similar and vary from about $0.015 \text{ m}^2 \text{ s}^{-2}$ at $x = -50 \text{ m}$ to $0.001 \text{ m}^2 \text{ s}^{-2}$ at $x = -140 \text{ m}$ (Figs. 1c,d). The eddy velocity standard deviation (U and V) exceeds 0.01 m s^{-1} out to $x = -270 \text{ m}$ or $3L_{sz}$. Onshore of $x = -50 \text{ m}$, V^2 is larger than U^2 , due to the shoreline limiting cross-shore velocities, and both decrease. The maximum cross-shore (U_{mx}^2) and alongshore eddy velocity variance (V_{mx}^2) occur at $x_{U_{mx}} = -40 \text{ m}$ and $x_{V_{mx}} = -20 \text{ m}$, respectively (Figs. 1c,d). The modeled eddy velocity variance cross-shore structure is qualitatively similar to observations (e.g., Lippmann et al. 1999; Noyes et al. 2004; MacMahan et al. 2010; Feddersen et al. 2011; Feddersen 2014).

An analytic form for $U^2(x)$ and $V^2(x)$ is used in later analysis and is chosen to be 1) simple with three parameters, 2) reasonably match both U^2 and V^2 from shoreline to offshore of the surfzone, and 3) have a power-law scaling at large $|x|$. The analytic $U_{an}^2(x)$ is

$$U_{an}^2(x) = \frac{U_{mx}^2}{1 + \left[\frac{(x - x_{U_{mx}})}{l_x} \right]^2}. \quad (5)$$

with $U_{mx}^2 = 0.015 \text{ m}^2 \text{ s}^{-2}$, and $l_x = 38 \text{ m}$ (approximately $|x_{U_{mx}}|$) is the cross-shore length-scale over which the turbulence varies. This analytic form $U_{an}^2(x)$ generally reproduces $U^2(x)$ from shoreline to offshore of the surfzone (dashed red curve in Fig. 1c), although it does not decay rapidly enough onshore of $x_{U_{mx}}$ and slightly overestimates U^2 offshore of x_{sz} . The $V^2(x)$ is also generally well-represented by the same analytic form but with different parameters (Fig. 1d). In the region offshore of the U^2 (or V^2) maximum where its gradient is large, the best-fit integer exponent in the denominator of (5) is two.

b. Drifter simulations

As the eddy (rotational) motions velocities dominate nearshore dispersion (e.g., Spydell and Feddersen 2009), observational nearshore drifters studies typically wave-average drifter trajectories to filter out surface gravity waves (Spydell et al. 2009, 2014) to link observed Lagrangian statistics to Eulerian eddy statistics. Similarly, here virtual drifters tracks $\mathbf{X}(t)$ are determined by advection from the eddy velocities $\mathbf{u}$,

FIG. 2. Virtual drifter positions (colored dots) for one of the 53 realizations at various times (indicated above each panel). Drifters are colored by the initial cross-shore position X_0 : offshore of the surfzone (blue), outer edge of the surfzone (green), and near the shoreline (red). The gray dash-dotted vertical line indicates the surfzone boundary x_{sz} . The background color is the instantaneous vorticity.

$$\mathbf{X}(t) = \mathbf{X}_0 + \int_0^t \mathbf{u}[\mathbf{X}(t'), t'] dt', \quad (6)$$

where $\mathbf{X}_0 = (X_0, Y_0)$ is the initial drifter position. Diffusivities derived from advection by the eddy or the full (rotational plus irrotational) velocities are very similar at time scales > 30 s, hence, eddy and low-passed (removing the waves) full velocities are similar. A fourth-order Runge–Kutta algorithm is used to integrate (6) with $\mathbf{u}(x, y, t)$ fields sampled at 1 Hz and with $\mathbf{u}(x, y, t')$ linearly interpolated to drifter positions $\mathbf{X}(t')$. To ensure stationarity, the model is spun up for 2000 s before virtual drifters are released over $-150 \leq x \leq 0$ and for all y on a regular grid of 31 drifters $\times$ 100 drifters (corresponding to $\Delta x = 5$ m and $\Delta y = 16$ m) in the cross- and alongshore directions, respectively. Every 250 s after the first release of 3100 drifters, 3100 additional drifters are released on the same regular grid. Each release is quasi-independent as the eddy Eulerian decorrelation time is ≈ 250 s, resulting in a total of 53 quasi-independent releases of 3100 drifters. The resulting drifter trajectories have duration from 5000 to 1.8×10^4 s.

For one of the drifter realizations, the drifter time evolution reveals a complex stirring field spanning from the surfzone to the inner shelf (Fig. 2). Drifters are initially released on a regular grid at $t = 0$ s (Fig. 2a). At $t = 100$ s (Fig. 2b), drifters released shoreward of $x \approx x_{U_{mx}}$ (red dots) have been noticeably stirred by the surfzone eddies whereas inner-shelf released drifters (blue dots) have barely moved. At $t = 250$ s (Fig. 2c), drifters released near the shoreline (red dots), sampling strong eddies (vorticity, see $y \approx 325, 550, 750$ m), have been strongly stirred to the surfzone boundary (gray dashed line, $x_{sz} = -90$ m) whereas inner-shelf released drifters are only somewhat distorted from their release pattern. At $t = 1000$ s, strong stirring is evident for surfzone-released drifters (red dots are seaward of yellow dots in Fig. 2d), but inner-shelf released drifters (blue dots) have only been weakly stirred. At $t = 4000$ s (Fig. 2e), surfzone released drifters are fully stirred within the surfzone, but only about half of the inner-shelf released drifters (blue dots) have entered the surfzone. Lagrangian statistics based on these trajectories quantify the cross-shore variability of the dispersion evident in Fig. 2.

c. Definition of Lagrangian statistics

Lagrangian statistics that quantify scalar transport and mixing are estimated from the drifter trajectories. The mean cross-shore trajectory (i.e., center of mass location) for drifters released from X_0 is given by

$$\bar{X}(t, X_0) = \langle X(t, X_0) \rangle, \quad (7)$$

where t denotes time from release, the mean $\langle \rangle$ is over all 100 drifters released from X_0 and over the 53 quasi-independent realizations. The mean drift velocity $\bar{U}$ is

$$\bar{U}(t, X_0) = \frac{d\bar{X}(t, X_0)}{dt}, \quad (8)$$

and equivalent to the average velocity over all drifters released from X_0 . The cross-shore dispersion is defined as

$$D_x^2(t, X_0) = \langle [X(t, X_0)]^2 \rangle - [\bar{X}(t, X_0)]^2, \quad (9)$$

and represents the variance of drifter positions about their mean position $\bar{X}(t, X_0)$ for all drifters released from X_0 . The Lagrangian velocity variance $\mathcal{U}_L^2$ is defined similarly

$$\mathcal{U}_L^2(t, X_0) = \langle [U(t, X_0)]^2 \rangle - [\bar{U}(t, X_0)]^2, \quad (10)$$

and is equal to the Eulerian velocity variance only initially, $\mathcal{U}_L^2(t=0, X_0) = \mathcal{U}^2(X_0)$, before drifters have dispersed from X_0 . For homogeneous turbulence, $\mathcal{U}_L^2$ does not depend on t or X_0 and is equal to the Eulerian velocity variance $\mathcal{U}^2$. The cross-shore eddy diffusivity $K_x(t, X_0)$ is related to the dispersion via

$$K_x(t, X_0) = \frac{1}{2} \frac{d}{dt} D_x^2(t, X_0). \quad (11)$$

and for isotropic and homogeneous turbulence is equivalent to (1). The alongshore dispersion $D_y^2(t, X_0)$, Lagrangian velocity variance $\mathcal{V}_L^2(t, X_0)$, and diffusivity $K_y(t, X_0)$ are similarly defined. Time derivatives are estimated as simple Euler forward differences. Note, that with the releases staggered by 250 s, the number of realizations used in the mean (and thus degrees of freedom) decreases at $t > 5000$ s. Because estimates of K using (11) are notoriously noisy (e.g., [Spydell et al. 2009](#); [LaCasce et al. 2014](#)), high-frequency noise in $K_x(t, X_0)$ and $K_y(t, X_0)$ [and $\bar{U}(t)$] is removed by time-smoothing $K_i(t)$ [and $\bar{U}(t)$] using a boxcar filter centered on t where the filter length increases linearly with time (the total length is equal to $0.4t$), accounting for the long-time decrease in degrees of freedom.

Lagrangian statistics are estimated for times up to 12 000 s (or 3.3 h). Other processes with tidal or Coriolis time scales can be important to drifter dispersion at longer time scales.

3. Lagrangian statistics

The release-location-dependent mean cross-shore trajectory $\bar{X}(t, X_0)$ and drift velocity $\bar{U}(t, X_0)$ are dominated by a few features ([Figs. 3a,b](#)). The first is the initial drift towards the location of the $\mathcal{U}^2$ maximum $x_{\mathcal{U}_{\max}} = -40$ m (gray line in [Fig. 3a](#)), that is shoreward ($\bar{U} > 0$) for $x < x_{\mathcal{U}_{\max}}$ m and offshore ($\bar{U} < 0$) for $x > x_{\mathcal{U}_{\max}}$ m ([Fig. 3b](#)). Onshore of $x_{\mathcal{U}_{\max}}$, the $\bar{U}$ decreases rapidly until reaching a relatively large minimum from -0.02 to -0.035 m s^{-1} at 100–300 s (maximum offshore velocity, open circles on red curves in [Fig. 3b](#)), with a generally deeper and earlier minimum for X_0 closer to the shore. Offshore of $x_{\mathcal{U}_{\max}}$, the $\bar{U}$ increases to a maximum (0.005 – 0.021 m s^{-1}) that generally is weaker and occurs later for larger X_0 . Subsequently, the drift velocity magnitude $|\bar{U}|$ decays with time. For $-125 < X_0 < x_{\mathcal{U}_{\max}}$ m, the onshore $\bar{U}$ transitions to offshore drift, that is, $\bar{U}(t)$ passes through 0, with the transition time occurring later for more offshore X_0 ([Fig. 3a](#)). For example, just offshore of $x_{\mathcal{U}_{\max}}$ ($X_0 = -55$ m, yellow curve [Figs. 3a,b](#)), the transition occurs at ≈ 400 s, whereas just offshore of the surfzone ($X_0 = -95$ m, cyan), the transition occurs at ≈ 3000 s. At long times, the mean drift velocities $\bar{U}$ collapse for all X_0 .

The cross- and alongshore Lagrangian velocity variances $[\mathcal{U}_L^2(t, X_0), \mathcal{V}_L^2(t, X_0)]$ are time dependent ([Figs. 3c,d](#)) due to drifters sampling varying eddy velocities as they disperse from X_0 . At very short times ($t \leq 20$ s), the cross- and alongshore Lagrangian and Eulerian velocity variance are very similar $\mathcal{U}_L^2(t, X_0) \simeq \mathcal{U}_i^2(X_0)$ as the drifters have not yet dispersed and the $\mathcal{U}_L^2$ are essentially constant in time. The time when $\mathcal{U}_L^2$ and $\mathcal{V}_L^2$ become time-varying depends on the distance between X_0 and $x_{\mathcal{U}_{\max}}$ or $x_{\mathcal{V}_{\max}}$ ([Figs. 3c,d](#)). For example, for a near-shoreline ($X_0 = 5$ m) release, $\mathcal{U}_L^2$ increases at approximately 30 s, reaching a maximum at about 600 s, that is, approximately 3 times the initial value (reddest curve, [Fig. 3c](#)). For the farthest offshore released drifters ($X_0 = -145$ m), $\mathcal{U}_L^2$ and $\mathcal{V}_L^2$ start increasing at about 1000 s to values approximately 6 times their initial values. For release locations onshore and offshore of $x_{\mathcal{U}_{\max}}$, $\mathcal{U}_L^2(t, X_0)$ increases from its initial value ([Fig. 3c](#)), and $\mathcal{V}_L^2(t, X_0)$ also similarly increases onshore and offshore of $x_{\mathcal{V}_{\max}}$ ([Fig. 3d](#)). The $\mathcal{U}_L^2$ and $\mathcal{V}_L^2$ differ because the Eulerian cross-shore $\mathcal{U}(X_0)$ and alongshore $\mathcal{V}(X_0)$ eddy velocities have different cross-shore structure ([Fig. 1c](#)). At long times, $\mathcal{U}_L^2$ (as well as $\mathcal{V}_L^2$) converge for all X_0 and have an approximate

FIG. 3. (a) The mean cross-shore trajectory $\bar{X}$ from (7), (b) mean cross-shore drift velocity $\bar{U}$ from (8), the (c) cross-shore $\mathcal{U}_L^2$ and (d) alongshore $\mathcal{V}_L^2$ Lagrangian velocity variance from (10), and the (e) cross-shore K_x and (f) alongshore K_y eddy diffusivities from (11) versus time t . Colors indicate initial cross-shore drifter location X_0 : from the shoreline (red) to offshore (blue). In (a), the location of the maximum Eulerian cross-shore eddy velocity $x_{U_{\max}}$ is indicated by the gray line. In (a) and (b), the dashed black is a $|\bar{X}(t)| \sim t^{1/4}$ and $|\bar{U}| \sim t^{-3/4}$, respectively, prediction. In (b), open circles indicate the time of strongest offshore velocity, i.e., the time the shoreline is felt. Various scaling laws are indicated in (c)–(f).

asymptotic time dependence of $t^{-1/4}$ (dashed line in Figs. 3c,d).

The cross-shore diffusivity and alongshore diffusivities (K_x , K_y) each have qualitatively similar time dependence across all X_0 (Figs. 3e,f). Initially ($t < 50$ s), the cross-shore diffusivity is ballistic for all X_0 with $K_x \propto t$ (Fig. 3e), as expected for homogeneous turbulence. Thereafter, the cross-shore diffusivity K_x reaches a maximum value K_x at time T_x , and subsequently decreases indicating a subdiffusive (as opposed to Brownian) regime (Fig. 3e). The K_x maximum and time to maximum (K_x , T_x) range from ($1 \text{ m}^2 \text{ s}^{-1}$, 150 s) for $X_0 = x_{U_{\max}}$ to ($0.45 \text{ m}^2 \text{ s}^{-1}$, 2000 s) for X_0 offshore of the surfzone for a factor of 2.2 times and 13 times variation in K_x and T_x , respectively. In general, T_x and K_x are proportional to and inversely proportional to the

distance from release to $x_{U_{\max}}$ ($|X_0 - x_{U_{\max}}|$). For times longer than T_x , the cross-shore diffusivity K_x is subdiffusive, scaling as $K_x \propto t^{-1/2}$ (dashed line in Fig. 3e). At even longer times, K_x becomes independent of X_0 (note collapse of red–cyan curves for $t > 1000$ s), with the time of lost X_0 dependence also depending on $|X_0 - x_{U_{\max}}|$. For the most offshore released drifters ($X_0 = -145$ m), the X_0 dependence is lost at $t > 6000$ s.

The alongshore diffusivity K_y is generally larger than K_x but has qualitatively similar time dependence (Fig. 3f). Like K_x , the initial K_y time dependence is ballistic. For offshore released drifters ($X_0 < -120$ m), after the initial ballistic growth, K_y growth noticeably slows but then subsequently accelerates due to the strong time-varying $\mathcal{V}_L^2(t)$ (Fig. 3d). Subsequently, the alongshore diffusivity $K_y(t, X_0)$ also reaches a maximum K_y at time

FIG. 4. (a) Cross-shore K_x and (b) alongshore K_y eddy diffusivity, scaled by the release location X_0 cross-shore u^2 and alongshore eddy velocity variance v^2 , respectively, versus time t . Colors indicate the initial release location and are the same as in Fig. 3. The dashed black line, $= t$, indicates ballistic dispersion.

T_y (Fig. 3f), ranging from $(K_y, T_y = 4 \text{ m}^2 \text{ s}^{-1}, 210 \text{ s})$ for $X_0 = x_{v_{\text{mx}}}$ to $(K_y, T_y = 1 \text{ m}^2 \text{ s}^{-1}, 10^4 \text{ s})$ for offshore X_0 (dark blue in Fig. 3f). Similarly, T_y and K_y are proportional to and inversely proportional to the release distance from $x_{v_{\text{mx}}}$ ($|X_0 - x_{v_{\text{mx}}}|$). At long times ($> T_y$), K_y decreases subdiffusively $K_y \propto t^{-1/4}$ (dashed lines, Fig. 3f), but decreases more slowly than K_x with its $t^{-1/2}$ scaling. At these long times, K_y also becomes independent of X_0 and the K_y curves collapse.

4. Scaling the Lagrangian statistics

a. Scaling the short-time diffusivity and mean drift

1) DIFFUSIVITY

For all X_0 , the initial ($t < 50 \text{ s}$) diffusivity growth $K_i \propto t$ (Figs. 3e,f) indicates ballistic dispersion [see (2a)]. For homogeneous turbulence, $K/u^2 = t$ for ballistic scaling. Similarly, K_x and K_y here are scaled by the eddy velocity variance (u^2, v^2) at the release location X_0 (Figs. 1c,d). For $t < 50 \text{ s}$, the time dependence of $K_x(t, X_0)/u^2(X_0)$ (Fig. 4a) and $K_y(t, X_0)/v^2(X_0)$ (Fig. 4b) scales as t and collapses for all X_0 . This indicates that the short-time drifter dispersion is ballistic with the local (release location) eddy velocity variance. The ballistic scaling breaks down earlier for K_x than for K_y (compare Figs. 4a,b). The K_x ballistic scaling breaks down sooner for X_0 near $x_{u_{\text{mx}}}$ (i.e., small $|X_0 - x_{u_{\text{mx}}}|$). For example, at $X_0 = x_{u_{\text{mx}}}$ (orange curve in Fig. 4a), the ballistic scaling is strictly applicable to about $t = 50 \text{ s}$, whereas for $X_0 = -145$ (far from $x_{u_{\text{mx}}}$), it is applicable to $\approx 100 \text{ s}$ (blue curve in Fig. 4a), highlighting the importance of the distance $|X_0 - x_{u_{\text{mx}}}|$. For K_y , a clear X_0 dependence for when the ballistic scaling breaks down is not evident (Fig. 4b).

2) MEAN DRIFT

The short-time mean drift velocity $\bar{U}$ is nonzero and directed towards $x_{u_{\text{mx}}}$ (Fig. 3b). For zero mean flow and homogeneous turbulence, the mean drift $\bar{U}$ is zero, whereas if the turbulence is inhomogeneous, gradients in the eddy diffusivity induce a nonzero mean drift velocity $\bar{U}$ (e.g., Davis 1991). For short times, before drifters have dispersed far from X_0 , this relationship is

$$\bar{U}(t, X_0) = \frac{d}{dX_0} K_x(t, X_0). \quad (12)$$

Because the short-time ($t < 50 \text{ s}$) K_x is ballistic for all X_0 (Fig. 4), (12) becomes

$$\bar{U}(t, X_0) = \frac{d u^2(X_0)}{d X_0} t, \quad (13)$$

and $\bar{U}$ grows linearly in time with constant mean drift acceleration of

$$\frac{d}{dt} \bar{U}(t, X_0) = \frac{d}{d X_0} u^2(X_0). \quad (14)$$

In the ballistic regime at $t = 10 \text{ s}$, (14) accurately gives the mean drift acceleration $d\bar{U}/dt$ (Fig. 5) with very high skill. The $\bar{U} \propto t$ scaling is consistent with the $\bar{U}$ time evolution (Fig. 3b). Because u^2 and ballistic K_x are maximum at $x_{u_{\text{mx}}}$, the short-time drift is towards $x_{u_{\text{mx}}}$ (Figs. 3a, b). At these short times, the presence of the shoreline does not impact the mean drift acceleration. At $t = 100 \text{ s}$, (12) breaks down for X_0 near $x_{u_{\text{mx}}}$ (not shown) because K_x is no longer ballistic (i.e., orange curve in Fig. 4a). In contrast, for X_0 far from $x_{u_{\text{mx}}}$, (12) still holds at $t = 100 \text{ s}$ because K_x is still largely ballistic (i.e., blue curves in Fig. 4a).

FIG. 5. The cross-shore drift acceleration $d^2\bar{X}/dt^2$ versus the cross-shore gradient of the cross-shore squared eddy velocity $dU^2(x)/dx$ at $t = 10$ s. Colors indicate initial cross-shore drifter location X_0 : from the shoreline (red) to offshore (blue). The 1-to-1 line is dashed.

b. Scaling the maximum and timing of the diffusivity $\mathcal{K}$ and $\mathcal{T}$

1) THE DIFFUSIVITY MAXIMUM $\mathcal{K}_x$ AND $\mathcal{K}_y$

The magnitude of the diffusivity maxima $\mathcal{K}_x$ and $\mathcal{K}_y$ can vary significantly (Figs. 3e,f). Over all release locations, $\mathcal{K}_x(X_0)$ varies by a factor of ≈ 2 , with maximum near $x_{U_{\max}}$ (the location of U^2 maximum) and minimum offshore (Figs. 3e, 6a). The $\mathcal{K}_y(X_0)$ varies similarly with release location, with maximum near $x_{V_{\max}}$ (Figs. 3f, 6b). Although the release location velocity variance [$U^2(X_0)$ and $V^2(X_0)$] determines the initial ballistic dispersion, it does not collapse $\mathcal{K}_x$ or $\mathcal{K}_y$ for longer times (Figs. 4a,b). As drifters cross-shore disperse from X_0 , they sample regions of differing eddy velocities resulting in a time-dependent Lagrangian velocity variance U_{Li}^2 (Figs. 3c,d). Here, an “effective” eddy velocity variance is sought that can scale $\mathcal{K}_x$ and $\mathcal{K}_y$ in a form analogous to Brownian diffusion [(2)]. Initially, drifters drift from X_0 towards $x_{U_{\max}}$ (Fig. 3a), hence, an effective Eulerian cross-shore eddy velocity variance $\bar{U}^2(X_0)$ is defined as the cross-shore average of $U^2(x)$ from X_0 to $x_{U_{\max}}$,

$$\bar{U}^2(X_0) = \frac{1}{X_0 - x_{U_{\max}}} \int_{X_0}^{x_{U_{\max}}} U^2(x') dx'. \quad (15)$$

The effective alongshore eddy velocity $\bar{V}^2(X_0)$ variance is defined similarly, but averaged between X_0 and the location of the maximum alongshore eddy velocity $x_{V_{\max}}$:

$$\bar{V}^2(X_0) = \frac{1}{X_0 - x_{V_{\max}}} \int_{X_0}^{x_{V_{\max}}} V^2(x') dx'. \quad (16)$$

The parameterization for $\mathcal{K}$ is then expressed as

$$\mathcal{K}_x(X_0) = \bar{U}^2(X_0) \bar{T}_{Lx}, \quad (17)$$

$$\mathcal{K}_y(X_0) = \bar{V}^2(X_0) \bar{T}_{Ly}, \quad (18)$$

where the time scales $\bar{T}_{Lx}$ and $\bar{T}_{Ly}$ are spatially uniform. These time scales are denoted “effective” cross- and alongshore Lagrangian time scales as they are associated with the maximum diffusivity in analogy with T_L being associated with K^∞ in homogeneous, isotropic turbulence (Taylor 1922). Note that $\bar{T}_{Lx}$ and $\bar{T}_{Ly}$ are not decorrelation time scales as T_L is understood to be.

With this simple effective cross-shore velocity variance $\bar{U}^2$, the parameterized $\mathcal{K}_x(X_0)$ in (17) represents well the drifter-derived $\mathcal{K}_x(X_0)$ with a constant best-fit $\bar{T}_{Lx} = 66$ s (Fig. 6a). Similarly, in the alongshore, the parameterized $\mathcal{K}_y(X_0)$ in (18) represents well the drifter-derived $\mathcal{K}_y(X_0)$ for $X_0 < x_{U_{\max}}$ with a constant best-fit $\bar{T}_{Ly} = 146$ s (Fig. 6b). However, in the inner surfzone for $X_0 > x_{U_{\max}}$, this $\mathcal{K}_y$ scaling breaks down with 30%–50% errors. In this near-shoreline region, a smaller $\bar{T}_{Ly}$ is required in this parameterization analogous to how atmospheric boundary layer, and turbulent channel flow, Lagrangian time scales decrease towards the boundary (Wilson and Sawford 1996; Choi et al. 2004).

2) THE TIME TO THE DIFFUSIVITY MAXIMUM $\mathcal{T}_x$ AND $\mathcal{T}_y$

The time of the diffusivity maximum $\mathcal{T}_x(X_0)$ and $\mathcal{T}_y(X_0)$ is hypothesized to be proportional to the time for drifters to disperse across the width of the strong eddying region, [across $U^2(x)$], approximately the surfzone width (Fig. 1c). This time is proportional to a length-scale $\mathcal{L}$ divided by a velocity scale. Because $\mathcal{K}_x$ is approximately ballistic up to $\mathcal{T}_x$ (Fig. 3e), the release location eddy velocity $U(X_0)$ is used as a velocity scale. The strong U^2 region has max at $x_{U_{\max}} = -40$ m with width $\approx 2|x_{U_{\max}}|$. For offshore release $X_0 < x_{U_{\max}}$, drifters must disperse from the release location to the shoreline resulting in the length-scale $\mathcal{L}_x = |X_0|$. For release onshore of $x_{U_{\max}}$, drifters must disperse from X_0 to $2x_{U_{\max}}$, so that $\mathcal{L}_x = -2x_{U_{\max}} + X_0$. For all X_0 , the length scale is $\mathcal{L}_x = |X_0 - x_{U_{\max}}| + |x_{U_{\max}}|$, and $\mathcal{T}_x$ is parameterized as

$$\mathcal{T}_x(X_0) = \frac{A_1}{U(X_0)} \left(|X_0 - x_{U_{\max}}| + |x_{U_{\max}}| \right), \quad (19)$$

where A_1 is a nondimensional constant. This parameterization (19) reproduces well the drifter-derived $\mathcal{T}_x(X_0)$ with an $O(1)$ best-fit $A_1 = 0.35$ (compare black

FIG. 6. The maximum (a) cross-shore diffusivity $\mathcal{K}_x$ and (b) maximum alongshore diffusivity $\mathcal{K}_y$ as a function of cross-shore release location X_0 (black curves). The red curves in (a) and (b) are the parameterized $\bar{U}^2 \bar{T}_{Lx}$ from (17) and $\bar{V}^2 \bar{T}_{Ly}$ from (20). The time to maximum (c) cross-shore diffusivity $\mathcal{T}_x$ and (d) alongshore diffusivity $\mathcal{T}_y$ versus cross-shore release location X_0 (black curves). The red curves in (c) and (d) are parameterizations given by (19). The surfzone extent x_{sz} , and the location of maximum cross- ($x_{u_{\max}}$), and alongshore ($x_{v_{\max}}$), eddy velocity variances are indicated by vertical dashed, dash-dotted, and dotted lines, respectively.

and red curves in Fig. 6c), suggesting that once drifters quasi-ballistically disperse across the strong eddying region, the maximum diffusivity is reached.

Here, $\mathcal{T}_y$ is similarly related to time for cross-shore drifter dispersion through the strong $\mathcal{V}^2(x)$ region, with similar parameterization. The cross-shore velocity scale is again $\mathcal{U}(X_0)$, as drifters must disperse across the strong $\mathcal{V}^2$ region with peak at $x_{v_{\max}}$ and half-width of $x_{v_{\max}}$ (Fig. 1c). This gives a length scale of $\mathcal{L}_y = |X_0 - x_{v_{\max}}| + |x_{v_{\max}}|$, and $\mathcal{T}_y(X_0)$ is then parameterized as

$$\mathcal{T}_y(X_0) = \frac{A_2}{\mathcal{U}(X_0)} \left(|X_0 - x_{v_{\max}}| + |x_{v_{\max}}| \right). \quad (20)$$

This parameterization reproduces reasonably well the drifter-derived $\mathcal{T}_y(X_0)$ structure (Fig. 6d). The $O(1)$ best-fit $A_2 = 2.5$ is about 8 times larger than A_1 as $\mathcal{T}_y$ is generally larger than $\mathcal{T}_x$. For X_0 offshore of x_{sz} , the parameterization results in times too small, likely because the assumption of cross-shore locally ballistic dispersion has already broken down (Fig. 3e).

c. Scaling the long-time Lagrangian statistics

1) MEAN DRIFT

At long times, after drifters have sufficiently dispersed so that the shoreline is felt, the shoreline limits onshore

displacements imparting a mean negative drift velocity $\bar{U}$ (e.g., [Spydell and Feddersen 2012b](#)). In general, the time when the shoreline affects $\bar{U}$, that is the maximum offshore drift velocity (open circles in Fig. 3b), increases with offshore X_0 . Thereafter, long-time offshore drift collapses for most X_0 , except for the farthest offshore X_0 . For homogeneous turbulence with Brownian dispersion ($D_x^2 \propto t$), but with a reflecting shoreline, the long-time cross-shore trajectory $\bar{X}$ scales as ([Spydell and Feddersen 2012b](#))

$$|\bar{X}(t)| \propto [D_x^2(t)]^{1/2} \propto t^{1/2}, \quad (21)$$

resulting in an offshore directed mean trajectory. As the long-time D_x^2 is related to K_x through (11), the long-time subdiffusive $K_x \propto t^{-1/2}$ found here corresponds to $D_x^2 \propto t^{1/2}$ (not t^1). Applying the scaling (21) gives $\bar{X} \propto t^{1/4}$ which matches $\bar{X}$ for mid-surfzone release (compare dashed black and yellow curve in 3a). Similarly, this scaling gives $\bar{U} \propto t^{-3/4}$, matching the long-time offshore velocity (compared dashed black and colors in Fig. 3b). Thus, the mean trajectory scaling $|\bar{X}(t)| \propto [D_x^2(t)]^{1/2}$ is also applicable in inhomogeneous turbulence with a boundary, although $\bar{X}$ moves offshore more slowly here than for homogeneous turbulence due to offshore decaying turbulence.

2) DIFFUSIVITIES

Here, the subdiffusive behavior of the long-time cross-shore ($K_x \sim t^{-1/2}$) and alongshore ($K_y \sim t^{-1/4}$) diffusivities (Figs. 3e,f) are investigated. For the cross-shore diffusivity, the approach to scaling the long-time behavior is analogous to that for scale-dependent relative diffusivities (Richardson 1926; SpydeLL et al. 2007). At long times, drifters are cross-shore well-mixed within the surfzone, so that the magnitude of K_x is a result of drifter dispersing farther offshore as D_x in (9) increases. At these long times, the local K_x is assumed Brownian [see (2b)] with form $K_x \propto \mathcal{U}_{\text{an}}^2(x) T_L$, such that offshore of the surfzone, the analytic $\mathcal{U}_{\text{an}}^2 \sim x^{-2}$ [(5)]. Here, T_L is assumed spatially uniform, analogous as was found for the effective $\bar{T}_{Lx}$ in the K_x scaling (Fig. 6a). Thus, only the $\mathcal{U}^2$ inhomogeneity affects the diffusivity. Assuming drifters are distributed from the shoreline to x , that is, D_x is proportional to $|x|$, results in

$$K_x \propto D_x^{-2}, \quad (22)$$

so that drifters feel weaker diffusivities the farther offshore they have dispersed. For a general power law $\mathcal{U}^2 \sim x^{-\beta}$, a general relationship between K and D can be obtained (section 5c). Note that $K \sim D^{4/3}$ for turbulent relative dispersion (Richardson 1926). Using (22) in (11) results in a long-time dispersion scaling of $D_x^2 \sim t^{1/2}$ and long-time subdiffusive $K_x \sim t^{-1/2}$ with power-law scaling consistent with the long-time K_x for surfzone X_0 (Fig. 3e). Thus, the subdiffusive power law for K_x is set by the spatial decay of the eddy velocities $\mathcal{U}^2$ (explored further in section 5c). Note, that well offshore of the surfzone, the analytic $\mathcal{U}_{\text{an}}^2(x) \sim x^{-2}$ does not decay as rapidly as $\mathcal{U}^2(x)$ (Fig. 1c). However, using x^{-3} in the scaling results in $K_x \sim t^{-3/5}$ similar to the $t^{-1/2}$.

The long-time alongshore diffusivity scaling $K_y \sim t^{-1/4}$ is now investigated. Unlike the cross-shore dispersion, which is uncoupled from the alongshore dispersion, the alongshore dispersion is coupled to the cross-shore dispersion because $\mathcal{V}^2$ is a function of x . Thus, K_y depends directly on the cross-shore distribution of drifters. For long times, and assuming a constant alongshore Lagrangian time scale T_{Ly} , the local (if drifters did not spread in x) alongshore eddy diffusivity would be $K_y(x) = \mathcal{V}^2(x) T_{Ly}$. However, because drifters spread in x , the eddy diffusivity K_y is the cross-shore position weighted average of $\mathcal{V}^2(x) T_{Ly}$, that is,

$$K_y(t, X_0) = T_{Ly} \int \mathcal{V}^2(x) p(x, t, X_0) dx, \quad (23)$$

where $p(x, t, X_0)$ is the x distribution of drifters released from X_0 at time t . For simplicity, drifters are assumed

uniformly distributed from the shoreline to $-D_x$, and $\mathcal{V}^2(x)$ has the functional form $\mathcal{V}_{\text{an}}^2(x)$ [Fig. 1d and Eq. (5)], thus,

$$\begin{aligned} K_y(t) &\approx \frac{V_0^2 T_{Ly}}{D_x(t)} \int_{-D_x(t)}^0 \frac{1}{1 + [(x - x_0)/l_x]^2} dx \\ &\approx V_0^2 T_{Ly} \frac{l_x}{D_x(t)} \left\{ \tan^{-1} \left[\frac{D_x(t) - x_0}{l_x} \right] + \tan^{-1} \left(\frac{x_0}{l_x} \right) \right\}. \end{aligned} \quad (24)$$

This can be asymptotically approximated for $D_x \gg l_x$, which to leading order is

$$K_y(t) \sim V_0^2 T_{Ly} \frac{l_x}{D_x(t)} \left[\frac{\pi}{2} - \frac{l_x}{D_x(t) - x_0} + \tan^{-1} \left(\frac{x_0}{l_x} \right) \right]. \quad (25)$$

For large t , $D_x \sim t^{1/4}$, so that the long-time K_y scales as

$$K_y(t) \sim \frac{1}{t^{1/4}} + O\left(\frac{1}{t^{1/2}}\right), \quad (26)$$

consistent with the drifter-derived scaling (dashed black line, Fig. 3b). Notice that the leading order scaling is $\sim 1/D_x(t)$ as long $\mathcal{V}^2(x)$ decays offshore faster than $1/|x|$. This ensures that $\int_{-D_x}^0 \mathcal{V}^2(x) dx$ increases slowly enough for increasing D_x to not affect the $K_y(t) \sim 1/D_x(t)$ scaling at leading order. Thus, unlike in the cross-shore where the offshore $\mathcal{U}^2$ power law determines the K_x long-time dependence, the specific offshore shape of $\mathcal{V}^2(x)$ does not determine the long-time alongshore diffusivity scaling $K_y \sim t^{-1/4}$, rather, cross-shore dispersion governs the long-time alongshore K_y scaling.

5. Fickian models of surfzone cross-shore dispersion

As material dispersion is often modeled using a diffusion equation, the ability of a diffusion equation to simulate the drifter dispersion in inhomogeneous turbulence with a boundary is examined. With zero mean Eulerian flow, the one-dimensional diffusion equation for conserved tracer $\phi(t, x)$ is

$$\frac{\partial}{\partial t} \phi = \frac{\partial}{\partial x} \left[\kappa_F(t, x) \frac{\partial}{\partial x} \phi \right], \quad (27)$$

where $\kappa_F(t, x)$ is the cross-shore- and time-dependent Fickian diffusivity. Analogous to drifter dispersion, the domain is $x \leq 0$, with a shoreline ($x = 0$) no-flux boundary condition $d\phi/dx = 0$, analogous to drifter reflection. A delta-function initial condition $\phi(0, x) = \delta(x - X_0)$

corresponds to drifters released from X_0 . Analogous to drifter statistics, the tracer center of mass $\bar{X}_\phi(t)$ and eddy diffusivity K_ϕ are estimated from ϕ moments as

$$\bar{X}_\phi(t, X_0) = \phi_1(t, X_0), \quad (28)$$

$$K_\phi(t, X_0) = \frac{1}{2} \frac{d}{dt} \{ \phi_2(t, X_0) - [\phi_1(t, X_0)]^2 \}, \quad (29)$$

where the n th moment is $\phi_n(t, X_0) = \int_{-\infty}^0 x^n \phi(t, x, X_0) dx$. Here, the effect of cross-shore uniform and cross-shore varying Fickian diffusivity κ_F on the tracer eddy diffusivity K_ϕ is examined to provide insight into the drifter-derived eddy diffusivity $K_x(t, X_0)$.

a. Cross-shore uniform, time-dependent Fickian diffusivity

In [Spydell and Feddersen \(2012b\)](#), surfzone drifter dispersion for drifters spanning a range of X_0 was simulated with a cross-shore uniform, but time-dependent Fickian diffusivity that allows for ballistic dispersion,

$$\kappa_F = \kappa(t) = \mathcal{U}^2 T_L [1 - \exp(-t/T_L)], \quad (30)$$

where $\mathcal{U}^2$ is the spatially uniform cross-shore eddy velocity variance and T_L is the spatially uniform Lagrangian (decorrelation) time scale in contrast to $\bar{T}_{Lx}$ which parameterizes the maximum diffusivity. The diffusion equation (27) is solved for three $X_0 = -40, -80, -120$ m, $\mathcal{U}^2$ that matches that at $X_0 = -80$ m, and $T_L = 120$ s. The choice of $T_L = 120$ s is discussed in [section 6a](#) and is approximately the surfzone-averaged Eulerian time scale. With this κ_F , the analytically derived $\bar{X}_\phi(t, X_0)$ and $K_\phi(t, X_0)$ have release location X_0 dependent offshore drift $\bar{X}_\phi$, and a shoreline induced subdiffusive regime with asymptotic diffusivity reduced [by factor $(\pi - 2)/\pi < 1$] relative to no shoreline ([Spydell and Feddersen 2012b](#)). There are no inhomogeneous eddy velocity variance effects.

Initially ($t < 100$ s), the center of mass position $\bar{X}_\phi$ is constant for all three X_0 , consistent with model-drifter-derived $\bar{X}$ ([Fig. 7a](#)). By design, the short-time K_ϕ matches well the drifter $K_x(t, X_0)$ at $X_0 = -80$ m ([Fig. 7c](#)), however the short-time K_ϕ does not match K_x at other locations because $\mathcal{U}^2(x)$ varies. For larger times, drifter and Fickian-diffusion statistics have significant differences. The tracer $\bar{X}_\phi$ monotonically drifts offshore eventually quite rapidly (colors in [Fig. 7a](#)) because the cross-shore constant $\mathcal{U}^2$ is relatively large. In contrast, at $X_0 = -80$ m and $X_0 = -120$ m, $\bar{X}$ is onshore for $t < 1000$ s before heading offshore at long times much more slowly than the tracer center of mass $\bar{X}_\phi$. The strong subdiffusive regime in the drifter-derived K_x is not present in the tracer K_ϕ , which asymptotes to a constant value larger than

the long-time K_x . Hence, a spatially uniform surfzone κ_F will overestimate material flux onto the inner shelf. These comparisons highlight the strong role that cross-shore inhomogeneous turbulence plays in drifter or tracer dispersion and thus cross-shore exchange.

b. A cross-shore- and time-dependent Fickian diffusivity

To represent the cross-shore inhomogeneous turbulence, a spatially varying and time-dependent $\kappa_F(x, t)$ is used with form

$$\kappa_F(x, t) = \mathcal{U}_{\text{an}}^2(x) T_L [1 - \exp(-t/T_L)], \quad (31)$$

where the analytic $\mathcal{U}_{\text{an}}^2(x)$ in (5) reasonably represents the cross-shore variable $\mathcal{U}^2(x)$ from shoreline to offshore of the surfzone ([Fig. 1c](#)). The diffusion equation (27) is solved numerically with $T_L = 120$ s and for $X_0 = -40, -80, -120$ m. The cross-shore variable $\kappa_F(x, t)$ results in much better agreement between $[\bar{X}(t, X_0), K_x(t, X_0)]$ and $(\bar{X}_\phi, K_\phi)$ ([Figs. 7c,d](#)) than that for cross-shore uniform κ_F ([Figs. 7a,b](#)). At short times ($t < 100$ s), the ballistic dispersion is reproduced for all X_0 , as $\mathcal{U}_{\text{an}}^2(x)$ reasonably matches $\mathcal{U}^2(x)$. At intermediate times ($10^2 < t < 10^3$ s), the K_x maximum and time to maximum T_x are largely reproduced by K_ϕ ([Fig. 7d](#)). As $\mathcal{U}_{\text{an}}^2(x)$ and $\mathcal{U}^2(x)$ are similar, this implies that with (31), the effective Lagrangian time scale $\bar{T}_{Lx} = 66$ s is reproduced. This reinforces the difference between $\bar{T}_{Lx}$ (which sets $\mathcal{K}$) and the decorrelation time T_L , and suggests that $\bar{T}_{Lx}$ is about half of T_L ($\bar{T}_{Lx} \approx T_L/2$). At intermediate times ($10^2 < t < 10^3$ s), the X_0 -dependent mean drift $\bar{X}$ is also reproduced by $\bar{X}_\phi$ ([Fig. 7c](#)). At long times ($t > 10^3$ s), the slow offshore drift velocity $\bar{U}$ and the long-time $K_x \sim t^{-1/2}$ is largely reproduced by $\bar{X}_\phi$ and K_ϕ , in contrast to long-time rapid offshore drift and constant K_ϕ ([Figs. 7a,b](#)) for a spatially uniform κ_F . This indicates that the Fickian diffusivity in (31) with cross-shore variable $\mathcal{U}_{\text{an}}^2(x)$ that reasonably matches $\mathcal{U}^2(x)$ from shoreline to offshore of the surfzone and spatially uniform T_L within a diffusion equation (27) can reproduce the drifter-derived Lagrangian statistics in regions of inhomogeneous turbulence. It also indicates that this simple and computationally efficient model can be used to test scalings.

c. The effect of cross-shore decaying eddy velocity variance

Here, the relationship between the offshore decay of $\mathcal{U}_{\text{an}}^2(x)$ and the long-time $K_\phi \sim t^\gamma$ power law is examined. The long-time drifter $K_x \sim t^{-1/2}$ scaling results from $\mathcal{U}_{\text{an}}^2 \sim x^{-2}$ [[section 4c\(2\)](#)]. For a generalized $\mathcal{U}_{\text{an}}^2 \sim x^{-\beta}$, similar reasoning leads to

FIG. 7. (a) The cross-shore mean position $\bar{X}_\phi$ and (b) the cross-shore diffusivity K_ϕ versus time t for the spatially constant Fickian diffusion model for three release locations X_0 (colors, see legend). Black curves are surfzone-turbulence-derived quantities. (c),(d) As in (a) and (b), but for the cross-shore-dependent Fickian diffusion model.

$$K_x \sim t^{-\beta/(2+\beta)}. \quad (32)$$

This is analogous to scale-dependent relative diffusivity, where $\beta = -4/3$ yields $K_x \sim t^2$ or $D_x^2 \sim t^3$ for drifter separations (Richardson 1926).

The long-time K_x scaling (32) is tested via numerical solution of (27) with Fickian diffusion of the form in (31), but with analytic $\mathcal{U}_{\text{an}}^2(x)$ with β dependence, that is,

$$\mathcal{U}_{\text{an}}^2(x) = \frac{\mathcal{U}_0^2}{1 + \left| (x - x_{\mathcal{U}_{\text{mx}}}) / l_x \right|^\beta}, \quad (33)$$

such that far-offshore $\mathcal{U}_{\text{an}}^2 \sim |x|^{-\beta}$ with larger β corresponding eddy velocities decaying more rapidly offshore. Numerical solutions use the same $T_L = 120$ s, $X_0 = -40$ m, $\mathcal{U}_0^2 = 0.015 \text{ m}^2 \text{ s}^{-2}$, and β varies from 0.5 to 3. For each β , $K_\phi(t)$ is estimated from (29). Assuming a long-time power law for $K_\phi \sim t^\gamma$, the exponent γ is estimated as $\gamma = d\{\log[K_\phi(t)]\}/d[\log(t)]$ averaged over $9000 \leq t \leq 1 \times 10^4$ s with derivatives estimated by finite difference.

The more rapidly $\mathcal{U}_{\text{an}}^2(x)$ decays offshore, the more negative the exponent γ (red dots in Fig. 8) and the more

strongly subdiffusive the long-time K_x . The exponent γ is largely consistent with the long-time K_x scaling (32) (compare red dots to the dashed curve). Differences between γ estimated using ϕ and the scaling (32) are due to differences between $\mathcal{U}_{\text{an}}^2(x)$ in (33) used in the diffusion equation, and the form $|x|^{-\beta}$ that leads to (32). Additionally, the diffusion equation includes shoreline effects not present in (32). Overall, however, the similarity between γ and the scaling (32) indicates that (32) can be used to estimate the long-time diffusivity in other regions with spatially inhomogeneous turbulence.

6. Discussion

For point drifter releases in an inhomogeneous turbulent surfzone, the short-time and long-time mean drift $\bar{X}$ and the short-time, maximum, and long-time cross- and alongshore diffusivities K can be scaled. These scalings require knowledge of the cross-shore structure of the cross-shore $\mathcal{U}^2(x)$ and alongshore $\mathcal{V}^2(x)$ eddy velocity variances, and the constant effective Lagrangian time scales ($\bar{T}_{Lx}$, $\bar{T}_{Ly}$). However, only one case of normally incident wave conditions with zero alongshore

FIG. 8. The power-law exponent γ of the long-time cross-shore diffusivity $K_\phi \sim t^\gamma$ versus the exponent of offshore eddy variance decay $\mathcal{U}^2 \sim x^{-\beta}$ for both analytic prediction [(32), dashed curve] and numerical solution (red dots).

current was examined here. The presence of a sheared alongshore current will increase K_y substantially via shear dispersion (Spydell and Feddersen 2012b), but potentially decrease K_x via mixing suppression induced by the eddy propagation speed and the mean current difference (Spydell 2016). The effect of other parameters, such as variable beach profiles, incident wave height H_s , period, or directional spread σ_θ , on $\bar{X}$ and K_i are not yet understood.

a. Scaling the “bulk” Lagrangian Timescale

In scaling $\mathcal{K}_x$ and $\mathcal{K}_y$, cross-shore uniform effective Lagrangian time scales $\bar{T}_{Lx}$ and $\bar{T}_{Ly}$ were appropriate but were found by a fit. However, the effective Lagrangian time scales are generally unknown for other surfzones or other applications. For example $\bar{T}_{Lx}$ may scale with incident H_s or beach slope. For the simulation presented here, the Eulerian time scale T_E is used to derive the effective Lagrangian time scale following concepts developed by Middleton (1985).

In homogeneous turbulence, dispersion can be in either the “frozen field” or “fixed float” limits (e.g., Middleton 1985; Lumpkin et al. 2002). In the frozen field limit, drifters sample multiple eddies before the eddy changes significantly, so that the ratio of Lagrangian to Eulerian timescales is small ($T_L/T_E \ll 1$), and $K^\infty \propto \mathcal{U}L_E$, where L_E is the Eulerian eddy size (Middleton 1985). In the fixed float limit, temporal Lagrangian fluctuations are caused by Eulerian velocity fluctuations and $T_L/T_E \approx 1$ resulting in $K^\infty = \mathcal{U}^2 T_E$. These limits are distinguished using the Eulerian “Middleton” parameter α (Middleton 1985) defined as $\alpha = \mathcal{U} T_E / L_E$, where

small $\alpha < 0.3$ indicates fixed float and larger $\alpha > 0.3$ represents frozen field limits (Lumpkin et al. 2002). Surfzone drifter observations spanning different beaches and different wave conditions suggest that there is not a single value of surfzone $\alpha_{x,y}$ and it can potentially vary between fixed float and frozen field regimes (Spydell et al. 2014). To assess whether for this simulation turbulent dispersion is in the fixed float or frozen field regimes, the cross-shore-dependent Middleton parameters

$$\alpha_x(x) = \mathcal{U}(x) \frac{T_{Ex}(x)}{L_{Ex}(x)}, \quad \alpha_y(x) = \mathcal{V}(x) \frac{T_{Ex}(x)}{L_{Ex}(x)} \quad (34)$$

are calculated where the Eulerian time scales $T_{Eu_i}(x)$ in (A1b) and length scales $L_{Eu_i}(x)$ in (A1a) are estimated as described in appendix A. The Eulerian length scales are mostly cross-shore uniform and are the order of the surfzone width (not shown). At all cross-shore locations both α_x and α_y are small (≤ 0.04 , Figs. 9a,b), much less than the 0.3 value separating fixed float and frozen field regimes. Although the Middleton parameter was developed for isotropic and homogeneous turbulence, this suggests that in the region spanning the inhomogeneous surfzone to inner shelf is in the fixed float regime, where the Lagrangian and Eulerian time scales are similar.

The cross-shore Eulerian velocity time scale T_{Ex} varies between approximately 120 and 200 s for regions not near the shoreline ($x < -25$ m, Fig. 9c). Although 2–3 times larger than the effective Lagrangian time scale $\bar{T}_{Lx}$ (Fig. 9c), T_{Ex} is similar to and slightly larger than the cross-shore constant Lagrangian time scale $T_{Lx} = 120$ s (a true decorrelation time) used in a diffusion equation that well reproduced the Lagrangian statistics (section 5). Thus, consistent with the small α_x found in the surfzone, the Lagrangian time scale will be approximately equal to the cross-shore-averaged (represented by a bar) Eulerian time scales, that is, $T_{Lx} \approx \bar{T}_{Ex}$. It also implies that the effective Lagrangian time scale, which sets $\mathcal{K}_x$, can be written as $\bar{T}_{Lx} \approx 0.5 T_{Lx} \approx 0.5 \bar{T}_{Ex}$. Recall $\bar{T}_{Lx}$ is not a decorrelation time. In the alongshore, the relationship between $\bar{T}_{Ly}$, T_{Ly} , and T_{Ey} is not clear because alongshore diffusion simulations with an associated T_{Ly} were not performed as alongshore dispersion cannot be represented as 1D diffusion due to coupling with cross-shore dispersion. However, within the surfzone, the alongshore Eulerian and effective Lagrangian time scale are similar ($T_{Ey} \approx \bar{T}_{Ly}$) (Fig. 9d). The small surfzone α_y (Fig. 9b) indicates fixed float conditions where $T_{Ly} \approx T_{Ey}$. This suggests that the time scale that sets the alongshore diffusivity maximum is closer to the decorrelation Lagrangian time scale, that is, $\bar{T}_{Ly} \approx T_{Ly}$, potentially because alongshore dispersion is unbounded.

FIG. 9. (a) The Middleton parameter α_x in (34) versus x . (c) The Eulerian cross-shore velocity time scale T_{Ex} from (A1b) (blue) and cross-shore uniform effective Lagrangian time scale $\bar{T}_{Lx}$ (black line) versus x . (b),(d) Alongshore versions of (a) and (c), i.e., α_y in (b) and T_{Ey} and $\bar{T}_{Ly}$ in (d). The surfzone extent x_{sz} (vertical dashed), and the maximum cross- and alongshore eddy velocities ($x_{u_{\max}}$, $x_{v_{\max}}$, dotted) are indicated.

Thus, surfzone dispersion on alongshore uniform coasts with no mean flows can be well simulated with knowledge of the cross-shore structure of the eddy velocity variance [$u^2(x)$ and $v^2(x)$] and Eulerian time scales.

b. Relationship to previous surfzone drifter studies

For short to intermediate times ($t \leq 1000$ s), the K_x estimated here are relatively consistent with K_x estimated from in situ (Spydell et al. 2007; Brown et al. 2009; Spydell et al. 2009) and numerical (Spydell and Feddersen 2009) surfzone drifter dispersion. Many of these studies had nonzero mean alongshore current and so the focus here is on K_x . In both in situ and numerical, K_x is initially ballistic, reaches a maximum, then decreases (i.e., is subdiffusive). In some cases, the diffusivity estimator had a bias towards constant long-time diffusivity (Spydell et al. 2007), that reduced subdiffusive K_x behavior. The shoreline was argued capable of inducing the observed subdiffusive K_x for $t < 10^3$ s (Spydell and Feddersen 2012b). However, in these works, drifters were released over many X_0 , and the derived K_x are an average over X_0 , blurring out inhomogeneous turbulence effects. The relatively short ($t < 10^3$ s) observed and modeled drifter trajectories resulted in unclear long-time diffusivity behavior, as discussed by Spydell and Feddersen (2012b). Here, long-time subdiffusive behavior is clear in the long (2×10^4 s) trajectories. Much longer in situ trajectories and many

more drifters are required to constrain in situ long-time diffusivity estimates.

c. Effect of neglecting irrotational velocities in advecting drifters

Here, drifters were advected with the rotational velocities only [(6)] to isolate the effects of eddy-induced dispersion particularly their cross-shore gradients. Although not classically turbulent, irrotational motions due to Stokes drift fluctuations (Herbers and Janssen 2016) in a random, irrotational, weakly nonlinear surface gravity wave field also can induce Lagrangian dispersion (e.g., Herterich and Hasselmann 1982; Bühler and Holmes-Cerfon 2009) that can potentially be sub- or superdiffusive (Balk 2002). This irrotational dispersion mechanism is neglected here. Previously, irrotational wave-induced diffusivity was found to be two orders of magnitude weaker than observed surfzone drifter dispersion, suggesting wave-induced dispersion is not applicable (Spydell et al. 2007). Similarly, advecting drifters with only rotational velocities resulted in nearly the identical diffusivity to drifters advected with the full velocity field within the surfzone (Spydell and Feddersen 2009). In regions seaward of the surfzone ($-200 < x < 100$ m), however, where eddy variance is weak, wave-induced dispersion cannot be ruled out a priori as irrotational surface gravity wave velocities in the sea-swell band ($f > 0.04$ Hz) are an order of magnitude larger than the rotational velocities. At lower

(infragravity) frequencies, the model rotational and irrotational velocities are of the same order of magnitude. Note that model infragravity energy does not necessarily represent realistic conditions because bound wave generation at the numerical wavemaker is lacking (Fiedler et al. 2018, 2019). Wave-induced asymptotic diffusivity can be estimated over all frequencies from the Eulerian irrotational velocity spectra following Herterich and Hasselmann (1982) as applied in Spydel et al. (2007). Offshore of the surfzone, this wave-induced diffusivity is two or more orders of magnitude smaller than K_x . Applying the calculation to only the infragravity frequencies yields wave-induced diffusivity four or more orders of magnitude smaller than K_x . Thus, including irrotational motions in advecting drifters would have no appreciable effect on the estimated diffusivities.

d. Relationship to shelf processes

Here, for an idealized 2D simulation that only included surfzone processes on an alongshore uniform beach, Lagrangian statistics for surface drifters are estimated for 12 000 s or 3.3 h. However, shelf processes can also induce 2D dispersion for surface drifters. Nearshore internal tides (e.g., Walter et al. 2012; Sinnett et al. 2018) can induce dispersion (e.g., Romero et al. 2013; Suanda et al. 2018). Wind-induced variability can also induce 2D dispersion. However, these processes will induce dispersion on the longer tidal and Coriolis time scales. Coastal submesoscale processes can also induce dispersion. However, the root-mean-square of the local Rossby number ζ/f (where ζ is the vorticity and f the local Coriolis parameter) is rarely >3 within a few kilometers of the coast (Dauhajre et al. 2017) indicating that for midlatitude regions, the relevant time scales are longer than those considered here. For three-dimensional drifters or tracers, both rip currents interacting with stratification (Kumar and Feddersen 2017) and internal waves (Suanda et al. 2018) can considerably affect 3D dispersion. Furthermore, a non-alongshore uniform bathymetry, explicitly not considered here, will also lead to significant eddy generation (e.g., MacMahan et al. 2004; Long and Özkan-Haller 2016) and dispersion.

e. Applicability to other oceanographic regions

The principal result here is that strong gradients in the eddy velocity variance have significant effects on the dispersion and mean drift of surface drifters. Although the inhomogeneous turbulence of the surfzone to inner shelf is considered here, these results may be more broadly applicable, as many regions of the ocean have gradients in eddy velocities that are order one relative to the eddy size. For example, a meridional (cross-mean flow) horizontal diffusivity maximum and subsequent decrease

(subdiffusion) in the Antarctic Circumpolar Current (ACC) was attributed (Klocker et al. 2012) to the mixing suppression induced by the mean current and eddy propagation velocity difference (e.g., Ferrari and Nikurashin 2010). Similarly, using Global Drifter Program data for the surface and mixed layer, diffusivity maxima K occurred between 3 and 10 days with a subsequent decrease (Zhurbas et al. 2014). Defining K_∞ as the average diffusivity over 15–20 days, Zhurbas et al. (2014) showed that K/K_∞ maps strongly resembled eddy kinetic energy maps, with both having the largest values in the ACC and western boundary current extensions, also attributed to the mixing suppression mechanism invoked by (Klocker et al. 2012). However, in both of these cases, the gradients in eddy kinetic energy were strong and thus inhomogeneous turbulence may have contributed to the observed subdiffusion. These regions did not have boundaries. In the Santa Barbara Channel, a region with boundaries and variations in eddy kinetic energy, diffusivity maxima and subsequent decrease also was noted (Dever et al. 1998). Nonzero and nonuniform mean currents also can alter dispersion. Shear dispersion induced by sheared mean currents enhance the along-flow diffusivity (e.g., Taylor 1953; Spydel and Feddersen 2012a). In contrast, mixing suppression by unequal eddy propagation and (potentially nonsheared) mean current speeds would decrease the diffusivity (e.g., Ferrari and Nikurashin 2010). Other mechanisms influencing the diffusivity include slow time modulations of the mean flow (Qian et al. 2014). Here, however, the mean currents were zero and these mechanisms were not present. In environments with both sheared mean currents and gradients in eddy velocities, shear dispersion, mixing suppression, and inhomogeneous turbulence all could affect the diffusivity, but the relative contribution of each mechanism is not understood.

7. Summary

Drifters were tracked in an idealized nearshore simulation with the 2D wave-resolving funwaveC model using random normally incident but directionally spread waves. This simulation had no mean currents but a strong cross-shore inhomogeneous eddy field. Lagrangian statistics were calculated with many more drifters and over much longer ($\approx 10^4$ s) trajectories than in previous observational or modeling studies, allowing for the cross-shore release location and time dependence to be accurately studied.

The cross-shore inhomogeneity of the eddy field affects both the mean drift $\bar{X}$, and the cross-shore (K_x) and alongshore (K_y) diffusivity. The short-time cross- and alongshore diffusivity is locally ballistic and the mean

FIG. A1. The spectrum of (a) cross-shore $S_u(k_y, x)$ and (b) alongshore $S_v(k_y, x)$ velocities versus cyclic alongshore wavenumber k_y . Spectra are taken at various cross-shore locations (colors): from the shoreline (red) to offshore (blue) where the edge of the surfzone is $x \approx -90$ m. Inverse-energy cascade ($k_y^{-5/3}$) and enstrophy cascade (k_y^{-3}) scalings are shown as solid and dashed lines, respectively.

cross-shore drift is toward the cross-shore eddy velocity variance maximum, consistent with theory. At intermediate times, K_x and K_y reach a maximum and subsequently decrease. Due to the inhomogeneous surfzone turbulence, maximum cross- and alongshore diffusivities vary by a factor of two, and the timing of the maximum varies by more than a factor of 10, depending on the initial drifter cross-shore release location. Cross-shore K_x and alongshore K_y diffusivity maxima are parameterized in terms of a release-location-dependent, cross-shore-averaged eddy velocity variance, taking into account cross-shore inhomogeneity, and a cross-shore uniform effective Lagrangian time scale. The time to the diffusivity maxima is related to the release location eddy velocity and the distance from release location across the maximum of the eddy velocity variance. At long times, the subdiffusive diffusivities scale as $K_x \sim t^{-1/2}$ and $K_y \sim t^{-1/4}$ independent of release location. This time decay is related to the cross-shore decay of the eddy velocity variance. Hence, inhomogeneous surfzone turbulence induces subdiffusive cross- and alongshore dispersion at long times. The different long-time K_x and K_y time dependence is because alongshore dispersion is coupled to cross-shore dispersion.

Cross-shore drifter transport and dispersion was consistent with a diffusion equation that included a time- and space-dependent Fickian diffusivity, which locally has ballistic to Brownian diffusion time dependence. The Fickian diffusivity is related to the cross-shore eddy velocity variance and with this spatial dependence, the short-, intermediate-, and long-time behavior of $\bar{X}$ and K_x are reproduced. The Eulerian Middleton parameter α is small in this region indicating fixed float dispersion and that the effective Lagrangian time scale is related to the (surfzone averaged) Eulerian time scale. In these

simulations there was no mean flow and so the effects of shear dispersion and mixing suppression was not present. The effects of inhomogeneous turbulence on drifter dispersion spanning from the surfzone to inner shelf are likely also relevant for other oceanographic regions such as the ACC and western boundary current extensions.

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APPENDIX

Eulerian Time Scales and Length Scales

From model output, cross- and alongshore Eulerian velocity alongshore cyclic wavenumber k_y and frequency f spectra, $S_{u_i}(k_y, x)$ and $S_{u_i}(f, x)$, respectively, are calculated at all cross-shore x locations from which Eulerian length scales L_E and time scales T_E are estimated. The frequency spectra $S_u(f, x)$ and $S_v(f, x)$ are red with f^{-2} dependence for $0.002 < f < 0.04$ Hz at all x (not shown). Thus, over these 25–500 s time scales, the Eulerian eddy field is roughly consistent with a simple AR1 stochastic process forced with white noise (i.e., the breaking wave eddy forcing, Clark et al. 2012), consistent with stochastic surfzone drifters simulations (Spydell and Feddersen 2012b). The magnitude of $S_u(k_y, x)$ and $S_v(k_y, x)$ depends strongly on cross-shore location (Fig. A1) with

maxima at $x_{U_{\max}}$ and $x_{V_{\max}}$, respectively, consistent with $U(x)$ and $V(x)$ (Fig. 1c). The $S_u(k_y, x)$ and $S_v(k_y, x)$ have similar spectral shapes (Figs. A1a,b) for $k_y > 0.01$ cycles per meter (cpm). However at small wavenumber ($k_y < 0.005$ cpm), particularly closer to the shoreline, $S_v(k_y, x)$ is larger and flatter than $S_u(k_y)$ indicating anisotropy. The alongshore wavenumber spectra $S_u(k_y, x)$ and $S_v(k_y, x)$ suggest the presence of both inverse-energy cascade ($\sim k_y^{-5/3}$) and enstrophy cascade ($\sim k_y^{-3}$) regions (Fig. A1, solid and dashed lines) expected for 2D turbulence forced at a single wavenumber (e.g., Salmon 1998).

The cross-shore-dependent Eulerian eddy time $T_E(x)$ and length scales $L_E(x)$ are estimated from the along-shore wavenumber and frequency spectra via

$$L_{E_{u_i}}(x) = 5U_i^2 \left[\int_0^\infty k_y S_{u_i}(k_y, x) dk_y \right]^{-1}, \quad (\text{A1a})$$

$$T_{E_{u_i}}(x) = \frac{1}{4} U_i^{-2} S_{u_i}(f = 0, x). \quad (\text{A1b})$$

The definition of T_E is the spectral version of the Eulerian velocity decorrelation time scale used in Middleton (1985). The length scale L_E in (A1a) differs from Middleton (1985), who used $L_E = 2U^{-2} \int k^{-1} S(k) dk$, because at small k , $S(k) \neq 0$ (Fig. A1) leading to large L_E . The value of 5 is included in (A1a) so that the L_E from (A1a) and that of Middleton (1985) are approximately equal for the same wavenumber spectra $S_u(k_y)$. Thus, the α calculated here can be directly compared to previously published (Middleton 1985; Lumpkin et al. 2002) values.

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