Application of Classical Coastal Trapped Wave Theory to High-Scattering Regions

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(Manuscript received 31 May 2018, in final form 5 June 2019)

ABSTRACT

Since the 1970s, analytical models of coastal trapped waves (CTWs) have been developed using a first-order wave equation in the long-wave limit. Formulations of this kind require assumptions of a straight coastline with similar shelf bathymetry. These assumptions prevent the models from capturing the scattering and backscattering behavior of propagating CTWs that encounter changing coastlines, bathymetry, or shelf width. CTW modes from two different analytical models, one of which includes friction and stratification, are compared with CTW observations of velocity and pressure from a study region near the Outer Banks off the North Carolina coast in the United States. The coastline in the study region is relatively straight locally but is bounded by an estuary to the north and shelf narrowing to the south, both of which induce scattering. The models suggest that the CTWs in this region are insensitive to changes in stratification, implying that observed seasonal differences in wave magnitude are due to seasonal wind forcing. Furthermore, friction is found to be important, particularly for mode-1 propagation, but higher-order modes are prevalent despite the importance of friction. There is very poor agreement between the observed and modeled free and forced CTWs because of scattering. This lack of agreement indicates that this is not a globally applicable theoretical formulation because many global coastlines violate the basic assumptions.

1. Introduction

Coastal trapped waves (CTWs) are subinertial, shallow water waves that are typically initiated by alongshore wind stress (Adams and Buchwald 1969), although other forcing mechanisms such as estuarine output, topographic irregularities, and tides are possible (Brink 1991). Conservation of potential vorticity considerations dictate that they propagate cyclonically with the coast on the right (left) in the Northern (Southern) Hemisphere (Longuet-Higgins 1965). These waves contribute a large portion of the coastal sea level and current low-frequency

variability along many coastlines but are generally understudied. This is due to a number of factors including the length of the waves relative to typical mooring or tide gauge spacing and other properties (i.e., period, phase speed, etc.) that change over relatively short distances as a result of shelf width, bathymetry, stratification, and bottom friction.

Because of these difficulties, theory utilizing a forced, first-order wave equation was first developed in the 1970s to try to describe CTWs analytically. The simplest, barotropic formulation of this problem in the long-wave limit (Buchwald and Adams 1968; Gill and Schumann 1974) has been expanded to include stratification (Gill and Clarke 1974), variable bathymetry (Clarke 1977), and bottom friction (Brink 1982b). Observations of

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along-shelf CTW sea level fluctuations compare reasonably well with theory for coastlines that are relatively straight and have consistent bathymetry (Brink 1982a; Battisti and Hickey 1984; Church et al. 1986; Chapman 1987). However, most global coastlines violate the assumptions of a straight coastline and shelf similarity that are required to solve these formulations in the long-wave limit. Even within the studies mentioned with a “well behaved” coastline, the poorest agreement arises as a result of violating those assumptions (Chapman 1987) or uncertainty in the wind forcing (Battisti and Hickey 1984).

It is known that changes in coastline, shelf width, and bathymetry can induce scattering into a greater proportion of higher-order modes or backscattering upstream (Wilkin and Chapman 1990). Theoretical experiments have been set up for specific, idealized shelf geometries to test this (see Wang 1980; Huthnance et al. 1986), but present comprehensive theory cannot capture this behavior. Additionally, much is still unknown about scattering of CTWs and the role of factors like stratification, friction, and wind forcing. There has been a historical lack of CTW studies along the east coast of the United States, with a few exceptions (e.g., Wang 1979; Schulz et al. 2012), because of the complex coastline and bathymetry in the Mid-Atlantic Bight. One such example is Hudson Canyon’s tendency to generate mode-2 CTWs resulting from the interactions among wind direction, propagation direction, and canyon geometry (Zhang and Lentz 2017, 2018). Hudson Canyon is only a single complicating factor; other complexities such as sharp changes in the coastline near Cape Cod, abrupt changes in bathymetry in Georges Bank, interruption of the waveguide by estuaries, and narrowing of the shelf near Cape Hatteras also likely play a role. Wang (1980) hypothesized that free waves might not be capable of existing in the New York Bight and North Carolina coast because of the complex geometry.

Present theory and understanding do not allow us to directly study CTW scattering, so the problem must be approached indirectly. In this paper, we use two different analytical models—the simplest, original model (Gill and Schumann 1974) and the most modern model (Brink 2006)—to determine the effect of seasonal stratification, bottom friction, and wind forcing on the cross-shelf structure of CTWs near the Outer Banks of North Carolina. The study region has a relatively straight coastline locally, but is bounded by an estuary (Chesapeake Bay) to the north and shelf narrowing to the south that may induce both refraction and backscattering. We use velocity and pressure observational data from a mooring array to verify the presence of CTWs in the study region and compare the cross-shelf structure to the analytical model results. Differences between observed and modeled CTWs provides

us insight about the possible influence of backscattering CTW propagation in our study region.

This paper is organized in the following way. Section 2 reviews the two theoretical models used to obtain analytical CTW solutions and observational data from a large mooring array for comparison. Section 3 describes the analytical free- and forced-wave modes for our study region. Section 4 provides a description of the observed CTW field and compares observed and modeled results. Section 5 discusses differences from this comparison, as well as identifying the source of these waves and their propagation along the larger U.S. East Coast continental shelf. Section 6 summarizes the findings of this study and presents possible future work.

2. Data and methods

a. Theoretical free-wave models

We use two different analytical models to determine the analytical CTW free-wave solutions for a cross-section of the North Carolina shelf. The first model is the simple formulation by Gill and Schumann (1974), and the second is an expansion of the first by Brink (2006) that includes additional features. Comparing the analytical solutions produced by both models allows us to determine the theoretical effect of stratification and friction on CTWs in our study region.

1) GILL AND SCHUMANN (1974) MODEL

We first use the formulation by Gill and Schumann (1974) to find an analytical solution for the CTW modes in our study region. We consider a locally straight coastline with shelf similarity in a right-handed coordinate system, such that the positive x axis points seaward and the y axis lies along the coastline at $x = 0$. This convention is the opposite of the one used by Gill and Schumann (1974), but we feel that a standard right-handed coordinate system makes more sense given the orientation of the coastline that we are interested in studying. We also note that there are a few errors in the original paper (e.g., normalization of the forced-wave solutions), and therefore we go through the full derivation in a complete, methodical manner. The inviscid, linearized, barotropic, vertically integrated momentum equations are

$$-fv = -g\frac{\partial\eta}{\partial x}, \quad (1)$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial\eta}{\partial y} + \frac{Y}{H}, \quad \text{and} \quad (2)$$

$$\frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0, \quad (3)$$

where $u(x, t)$ and $v(y, t)$ are the depth-integrated velocities, $\eta(x, y, t) = \eta'(x, y, t) + \eta_a(x, t)$ is the sea surface perturbation as a combination of sea surface height above equilibrium (η') and sea surface response to atmospheric pressure (η_a), $H(x)$ is the water depth, $Y(y, t)$ is the meridional wind stress $\tau^{wy}(y, t)$ divided by the water density ρ_w ($\rho_w = 1027 \text{ kg m}^{-3}$), f is a constant Coriolis parameter, and g is gravity. The $\partial u/\partial t$ term in Eq. (1) has been ignored because v is much larger than u in the long-wave limit. Likewise, the zonal wind stress term X/H has also been removed from Eq. (1) because we are only interested in wind forcing in the alongshore direction. Last, satisfaction of the condition $\varepsilon = f^2 L^2 / (g H_0) \ll 1$, where L is the shelf width and H_0 is the average shelf depth, allows us to neglect the surface divergence term from Eq. (3).

To find a vorticity equation, we employ the standard definition for the streamfunction ψ :

$$u = \frac{1}{H} \frac{\partial \psi}{\partial y} \quad \text{and} \quad (4)$$

$$v = -\frac{1}{H} \frac{\partial \psi}{\partial x}. \quad (5)$$

Eliminating η from the momentum equations and then using this streamfunction definition and the continuity equation, we obtain the vorticity equation

$$\frac{\partial^2}{\partial t \partial x} \left(\frac{1}{H} \frac{\partial \psi}{\partial x} \right) + \frac{f}{H^2} \frac{dH}{dx} \frac{\partial \psi}{\partial y} = \frac{dH}{dx} \frac{Y}{H^2}. \quad (6)$$

This problem can be solved with appropriate definitions for the shelf profile $H(x)$ and wind forcing $Y(y, t)$ as well as boundary conditions at the coast and shelf edge. We impose a condition of no flow across the boundary at the coast ($\psi = 0$ at $x = 0$) and flow vanishing at the shelf edge in the long-wave limit ($\partial \psi / \partial x = 0$ at $x = L$).

Equation (6) can be solved via separation of variables by further defining the streamfunction as $\psi(x, y, t) = F(x)\Phi(y, t)$, where $F(x)$ is the free-wave solution and $\Phi(y, t)$ is the forced-wave solution, which is considered in section 2b. By setting the right-hand side of Eq. (6) or, equivalently, the forcing equal to zero and implementing the separation, we get a second-order ordinary differential equation for the free-wave solution F :

$$\frac{d}{dx} \left(\frac{1}{H} \frac{dF}{dx} \right) - \frac{fF}{cH^2} \frac{dH}{dx} = 0, \quad (7)$$

where c is an unknown separation parameter but physically represents the phase speed. For a well-behaved shelf profile, such as the exponential shelf profile used in Buchwald and Adams (1968) and Gill and Schumann

(1974), Eq. (7) can be solved analytically to obtain the free-wave solution modes F_n and the corresponding phase speeds c_n . However, for a more complex shelf profile, such as the real bathymetry off the coast of North Carolina that we use here, a finite-difference scheme can be used to solve the equation numerically. By setting Eq. (7) equal to 1 rather than to 0 and looping through appropriate guesses for c , we identify the modes and corresponding phase speeds by resonant peaks in the variance of F . This small change allows us to essentially identify resonant wave modes with the same frequency as their forcing.

The free-wave alongshore velocity v and sea surface perturbation η can then easily be calculated using Eq. (5) and Eq. (1), respectively. This requires us to assume that ψ is proportional to F at this point.

2) BRINK (2006) MODEL

We also use a more complex formulation by Brink (2006) that includes some important features—such as stratification, bottom friction, and vertical variations—that the formulation by Gill and Schumann (1974) is missing. We again go through the momentum equations and assumptions for consistency and comparison with the above model. Using the same coordinate system as above and used by Brink (2006) with z positive upward, the updated momentum equations become

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} - f v = -\frac{1}{\rho_0} \left(\frac{\partial P}{\partial x} - \frac{\partial \tau^{tx}}{\partial z} \right), \quad (8)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v_0}{\partial x} + v_0 \frac{\partial v}{\partial y} + w \frac{\partial v_0}{\partial z} + f u = -\frac{1}{\rho_0} \left(\frac{\partial P}{\partial y} - \frac{\partial \tau^{ty}}{\partial z} \right), \quad (9)$$

$$\frac{\partial P}{\partial z} = -g \rho_2, \quad (10)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad \text{and} \quad (11)$$

$$\frac{\partial \rho_2}{\partial t} + u \frac{\partial \rho_1}{\partial x} + v_0 \frac{\partial \rho_2}{\partial y} + w \frac{\partial \rho_1}{\partial z} = 0, \quad (12)$$

where $u(x, z, t)$, $v(y, z, t)$, and $w(x, y, z, t)$ are the velocities; $v_0(x, z)$ is the mean flow; $\rho(x, y, z, t) = \rho_0 + \rho_1(x, z) + \rho_2(x, y, z, t)$ is the density as a combination of the mean density ρ_0 , background density ρ_1 , and wave perturbation density ρ_2 ; $P(x, y, z, t)$ is the perturbation pressure; and $\tau^{tx}(x, z)$ and $\tau^{ty}(y, z)$ are the turbulent stresses. Bottom turbulent stresses τ_0^{tx} and τ_0^{ty} are related to the interior bottom velocities u_B and v_B as $\tau_0^{tx} = \rho_0 r_B u_B$ and $\tau_0^{ty} = \rho_0 r_B v_B$, where $r_B(x) = C_D \sigma_v$ is a bottom resistance or friction coefficient based on drag coefficient C_D ($C_D = 3.0 \times 10^{-3}$) and standard deviation of alongshore velocity $\sigma_v(x)$ (Brink 1982a,b).

The problem can be solved by assuming solutions of the form $P = p(x, y) \exp[i(\omega t + ly)]$, where ω is the frequency and l is the alongshore wavenumber, and introducing the square of the Brunt–Väisälä frequency,

$$N^2 = -\frac{g}{\rho_0} \frac{\partial \rho_1}{\partial z} \quad \text{and} \quad M^2 = -\frac{g}{\rho_0} \frac{\partial \rho_1}{\partial x}, \quad (13)$$

to simplify the resulting partial differential equation for pressure. At the time of writing, K. Brink has conveniently made his dynamic mode analysis code (called “bigr” for real ω and “bigc” for complex ω) available online (<http://www.whoi.edu/page.do?pid=23361>) to solve this problem based on user input to define the environment. The code calculates and displays P , u , v , and w modes as well as the corresponding phase speed, wind coupling coefficient, friction damping coefficient, frictional decay time, and energy diagnostic parameters.

b. Theoretical forced-wave model

We return to the formulation used by Gill and Schumann (1974) to find the forced-wave solution and to obtain the full CTW solution from Eq. (6). The identified free-wave mode solutions F_n can be substituted into the streamfunction definition $\psi(x, y, t) = F(x)\Phi(y, t)$ to express it as the sum of modes; that is,

$$\psi(x, y, t) = \sum_n F_n(x) \Phi_n(y, t). \quad (14)$$

Note that we utilize the free-wave mode solutions $F_n(x)$ from the Gill and Schumann (1974) model for simplicity, but the solutions $F_n(x, z)$ from the Brink (2006) model are equally valid and can be used to find a full solution ψ that is a function of x , y , z , and t . Substituting this expression into Eq. (6) results in a set of first-order wave equations for the forced-wave solution Φ_n :

$$\frac{1}{c_n} \frac{\partial \Phi_n}{\partial t} + \frac{\partial \Phi_n}{\partial y} = b_n f, \quad (15)$$

where b_n is a wind coupling coefficient whose value arises from the normalization of F_n . The free-wave modes are normalized as $\mathcal{F}_n = F_n/(C_{mn})^{1/2}$, where

$$C_{mn} = \int_0^L \frac{1}{H^2} F_m F_n dx. \quad (16)$$

The wind coupling coefficient then becomes

$$b_n = \int_0^L \frac{1}{H^2} \frac{dH}{dx} \mathcal{F}_n dx. \quad (17)$$

With known right-hand side, Eq. (15) can also be solved using a finite-difference scheme to determine

the forced-wave solutions Φ_n , which can be used in conjunction with the free-wave solutions F_n to obtain the full solution ψ , as in Eq. (14). As before, the alongshore velocity v and sea surface perturbation η can easily be calculated.

c. OMP observational data

The Ocean Margins Program (OMP) was a multidisciplinary project that set out to examine carbon cycling in the North Atlantic Ocean, specifically focused on the area between the mouth of the Chesapeake Bay and Cape Hatteras—an area that also happens to be a likely hotspot of CTW scattering. A large array of 27 moorings was deployed on the continental shelf within the study region (see Fig. 1 for locations) during two separate time periods in 1996: “Deployment A” from February to May and “Deployment B” from June to October. Although many data were collected, they were never analyzed and results were not published; they therefore remain a valuable, untapped resource more than 20 years later (the data are available at <http://po.msfc.sunysb.edu/omp/>).

For this study, we focus on the northern cross-shelf transect of moorings because it is the widest and has the highest density of mooring coverage. We use hourly surface and near-bottom velocity data from six stations [stations 1–5 and 7 for Deployment A, and stations 1–5 and 9 (station 9 has no bottom data) for Deployment B] and bottom pressure data from three stations (Stations 5–7 for Deployment A and stations 5–6 and 10 for Deployment B) on this transect. Bottom pressure data are supplemented with hourly tide data from nearby NOAA tide gauge 8651370 (identifier DK) in Duck, North Carolina (the data are available online: <https://tidesandcurrents.noaa.gov/stationhome.html?id=8651370>). These data allow us to study the cross-shelf structure of velocity and sea surface height (SSH) of the CTWs in this region. To determine the seasonal effect of stratification and wind forcing on CTWs, we consider Deployment A from February to May to be a winter and transitional spring case with low stratification and high wind. Likewise, we consider Deployment B from June to October to be a summer and transitional autumn case with high stratification and low wind.

3. Model results

a. Free-wave solutions for OMP cross section

We utilize the above models—hereinafter referred to as the GS74 and B06 models as the models of Gill and Schumann (1974) and Brink (2006), respectively—to calculate the theoretical velocity and sea surface perturbation solutions for a cross section of the North Carolina coast

FIG. 1. Locations of moorings (black and yellow dots), tide gauges (red dots), and buoys (cyan dots) within (a) the larger northwestern Atlantic continental shelf and (b) the OMP study region. Black dots are moorings deployed during Deployment A and B; yellow dots are moorings deployed during only Deployment A. The black box in (a) outlines the study region in (b). Gray lines are bathymetric contours at 20, 40, 60, 80, 200, 1000, 2000, 3000, and 4000 m. Tide gauges from north to south are FP = Fort Point, New Hampshire (8423898); NI = Nantucket Island, Massachusetts (8449130); MK = Montauk, New York (8510560); SH = Sandy Hook, New Jersey (8531680); AC = Atlantic City, New Jersey (8534720); CM = Cape May, New Jersey (8536110); LS = Lewes, Delaware (8557380); OC = Ocean City Inlet, Maryland (8570283); KP = Kiptopeke, Virginia (8632200); CB = Chesapeake Bay Bridge Tunnel, Virginia (8638863); DK = Duck, North Carolina (8651370); WB = Wrightsville Beach, North Carolina (8658163); SP = Springsmaid Pier, South Carolina (8661070); CR = Charleston, Cooper River Entrance, South Carolina (8665530); SV = Fort Pulaski, Savannah, Georgia (8670870); PC = Trident Pier, Port Canaveral, Florida (8721604); LW = Lake Worth Pier, Atlantic Ocean, Florida (8722670); and VB = Virginia Beach, Biscayne Bay, Florida (8723214).

at 36.7°N ($f = 8.69 \times 10^{-5} \text{ s}^{-1}$), through the northern cross section of the OMP mooring array (Fig. 1). The models provide solutions with arbitrary amplitude, and therefore all velocity and sea surface perturbations are normalized by their absolute maximum. We use bathymetry data with a spatial resolution of 3 arc s from the National Centers for Environmental Information Coastal Relief Model of the southeastern Atlantic Ocean to define the shelf profile $H(x)$ within both models (Fig. 2a; the data are available online: <https://www.ncei.noaa.gov/metadata/geoportal/rest/metadata/item/gov.noaa.ngdc.mgg.dem:305/html>). The data are smoothed using the MATLAB software's "smoothdata" function with the moving-median method and default window size, which algorithmically calculates the necessary window size to attenuate 25% of the energy. The neglect of the surface divergence condition, $\varepsilon \ll 1$, is satisfied at $O(10^{-5})$ for this cross section, with L approximately equal to 150 km and H_0 of approximately 0.445 km.

Additional inputs to define the environment are required for the B06 model dynamic mode analysis. We seek solutions in the long-wave limit with a free surface, closed coastal boundary, and open offshore boundary on a grid of 20×100 points. We impose a flat bottom at the offshore boundary to satisfy the computational boundary condition. Horizontally uniform buoyancy frequency profiles interpolated to the model grid to describe the average stratification for winter (February and March) and summer (July and August) (Fig. 2b) are calculated from the NODC northwestern Atlantic regional climatology of salinity and temperature of the study region (the data are available online: https://www.nodc.noaa.gov/OC5/regional_climate/nwa-climate/) because our observational data for these fields are too coarse. We impose constant bottom friction $r_b = 4.48 \times 10^{-2} \text{ cm s}^{-1}$, with average standard deviation of alongshore bottom velocity $\sigma_v = 14.83 \text{ cm s}^{-1}$ based on mooring observations.

FIG. 2. Regional inputs used to describe the local environment: (a) actual (gray line) and smoothed (black line) bathymetry; (b) actual and interpolated winter (cyan and blue lines, respectively) and actual and interpolated summer (orange and red lines, respectively) buoyancy frequency.

The first three surface free-wave modes from the GS74 model have phase speeds of $c = -7.75$, -0.96 , and -0.62 m s^{-1} , where the negative sign indicates southward propagation with the coast on the right as expected from conservation of potential vorticity. Maximum southward velocity is found at the coast for all modes, with modes 2 and 3 having secondary northward maxima about 90 and 80 km from the coast, respectively (Fig. 3b). Maximum (positive) sea surface perturbation is also found at the coast for mode 1, but maximum (negative) sea surface perturbation is found approximately midshelf at 60 and 50 km from the coast for modes 2 and 3 with secondary (positive) maxima at the

coast (Fig. 3a). All modes decay to and approach zero at the shelf edge.

Similarly, the first three free-wave modes from the B06 model have phase speeds of $c = -7.18$, -1.81 , and -0.93 m s^{-1} in winter and $c = -7.35$, -2.05 , and -1.00 m s^{-1} in summer. We also find perturbation frictional decay scales of $T_f = 0.72$, 4.07, and 3.93 days in winter and $T_f = 0.77$, 4.21, and 4.63 days in summer. Surface velocity and sea surface perturbation structures (Fig. 3b) are very similar to those from the GS74 model, as well as between winter and summer. In summer, southward velocities are higher, northward velocities are lower, and sea surface perturbations are larger than in winter. These differences are insignificant for mode 1 ($<5\%$) but are greater for higher-order modes ($\sim 8\%$ – 15%). Secondary maxima are pushed farther offshore relative to the GS74 model, a known effect of friction (Brink 1982a), and undulations from the bathymetry are more pronounced in the velocity structures.

Furthermore, the vertical structure of the free-wave modes from the B06 model are barotropic, with little change in the velocity and pressure from the surface to the bottom on the shelf (Fig. 4). Differences between winter and summer are also small with depth. The relative strength of local maxima is less in summer, particularly in the upper few meters. The most noticeable difference is the shape of the mode 3 shelf-trapped positive pressure perturbation. The effect of stratification and friction will be discussed further in section 5.

b. Forced-wave solution for OMP cross section

We use hourly wind velocity data that have been averaged over 6-h periods from NOAA NDBC buoy 44014 (the data are available online: http://www.ndbc.noaa.gov/station_page.php?station=44014). The wind stress is then calculated as $\tau^{wy} = \rho_a C_D v_{\text{wind}}^2$, where $\rho_a = 1.225 \text{ kg m}^{-3}$ is the air density and v_{wind} is the alongshore buoy wind velocity. The wind forcing is applied as a square wave distribution in a small forcing region between y_1 and y_2 ; that is,

$$Y(y, t) = \begin{cases} 0 & y < y_1 \\ \tau^{wy}/H & y_1 < y < y_2, \\ 0 & y > y_2 \end{cases} \quad (18)$$

as used in Adams and Buchwald (1969) and Gill and Schumann (1974), to allow propagation of the free waves into the study region. We focus on the time period 1–14 March 1996 because it is a period during which we observe strong CTWs (see section 4).

Amplitude of the forced-wave solution depends on both the amplitude of the wind forcing and the wind coupling coefficient, while distance traveled within a particular amount of time depends on the phase speed.

FIG. 3. Free-wave surface structure modes (mode 1, black line; mode 2, red line; and mode 3, blue line) of normalized (top) sea surface perturbation η and (bottom) alongshore velocity v for a cross section along 36.7°N latitude using (a),(b) the simple GS74 model and (c),(d) the B06 model. In (c) and (d), the effect of summer (dashed lines) and winter (solid lines) stratification reflected from dynamic mode analysis is shown.

For this particular case, we find that at the end of the 2-week period in March the mode 1 signal has propagated nearly 2000 km south of the forcing origin with a maximum amplitude of about 1 ($b_{n1} = -2.15$). Mode 2 has only propagated 700 km with maximum amplitude of 0.1 ($b_{n2} = -0.75$) and mode 3 has propagated 450 km with a maximum amplitude of 0.056 ($b_{n3} = -0.39$) (Fig. 5). The distance traveled by the higher-order modes is much less than mode 1 because of the discrepancy in their respective phase speeds. It is important to note that CTWs in our study region can only propagate about 100 km southward to Cape Hatteras before being either scattered by the narrowing shelf (Wilkin and Chapman 1990) or overwhelmed by the Gulf Stream (Ezer 2016), and the models cannot capture this behavior.

We then calculate the total solution using Eq. (14) and plot the velocity and sea surface perturbation solutions at a fixed y location 2 km south of the forcing origin at 30-h intervals from 0000 1 March 1996. This evolution of the CTW signal is shown in Fig. 6. The passage of two large CTWs is seen during this time period (from 62 to 92 h and from 272 to 332 h). The interaction between fast mode 1 and much slower higher-order modes results in southward velocity over the shelf that reaches a maximum at midshelf rather than at the coast and a small, weak northward velocity jet at the shelf break. Likewise,

this interaction results in a sea surface perturbation that is positive over the shelf with a maximum at the coast and a gentler decay toward zero when compared with just mode 1. We will compare these model results with the observations and test the impact of local versus remote wind forcing in the following section.

4. Observed coastal trapped waves

a. Description of the CTW field

We identify the range of periods for CTWs in our study area using continuous wavelet power spectra analysis on the mooring velocity data as it is capable of handling shifting or changing periods (Camayo and Campos 2006). We then apply a Butterworth bandpass filter with period cutoffs at 3 and 12 days to the pressure and velocity data based on identified periods to isolate the CTW signal and remove signals outside of our focus (i.e., tides, Kelvin waves, very long period variability, etc.). Figure 7 shows that the CTW signal is coherent across the shelf and that the velocity and pressure fields are in agreement with each other, noting that pressure has been multiplied by -1 for a more direct comparison with velocity. The pressure and velocity fields are in phase which is indicative of a progressive wave. Shallow station 1 (water depth = 7 m) shows damping of velocity

FIG. 4. The first three free-wave vertical structure modes of normalized pressure and meridional velocity for a cross section along 36.7°N latitude using (a) winter and (b) summer stratification from the B06 model. Depth is plotted logarithmically to emphasize the vertical structure in shallow shelf water. Solid black lines indicate the zero crossings.

with depth during Deployment B, but otherwise the surface and bottom velocities are remarkably similar indicating that the wave is barotropic and in phase, in agreement with the B06 model free-wave modes. Station 1 is likely more stratified than the other stations because of the combined effect of weak tidal mixing (tidal currents $< 10 \text{ cm s}^{-1}$) and freshwater stratification from Chesapeake Bay outflow (Jiang and Xie 2016).

The CTW velocity signal is much stronger during Deployment A (Fig. 7a) than during Deployment B (Fig. 7b). We see average and maximum velocity amplitudes of ± 8 and $\pm 30 \text{ cm s}^{-1}$ for Deployment A but only ± 4 and $\pm 17 \text{ cm s}^{-1}$ for Deployment B. Velocity amplitude and activity decreases toward the end of Deployment A as wind forcing decreases, whereas it increases toward the end of Deployment B as wind forcing increases (Fig. 7). We find that the bandpass-filtered meridional velocity is best explained by the meridional wind component. Velocity from both Deployments A and B have a statistically significant correlation $r = 0.41$ (lag = 0, with p value $\ll 0.05$) with the v -component of wind. We further find that the CTW velocity magnitude is highly correlated with the wind magnitude ($r = 0.83$ and lag = 0, with p value $\ll 0.05$). This indicates that not only are CTWs initiated by the alongshore wind, but that the strength (or magnitude) of the waves is directly related to the strength of the wind.

This follows what we expect based on the simple forced-wave model and motivates us to compare the observational and model results.

b. Comparison between observed and modeled CTWs

We compare the surface velocity free-wave modal structures from the B06 model with ones obtained using standard EOF analysis (North 1984) on the observational data. The first three successive EOF modes explain 88.8%, 7.0%, and 2.7%, respectively, of the variability for winter Deployment A and 67.1%, 12.3%, and 8.4% of the variability for summer Deployment B. These percentages are consistent with an increase of scattering into higher-order modes with stronger stratification (Wilkin and Chapman 1990). The magnitude of explained variability in the PCs (not shown) is also about 2 times as high for Deployment A as for Deployment B, consistent with the velocity amplitude observations. However, modes obtained from EOF analysis do not show good agreement with those obtained from the B06 model (Fig. 8) and seem to represent much higher-order modes. Summer mode 2 from EOF analysis is the exception and shows reasonable agreement with B06 modes 2 and 3.

We also compare the modeled forced waves and observations, focusing on the time period 1–14 March 1996 as in section 3b. However, we use three different wind forcings from buoys 44025, 44009, and 44014 as well as

FIG. 5. Resultant propagation of the forced-wave modes (mode 1, black line; mode 2, red line; and mode 3, blue line) using wind forcing from buoy 44014 after 2 weeks from the GS74 model. Note the different vertical axis scale for each of the panels.

their ensemble to test the effect of local versus remote wind forcing. Buoys 44025 and 44009 are respectively approximately 400 and 200 km north of the OMP study region, while buoy 44014 is alongside the mooring array. Therefore, 44025 and 44009 are considered as remote wind forcing and 44014 is local. Analytical phase speeds from the GS74 model are used to get the theoretically correct phase lag for the propagation of the signal into the study region.

Agreement between observed and modeled CTW velocities is also very poor for this analysis (Fig. 9) and the modeled velocity magnitude is severely overestimated at certain time points. Using remote versus local wind forcing or an ensemble of the three different forcings does not improve the agreement, suggesting that incorrect wind forcing is not an issue as in Battisti and Hickey (1984). Furthermore, the modeled combined CTW modes still only capture single signed (negative) velocity and, therefore, fail to capture observed northward (positive) velocities. This suggests that the modeled contribution from higher-order modes, which have lower wind coupling

coefficients and segments of northward velocity, is underestimated in either amplitude, contribution, or both.

Brink (1982b) and Battisti and Hickey (1984) comment that it is more difficult to get good agreement between observed and modeled CTW velocity than pressure or sea surface perturbation. We are unable to repeat the above cross-shelf observation and model comparison for sea surface perturbation as we have data from too few pressure gauges. We can, however, compare the CTW pressure signal at the coast from DK tide gauge 8651370 with the modeled time-dependent CTW mode following Halliwell and Allen (1984) and others (Springer et al. 2009; Rivas 2017). Using mode 1 c_n and T_f from the B06 model, we find reasonable agreement between the model and observations (Fig. 10). The model captures some events very well, including the event during the first two weeks of March, but also tends to overestimate others in the fall and winter. In general the phase is correct, and it captures the seasonality of the CTW amplitude.

FIG. 6. Full solution for normalized velocity (black line) and sea surface perturbation (gray line) at 30-h intervals from $t = 0000$ 1 Mar 1996 from the GS74 model.

5. Discussion

a. Differences between theory and observations

While we have found clear evidence of the presence of CTWs within our study region, there are discrepancies between our observations and the full GS74 model results. We perform additional B06 model runs to determine to what extent the effects of stratification and friction might impact these differences as these factors are not included in the GS74 model. These additional model runs include a strong summer stratification case where N^2 has been doubled in the upper 50 m of the water column and an essentially frictionless case ($\sigma_v = 0.01 \text{ cm s}^{-1}$) with winter stratification. We can now identify clear patterns for the effect of stratification and friction with these additional B06 model runs (Table 1). In all cases, cross-shelf structure is largely unaffected, and therefore we focus on changes to the output parameters.

We find that increasing stratification increases phase speed, decreases wind coupling, and increases decay time [therefore, decreasing friction damping as $T_f = (c_n a_n)^{-1}$]. Phase speed changes are only greater than 5% for higher-order modes as phase speed is a function of stratification (Battisti and Hickey 1984). Mode 3 is most affected by changes in stratification, with T_f increasing a full day from winter to strong summer stratification. We further find that decreasing friction from $\sigma_v = 14.83$ to 0.01 cm s^{-1} does not change the phase speed or wind coupling but greatly increases the decay time by a factor of $O(10^3)$, approaching infinity for a truly frictionless case.

Most changes from winter to summer stratification are small, suggesting results are largely insensitive to stratification and using a barotropic case is not a large source of error. This is supported by good agreement between the c_n and b_n output by the GS74 model and the winter

FIG. 7. CTW velocity amplitude (cm s^{-1}) and negative pressure amplitude (dbar) for (a) Deployment A and (b) Deployment B stations. Displayed are filtered values for the surface velocity (blue or red lines), bottom velocity (cyan or orange lines), and bottom pressure (black lines). Note that SSH from DK tide gauge 8651370 was used in place of bottom pressure at station 01. Zonal (gray line) and meridional (black line) wind components from buoy 44014 are shown in the bottom plot of (a) and (b).

FIG. 8. EOFs for (a) winter Deployment A and (b) summer Deployment B. Displayed are the first three modes (mode 1, black lines; mode 2, red lines; and mode 3, blue lines) of normalized surface velocity patterns from the B06 model (thin, dashed lines) and EOF analysis (thick solid lines). The x axis ends at the shelf break.

stratification case of the B06 model (Table 1) as well as Burger number $Bu = N^2 H^2 / (f^2 L^2) < 1$ using peak summer N^2 and a length scale characteristic of the shelf. Therefore, differences between observations and model results are not due to neglecting stratification. This further suggests that observed differences between winter and summer velocity amplitude (Fig. 7) are mostly due to seasonal changes in wind magnitude as the wind coupling coefficient changes insignificantly with stratification. Furthermore, the time-dependent CTW mode model is able to capture the seasonality of the CTW amplitude from the wind forcing (Fig. 10).

The large changes in the decay time with the inclusion of friction indicate that it is an important factor to consider. This particularly applies to mode 1, which has decay times of less than a day for all model runs including friction. However, agreement between observed and modeled CTWs does not improve with the inclusion of friction and discrepancies are likely due to scattering. Huthnance et al. (1986) suggested that strong friction may prevent the growth and transmission of scattered CTWs, but the apparent prevalence of higher-order modes within the observations indicates that it does not.

The simple models used also assume a straight coastline with self-similar bathymetry. This means that the analysis does not include the effect of scattering into higher-order modes that results from narrowing of the

shelf south of our study region and complex bathymetric features north of our study region, including the Chesapeake Bay mouth to the immediate north. The backscattering influence of a narrowing shelf can be seen as much as 200 km upstream of a scattering region (Wilkin and Chapman 1990). Convergence of phase speeds as the shelf narrows leads to amplification of the amplitude and eventually leads to a reflection (Wang 1980). The reflected wave is transmitted above the critical frequency as a short wave and, therefore, cannot be captured by theory regulated by the long-wave limit. The outlined shortcomings of current theoretical formulations and disagreement with our observations indicate that this idealized theory cannot be used globally. Indeed, it will not be able to reproduce or predict CTWs in regions with high scattering probability based on their shelf geometry.

b. CTW propagation on the U.S. East Coast continental shelf

To provide further evidence of scattering and to determine the source and fate of CTWs observed in our study region, we create a Hovmöller plot of bandpass-filtered 6-min-resolution SSH from 18 tide gauges along the U.S. East Coast for 2011 (Fig. 11). We find that CTWs that originate from Maine and points north in Canada become weak and dissipate between Nantucket,

FIG. 9. Comparison between the full solution for normalized velocity from the GS74 model (black lines) and observations (black diamonds) using wind forcing from buoys (left) 44025, (left center) 44009, and (right center) 44014 and (right) an ensemble of those buoys at specified time intervals (rows) from $t = 0000$ 1 Mar 1996.

Massachusetts, and Montauk, New York, likely because of the sharp bends in the coastline and abrupt changes in bathymetry around Nantucket Shoals. Stronger waves are then reinitiated between Montauk and Sandy Hook, New Jersey, around Hudson Canyon, which has been suggested to induce CTWs during down-valley wind (Zhang and Lentz 2017). CTWs are strongest along the

uninterrupted New Jersey coastline and near Duck, North Carolina, close to our study region, before dissipating between Duck and Wrightsville Beach, North Carolina. We propose that CTWs near Duck are enhanced by backscattering from the narrowing shelf and that the presence of the Gulf Stream separation near Cape Hatteras, North Carolina, leads to their

FIG. 10. Comparison between modeled time-dependent CTW mode (thin black line) and observations from DK tide gauge 8651370 (thick black line) for normalized pressure $P = (p - \mu)/\sigma$.

termination. Furthermore, weaker CTWs observed south of the Gulf Stream separation point tend to travel opposite to their native direction because of the influence of the Gulf Stream on the shelf flow.

A strong seasonal influence on the CTW activity is seen over the entire shelf. Most activity is observed between January and mid-April as well as for October–December when stratification is weak and wind forcing is strong. Weak summertime (June–September) activity is mostly seen south of the Gulf Stream separation where stratification is more constant year-round. A strong event toward the end of August is an exception; this CTW is largely uninterrupted from Montauk all the way to southern Florida, although with a phase speed that clearly changes as it progresses down the shelf and a reversal in propagation direction around Springsmaid Pier, South Carolina. This strong event was initiated by Hurricane Irene, which made landfall in southern New Jersey on 28 August 2011. This highlights the influence strong storms can have on initiating CTWs even during unfavorable summertime conditions.

Last, we estimate a phase speed of $c \sim 9.9 \pm 1.8 \text{ m s}^{-1}$ near Duck from strong banded activity in November (Fig. 11b). While slightly higher than the mode-1 phase speed from both the GS74 and B06 models, it is of the same order of magnitude and subject to large temporal variability. The plot additionally shows that there are large variations in phase speed even over short distances. This feature is likely due to changes in shelf width and bathymetric slope (Woodham et al. 2013), which are known to influence the phase speed.

6. Conclusions

Two deployments of a mooring array off the coast of the Outer Banks as part of the Ocean Margins Program provide observations of velocity and pressure among other variables throughout two seasons. These observations, which are being analyzed for the first time, allow us to extract the coastal trapped wave signal from the data and determine how seasonal stratification and wind forcing affect the observed CTW properties. We find

TABLE 1. Parameter outputs from the GS74 model (“barotropic” column) and B06 model (the other columns) for modes 1, 2, and 3 (first, second, and third rows, respectively, for each variable) for different stratification and friction environments.

	Barotropic	Winter stratification	Summer stratification	Doubled summer stratification	Frictionless, winter stratification
Phase speed	775	718.3770	734.5254	744.9609	718.3770
$c_n \text{ (cm s}^{-1}\text{)}$	96	180.8701	205.1689	221.3174	180.8701
	62	93.0916	99.7199	102.7939	93.0916
Wind coupling coef	−2.1540	−2.2933	−2.2407	−2.1383	−2.2933
$b_n \text{ [(cm s}^{-1}\text{)}^{1/2}\text{]}$	−0.7462	−0.4588	−0.4643	−0.4449	−0.4588
	−0.3949	−0.3754	−0.3396	−0.3191	−0.3754
Friction damping coef		2.2302	2.0533	1.8314	1.4868
$a_n \text{ (}\times 10^{-8} \text{ cm}^{-1}\text{)}$		1.5706	1.3392	1.1139	1.0471
		3.1636	2.5093	2.2418	2.1091
Frictional decay time		0.7224	0.7674	0.8483	1.0836×10^3
$T_f \text{ (days)}$		4.0743	4.2124	4.6949	6.1113×10^3
		3.9300	4.6253	5.0225	5.8949×10^3

FIG. 11. Hovmöller plot of squared, de-meaned, bandpass-filtered MLLW SSH (m^2) from tide gauge observations. The black box in (a) outlines the zoomed-in area in (b). The black line in (b) indicates the slope used to calculate phase speed. Tide gauge station abbreviations are as defined in Fig. 1.

that CTW velocity amplitude is about 2 times as high in winter as in summer with average $v = \pm 8$ and $\pm 4 \text{ cm s}^{-1}$, respectively, and is highly correlated with changes in wind forcing.

Two different analytical models, one based on Gill and Schumann (1974) and another provided by Brink (2006), allow us to determine the theoretical cross-shelf modal structure and the effect of stratification and friction on propagation properties. Analysis indicates that the solutions are rather insensitive to stratification, but friction plays a large role as the mode 1 decay time is less than a day. A comparison of the CTW modes from observations and the B06 model generally shows poor agreement, as does a comparison of the full forced model solution for the cross-shelf structure from the GS74 model with the observations. The models used neglect the effect of scattering or backscattering from a narrowing shelf.

Although other studies have had success in comparing similar models with observations (Brink 1982a; Battisti and Hickey 1984; Church et al. 1986; Chapman 1987), our lack of agreement indicates that this is not a universally

applicable theoretical formulation. Many sections of global coastline do not meet the required assumptions of a straight coastline with shelf similar bathymetry so theoretical CTW modes will show equally poor agreement with observations. There is a need to significantly update the theoretical formulation to allow for scattering or abandon it in favor of numerical hydrodynamic models.

Acknowledgments. We thank Charles Flagg for providing us with the OMP data and helpful discussions during the analysis phase. We also thank Dong-Ping Wang and Christopher Wolfe for their assistance with the Gill and Schumann (1974) analytical model.

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