Entrainment and Diffusion in a Gulf Stream Cyclonic Ring

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ABSTRACT

The mixing and entrainment processes present in a cyclonic ring are investigated by means of a parametric model which is fitted to serial temperature data for a North Atlantic ring. The physical model assumes an axially symmetric ring in which the temperature is presumed to be governed by

$$\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK_h\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right)+\frac{1}{r}J(\psi,T),$$

where K_h and K_z are the horizontal and vertical eddy diffusivity coefficients and $J(\psi,T)$ is the (r,z) Jacobian of the transverse streamfunction ψ and the temperature T. Data from two cruises during the 1967 observation of a ring by the Woods Hole Oceanographic Institution provided estimates of the derivatives of the temperature. Regression analysis was used to determine the coefficients for polynomial representations of $\psi(r,z)$ for selected combinations of K_h and K_z. The study indicates upper bounds on the order of magnitudes for the diffusivities $(K_h,K_z)=(10^5,10)$ cm^2 s^{-1} based upon near-minimum least-squares error estimates from the regression analysis. An important result is that little differentiation exists between a purely advective entrainment regime and those regimes including both entrainment and diffusion; i.e., the entrainment circulation appears to be the dominant mechanism in the temporal changes of the ring for a time scale of at least two months. The results provide streamline patterns for the transverse flow from the surface to 1000 m depth consistent with isotherm movement and changes in the ring water masses.

1. Introduction

Gulf Stream cyclonic rings first came to the attention of investigators as large cold water anomalies in the Sargasso Sea (Iselin, 1936). Rings have been observed in the western Sargasso Sea between 60–70W longitude southward to 25N latitude (Parker, 1971). These features are readily identifiable from temperature fields and generally exhibit a dome-shaped structure (Fig. 1) perturbing isotherms over a depth range from the surface to more than 2000 m. The dome shapes have remarkable axial symmetry which suggest the use of cylindrical coordinates in physical models. The temperature sections in Fig. 2 clearly show this shape.

Rooth (1967) examined hydrographic sections through a cyclonic ring that were made by Fuglister and suggested that mixing along isopycnal surfaces controls the ring decay process. In a recent study of the dynamics of cyclonic ring spindown, Molinari (1970) allowed for circulation in the meridional plane of the ring as well as mixing of momentum along isopycnal surfaces. Such studies require specification of pivotal coefficients, such as the viscosity and diffusivities, which control the interactions of physical mechanisms. In the absence of diffusivities derived from field studies, models rely on estimates based on spatial and temporal scales associated with rings. Indeed, the field data assembled so far preclude much more than just such estimates. The present effort is an attempt to bring data taken by the Woods Hole Oceanographic Institution (WHOI) to bear on the relations between physical mechanisms which can bring about ring decay.

The representation of diffusion processes in theoretical models of mixing is normally achieved by relating the covariance of fluid velocity and conservative property fluctuations to the spatial gradients of mean property concentrations. Coefficients governing the turbulent portion of mixing are introduced in this manner. Advective terms in a model bring about changes in concentration due to the regular motions of the fluid and are expressed in terms of mean fluid velocities and mean property gradients (Okubo, 1970). The words mean, variation and gradient all imply spatial and temporal averaging procedures which serve to delineate diffusion and advection in an objective way for each investigator. Mixing models are traditionally used to study the distributions of some conservative property. Implicitly, the concentrations represented in the solutions are mean concentrations developed in response to mean fluid velocities. In this investigation the averaging procedures are explicitly employed in the preparation of gradient estimates for the conservative property. A regression procedure has been imposed which inverts the traditional roles of fluid velocity and property concentration and the mixing model is used to

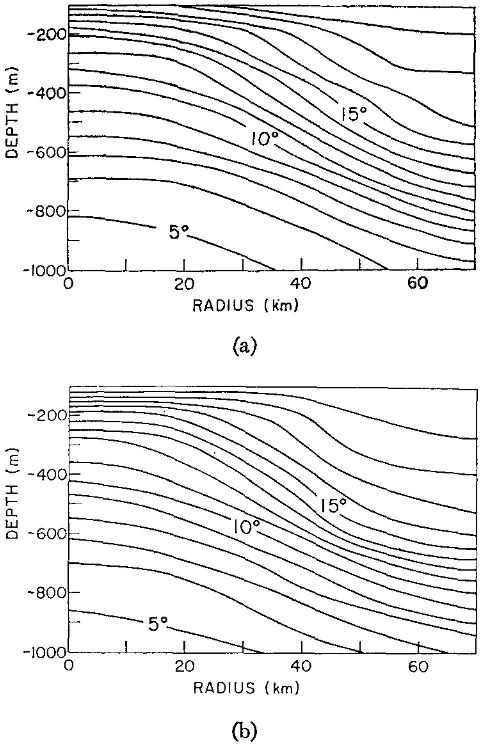

FIG. 1. Radial section through ring showing observed temperature field (°C): (a) *Atlantis II* 35 cruise, (b) *Atlantis II* 38 cruise.

solve for a mean velocity field. The diffusivity coefficients enter as parameters in the examination of the velocity distributions which satisfy the model in a least squares sense.

2. The physical model

A considerable abstraction has been made relative to the full spectrum of processes representing the dynamic features in a cyclonic ring. The mixing model includes radial and vertical advection and diffusion while omitting the principle balance between pressure gradients, Coriolis and centripetal accelerations. The rotational modes of motion have been eliminated from direct consideration in the problem. However, the influence that these modes have on the mixing processes are present in the analysis through the observational data.

An examination of the temperature fields for cruise 35 and cruise 38 of the WHOI research vessel *Atlantis II* across the same cyclonic ring form the basis of the study. Using data collected from 31 August to 6 September 1967, and from 26–28 October 1967, cruise tracks were drawn and hydrographic station temperatures were plotted at 100 m intervals from the surface to 1000 m. Linear interpolation was used to locate selected temperatures along the tracks. With the exception of the surface, concentric circles adequately describe the temperature distribution at each level. The distribution for each cruise is shown at the 500 m level in Fig. 2. The asymmetric surface temperature distribution was not considered in the subsequent analysis and the physical model does not explicitly account for the changes in thermal energy due to air-sea interaction over the period between the two cruises.

As a result of the symmetry in temperature, the physical model is expressed in terms of a cylindrical coordinate system (r,z) centered in the ring and translating with it. In this way the terms regarded as representing advection may be considered as those which model the horizontal and vertical components of entrainment. In accord with the observational data and dynamic assumptions, the general mixing equation is given by

$$\frac{DT}{Dt}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK_h\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right), \qquad (1)$$

where T is temperature, K_h and K_z are the horizontal and vertical diffusivities respectively, and DT/Dt indicates a material derivative.

The approach requires the evaluation of spatial and temporal derivatives. These quantities are the local temperature change with time and the first and second derivatives of the mean temperature field. Graphic techniques were applied to approximate the time derivative from observed data. Temperature was plotted against radius for each cruise at 100 m intervals and a smooth curve drawn to the distribution. Temperatures were then read at 10 km intervals for each cruise and the difference used to calculate the time rate of change at those radii. A smooth curve drawn through these points was used to provide the derivative for each level. These local derivatives are estimated on the basis of the 52-day interval between the last survey day of each cruise. A 1000 m depth limitation was placed on the model analysis although the majority of hydrostations extended to 1100 m.

By using an average of the temperature fields from the two cruises, a mean temperature field, shown in Fig. 3, was obtained. This mean field was fitted by regression analysis for a two-dimensional (r,z) polynomial representation. The polynomial, a compromise between goodness of fit and smoothing, was generated to provide an analytic expression of sufficient order (fourth degree in r and z) for adequate estimates of the derivative quantities. The mean temperature field approximation is shown by the dashed lines in Fig. 3.

The mixing equation may be expanded and combined with the mass conservation relation for an incompressible fluid to yield

$$\frac{1}{r}\frac{\partial(ur)}{\partial r}+\frac{\partial w}{\partial z}=0, \qquad (2)$$

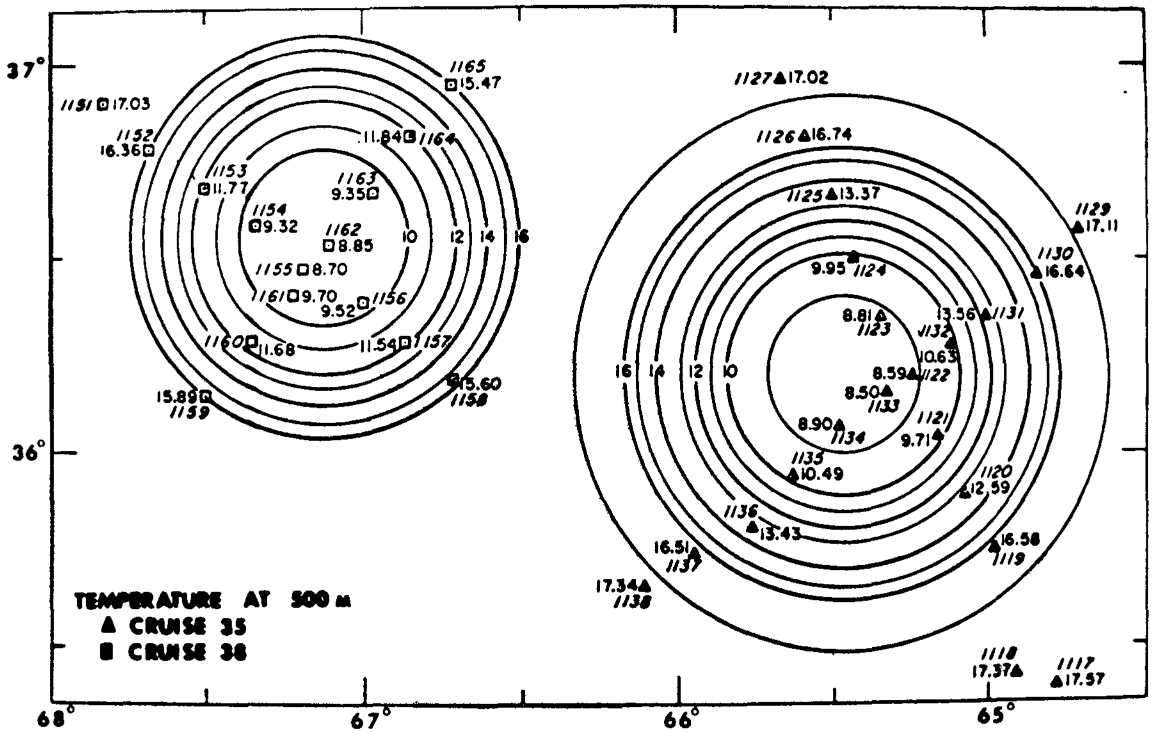

FIG. 2. Contoured temperature sections at a depth of 500 m for *Atlantis II* cruises 35 and 38.

and rewritten as

$$\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rK_h\frac{\partial T}{\partial r}-urT\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}-wT\right), \qquad (3)$$

where u and w are the mean horizontal and vertical velocity components respectively. A streamfunction ψ, defined such that

$$ur=\frac{\partial\psi}{\partial z} \quad\text{and}\quad wr=-\frac{\partial\psi}{\partial r}, \qquad (4)$$

is assumed to have a polynomial representation

$$\psi=r^2z(A_0+A_1r+A_2z+A_3r^2+A_4rz+A_5z^2) \qquad (5)$$

which satisfies the boundary conditions

$$\psi(0,z)=0, \qquad (6)$$

$$\psi(r,0)=0. \qquad (7)$$

These expressions imply the absence of radial velocity at the center and vertical velocity at the surface of the ring. Introduction of the streamfunction, differentiation, and regrouping of terms renders the mixing model in the form

$$\frac{\partial T}{\partial t}-\left(\frac{1}{r}K_h\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}\right)-K_z\frac{\partial^2 T}{\partial z^2}-\frac{1}{r}J(\psi,T)=0, \qquad (8)$$

where $J(\psi,T)$ is the Jacobian of streamfunction and temperature. By regression analysis, Eq. (8) together with (5) is solved for the coefficients of the streamfunction with the additional computation of a least-squares error integral over (r,z). The error residual is a measure of the degree to which (8) is satisfied with chosen diffusivities and known gradient quantities. A parametric study of possible velocity distributions was conducted with sets of (K_h,K_z) selected over a range suitable for oceanic mixing of this scale (Bowden, 1964).

3. Model results

The diffusivity parametric study yielded a family of streamlines and a set of error residuals which show the effect of K_h and K_z variations on the model mixing process. The selection of a representative (K_h,K_z) pair for the observation interval relies primarily on the hydrostation data.

Direct field measurements of radial and vertical movement within a cyclonic ring have not been made for periods of time comparable to the one under study nor is it certain that these motions could adequately be detected. A qualitative estimate of the effect of the mixing process can be made on the basis of the *T-S* relations for hydrographic stations taken during the two cruises. The similarity between *T-S* curves of a station within the ring and one outside is dependent on the radial distance between the two stations and on

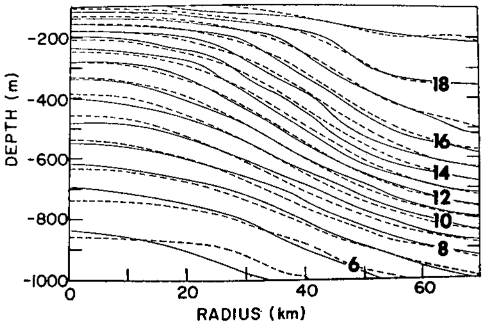

FIG. 3. Radial section through ring showing averaged temperature field (solid lines) based on *Atlantis II* 35 and 38 cruises. Approximated temperature field is shown by the dashed lines.

the age of the ring. For smaller spatial separations and for greater age the similarity is more marked. Examination of the T-S curves for the 52-day period indicates changes to warmer and more saline waters within the ring core that decrease monotonically from 300 to 1000 m depth. The associated water movement then appears to bring Gulf Stream and North Atlantic Central Water into the ring.

Least-squares error residuals or variances were computed as

$$E=\int\int\left[r\frac{\partial T}{\partial t}-r\text{ (Diffusive terms)}-J(\Psi,T)\right]^2 drdz \quad (9)$$

over the ring and were normalized using the variance of 0.2528×10^{11} for the advective case. For comparison, the value of the integral

$$I=\int\int\left[r\frac{\partial T}{\partial t}\right]^2 drdz \quad (10)$$

over the ring is 2.182×10^{11}°C^2 cm^4 s^{-2}. The normalized variances are shown in Table 1 with associated (K_h,K_z) pairs. Computations in which K_h was greater than 10^5 cm^2 s^{-1} have not been included as the variances are orders of magnitude larger than those listed and the models exhibit streamline patterns predicting water movement up and out of the ring. The initial regression analysis tested a simple advective regime for which $(K_h,K_z)=(0,0)$. In this case, advection into the ring was observed above the 600 m water level with a return flow taking place between 600 and 1000 m that balances less than half of the inward flow. The remainder was advected toward the center of the ring and to greater depths.

TABLE 1. Normalized least-squares error variances.

Diffusivity (cm^2 s^{-1}) K_h	K_z	Normalized variances	$\Psi_{\max}$ (10^8 cm^3 s^{-1} rad^{-1})
0	0	1	55.33
10^4	0	0.9786	54.10
10^5	0	1.2460	45.24
10^4	1	0.9830	54.27
10^5	1	1.2694	45.51
10^4	10	1.0716	55.95
10^5	10	1.5285	48.01

TABLE 2. Coefficients of $\psi=r^2z(A_0+A_1r+A_2z+A_3r^2+A_4rz+A_5z^2)$ for optimum case.

Coefficient	Value
A_0	-0.1369×10^{-7}
A_1	0.1625×10^{-14}
A_2	-0.1107×10^{-12}
A_3	-0.3418×10^{-22}
A_4	0.1011×10^{-19}
A_5	-0.6869×10^{-19}

The remaining cases include both diffusive and advective terms. Based on the error function, the pair $(K_h,K_z)=(10^4,0)$ cm^2 s^{-1} is the best choice for horizontal and vertical diffusivity coefficients. The streamline polynomial coefficients for this case are given in Table 2. The flow pattern for this case is shown in Fig. 4 and the associated horizontal and vertical mean velocity components are shown in Figs. 5 and 6. The streamlines for this pair indicate a flow field entering the ring above 600 m and directed outward below. The radial velocity component reflects this flow while a vertical component is almost entirely directed downward and is strongest near the ring center. A comparison of the pure advective regime and the $(K_h,K_z)=(10^4,0)$ cm^2 s^{-1} regime suggests that the horizontal diffusion reduces the net flow of water into the ring. The internal zero isotach of u displays a shift confining the inward velocities to a shallower layer. Changes produced in the vertical velocity component are similar in that the isotachs are displaced toward the center suggesting retarded sinking below 600 m. As a result of the relative changes in the velocity fields, less water is transported into the ring model which incorporates diffusion.

Negative values of eddy diffusivity were also tested in the model. These cases produce reasonable flow patterns within the previously mentioned restrictions on

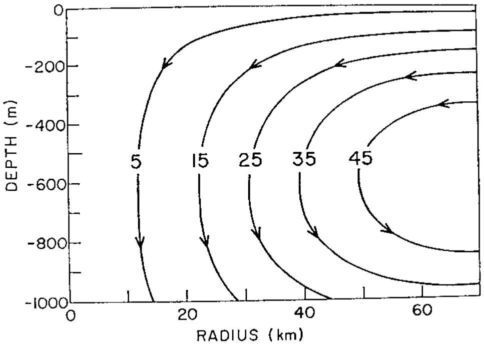

FIG. 4. Field of streamlines (10^8 cm^3 s^{-1} rad^{-1}) for $K_h=10^4$ cm^2 s^{-1} and $K_z=0$.

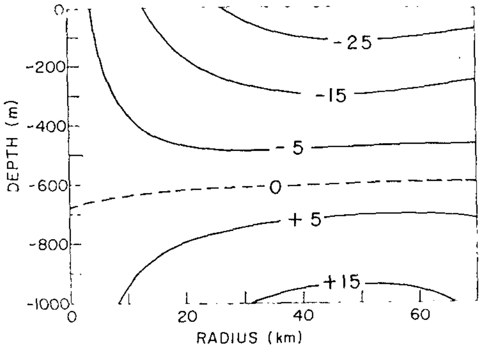

FIG. 5. Isotachs of the radial velocity component u (10^{-3} cm s^{-1}) for $K_h=10^4$ cm^2 s^{-1} and $K_z=0$.

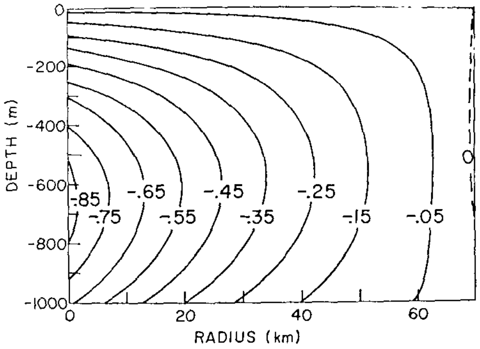

FIG. 6. Isotachs of the vertical velocity component w (10^{-3} cm s^{-1}) for $K_h=10^4$ cm^2 s^{-1} and $K_z=0$.

order of magnitude and demonstrate a relative insensitivity of the approach to the diffusion parameters. Even so, the minimum least-squares error residual still occurs for positive diffusivities at $K_h=10^4$ cm^2 s^{-1}. This value of K_h was quoted by Rooth (1967) as being a suitable one for such a strong shear zone.

4. Summary

The physical model represented in (8) has been employed to examine the relative importance of advective and diffusive terms as deduced from the observed changes in a cyclonic ring. The analysis of variations in eddy diffusivities has shown there is little distinction in the (K_h, K_z) values within the range yielding physically acceptable results. Comparisons with the purely advective case suggest that entrainment was the dominant mechanism for changing the central water mass over the observed period.

The streamline pattern shown in Fig. 4 represents a realistic one. The evidence supporting the horizontal velocity distribution is drawn from the comparison of water types for the two cruises which infer mixing that proceeds from outside the ring toward the center in the upper regions. The return flow in the lower region is also supported by the T-S relations. The vertical velocity distribution agrees with field evidence demonstrating the sinking of isotherms with age. Parker (1971) reported a sinking rate of 0.6 m per day or 0.69×10^{-3} cm s^{-1} for the 17°C isotherm as an average value within cyclonic rings in the western Sargasso Sea. This order of magnitude is reproduced in the mean vertical components (Fig. 6) derived from the model streamfunction.

An estimate of the net exchange of water for the 52-day observation interval has been made using the maximum streamfunction value as representing water movement into the ring. The net transport across a cylindrical surface at a radius of 70 km is 34×10^3 m^3 s^{-1} toward the ring center from the surface to 1000 m. Therefore, on the basis of this physical model, a complete replacement of the core water mass will occur within a 15-year period. This figure further substantiates the speculation on lifetimes, such as the 3–5 year estimate by Barrett (1971) which is based on degradation of potential energy, and offers evidence for the physical interactions which would contribute to the relative longevity of these mesoscale features.

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