Submesoscale Vortical Wakes in the Lee of Topography

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ABSTRACT

An idealized framework of steady barotropic flow past an isolated seamount in a background of constant stratification (with frequency N) and rotation (with Coriolis parameter f) is used to examine the formation, separation, instability of the turbulent bottom boundary layers (BBLs), and ultimately, the genesis of submesoscale coherent vortices (SCVs) in the ocean interior. The BBLs generate vertical vorticity ζ and potential vorticity q on slopes; the flow separates and spawns shear layers; barotropic and centrifugal shear instabilities form submesoscale vortical filaments and induce a high rate of local energy dissipation; the filaments organize into vortices that then horizontally merge and vertically align to form SCVs. These SCVs have $O(1)$ Rossby numbers ($Ro_\zeta = \zeta/f$) and horizontal and vertical scales that are much larger than those of the separated shear layers and associated vortical filaments. Although the upstream flow is barotropic, downstream baroclinicity manifests in the wake, depending on the value of the nondimensional height $\hat{h}$, which is the ratio of the seamount height h_s to that of the Taylor height $h_T = fL/N$, where L is the seamount half-width. When $\hat{h} < 1$, SCVs span the vertical extent of the seamount itself. However, for $\hat{h} > 1$, there is greater range of variation in the sizes of the SCVs in the wake, reflecting the wake baroclinicity caused by the topographic interaction. The aspect ratio of the wake SCVs has the scaling $L_z/L_h \sim \sqrt{f/N}$, instead of the quasigeostrophic scaling $L_z/L_h \sim f/N$.

1. Introduction

The interaction of ocean currents with topography is a significant source of vertical vorticity influx into the flow. In the Southern Ocean a local enhancement of baroclinic instability due to increased baroclinicity in the lee of topography (Pierrehumbert 1984), causes the eddy generation sites to be located at prominent topographic features (Bischoff and Thompson 2014; Abernathey and Cessi 2014). In most of the world's oceans, however, topography-induced vortices are formed from the separation and roll-up of bottom drag-generated shear layers (D'Asaro 1988; Dong and McWilliams 2007; Dong et al. 2007; Vic et al. 2015; McWilliams 2016; Srinivasan et al. 2017). The effect of bottom drag in the ocean is strongest where the ocean currents are strongest, as is common in the strongly

stratified upper thermocline that contains the fast eastern (Molemaker et al. 2015) and western boundary currents (WBCs) (Gula et al. 2016) as they interact with islands, seamounts, headlands, slopes, and shelves along their path. Weaker currents like the deep WBCs and mesoscale eddies similarly generate vorticity, but do so in lower stratification environments typical of the deep ocean (Armi 1978; Armi and D'Asaro 1980). This bottom drag-mediated vorticity injection is significantly different from what is commonly seen in the atmosphere in the form of lee vortices (Epifanio 2003) that result from potential vorticity (PV) generated in hydraulic jumps in a mountain wake (Epifanio and Durran 2002).

Flow-topography interactions are also a source of internal gravity waves, both when the currents are tidally driven, forming internal tides (Nycander 2005), or when lee waves are generated by geostrophic currents (Nikurashin et al. 2014; Wright et al. 2014). When upstream currents are geostrophic and slowly varying in time, the relative prevalence of wave or vortical regimes is determined by two nondimensional parameters: the bulk Rossby number (Ro_L), and the steepness parameter s ($=Fr^{-1}$, where Fr is the Froude number), defined as

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$$\text{Ro}_L = \frac{U}{fL}, \quad s = \frac{Nh}{U}. \quad (1)$$

Here U is the flow speed upstream of the topography, f is the local Coriolis parameter, N is the associated buoyancy frequency of the incoming stratified flow, and L and h are the horizontal and vertical length scales that characterize the topography. Vortical generation dominates when $\text{Ro}_L \ll 1$, while wave generation typically requires $\text{Ro}_L > 1$ (Schär 2002; McWilliams 2016). The steepness parameter s determines whether or not the flow goes mostly over the topography ($s < 1$), in the process leading to the formation of lee waves, or if the flow undergoes splitting and goes around the topography ($s > 1$) (Epifanio 2003). Here we only consider cases with $s < 1$, a consequence of which is the suppression of lee wave generation. This happens when the topography is tall or the flow slow, in which case the flow does not have the kinetic energy to surmount the obstacle and overcome the effects of gravity. In the upper ocean thermocline, stratification is typically large (leading to $s \gg 1$), and the same is often true even in the abyss. The $s > 1$ regime is less understood in the oceanic context because the historical literature has focused more on lee wave generation and breaking.

Previous studies have examined topographic vortex generation in the context of island wakes (Dong et al. 2007; Dong and McWilliams 2007), the subsurface headlands in the path of the California Undercurrent (CUC) (Molemaker et al. 2015; Dewar et al. 2015), the Gulf Stream separation (Gula et al. 2015b, 2016), the Persian Gulf Outflow (Vic et al. 2015), off a headland in Point Barrow, Alaska (D'Asaro 1988), and in the Southern Ocean, when the Antarctic Circumpolar Current interacts with the Kerguelen Plateau (Rosso et al. 2015). A key finding of these studies is that irrespective of the small values of Ro_L , the ejected vortices can have large vortical Rossby numbers $\text{Ro}_\zeta/f > 1$, where $\zeta = v_x - u_y$ is the vertical vorticity. A primary reason for this is that the vorticity generation mechanism is a consequence of the separation of bottom shear layers from slopes, with horizontal scales ranging from ~ 100 m to 1 km, a range typically associated with submesoscale currents in the ocean (Thomas et al. 2008; McWilliams 2016). In particular, $\text{Ro}_\zeta \sim 1$ implies that the induced vortices are dynamically submesoscale because they are no longer geostrophic but gradient wind balanced.

This study generalizes and expands the scope of Dong et al. (2007) that considered vorticity structure and evolution of idealized island wakes. The idealized configuration of Dong et al. (2007) consisted of surface intensified flow past an island with vertical sides in

(Ro_L, Fr) parameter space where the vertical scale in the Froude number was the vertical extent of the surface-intensified flow. A pattern of alternating cyclonic and anticyclonic vortices were found to form through barotropic instability of the shear layers on the sides of the island (relative to the incoming flow). However, for sufficiently large Ro_L , a strong cyclonic–anticyclonic asymmetry was observed, owing to centrifugal instabilities in the anticyclonic part of the wake, leading to a prominence of coherent cyclones. However, real ocean topographic features like seamounts and headlands have finite slopes. As shown by Molemaker et al. (2015), using a realistic flow configuration of the separation of the CUC off a slope, the vortex generation process in the bottom boundary layer (BBL) instead proceeds by the generation and separation of nearly horizontal drag-generated shear layers with a significant vertical vorticity ($\zeta = v_x - u_y$) component. The strength of the induced vorticity is directly proportional to the local speed of flow and the slope of the topography where the flow intersects, and it varies inversely with the BBL thickness. When Ro_ζ is large, a strong parity (cyclonic/anticyclonic) asymmetry develops in the separated flow, based on whether the current is against/toward the direction of Kelvin wave propagation. The asymmetry in the CUC is anticyclonic, which leads to centrifugal instabilities in the wake (CI); this has the effect of relaxing the high vorticity values ($|\zeta|/f > 1$) to near-inertial values ($|\zeta| \sim f$), as expected from the dynamical progression of CI (Carnevale et al. 2011; Dewar et al. 2015). It also leads to the formation of anticyclonic submesoscale coherent vortices (SCVs; called Cuddies, for California Undercurrent eddies). CI is also associated with an enhanced local energy dissipation rate.

A significant difference between the present study and previous ones is our focus in understanding the vertical and horizontal structure of the wake and SCVs. Following the generation of BBLs on the topography, wake separation, and its subsequent instability, we trace the life cycle of the unstable filaments as they upscale in both vertical and horizontal directions to form stable long-lived SCVs. Our view here is that currents on slopes are generic in the ocean, not uniquely confined to particular boundary currents, and therefore so is the submesoscale population they engender.

2. Methods

a. Idealized flow configuration

The ocean is wind driven and strongly vertically sheared with the strongest flow velocities typically found close to the surface, although bottom (Beal and

Bryden 1999) and subsurface-intensified currents (Hristova et al. 2014) are also found. Depending on the local ocean depth and regional flow characteristics, surface currents that encounter topography have $O(1) \text{ m s}^{-1}$ values in the WBCs and in the Southern Ocean, while bottom currents in the same region can reach 0.1 m s^{-1} (Csanady et al. 1988; Nikurashin et al. 2014). Strong subsurface boundary currents with maxima in the thermocline, instead of at the surface, include the CUC and the equatorward WBC in and around the Solomon Sea (with speeds in a similar range as the other prominent WBCs) (Hristova et al. 2014).

Idealized configurations for the flow past isolated topography can be constructed by approximately matching the topography shape, upstream stratification, and vertical and horizontal shear of the upstream flow to a localized oceanic configuration. Alternatively, the approach we pursue here is to simplify the topographic shape (chosen to be a circular seamounts), the upstream stratification (assumed to be uniform), and the vertical shear (set to zero, implying an incoming barotropic flow). This level of idealization allows us to isolate the flow–topography interaction, while permitting a traversal of the parameter space of the problem in the absence of complicating details. First, we set the incoming flow velocity to be $U = 0.1 \text{ m s}^{-1}$, which is in line with the largest values seen in the deep ocean bottom, except the Southern Ocean, but small for eastern and western boundary currents that can have $O(1) \text{ m s}^{-1}$ currents on topographic slopes. The stratification, specified by the Brunt–Väisälä frequency, varies between $N = 1 \times 10^{-3} \text{ s}^{-1}$ and $N = 6 \times 10^{-3} \text{ s}^{-1}$ and spans a range of values between those found in the deep ocean and those in the ocean thermocline. A midlatitude value of $f = 7 \times 10^{-5} \text{ s}^{-1}$ is set, and the seamount shape is chosen to be Gaussian,

$$h_b(x, y) = h_s e^{-(x^2+y^2)/L^2}, \quad (2)$$

where the seamount half-width $L = 15 \text{ km}$ and height $h_s = 800 \text{ m}$ are fixed in all the results shown here; N is therefore the only variable. The various simulations and their respective dimensional and corresponding nondimensional parameters are shown in Table 1.

b. Nondimensionalization

We nondimensionalize space and time dimensions in the form

$$\hat{x} = x/L, \quad \hat{y} = y/L, \quad \hat{z} = z/h_T, \quad \hat{t} = Ut/L, \quad (3)$$

where the incoming background barotropic flow U is along the y direction and

TABLE 1. Table showing the parameters of individual simulations analyzed in this manuscript. Multiple entries for D and γ represent distinct runs that correspond to one another.

$f (\text{s}^{-1})$	$N (\text{s}^{-1})$	$D (\text{km})$	Ro_L	$\hat{h}$	γ
7×10^{-5}	0.000 13	[1, 2, 4]	0.1	0.1	[0.8, 0.4, 0.2]
7×10^{-5}	0.000 26	[1, 2, 4]	0.1	0.2	[0.8, 0.4, 0.2]
7×10^{-5}	0.000 53	[1, 2, 4]	0.1	0.4	[0.8, 0.4, 0.2]
7×10^{-5}	0.000 66	[1, 2, 4]	0.1	0.5	[0.8, 0.4, 0.2]
7×10^{-5}	0.000 8	[1, 2, 4]	0.1	0.6	[0.8, 0.4, 0.2]
7×10^{-5}	0.000 9	[1, 2, 4]	0.1	0.7	[0.8, 0.4, 0.2]
7×10^{-5}	0.002	[1, 2, 4]	0.1	1.4	[0.8, 0.4, 0.2]
7×10^{-5}	0.002 5	1	0.1	1.9	0.8
7×10^{-5}	0.004	1	0.1	2.9	0.8
7×10^{-5}	0.006	1	0.1	4.6	0.8

$$h_T = fL/N \quad (4)$$

is the Taylor height (Hogg 1973a). The maximum nondimensional height of the seamount can now be written as

$$\hat{h} = h_s N / fL = \text{Ro}_L s = h_s / h_T. \quad (5)$$

The nondimensional height is related to the Burger number for this problem, as $\text{Bu} = (\text{Ro}_L s)^2 = \hat{h}^2$, where Ro_L and s have been defined earlier in (1). A local measure of the nondimensional height can also be constructed with the local topographic slope $\theta_b = \tan^{-1}(|\nabla h_b|)$ as

$$\hat{h}_\theta = N \theta_b / f, \quad (6)$$

which reduces to the “global” estimate in (5), given the approximate relationship, $\theta_b \sim h_s / L$. For the choice of the seamount in this paper, $\theta_b \sim 3^\circ$, which is similar to the values seen along the Florida slope before Gulf Stream separation off the Florida Straits (Gula et al. 2015a). The depth parameter, measuring the height of the seamount from the bottom relative to the total water depth D is

$$\gamma = \frac{h_s}{D}, \quad (7)$$

so that the “island” limit corresponds to $h_s = D$. Another nondimensional parameter of relevance is the nonhydrostatic parameter,

$$\Gamma = \frac{U}{NL}, \quad (8)$$

which measures the importance of the nonhydrostatic terms in the vertical momentum equation (Schär 2002; Gill 1982). In all the cases examined in this manuscript, we have $\Gamma \ll 1$, thereby ensuring that the hydrostatic

primitive equations govern the dynamics (rather than the full Boussinesq equations), and nonhydrostatic effects are less important. Because the focus of this paper is understanding predominantly vortical regimes, which are found for $Ro_L \ll 1$, we present results for a specific value, $Ro_L = 0.1$, and vary other parameters in the problem, in particular the depth ratio γ and the steepness parameter s (or equivalently $\hat{h} = Ro_L s$). Parameter $\hat{h}$ is varied by changing only N . The seamount dimensions, h_s and L , are always held constant, while γ is varied by making the water deeper or shallower, through D (assuming $D > h_s$, so we never reach the “island” limit).

c. Computational model

We employ ROMS, a primitive equation, split-explicit, hydrostatic, terrain-following σ -coordinate oceanic model (Shchepetkin and McWilliams 2003, 2005). Momentum advection is computed using a third-order upwind-biased scheme (Shchepetkin and McWilliams 2003, 2005) that is equivalent to a fourth-order central-difference scheme supplemented by a biharmonic diffusion operator whose local diffusivity depends both on velocity and the grid spacing Δx , such that it vanishes as $\Delta x \rightarrow 0$ (Shchepetkin and McWilliams 2005). The turbulent bottom drag is parameterized as

$$\tau_b = \rho_0 c_d \|\mathbf{u}_b\| \mathbf{u}_b, \quad (9)$$

where ρ_0 is a reference density and $\mathbf{u}_b$ is the velocity in the bottommost σ layer. The drag coefficient c_d uses the Von Kármán–Prandtl logarithmic form,

$$c_d = [\kappa / \log(\Delta z_b / z_{ob})]^2, \quad (10)$$

where $\kappa = 0.41$ is the Von Kármán constant, Δz_b is the thickness of the bottommost σ layer, and $z_{ob} = 1$ cm is the roughness parameter. The BBL is parameterized with the K -profile parameterization (KPP) (Large et al. 1994), in particular using the formulation detailed in McWilliams et al. (2009). The domain, which has open boundaries on all sides, is 320 km ($\approx 21L$) in the along-flow direction and 260 km ($\approx 17L$) in the cross-flow direction. The seamount is placed in the middle of the domain along the x axis, but is located at $y = 7L$ from the inflow boundary, giving it about $13L$ downstream development. Radiative open boundary conditions are used with adequate sponge layers as detailed in Mason et al. (2010). Tests were performed by doubling the flow domain size, while keeping the resolution fixed; this resulted in marginal differences in the wake structure, thereby ensuring that the upstream and side boundary conditions do not affect the flow in the domain. The

horizontal resolution is fixed at $\Delta x = 350$ m for all solutions discussed in this study, resulting in a grid size of 915×740 points in the horizontal. The vertical ROMS grid is bottom stretched to ensure that the solution is BBL resolving. Because the model is terrain following, the bottommost σ level ($\sigma = 0$) corresponds to the ocean bottom, $z(\sigma = 0, x, y) = h_b(x, y)$, while the topmost σ level matches the free surface. Seventy σ levels are used resulting in a vertical grid spacing that varies from 1.8 m on the flat bottom to around 1 m at the top of the seamount itself. Finally, we note that the ROMS vertical grid parameters for the surface and bottom stretching are $\theta_s = 1$ and $\theta_b = 6$, respectively [for a description of the ROMS vertical grid structure and the meaning of these parameters, see Shchepetkin and McWilliams (2009)].

d. Eddy integral scales

To evaluate the spatial extent of the vortical filaments and eddies generated by the flow–topographic interaction, we introduce eddy integral scale measures $L^I(y)$ and $H^I(y)$ that capture the average horizontal and vertical scale of vortical features at a given y location. The inverses of these integral scales are defined as

$$\frac{1}{L^I(y)} = \frac{1}{2\sqrt{2}} \frac{1}{T} \int_0^T \left[\frac{\int \zeta'^2 dx dz}{\int \text{EKE} dx dz} \right]^{1/2} dt, \quad \text{and} \quad (11)$$

$$\frac{1}{H^I(y)} = \frac{1}{2\sqrt{2}} \frac{1}{T} \int_0^T \left[\frac{\int U_z'^2 dx dz}{\int \text{EKE} dx dz} \right]^{1/2} dt. \quad (12)$$

Here, the eddy vorticity $\zeta' = v'_x - u'_y$, eddy vertical shear $U_z' = u_z'^2 + v_z'^2$, and eddy kinetic energy $\text{EKE} = u'^2 + v'^2$ are based on velocity fields computed relative to a time-mean, $(u', v') = (u, v) - (\bar{u}, \bar{v})$, where $(\bar{u}, \bar{v}) = (1/T) \int_0^T (u, v) dt$. The factor of $2\sqrt{2}$ is based on the expected result for an idealized, localized Gaussian eddy [i.e., for an idealized two-dimensional flow $U(x, z) = U_0 e^{-(x^2/l_x^2 + z^2/l_z^2)}$, we recover $L^I = 2l_x$ and $H^I = 2l_z$]. Integral measures (11) and (12) are direct extensions of those used in idealized homogeneous turbulence studies and usually computed in spectral space [for a specific use case in a closely related problem of homogeneous rotating-stratified turbulence, see Waite and Bartello (2006)]. The definition of these length scales as an inverse is governed by numerical stability: both the vertical shear and vorticity are more localized (i.e., at smaller scales) than the EKE and are more likely to have extremely small values that cause divergences on division.

e. Scale-to-scale eddy kinetic energy flux

The energy fluxes between submesoscale vortical components of different spatial scales in the wake are computed by a coarse-graining approach employing Gaussian filters (Eyink and Aluie 2009). The filter-based approach is suitable for spatially inhomogeneous flows, unlike conventional scale-to-scale energy fluxes using spectral methods (Kraichnan 1971; Molemaker and McWilliams 2010; Arbic et al. 2013). Originally derived in the context of large-eddy simulations, per this approach, a low-pass spatial filter is applied to the energy equations (derived directly from the primitive equations). The resulting equations have terms corresponding to each term in the unfiltered energy equation except for a new term that couples the filtered term shears with the residual term stresses and represents the flux of energy from smaller high-pass-filtered scales to the coarse-grained low-pass-filtered scales. Thus for a given filter scale ℓ (where we only consider horizontal isotropic filters), if (u^ℓ, v^ℓ, w^ℓ) is the low-pass-filtered eddy velocity ($u^\ell \equiv u^\ell - u'^\ell$), then the flux of energy from scales larger than ℓ (the coarse-grained filtered fields) to scales smaller than ℓ (the “residual” components) can be shown to be [see Aluie et al. (2018) and Barkan et al. (2017) for details]

$$\Pi^\ell(x, y, z) = \Pi_h^\ell + \Pi_v^\ell, \quad (13)$$

where

$$\begin{aligned} \Pi_h^\ell = & -[\tau^\ell(u', u')u_x'^\ell + \tau^\ell(u', v')u_y'^\ell \\ & + \tau^\ell(v', v')v_y'^\ell + \tau^\ell(u', v')v_x'^\ell], \quad \text{and} \end{aligned} \quad (14)$$

$$\Pi_v^\ell = -[\tau^\ell(v', w')v_z'^\ell + \tau^\ell(u', w')u_z'^\ell], \quad (15)$$

and the Leonard stress term (Leonard 1975) is

$$\tau^\ell(u, v) = (uv)^\ell - u^\ell v^\ell. \quad (16)$$

The filtered instantaneous eddy kinetic energy can be written in the form $\text{EKE}^\ell = \tau^\ell(u', u')/2$ and represents the EKE in scales larger than ℓ .

3. Vortical flow regimes

We now offer a broad overview of the vortical flow regimes that manifest under our idealized framework in a $(\hat{h}, \gamma)$ parameter space. Starting from the quasi-geostrophic (QG) limit given by $\hat{h} \ll 1$, $\gamma \ll 1$, and increasing $\hat{h}$ and γ , we find strong eddying regimes as $\hat{h} \rightarrow 1$, with the additional result that the wake structure becomes largely independent of γ for $\hat{h} > 1$.

a. Steady wakes ($\hat{h} \ll 1$)

In the limit of $\hat{h} \rightarrow 0$, the computed wake solutions are found to be time independent and stable, although the spatial structure of these steady solutions is a strong function of γ . The limit $\gamma \rightarrow 0$, leads to the QG solution of flow past a seamount, which has been the subject of previous theoretical (Hogg 1973a; Smith 1979; Schär and Davies 1988) and numerical studies (Chapman and Haidvogel 1992). The steady QG solution for a constant inflow in a linearly stratified environment consists of a single isolated anticyclone situated symmetrically on top of the seamount (Hogg 1973a,b). This anticyclone, also referred to as a “Taylor cap” or “Taylor cone,” is generated by vortex stretching of fluid elements as they flow over the seamount, as a consequence of a lack of PV sources or sinks within the domain in the absence of bottom drag or wave effects.

The Taylor cap is clearly captured in our ROMS solutions even with bottom drag present (Figs. 1a, 2a,b), displaying the smooth deflection of isopycnals over the seamount (Figs. 2a,b), and the symmetrical velocity anomaly and vorticity fields closely track the theoretical QG solution derived by Schär and Davies (1988) in isopycnal coordinates. A minor difference is the appearance of a weak cyclonic wake (Fig. 1a) that we attribute to the effects of bottom drag, because ROMS solutions for this parameter value, in the absence of bottom drag, do not show this cyclonic wake. The no-drag cases are, however, not explored further for lack of physical relevance. The cyclonic wake strengthens when either γ is increased (Figs. 1e,i) or $\hat{h}$ is increased (Fig. 1b), both indicating a departure from the QG regime. The analytical solution of Schär and Davies (1988) formally breaks down at $\hat{h} \approx 0.5$ (for a somewhat similar seamount shape to the one chosen here) when one of the isopycnals in the solution intersects the bottom. The breakdown with increasing γ is, however, not explored. The increase in γ in particular introduces a prominent stagnation region in the midst of the Taylor cap on the top of the seamount, flanked by a strong cyclonic wake whose vertical extent spans the water column itself (as seen in the $\gamma = 0.8$ case in Figs. 2c,d). This stagnation region is a consequence of the isopycnals being prevented from deflecting in the vertical over the seamount due to the presence of the free surface, which flattens them and results in reduced flow close to the top of the seamount (from geostrophy). A comparison of Figs. 2a–d highlights the asymmetries induced in the vertical structure on increasing γ .

b. Wake instability and eddy generation [$\hat{h}_{\text{uns}}(\gamma) < \hat{h} < 1$]

For each of the three values of γ that we compute solutions for, we find an approximate value $\hat{h} = \hat{h}_{\text{uns}}$ beyond

FIG. 1. (a)–(l) Wake snapshots: barotropic vorticity $\bar{\zeta}^z/f$ for four different seamount nondimensional heights $\hat{h}$ and depth ratios γ as indicated. Columns share values of $\hat{h}$, while rows share γ values. The background barotropic flow is along the y axis from south to north, and $Ro_L = 0.1$ for all the cases. The corresponding values of s are (from left to right) 1, 2, 6, and 14. Note that the range of the x axis is chosen to be asymmetric around the seamount center to capture the east–west asymmetry in the wake.

which the flow becomes unstable, leading to eddying wake solutions. The $\hat{h}_{\text{uns}}(\gamma)$, which represents the “neutral curve” of wake instability in $(\hat{h}, \gamma)$ parameter space, is found to be a decreasing function of γ . This can be evinced by comparing the eddying wake in Fig. 1j when $(\hat{h}, \gamma) = (0.2, 0.8)$ with the steady wakes found for smaller

values of γ in Figs. 1b and 1f. Further, the nature and character of the wake eddies in the vicinity of $\hat{h}_{\text{uns}}(\gamma)$ are a strong function of γ , as detailed subsequently in this section. For $\gamma = 0.2$, and with increasing $\hat{h}$, the cyclonic wake remains stable until the flow on the anticyclonic side separates, resulting in a weak anticyclonic

FIG. 2. Vertical structure for $\hat{h} \ll 1$: x - z sections of the (a),(c) normalized velocity and (b),(d) Ro_g , with overlaid isopycnals for $\hat{h} = 0.1$ and two different values of the depth ratio γ [$\gamma = 0.2$ in (a) and (b); $\gamma = 0.8$ in (c) and (d)] at $\gamma = 0.1$, i.e., just past the seamount centerline. Here $Ro_L = 0.1$ and $s = 1$ for all cases. (top) The bottom 1000 m of the $D = 4000$ m deep ocean, whereas (bottom) the ocean is 1000 m deep and shows the full depth.

eddy dominance (not shown visually), which can be identified from the ratio of cyclonic/anticyclonic enstrophies plotted as a function of $\hat{h}$ in Fig. 3. The stability of the cyclonic wake when $\gamma \ll 1$ results from its confinement close to the ocean bottom; this in turn results in the stabilization by vertical mixing in the BBL (vertical structure not shown). The cyclonic wake in the $\gamma = 0.8$ case, however, extends all the way up to the ocean surface as previously discussed, and is unstable and subject to barotropic inflectional instabilities, resulting in a cyclonic eddy dominance (see Fig. 1j and the blue curve in Fig. 3). However, the significance of this asymmetry is tempered by the fact that the $\gamma = 0.8$ cyclonic eddying solution has significantly lower EKE compared to smaller γ values

(Fig. 3) that display a weak anticyclonic dominance. This fact can also be seen by comparing Figs. 1c and 1k. As an aside, it should be noted that the wake instabilities of the inflection-point type are inviscid instabilities and should be sensitive to the choice of the viscosity used. Because the internal biharmonic viscosity in ROMS decreases with increasing resolution, this sensitivity can be tested by recomputing the solutions at higher resolutions. However, our interest is in analyzing and understanding robust eddying wakes typical in the interaction of strong ocean currents with topography (Molemaker et al. 2015; Gula et al. 2016), and the wake stability problem is ancillary to the present study and consequently not explored further.

FIG. 3. (top) Time- and volume-averaged EKE, $(1/V)\int\langle u^2 + v^2 \rangle dV$ ($\text{m}^2 \text{s}^{-2}$). (bottom) Time- and volume-averaged eddy enstrophy parity ratio $Z_{cy}/(Z_{cy} + Z_{ac})$, where $Z = (1/V)\int\langle \zeta^2 \rangle dV$ and subscripts “cy” and “ac” refer to cyclonic and anticyclonic, respectively. A ratio value of 0.5 implies cyclone–anticyclone symmetry. Here V represents the entire flow volume excluding the bottom boundary layer.

c. The tall seamount regime ($\hat{h} > 1$) and γ independence

The previous sections highlight the strong dependence of the wake on γ for $\hat{h} < 1$, whether in the spatial structure of the wake, the nature of the cyclonic–anticyclonic asymmetry of eddies in the wake, or in the actual magnitude of EKE. For example, from Fig. 3, for $\hat{h} = 0.6$, the EKE in the $\gamma = 0.2$ case is about a factor of four larger than the $\gamma = 0.8$ case. This strong dependence on γ , however, is reduced as $\hat{h} \rightarrow 1$ and, in particular, for $\hat{h} > 1$. Figures 1d, 1h, and 1i display a striking degree of similarity in both the magnitude and structure of the barotropic vorticity for different values of γ (note that the three plots are shown at different time steps in their respective computations). The vertical structure of the time-mean velocity and vorticity, shown just downstream of the seamount in Fig. 4 for the $\gamma = 0.2$ and $\gamma = 0.8$ cases, underscores this similarity, especially compared to the significant differences at small values of $\hat{h}$. The key to understanding this γ independence is in the isopycnal structure in Fig. 4 compared to that in the QG case, $\hat{h} = 0.1$ in Fig. 2. In QG dynamics ($\hat{h} \ll 1$), the vertical deflection of isopycnals over the seamount is the height

itself $O(\hat{h})$, while the height of the Taylor cap, which represents the vertical extent over which the flow feels the topography, is precisely $O(1)$ [$O(h_T)$ in dimensional coordinates]. Consequently, when $\gamma \sim 1$, the isopycnal deflection is suppressed, resulting in strong confinement effects and significant structural differences in the flow. When $\hat{h} \sim 1$, the isopycnal deflection in the vertical is still much smaller than the Taylor height itself, which is now similar order as the height of the seamount (rather than much larger when $\hat{h} \ll 1$). Thus, relative to the height of the seamount, vertical isopycnal deflections, evident in the nearly horizontal isopycnals in Fig. 4, are negligible, and the effects of the seamount in the vertical are not felt beyond the seamount itself, minimizing the effect of changing γ . This result is also captured in the variation of integral properties, EKE, and the enstrophy ratio, both of which converge in the limit $\hat{h} \rightarrow 1$ (see Fig. 3). Furthermore, because the isopycnals do not deflect much over the topography, vortex stretching effects are minimal; this explains why the anticyclonic vorticity over the seamount in Fig. 4 is much weaker than that in Fig. 2. The generation and injection of PV in the bottom boundary layer is, however, much stronger when $\hat{h} > 1$, as detailed in subsequent sections. Vortex stretching effects on the topography can induce dynamical changes in the wake in certain situations, for example, by enhancing preexisting upstream baroclinicity, as discussed by Abernathy and Cessi (2014). In the present paradigm the upstream flow is barotropic, but baroclinicity is induced purely as a consequence of interaction with the topography. In Fig. 4 this is visible in the form of a single lobe of anticyclonic vorticity below $z \sim 1$ and a second symmetrical lobe close to the top of the seamount. The dependence of the downstream wake structure as a function of $\hat{h} > 1$ is also discussed in subsequent sections. The independence of $\hat{h} > 1$ dynamics from γ allows us to choose a single value of γ (while varying $\hat{h} > 1$) for further analysis. We choose, $\gamma = 0.8$ primarily for reasons of numerical efficiency: this case has a shallower ocean ($D = 1000$ m) leading to higher vertical resolution for the same number of σ levels.

4. Phenomenology of SCV generation

We now characterize the generation pathway of wake SCVs and their spatial structure as a function of $\hat{h}$. We start by examining the BBL structure on the seamount and its subsequent separation to form tilted shear layers with high vorticity and vertical shear. A horizontal shear instability of these tilted shear layers on the slope generates submesoscale vortical filaments that rapidly combine together to form wake SCVs.

FIG. 4. Vertical structure for $\hat{h} > 1$ with the same format as in Fig. 2.

a. Spatial structure of the BBL on the seamount

The spatial structure of the BBL on and around the seamount is a strong function of the nondimensional height $\hat{h}$. In particular, for $\hat{h} < 1$ and $\gamma \ll 1$, the quasi-geostrophic anticyclone on top of the seamount induces asymmetries in the flow (Figs. 2a,b), accelerating the flow west ($\hat{x} < 0$) of the centerline and decelerating it to the east ($\hat{x} > 0$). The BBL thickness in ROMS is parameterized through KPP to encapsulate stratified Ekman layer turbulence (Large et al. 1994) near the bottom, which in general displays the scaling law (Taylor and Sarkar 2008),

$$h_{\text{bbl}} \sim \frac{u^*}{f} G\left(\frac{N}{f}\right), \quad (17)$$

where the friction velocity is $u^* = \sqrt{\tau_b/\rho}$ and τ_b is bottom stress, defined in (9). Parameter $G(x)$ is a function of turbulence properties in the BBL and is typically determined through experimental or large-eddy simulation studies (see Taylor and Sarkar 2008, and references therein). In the absence of stratification $G(x) = 1$, resulting in the simple form, $h_{\text{bbl}} \sim u^*/f$. For finite values of N , however, h_{bbl} displays a range of scaling laws, ranging from $h_{\text{bbl}} \sim u^*/\sqrt{Nf}$ (Pollard et al. 1972; McWilliams et al. 2009) [corresponding to $G(x) = x^{-1/2}$] to $h_{\text{bbl}} \sim u^*/N$ (Kitaigorodskii 1960) [$G(x) = x^{-1}$], as the Prandtl ratio N/f increases. In general, stronger bottom velocities are generated by faster currents and lead to deeper BBLs. Consequently, the east–west asymmetry in the QG-based velocity field on the seamount causes an east–west asymmetry in the BBL thickness: deeper BBLs

FIG. 5. (a)–(d) The bottom-stress curl $\mathbf{k} \cdot \nabla \times \tau_b / f$ ($\text{kg m}^{-2} \text{s}^{-1}$), overlaid with the contours of BBL thickness h_{bbl} (m), in particular showing the contours of 5 and 40 m. The time-mean velocity (m s^{-1}) just outside the boundary layer is also shown in arrows. The value of $\hat{h}$ for each case is indicated, while the seamount is highlighted in green. Here $\gamma = 0.2$.

to the west, and shallower BBLs to the east of the centerline. Figures 5a–c support these expectations. The eastward deceleration and subsequent bottom stress reduction and boundary layer shallowing are also the reason why the cyclonic boundary layer separates early. This early separation can also be understood in terms of classical ideas of boundary layer separation, where a vanishing bottom shear stress leads to flow separation on the cyclonic side. Regions of separated flow on the seamount also show vanishing values of the bottom stress curl $\mathbf{k} \cdot \nabla \times \tau_b$ (see Fig. 5), which measures the flux of vorticity into the ocean due to bottom shear stress (Molemaker et al. 2015).

The case $\hat{h} > 1$ is different because, as explained in section 3, the QG anticyclone has a vertical extent that varies as $O(1)$ [$O(h_T)$ in dimensional coordinates], leading to the emergence of more complex vertical flow structure when $\hat{h} > 1$. This, in turn, induces greater complexity in the BBL structure on top of the seamount.

Figure 5d illustrates this added complexity of BBL structure for larger values of $\hat{h}$.

The preceding argument changes for larger values of γ when $\hat{h} < 1$ but as discussed in the previous section, these differences are less significant when $\hat{h} > 1$. Because larger $\hat{h}$ values manifest stronger eddying regimes, we examine the structure of BBL in greater detail for $\hat{h} > 1$ seamounts, for the particular value of $\gamma = 0.8$ (which is also the choice for all results in this manuscript from here on).

Figure 6 displays the vertical (x – z) structure of the BBL and flow fields just upstream of the seamount centerline ($\hat{y} = -0.1$, in nondimensional coordinates). As explained in the section above, the cyclonic-side BBL has already separated, resulting in $h_{\text{bbl}} \approx 0$ (second row of Fig. 6) and separated shear layers (bottom two rows of Fig. 6) for $0 < \hat{x} < 2$, but the anticyclonic side BBL is still attached to the bottom ($\hat{x} < 0$, bottom panel of Fig. 6). For the case, $\hat{h} = 0.7$ and $\hat{h} = 1.4$, the

FIG. 6. Vertical structure of the flow and BBL before separation on the anticyclonic side ($\hat{y} = -0.1$) for increasing $\hat{h}$, in nondimensional x - z coordinates. (top) Normalized time-mean velocity $[(v(x, z) - U)/U]$, with overlaid time-mean isopycnals in white. (second row) Time-mean vertical mixing coefficient from KPP κ ($\text{m}^2 \text{s}^{-1}$), where the vertical axis is dimensional depth relative to the ocean bottom $z - h_b(x, y)$ (m). (third row) Along-flow vertical shear v_z (s^{-1}) and (bottom) vorticity Ro_g , both plotted on the same $(\hat{x}, z - h_b)$ plane as κ . Note that the κ color bar is saturated to highlight the smaller values on the slope. Here $\gamma = 0.8$.

QG-induced asymmetry is evident with a thickening of the BBL on the slope that is induced by flow acceleration ($\hat{x} < 0$). Yet for larger $\hat{h}$ values, the picture changes on the slope, as discussed in the subsequent subsection. Away from the slope, the BBL decreases with stratification, approximately as $h_{\text{bbL}} \sim 3u^*/\sqrt{Nf}$ (dashed blue line in Fig. 7) suggested above in the discussion surrounding (17). (As a reminder, the $\hat{h}$ in Fig. 6 is varied by only increasing the stratification and with fixed $\text{Ro}_L = 0.1$.) This h_{bbL} dependence on N is approximately supported in Fig. 7.

b. BBL on the slope and Ekman buoyancy effects

A striking observation from Fig. 6 is the sudden shallowing of the BBL on the slope relative to the abyssal bottom, an effect that is more prominent at larger values of $\hat{h}$. The shallow BBLs on the slope ($-2 < \hat{x} < 2$) are associated with high vertical and horizontal shears (bottom two rows of Fig. 6). This differs from predictions of BBL structure on a topographic slope in simplified models (MacCready and Rhines 1993; Brink and Lentz 2010) that can be distilled for the current problem as follows: flow along the slope on the anticyclonic side ($-2 < \hat{x} < 0$ in Fig. 6) should generate a

cross-flow, Ekman-induced downwelling. Since downwelling on the slope transports lighter fluid under heavier fluid, the ensuing gravitational instability should induce vertical mixing and deepen the mixed layers on the slope relative to the abyss. These well mixed layers on the slope possess a horizontal buoyancy gradient and a corresponding geostrophic flow that opposes the background flow, weakening the bottom stress and consequently suppresses the generation of horizontal vorticity.

The results in Fig. 6 do not show deepening of mixed layers on the downwelling side or any evidence of enhanced vertical mixing on the slope due to gravitational instabilities. Further, the vertical shear (third row of Fig. 6) is ageostrophic (not shown) and does not oppose the background flow. In fact whenever the BBL deepens on the slope relative to the abyss (the first two columns of Fig. 6), it seems to be primarily determined by the local flow outside the BBL: that is, faster currents make for deeper boundary layers. Section 6 offers possible reasons for why the Ekman-induced buoyancy shutdown seen in simplified one- and two-dimensional models might not be significant in the more complex flows observed here.

FIG. 7. BBL depth h_{bbl} (m) (solid blue line) as a function of background stratification N (s^{-1}) away from the seamount with the empirical estimate $3u^*/\sqrt{Nf}$ (blue dashed line) overlaid. Also shown is the volume-averaged, root-mean-square vorticity, $\zeta_{\text{rms}} = \sqrt{(1/V)\int \zeta^2 dV}$ normalized by f , at four values corresponding to Fig. 6. Here V represents the entire flow volume excluding the bottom boundary layer.

c. Vertical and horizontal shear in separated BBLs

Molemaker et al. (2015) had a simple hypothesis for the boundary layer structure as it separates from the slope. They suggested that the boundary layer structure after separation can be predicted based on heuristic considerations of no-slip at the bottom prior to separation. This hypothesis can be understood in a plane that is normal to the local flow direction (in this case assumed to be along the y direction). We start by assuming that the flow outside the bottom boundary layer of depth h_{bbl} is $V_0(x, z)$. Then ignoring the details of the turbulent stratified Ekman profile of flow within the BBL itself, scale estimates for the horizontal and vertical shears in the BBL are

$$(v_x, v_z) \sim \frac{V_0}{h_{\text{bbl}}} (-\sin\theta_b, \cos\theta_b), \quad (18)$$

consequent of which

$$v_x \approx -v_z \tan\theta_b, \quad (19)$$

where $\tan\theta_b = |\nabla h_s|$ is the bottom slope. The BBL has an associated horizontal scale, which from geometry is just

$$l_{\text{bbl}} \sim h_{\text{bbl}}/\tan\theta_b. \quad (20)$$

Because $\theta_b \ll 1$, corresponding to gentle ocean slopes, we have the relations

$$(v_x, v_z) \sim \frac{V_0}{h_{\text{bbl}}} (-\theta_b, 1), \quad (21)$$

and

$$v_x \approx -v_z \theta_b. \quad (22)$$

The slope $\theta_b = \tan^{-1} dh_s/dx$ is the local value evaluated at that depth. Here, $\theta_b(z) \in [0, 1]$ for the case of the

Gaussian seamount in question. Now, while (21) constitutes scale estimates and is only expected to hold approximately, (22) is expected to be more accurate because they include the same boundary layer structure assumptions. This constraint in (22) is thus imposed purely by the topographic structure on the postseparation boundary layer. It is a local constraint and is independent of spatial variations of h_{bbl} caused by flow inhomogeneities that exist either independent of or as a consequence of interaction with the topography, as highlighted in sections 4a and 4b. However, the actual magnitudes of h_{bbl} and $V_0(x, y)$ help determine the shear values of the separating boundary layer, which can be interpreted as the topographic forcing of the fluid, specified by (21). We evaluate these relations with reference to the seamount solutions when the corresponding BBL initially begins to separate. Figure 8a–c show the time-mean vertical and horizontal shears and the difference between the two immediately downstream of the seamount centerline (at $\hat{y} = 0.2$), justifying the validity of (22). Further support for (22) is offered by varying the ratio $\langle |\bar{v}_x| \rangle / \langle |\bar{v}_z| \theta_b \rangle$ as a function of $\hat{h}$, where the angular brackets denote an average over the x - z plane (at $\hat{y} = 0.2$) and the overbar denotes the usual time average. As seen in Fig. 8c, this ratio is approximately equal to unity for $\hat{h} > 1$.

Because we are examining the shear layer just downstream of separation, the along-flow derivatives ∂_y are negligible, and thermal wind balance,

$$fv_z \sim b_x, \quad (23)$$

might be expected to hold. However comparing Figs. 8b and 8d, we find that the buoyancy gradient term is in fact weaker. This is because in Fig. 8, the vorticity $\zeta \approx v_x$ and $|\text{Ro}_\zeta| \approx |v_x|/f > 1$. Consequently, the downstream evolution of the shear layer is expected to be governed

FIG. 8. Shear layer structure downstream of the separation ($\hat{y} = 0.2$), showing normalized time-mean (a) horizontal shear, (b) vertical shear times the bottom slope angle θ_b , (c) the difference between the two, supporting (22), (d) the horizontal buoyancy gradient, and (e) the ratio $\langle |\bar{v}_x| \rangle / \langle |\bar{v}_z \theta_b| \rangle$ in the shear layers as a function of $\hat{h}$, where the operator $||$ denotes the absolute value and the bar the time mean, while the angular bracket denotes the spatial average $\langle f \rangle = \int f(\hat{x}, \hat{z}) dx dz$ at $\hat{y} = 0.2$. Note that $\theta_b > 0$ for $\hat{x} < 0$, and $\theta_b < 0$ for $\hat{x} > 0$. Here $(\hat{h}, s, \text{Ro}) = (4.6, 46, 0.1)$.

by gradient wind balance, rather than thermal wind balance.

An important realization is that the vorticity injection into the ocean due to this seamount is

$$\zeta_b \sim V_o \theta_b / h_{\text{bbl}}. \quad (24)$$

In section 4a, the depth of the BBL was found to decrease with increasing stratification (or $\hat{h}$) as $h_{\text{bbl}} \propto N^{-1/2}$, and consequently we expect a corresponding increase in

the vorticity flux into the fluid with increasing $\hat{h}$. Yet because $\theta_b \sim h_s/L$, we can write the normalized shear layer vorticity as

$$\frac{\zeta_b}{f} \sim \text{Ro}_L \frac{h_s}{h_{\text{bbl}}} \sim \text{Ro}_L O(h_s \sqrt{N}), \quad (25)$$

which illustrates the point that, while increasing N or h_s are equivalent means of increasing $\hat{h}$, the vorticity injection has a different dependence on the

FIG. 9. (a)–(d) Downstream evolution of the separated shear-layer instability: $\hat{x}$ – $\hat{z}$ sections of a single ζ/f snapshot in time at different $\hat{y}$, illustrating BBL separation and instability. The flow is along the direction of increasing $\hat{y}$, and $(\hat{h}, \hat{s}, \text{Ro}) = (4.6, 46, 0.1)$.

respective quantities. The total volume averaged vorticity $\zeta_{\text{rms}} = \sqrt{(1/V) \int \zeta^2 dV}$ plotted in Fig. 7 has a somewhat weaker dependence than the $\sqrt{N}$ behavior suggested in (25). The strength of the BBL vorticity flux into the fluid is revisited in section 5c.

d. Shear layer instability

The tilted shear layers formed through BBL separation are subject to both horizontal barotropic inflection-point instabilities and vertical Kelvin–Helmholtz instabilities. The latter cannot be resolved in a primitive equations model and are instead parameterized as part of the vertical mixing scheme (Large et al. 1994), though this does not seem to be a significant effect here (The vertical shear effects are more important in the $\text{Ro}_L \rightarrow 0$ limit and will be discussed in a forthcoming study). A simple analysis of the eddy to time-mean energy fluxes in the shear layers shows a dominant horizontal shear instability (i.e., the energy fluxes are dominated by the horizontal Reynolds stress terms). This mirrors previous results on submesoscale topographic wakes (see Dong et al. 2007; Gula et al. 2015b; Srinivasan et al. 2017) and is consequently not discussed here.

Figure 9 highlights the evolution of the separated shear layer vorticity (corresponding to the time-mean plots shown in Fig. 8) at successive downstream locations for a single snapshot in time. The instability of the shear layers has a pronounced cyclonic–anticyclonic asymmetry (Figs. 9a,b) with the anticyclonic side displaying smaller vertical and horizontal scales. Previous work on QG vortex instabilities (Gent and McWilliams 1986) suggests an *internal* barotropic instability with the ratio of vertical and horizontal instability scales satisfying $l_z^2/l_h^2 \sim f/N$. This scaling is however not found here, likely because large Ro_ζ values in the shear layer violate the QG approximation also causing the cyclonic–anticyclonic asymmetry in the instability (model-based evidence for a failure of this QG scaling is presented in section 5a).

Further, visualizing the vorticity isosurfaces in three dimensions (Fig. 10) highlights the instability as taking the form of distinctive and elongated strands of vorticity that extend downstream from the separation line on the seamount. A corresponding movie highlighting the time evolution of these isosurfaces (3disosurface.mp4 in the supplemental material) shows the vorticity strands spiraling downslope and counterclockwise, suggesting

FIG. 10. Three-dimensional wake structure: ζ/f isosurfaces at a single snapshot of the flow for (a) $\hat{h} = 1.4$ and (b) $\hat{h} = 4.6$. The specific values of vorticity on the isosurfaces are $\zeta/f = \pm 0.45$ in (a) and $\zeta/f = \pm 0.65$ in (b). The corresponding movie in the supplemental material (3disosurface.mp4) shows the vorticity strands with a propagation downhill and downstream.

that the instability has a wave-like character with attributes of a topographic Rossby wave. A detailed analysis of the instability of tilted shear layers on a slope will be addressed in the future.

e. Merger of vortex filaments and SCV generation

When $\hat{h} = 1.4$ (see Fig. 10a and the corresponding movie in the supplemental material), the vortical strands formed in the near wake rapidly combine to form stable columnar vortices that are approximately as tall as the seamount itself. An alternative visualization, through a cross-stream (along x) average of vorticity (Fig. 11a), similarly highlights the change in vertical scale from small scale vortical filaments to tall columnar vortices. The case $\hat{h} = 4.6$ also forms downstream vortices but with vertical scales that are consistently smaller than the seamount height (Figs. 10b, 11b). There is also greater vertical structure in the wake when $\hat{h} = 4.6$, with smaller vortices observed closer to the top of the seamount. Similar to the increase in vertical scale, is the change in horizontal scale of the downstream eddy vorticity, from small vortical filaments as they break off from the vortical strands (the near wakes in Figs. 11c,d) to larger coherent vortices further downstream. This rapid merger

of the vortical filaments in the wake can be visualized in the movie corresponding to the single snapshot in Fig. 11 (VortXY_YZ.mp4 in the supplemental material).

The merger of like-signed vortices to form larger vortices is a well-explored feature of two-dimensional and rotating stratified flows (Dritschel 2002; Von Hardenberg et al. 2000, and references therein). The horizontal merger of vortex filaments to form topographic wake SCVs has been noted by Molemaker et al. (2015) and Southwick et al. (2016), similar to the flow evolution at a single horizontal section in Figs. 11c and 11d. While a majority of studies explore the merger of vortices that lie in the same horizontal plane, the merger process has also been found in vertically offset vortices, provided the vortices still overlap in the vertical (Reinaud and Dritschel 2002). In a related set of studies, McWilliams (1989) and McWilliams et al. (1994) observe an evolution of tall columnar vortices in decaying geostrophic turbulence and identify this process as an “alignment” of vertically offset vortical blobs of the same sign (Reasor and Montgomery 2001), rather than a merger. We take the latter perspective as being responsible for the emergence of vertical coherence in topographic wakes studied here. The energetic implications of the

FIG. 11. Another view of SCVs in the wake for different values of $\hat{h}$, corresponding to the isosurface plots in Fig. 10. (a),(b) Cross-stream averaged vorticity $\bar{\zeta}^x/f$ snapshots for $\hat{h} = 1.4$ in (a) and $\hat{h} = 4.6$ in (b). (c),(d) Corresponding horizontal ζ/f section at a depth of $z = 500$ m, which is half the depth of the water column in each case. The nondimensional height varies due to increasing stratification. Here $Ro_L = 0.1$ and $\gamma = 0.8$. The time evolution corresponding to this snapshot is shown in the supplemental material (VortXY_YZ.mp4).

downstream increase in the scale of eddy field are discussed in the next section.

5. Energetics of SCV generation

In this section, eddy integral scales L^I and H^I [defined in (11) and (12)] are used to quantify the increase in

vertical and horizontal eddy scales as a function of the downstream distance $\hat{y}$, for different values of $\hat{h}$. Further, by computing the scale-to-scale energy fluxes from (13), we explicitly show that SCVs are formed through an upscale transfer of eddy kinetic energy from smaller vortical filaments. Concurrently, high values of Ro_ζ in the separated shear layers (sections 4c and 4a) result

FIG. 12. Eddy integral scales (a) L^I/L and (b) H^I/h_s as a function of downstream distance $\hat{y}$ with the seamount center at $\hat{y} = 0$. (c) The Prandtl ratio fL^I/NH^I and (d) the empirical ratio $\sqrt{f}L^I/6\sqrt{NH^I}$ are also plotted as a function of $\hat{y}$. Four different values in the range $1.4 \leq \hat{h} \leq 4.6$ are included.

in the generation of negative PV on the anticyclonic side, leading to enhanced dissipation through centrifugal instabilities. Thus the energetics of SCV formation involve *local* dissipation (on the anticyclonic side) and *nonlocal* upscaling (defined as the transfer of EKE to larger scales).

a. Evolution of eddy integral scales downstream of the seamount

Figure 12 shows the along-flow evolution of the eddy integral scales, L^I and H^I (normalized with the seamount half-width L and height h_s , respectively). These plots show the eddy field to be dominated by larger scales further downstream of the seamount, as expected from the visual perspectives afforded in Fig. 11. In Figs. 12a and 12b we find that for each value of $\hat{h}$, both vertical and horizontal eddy scales increase by a factor of approximately 5 from the unstable vortical filaments in the near wake to coherent SCVs in the far wake. The horizontal scales in the far wake decrease only marginally with increasing $\hat{h}$, while the far-wake vertical scales suggest an approximately inverse relationship, that is, $H^I \propto \hat{h}^{-1}$. This relative dependence of the vertical and horizontal scales on $\hat{h}$ can be assessed by plotting the ratio

$$r_{\text{QG}} = \frac{fL^I}{NH^I} \quad (26)$$

as a function of $\hat{y}$ in Fig. 12c. This allows us to compare the SCV structure relative to that of eddies in QG balance, satisfying $H^I/L^I \sim f/N$. This QG Prandtl scaling is found to describe eddy aspect ratios in equilibrated, idealized, three-dimensional, triply periodic, forced-dissipative models (Waite and Bartello 2006) and is a natural point of reference. It is, however, clear from Fig. 12c that QG scaling is in general not valid in topographic wakes because of the strong dependence of the ratio fL^I/NH^I on $\hat{h}$ (and consequently f/N). A bit of guesswork allows us to construct an alternative empirical ratio,

$$r_{\text{emp}} = \frac{1}{6} \sqrt{\frac{f}{N}} \frac{L^I}{H^I}, \quad (27)$$

displaying a significantly weaker dependence on $\hat{h}$ (and f/N) (Fig. 12d), and consequently allowing us to infer the scaling,

$$\frac{H^I}{L^I} \sim \sqrt{\frac{f}{N}}, \quad (28)$$

with the obvious caveat that model runs over a wider range of Ro_L and $\hat{h}$ values are required to support its validity; the numerical factor of 6 is chosen just to

bring r_{emp} to $O(1)$ values. The inverse dependence on $\sqrt{N}$ suggests a possible connection with the shear-layer vorticity scaling in (25), derived in section 4c. In this context, we note that a scaling for the aspect ratio of radially symmetric, finite Ro_ζ vortices in gradient wind balance were recently found by Hassanzadeh et al. (2012). A comparison of their suggested scaling (a function of Ro_ζ , f/N , and the stratification within the vortex) with (28) is of interest, although it is unclear if their simplifying assumptions about vortical structure apply to the wake SCVs found here.

It should be emphasized that the downstream increase in the integral scales means a transfer of EKE from small to large eddy components in the flow. This is because the EKE does not change appreciably (due to abyssal bottom drag) as a function of $\hat{y}$, from the point of the fully developed shear layer instability in the near wake to the far wake (not shown). Therefore changes in eddy scale imply a flux of energy from small to large scales, as the flow evolves downstream (explicitly shown in the next section).

b. Scale-to-scale flux of eddy kinetic energy

To shed light on this upscaling mechanism, we trace the average energy flux from the small vortical filaments in the near wake to coherent SCVs in the far wake. For this purpose, we examine time-averaged fluxes $\langle \Pi^\ell \rangle$ [defined in (13), section 2e] that measure the average energy flux from eddy scales larger than ℓ to smaller scales. Horizontal x - y maps of $\langle \Pi^\ell \rangle$ at a depth $z = 500$ m from the bottom are shown for different values of ℓ in Fig. 13. This figure illustrates the key fact that the average energy transfer is from the smaller-scale eddy components to the larger-scale eddy components for all the values shown. It is clear that the direction of transfer is almost everywhere upscale, as evinced from the negative values of $\langle \Pi^\ell \rangle$ seen in the wake, for a range of filter scales $0.25L \leq \ell \leq 2.47L$ (Fig. 13). This offers dynamical support that the visual vortex mergers seen in Figs. 11c and 11d represent an upscale energy transfer. Further, for increasing values of ℓ the maximum upscale flux shifts downstream, from the near wake when $\ell = 0.25L$ (Figs. 13a,e) to the far wake when $\ell = 2.47L$ (Figs. 13d,e). This is expected because the scales of the eddies get larger after undergoing successive mergers as they advect downstream. Once the eddies form coherent SCVs, the flux of energy stops. Further, the largest values of x -averaged energy flux in Fig. 13e lie in the downstream range $1 < \hat{y} < 4$, consistent with the location of fastest increase in integral scales (Figs. 12a,b).

The previous analysis elucidates the dynamics of the upscaling mechanism in the horizontal. Whether this upscaling is a consequence of merger of vertically offset

FIG. 13. (a)–(d) Time-averaged filter scale flux $\langle \Pi^\ell \rangle$ ($\text{m}^2 \text{s}^{-3}$) for increasing values of the filter-scale ℓ at a depth of 500 m. (e) The x-averaged $\langle \Pi^\ell \rangle$ for the same values of ℓ . Note that the maximum value of the flux shifts downstream for increasing ℓ . Here $(\hat{h}, \text{Ro}_L, s) = (4.6, 0.1, 46)$.

vortical filaments (Reinaud and Dritschel 2002), or alignment of vortical motions (McWilliams et al. 1994; Reasor and Montgomery 2001), remains undetermined. An extension of the scale-to-scale transfer approach in the vertical is a possible route of investigation.

c. Dissipation from negative PV in shear layers

The interaction of the flow with bottom drag is the only source of PV injection in this idealized framework, which for the hydrostatic primitive equations has the form

$$q = fb_z + \underbrace{(v_x - u_y)b_z}_{q_v} - \underbrace{v_z b_x + u_z b_y}_{q_h}, \quad (29)$$

where the subscripts refer to the horizontal and vertical components of the shear-associated PV. Figure 14 highlights the PV in the separated BBLs for $(\hat{h}, \text{Ro}, s) = (4.6, 0.1, 46)$. First as expected, q_v is antisymmetric due to the dependence on the vorticity; it is negative in the anticyclonic side and positive on the cyclonic side. Secondary shear layers are also visible between the

separating BBLs and the seamount, whose q is opposite to those of the primary layers. These secondary BBLs are induced by SCVs formed downstream of the seamount that result in opposite signed q on the seamount itself. The horizontal component, q_h is, however, always negative at this y location (not shown), which can be derived by assuming weak along-flow variations $b_y \approx 0$ (section 4c) and balance (23), so that $q_h \approx -fv_z^2$ (Thomas et al. 2013). However, in the entire range of parameters considered in this manuscript, $q_v \gg q_h$, in line with previous studies (Molemaker et al. 2015). A key observation in Fig. 14 is the presence of negative q in the anticyclonic regions of the separated BBL. The presence of negative q is not stable and leads to a class of three-dimensional inertial instabilities that tend to homogenize q to a state where only nonnegative values of q exist in the flow (Dong et al. 2007; Molemaker et al. 2015). When the negative q is caused by the vertical vorticity, that is, q_v , these inertial instabilities are referred to as centrifugal instabilities and are most relevant for submesoscale topographic wakes. This can be illustrated visually by examining the evolution of the

FIG. 14. (a) Snapshot of the vertical component of PV vorticity q_v , as defined in (29), and (b) the total negative q just downstream of the seamount centerline, as defined in (29). Both are normalized with the background value fN^2 . Isopycnal contours are overlaid. Here $(\hat{h}, s, \text{Ro}) = (4.6, 46, 0.1)$.

extrema of vorticity found in filaments and vortices in the wake region. Vortical elements in the flow are identified as distinct regions where the Okubo–Weiss criterion $S^2 - \zeta^2 < 0$ holds, while also satisfying $|\zeta|/f > 0.1$. Here S is the magnitude of the principal strain tensor. Figures 15a–d highlight the downstream evolution of the vorticity extrema of each vortical element over the entirety of each model run (barring the initial startup). Two prominent details stand out as we examine this evolution for increasing values of N (shown as $\hat{h}$). First, as hypothesized in (25) the maximum vorticity of the filaments just ejected from the seamount increases approximately as $\sqrt{N}$. Second, $\zeta = -f$ is a strong lower bound for the anticyclonic eddies supporting the hypothesis that negative q instabilities can be equated with $\zeta < -f$ (or negative q_v) instabilities in topographic wakes. Filaments with values $\zeta < -f$ are mixed away within only a few downstream seamount lengths, inducing cyclonic–anticyclonic asymmetries in the wake structure when $\hat{h}$ is large. Another consequence of larger anticyclonic vorticity generated with increasing N is that more anticyclonic vorticity satisfying $\zeta < -f$ must be mixed by CI to $\zeta > -f$ values, suggesting stronger dissipation in the near wake with increasing N (and correspondingly $\hat{h}$). Figure 15e confirms this expectation showing a monotonic increase of the volume-integrated eddy dissipation $\langle \epsilon \rangle = \int \rho \epsilon dV$ with $\hat{h}$.

It should be noted that CIs transfer energy from vortical motions to small-scale three-dimensional motions and are in general nonhydrostatic. As such their dynamical evolution cannot be exactly captured by the hydrostatic model used here. However, Dewar et al. (2015) showed by employing the nonhydrostatic MITGCM model, that ROMS accurately captured the dissipation and mixing induced by CI, in spite of being a hydrostatic model. In ROMS, negative q is

destroyed when the centrifugal instability creates smaller scales that are either mixed away by the parameterized vertical mixing or by horizontal hyperdiffusion that acts at grid scales. Figure 16 displays the spatial collocation of negative q regions in the anticyclonic shear layers and enhanced values of ϵ generally in line with recent studies (Gula et al. 2016; Molemaker et al. 2015), although the actual magnitude of wake dissipation in the anticyclonic wake here is lower than those studies find. The reason is that, while the f value is similar to the f values that those process studies are based in [the CUC (Molemaker et al. 2015) and the Gulf Stream region (Gula et al. 2016)], the largest stratification choice here, $N = 0.006 \text{ s}^{-1}$, is somewhat lower than the values in those studies ($N \in [0.008, 0.12]$) but more importantly, the current speeds are many times faster in those studies than the $V_0 = 0.1 \text{ m s}^{-1}$ chosen here (especially in the Gulf Stream). Because BBL vorticity decreases with N and increases with V_0 [see (25)], those studies have anticyclones with larger values of vorticity and correspondingly stronger dissipation.

6. Discussion and conclusions

Oceanic topographic features like islands, headlands, depressions, and seamounts generally have finite but small bottom slopes ($\theta_b \ll 1$), which has important implications for the turbulent–drag interaction between currents and topography. In particular, the generation and injection of vertical vorticity into the ocean is accomplished through formation and separation of bottom boundary layers, instead of lateral boundary layers that would predominate if the oceanic side boundaries were nearly vertical. BBLs have a vertical extent of tens of meters, and on topographic slopes, an associated horizontal scale as small as hundreds of meters, hence Ro_ζ values larger than unity that on separation are injected into the oceanic interior.

To study this process, a shallow-slope seamount in constant background stratification subject to a steady barotropic inflow is examined here. The value of the Coriolis parameter f is set to values seen in the mid-latitudes, while stratification values are chosen to vary between those found between the deep ocean and the thermocline. An intrinsic length scale in this problem is the Taylor height $h_T = fL/N$, and the wake structure is found to be strong function of the ratio of the seamount height h_s to h_T , $\hat{h} = h_s/h_T$. The wake generation process starts with the formation of BBLs on the seamount slope. When $\hat{h} \ll 1$, the BBL does not separate and consequently no eddying flow need arise. In this nearly QG regime, the equilibrium state is steady with a single isolated anticyclone situated on top of the seamount.

FIG. 15. (a)–(d) Downstream evolution of vorticity extrema in the coherent filaments and eddies isolated through the procedure described in the text. The horizontal black lines are for $\zeta = \pm f$. For each case, $Ro_L = 0.1$, while the nondimensional height varies due to increasing stratification. (e) Volume-integrated dissipation [watts (W)] for the cases shown in (a)–(d).

By increasing $\hat{h}$ through either the h_s or N , the steady QG regime transitions to an eddying wake regime, as a consequence of BBL separation in the seamount lee. The BBLs on the slope and subsequently the shear layers that form after separation have strong vertical and horizontal shears that are geometrically constrained as $v_z \approx v_x \theta_b$, where θ_b is the bottom slope.

For the model runs shown here, we find that Ekman buoyancy effects of cross-slope advection, which can weaken

bottom stress and accompanying vorticity generation, are not strong in boundary layers on the slope. To reconcile the results here from recent work (Trowbridge and Lentz 2018, and references therein), we note the simplified theoretical and modeling configurations that those studies are based on. Almost all the studies approximate flow on the slope to be one-dimensional, with variations only in the direction normal to the slope (Garrett et al. 1993; MacCready and Rhines 1993;

FIG. 16. Negative q regions and enhanced dissipation. (a)–(c) Snapshots of negative q [normalized by fN^2 , where $(f, N) = (7 \times 10^{-5} \text{ s}^{-1}, 0.006 \text{ s}^{-1})$] at three different downstream sections illustrating the destruction of the negative q in the downstream evolution. (d)–(f) Time-averaged interior eddy dissipation ε ($\text{m}^2 \text{ s}^{-3}$). Here $(\hat{h}, s, \text{Ro}_L) = (4.6, 46, 0.1)$.

Brink and Lentz 2010) or two-dimensional (variations in cross-flow and slope-normal directions) (Benthuyssen et al. 2015; Ruan and Thompson 2016); this allows an indefinitely long evolution period for flow along the slope. Here, however, the flow has a strong along-flow pressure gradient that accelerates it from the stagnation region when the flow first encounters the seamount to $\sim U$ (the imposed barotropic velocity) values close to the point of separation. Further, curvature effects cause a cross-flow pressure gradient, also not present in the simplified models, and would appear as significant addition to the BBL momentum balance relative to the simplified configurations. In particular, along-flow acceleration could cause stronger bottom stress and counteract Ekman buoyancy weakening depending on the adjustment time scale of the downslope Ekman flow [which even in the case of no acceleration can be many inertial periods (Brink and Lentz 2010)]. Further investigation of the impact of along-flow acceleration and curvature on Ekman buoyancy fluxes is currently underway.

In general the presence of high vertical vorticity in the separated shear layers ($\zeta/f \sim 1$) is expected to lead to a combination of barotropic and centrifugal (when $\zeta/f < -1$) instabilities, the latter of which result in enhanced dissipation in the anticyclonic shear layers. Larger values of the background N (and correspondingly, $\hat{h}$) induce shallower BBLs with larger values of ζ/f ; the BBLs on separation generate shear layers with higher vortical vorticity leading to stronger centrifugal (or negative potential vorticity) instabilities and stronger dissipation on the anticyclonic side. Thus the near-wake

dissipation monotonically increases with increasing N (and correspondingly, $\hat{h}$). Furthermore, these shear layer instabilities lead to the formation of vortical filaments of similar scales as the shear layer width, which subsequently upscale to form larger, $\text{Ro}_\zeta \sim 1$, stable, submesoscale coherent vortices (SCVs). The upscaling mechanism involves a merger of filaments in the horizontal and an alignment in the vertical. The wake SCVs are found to not obey the classic Prandtl (QG) scaling for the eddy aspect ratio $L_z/L_h = f/N$, because of their $O(1)$ Rossby numbers. For the midlatitude f values considered here, by employing eddy integral scale measures, the eddy aspect ratios are instead shown to follow the scaling $L_z/L_h \sim \sqrt{f/N}$, although a larger parameter sweep would be required to test the generality of this scaling for topographic SCVs.

For small values of $\hat{h}$ (i.e., for weak stratification), vertically homogeneous Taylor column-like vortices are observed in the wake. However, as $\hat{h}$ is increased, SCVs at different vertical levels begin to emerge, in spite of the barotropic upstream flow (see Fig. 11). This can be understood as follows. For weak stratification, strong rotation couples fluid layers in the vertical, resulting in vertical coherence. When stratification becomes larger, this rotation-induced constraint only extends over a finite height, which for the seamount is given by h_T . Thus, when $\hat{h} \gg 1$ or equivalently $h_s \gg h_T$, SCVs of vertical sizes as small as h_T are present in the flow.

A key question is whether the results found here generalize to more realistic current and stratification profiles found in the ocean. This is relevant because the

boundary layer vorticity generation is a highly local process that depends on the local flow velocity, BBL depth, and slope θ_b . One avenue is to examine the SCV generation as a function of a local nondimensional height $\hat{h}_\theta = N\theta_b/N$ ($\approx \hat{h}$ for the seamount, with $\theta_b \sim h_s/L$). Furthermore, this study primarily focuses on isolating the SCV generation process in midlatitudes with small Ro_L , and there are other important physically relevant parameter regimes in faster currents (common in mean boundary currents) or closer to the equator that will be addressed in future studies.

Bottom currents, density stratification, and topographic slopes are universal features of the ocean. SCVs are widely found and therefore, submesoscale wake flows should be common, by the mechanisms of vortical generation demonstrated in this manuscript.

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