Trajectories and Speeds of Wind-Driven Currents Near the Coast

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ABSTRACT

Detailed observation of drogue movements within 800 m of a straight shoreline indicates the primary current generated by local winds to be directed within a few degrees of parallel to the shore nearly independently of wind direction. Subtle vertical structure in the onshore-offshore speed component is dependent on vertical stratification such that unstratified water produces a two-layered flow (onshore in the surface layer, offshore in the bottom layer), and moderately stratified water produces a three-layered flow (onshore in surface and bottom layers, offshore at intermediate depths). Theoretical conclusions from Jeffreys’ constant eddy viscosity theory support the unstratified velocity profile and accurately predict the alongshore current speeds. Numerical solutions with a depth-dependent eddy viscosity indicate that the three-layered flow pattern is a direct result of the density gradient. Even in these shallow waters the inclusion of Coriolis effects in the theory is necessary for a complete understanding of the current observations. Simple theoretical calculations on the response characteristics of various sized surface-tracked drogues as a function of wind speed indicate that drogue size should be carefully selected in terms of expected magnitudes of wind and current speeds.

1. Introduction

In contrast to recent advances in our knowledge of both wave-driven currents inside the breaker zone (see Longuet-Higgins, 1972) and currents on the continental shelf (Smith, 1974), wind-driven currents in the intermediate zone immediately seaward of the breaking point, where the effect of the coast is still of great importance, remain poorly understood.

As part of a larger effort to obtain a coordinated picture of the dynamical processes operating in a coastal environment, detailed observations were taken on the movement of drogues in locally generated wind drift within 800 m of the shoreline. In the study area at the Eglin Air Force Base missile test range 65 km east of Pensacola, Fla., the bottom topography is fairly uniform (see Fig. 1). In the summer months a typical Gulf of Mexico sea breeze system (Hsu, 1970) normally develops daily; the morning south-easterly winds (2-3 m s⁻¹) have rotated to the southwest by 1200 CST, by which time the wind speed has generally accelerated to 5-6 m s⁻¹. During most of the observation interval in 1971, however, interference from larger scale meteorological systems modified the sea breeze system, and winds generally came up daily out of the south-southwest. Tidal range is 0.3-0.5 m, but analysis has shown tidal currents in the immediate vicinity of the coast to be negligible compared to local wind drift. The nearest major inlet, Destin Pass into Choctawhatchee Bay, lies 34 km to the east.

Calculations by Ekman (1905) long ago suggested that the dominant flow direction in wind-driven shallow waters near coasts should be primarily along the coast except under nearly perpendicular winds. Little quantitative work has ensued to test these relations, excepting, of course, the great volume of work on deepwater currents in open waters and the Ekman spiral.

The objective of the present study was to determine the strength and the direction of the circulation induced in the prism of inshore water by surface wind stress applied at various angles to the shoreline. Current meter data (Murray, 1969) hinted at a very subtle vertical velocity structure which it was not possible to resolve directionally with available current meters owing to persistent wave orbital motions. Accordingly, precision theodolites were employed to track drogues emplaced throughout the study area at prescribed depths. Concurrent measurements were taken of the surface wind stress (Hsu, 1973) by the profile method. Moored current meters (Marine Advisers Q-15R) operating concurrently showed that the locally generated wind drift was on the average about four times the magnitude of the prevailing background current.

2. Experimental techniques

The drogues, constructed of two mutually orthogonal 0.6 m square, 3 mm thick polyvinyl chloride (pvc) sheets, were weighted to assure a vertical attitude and suspended from a 2 l float trimmed to minimal free-
board. Color-coded flags on buoyant poles attached to the floats identified individual drogues set at different depths for optical tracking. Deployment consisted simply of dropping the drogues into the ambient current off the stern of a 20 m boat used as the tracking control station.

Neglecting the drag on the suspension line and the pole, and assuming a vertical pole, the horizontal balance of forces on the drogue can be estimated by writing

\[
\frac{du_d}{dt} = \frac{F_1 + F_2}{\rho V},
\]

where \(u_d\) is the speed of the drogue relative to the ground, \(V\) the volume of water trapped by the drogue (\(\approx L^3\)), and

\[
F_1 = -\frac{1}{2} C_1 \rho L^2 u_0 |u_0|, \\
F_2 = \frac{1}{2} C_2 \rho L D h W^2 \sin \theta,
\]

where \(F_1\) is the drag force of the water on the drogue and \(F_2\) that component of the drag force of the wind on the marker pole which is transmitted as a horizontal force to the drogue; \(\theta\) then is the tilt angle of the line to the vertical (\(\theta = 15^\circ\) would be a maximum value according to our observations). \(C_1\) is the drag coefficient of the drogue (1.2), \(\rho\) water density, \(L\) the length of a side of the drogue (0.6 m), \(u_0\) the relative speed between drogue and water (\(u_0 = u_d - u\)), \(u\) water speed, \(C_2\) the drag coefficient for the pole, a circular

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**Fig. 1.** Nearshore bathymetry at the study site off the Eglin Air Force Base Reservation on Santa Rosa Island along the Florida Gulf Coast.

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**Fig. 2.** Speed error of rectangular cross section drogues resulting from wind drag on cylindrical marker poles as a function of wind speed. Note significant error in commonly used 0.3 m biplanar drogue.
cylinder (\(\sim 1.0\)), \(\sigma\) air density, \(D\) the diameter of the pole (0.03 m), \(h\) the exposed pole length (1.5 m), and \(W\) wind speed in the current direction.

Once the drogue has reached a steady speed \((du_0/dt = 0)\), Eq. (1) simplifies to

\[
\frac{u_0}{\frac{C_0\sigma D h \sin \theta}{C_1 \rho} \frac{W}{L}} = \left( \frac{u_t}{u_t + \frac{C_1 T}{2L}} \right)^{-1}
\]  

Fig. 2 gives the speed error resulting from wind drag on a pole 0.03 m in diameter and 1.5 m long at \(\theta = 15^\circ\), predicted for three sizes of square-sided drogues. The largest drogue, of course, has the smallest error, and for the drogues used in this study \((L=0.6\) m\) at the observed wind speeds \((2-5 \text{ m s}^{-1})\) the maximum error in speed is about 0.01-0.02 m s\(^{-1}\).

The time required for a drogue to accelerate to the ambient current speed can also be estimated from (1) by neglecting \(F_0\) and integrating:

\[
\int_{u_0-u}^{u_0} dt_0 = \frac{C_1}{2L} \int_{t=0}^{T} dt.
\]  

Since \(u_0 \leq 0\), this gives

\[
U_0 = \left( u_t^{-1} + \frac{C_1 T}{2L} \right)^{-1},
\]  

where \(U_0\) is the speed of the drogue relative to the current at a time \(t=T\). As \(u_0 = U_0 + u_\) the response time of any sized drogue to any value of current speed can be calculated from (4). Results for our drogue \((L=0.6 \text{ m})\) are shown in Fig. 3 for the range of current speeds appropriate to this study. The drogues are seen to reach \((1-e^{-t}) = 62.3\%\) of the ambient current speed within 15-30 s. Either an increase in current speed or a decrease in drogue size results in a faster acceleration to ambient speed.

During operations the location of each drogue was recorded at intervals of 5 min by two shore-based theodolites at a separation of 500 m. The theodolite reading accuracy of 1 min of arc has a spot width of \(0.15 \text{ m}\) at a distance of 514 m offshore at the outer limit of our range. Estimating a maximum of a 15
Fig. 4. Drogue trajectories on 20 June during a one-point drop experiment at the Florida Gulf Coast study site. Each track is labeled with drogue depth setting, speed, standard deviation of speed measurements (m s⁻¹), and number of observations per track. Depth contours are shown in meters.

±0.15 m separation over 300±5 s gives a ±0.002 m s⁻¹ maximum error.

3. Observations—Steady state

In the late afternoon when the wind has been blowing from the south-southwest for 2–3 h a steady velocity field, as evidenced by both the current meter and drogue data, forms near the coast. Figs. 4–6 are typical examples of the drogue trajectories during the late afternoon steady-state conditions.

Fig. 4 shows the movement of a group of six drogues with depth settings of 1.5, 3.0, 4.5, 6.0, 7.5 and 9.0 m released near the 12.5 m depth contour on 20 June after the wind has been blowing from an angle 60°–70° to the coastline (SSW) for several hours. It is im-

Fig. 5. As in Fig. 4 except for trajectories on 25 June during a two-point drop experiment.
Fig. 6. As in Fig. 4 except for trajectories on 27 June during a three-point drop experiment.

Immediately apparent that, despite the large component of wind stress normal to the coast, the primary flow is directed along the coast. Close inspection of Fig. 4 further reveals a subtle three-layered velocity structure, with water between the surface and about 2 m moving onshore at about a 10° angle to the coast; water between the depths of approximately 2 and 5 m moving seaward at about 60° to the coast; and waters below 5 m strikingly reversing this seaward rotation with depth and moving back onshore (in ordered rotation) at angles of up to 25° for the 9 m drogue. Note that speeds are fairly uniform at about 0.1 m s⁻¹ below the 1.5 m value of 0.207 m s⁻¹.

Fig. 5 shows the velocity field at about 1400 CST 25 June to be similar to that in Fig. 4. Two sets of drogues were dropped in that experiment, however: one set near the 6.5 m contour, the other near the 8.5 m contour. Although during the preceding 2.5 h the wind had been blowing from a direction 60° to the coast, the primary flow is again alongshore. The near-surface current (1.5 m level) moves onshore at about a 10° angle to the coast. Current vectors rotate seaward with increasing depth down to the 4.5 m level, below which the rotation reverses to include an onshore flow component. Note the marked lack of horizontal divergence exhibited by the three pairs of drogues at the 1.5, 3.0 and 4.5 m levels. Notable divergence is shown by the 6.0 m drogues, which is seemingly balanced by the influx of water shown by the strong onshore component of the 7.5 m drogue. Resultant current speeds decrease almost linearly with depth and show a slight decrease offshore in the upper two levels (1.5, 3.0 m) but a slight increase offshore in the 4.5 and 6.0 m levels.

In the experiment shown in Fig. 6 three sets of drogues were dropped: three drogues (1.5, 3.0, 4.5 m) on the 5 m depth contour, three (1.5, 3.0, 4.5 m) on the 6 m contour, and four (1.5, 3.0, 4.5, 6.0 m) on the 7 m contour. The wind had been blowing onshore at 4 m s⁻¹ for as long as 4.5 h from a direction 60° to the coast. Again the primary flow direction is along the coast, but the associated secondary flow (onshore-offshore components) is reduced. The threelayered flow pattern is subdued but clearly present along the 7 m contour. At the 6 m contour the velocity vectors are more or less parallel with depth, but inside the 5 m contour the drogues give a picture of net movement onshore.

Study of the complete set of 12 drogue experiments indicates that in the vicinity of the outer bar the drogue motion, in general, has an onshore component. Inside the 5 m contour on the outer bar, shoaling wave effects suddenly become important. The amplitude of the near-bottom orbital speed for a nominal afternoon wave (length 12 m, height 0.45 m, and period 3 s) increases, according to linear wave theory, from 0.02 m s⁻¹ on the 7 m contour to 0.07 m s⁻¹ on the 5 m contour to 0.2 m s⁻¹ on the 3 m contour. Because there was never any indication by any of the drogue tracks of a seaward flow across the bar, it is felt that the well-known asymmetry in the force field of shoaling waves produced a net onshore force which
caused the drogues near the crest of the outer bar to move shoreward.

One experiment, unusual in that the wind blew steadily for at least 3 h from a direction other than SSW, is shown in a preliminary report (Sonu et al., 1973). In that case, steady onshore winds blew from the south (about 10° clockwise from the normal to the coast) for 3 h prior to the experiment and caused a slow onshore movement of drogues throughout the water column even in 7 m of water. Consideration of the drag force of the wind on the marker pole, however (see Fig. 2), suggests that the waters below the 1.5 m level were moving seaward at slow speeds of 0.01–0.02 m s⁻¹. Although the near-surface water seems to be spilling over the sill created by the crest of the outer bar, the general picture of the velocity field produced by this near coastal normal wind is one considerably reduced in magnitude and oriented more in an offshore-onshore direction than those previously discussed.

To summarize the pertinent observations up to this point, onshore winds in the speed range of 3–5 m s⁻¹ blowing at angles of ~60° to the coast produce currents that move predominantly along the shore rather than onto the shore following the wind. A subtle vertical structure in the coastal normal component of the speed is frequently observed such that onshore components exist in the near-surface and near-bottom layers and a seaward-directed component is present in the interior of the water column. As water depth decreases, the onshore components lessen in importance. The wind-induced velocity field is apparently very sensitive to wind angle to the coastline. When the wind vector was within 10° of the coastal normal, the currents were markedly weaker and began to resemble a two-dimensional flow channelized in the onshore-offshore direction.

It is very instructive, especially for theoretical comparisons, to construct vertical profiles of the velocity from the drogue data. Fig. 7 shows the speeds $S$ and directions $\theta$ with respect to the shoreline from two of our drogue operations on 27 June. Note that in the morning profile (solid dots) the speed drops quasi-linearly with depth and the current direction (solid triangles) is within a degree of being parallel to the coast. In the afternoon (open dots) a severe kink develops in the speed profile, indicating a high-speed surface layer not unlike the slippery-sea effect described by Houghton and Woods (1969) as being due to intense solar heating of surface waters in the tropics. The afternoon direction profile (open triangles) is clearly organized into three zones, an offshore-flowing zone sandwiched between surface and bottom zones of onshore flow.

Fig. 8, based on drogue data from 1400 CST 25 June, again shows the three zones of flow direction, with speeds dropping from 0.18 m s⁻¹ near the surface down to about 0.06 m s⁻¹ near the bottom. Another drogue experiment at 1430 CST 20 June, shown in Fig. 9, not only repeats the three-zoned direction profile but
also suggests (when compared to Figs. 7 and 8) that the increased water depth greatly amplified the onshore-offshore fluctuation of the current direction. A high-speed layer in the upper 3 m is also apparent in the speed profile, as in Fig. 8.

In fact, numerous velocity profiles made in the summer of 1968 in the same location with a suspended current meter (Marine Advisers Q-13) repeatedly showed the upper 3–4 m of water moving relatively swiftly (0.15–0.2 m s\(^{-1}\)) over sluggish (0–0.05 m s\(^{-1}\)) bottom waters as a result of the local density stratification. Fig. 10 gives an extreme example of this effect: a wind blowing for several hours at 8 m s\(^{-1}\) fails to generate a current below the 6 m level owing to the sharp density gradient in that region. The local Richardson number,

\[
Ri = \frac{-g \partial \rho / \partial z}{\rho (\partial u / \partial z)^2},
\]

in the pycnocline is calculated at Ri=0.6, which satisfies the criterion of Taylor (1931) and Miles (1961) for a stable, stratified shear flow that Ri must be greater than \(\frac{1}{4}\). In this case, the density difference from surface to bottom is caused by a temperature drop from 27 to 25°C and a salinity increase from 33 to 36‰. I believe this stratification, although reinforced by afternoon heating, is largely due to advection from the east of warm brackish effluent from Choctawhatchee Bay, which was frequently observed trending in this direction during the daily boat run from Choctawhatchee Inlet to the study area. The intermittent presence along the coast of the mixed bay effluent in the surface layer is a function of the phase and magnitude of the tidal current near the inlet and the direction of the prevailing coastal currents.

In contrast to the Florida data, Figs. 11–13 show velocity profiles reconstructed from the drogue trajectories obtained by Saylor (1966) in Lake Superior during September 1964. Because of the fresh water, the latitude, and the time of year, we expect no significant density gradient in these shallow waters, and indeed none is mentioned by Saylor. The primary differences exhibited by the Lake Superior profiles are that the speed profile is considerably more uniform in the vertical and the direction profile is consistently only two layered, onshore in the upper layer and offshore in the lower layer. The Lake Superior data, however, do clearly agree with the Florida data in that the primary flow direction is alongshore to within a few degrees, despite wind angles of 30° to the coast in Figs. 11 and 12. Note especially in Fig. 13 that a relatively strong wind of 10 m s\(^{-1}\) blowing at an angle of 73° to the coast drives a current of only 0.14 m s\(^{-1}\), again directed alongshore to within a few degrees and as usual in the downwind component direction (in this case, to the left of the wind). The absence of both anomalous high-speed zones near the surface and the third (near-bottom) zone of onshore-moving water.

**Fig. 10.** Variation of current speed and density \(\sigma_1\) as a function of depth at the study site. Winds had been blowing for several hours at 8 m s\(^{-1}\) at angles of 55°–65° from the shoreline (southwest). Current direction is along the coast (east).
appears to be the result of the apparently weak stratification in the water column.

Sonu (1960) presents data also indicating nearshore wind drift in depths of water of about 10 m that are in agreement with the above observations.

4. Theoretical consideration

As a first step in understanding the mechanics of the processes which drive these nearshore currents, we consider the three-component equations of motion with friction parameterized as an isotropic eddy viscosity:

\[
\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} - f_v + K \frac{\partial v}{\partial x}, \quad (5)
\]

\[
\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial z^2} - f_u + K \frac{\partial u}{\partial y}, \quad (6)
\]

\[
\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial z^2} + g + K \frac{\partial u}{\partial z} + K \frac{\partial v}{\partial z}, \quad (7)
\]

where \(x\) is the direction parallel to shore, positive to the right looking onshore; \(y\) is the corresponding speed component; \(z\) is the direction normal to the shore, positive onshore; and \(w\) is the corresponding speed component; \(\rho\) is the pressure; and \(\rho\) is the water density. In these shallow depths and in such close proximity to the coastline the assumption of an isotropic eddy viscosity is probably not overly restrictive. In a recent numerical study of storm-generated currents Foristall (1974) imposed essentially the same restrictions on the eddy viscosity, including time invariability.

Assuming a steady state, neglecting the vertical speed component and the field accelerations, and assuming a homogeneous water mass permits (5), (6) and (7) to be combined and written

\[
K \frac{\partial^2 u}{\partial z^2} = -g - f_v, \quad (8)
\]

\[
K \frac{\partial^2 v}{\partial z^2} = -g + f_u, \quad (9)
\]
where $\xi$ is the elevation of the free surface above the undisturbed level. Following Jeffreys (1923), if the wind field is sufficiently uniform along the coast, we may expect no surface slopes in that direction, i.e., \( \partial \xi / \partial x = 0 \); then using the complex velocity and 
\[ W = u + i v, \]
Eqs. (8) and (9) may be written
\[ \frac{\partial^2 W}{\partial z^2} - j W = j G, \]
(10)
where $G = -(g/f) \partial \xi / \partial y$ is independent of $z$.

The solution of (10) is then
\[ W = G + A \sinh j q z + B \cosh j q z, \]
(11)
where $q^2 = g/f$, $j = \imath + 1$, $A$ and $B$ are complex constants, and $G$ is real. Hsueh and O'Brien (1971), in an updward study, solved a more complex version of (10) including offshore-dependent current speeds for the case of current forcing at the edge of a broad, flat continental shelf by using a Fourier series representation of the complex velocity $W$. If the presence of the coast is taken into account by setting
\[ \int_{-h/2}^{h/2} W dz = 0, \]
(12)
i.e., there is no net transport toward or away from the shore, Jeffreys shows that (11), in the case of a flat bottom, can be written
\[ W = G + A \sinh j q z + j C \frac{\cosh j q z}{\sinh j q z}, \]
(13)
where $\lambda = \frac{1}{2} q h$ and $C$ is real.

The surface boundary condition is the usual quadratic wind stress rule:
\[ \tau_s = K \rho \frac{\partial W}{\partial z} |_{z_s} = -\kappa_s \rho Q e^{i \alpha}. \]
(14)

The bottom boundary condition is not the usual Ekman "no-slip" condition, which tends to considerably underestimate current speeds in shallow water, but rather a quadratic bottom friction rule (Charnock, 1959; Bowden et al., 1959):
\[ \tau_b = K \rho \frac{\partial W}{\partial z} |_{z_b} = -\kappa_b \rho R^2 e^{i \gamma}, \]
(15)
where $\kappa_s$ and $\kappa_b$ are friction coefficients for air-water and water-sea bottom, respectively, $\rho$ is air density, $Q$ the wind speed blowing at an angle $\alpha$ to the shoreline, and $R$ the current speed making an angle $\gamma$ to the shoreline just above a frictional logarithmic bottom boundary layer, generally conceived as occupying the meter nearest the bottom (Bowden et al., 1959).

Owing to the complexities of the resulting analytical solution, Jeffreys made calculations only for very shallow and very deep water approximations of the full equations. These appear to give misleading results. Our procedure was to solve directly for $u$ and $v$ from the general solution on an IBM 360 computer. The principal computational steps are repeated here because there are several typographical errors in Jeffreys' paper, and in addition I have allowed for $\kappa_s \neq \kappa_b$.

First, the angle $\gamma$ which the bottom current makes with the shoreline is determined as the root of
\[ (\cosh 4 \lambda - \cos 4 \lambda) \sin\gamma \left( \frac{\rho}{\gamma} - \cos \alpha \cos \gamma \right) \frac{1}{\kappa_s} \]
\[ + Q \left[ \cos \left[ \sin (\gamma - \pi/4) \sinh \gamma - \cos (\gamma - \pi/4) \sin 4 \lambda \right] \right. \]
\[ - 2 \cos \gamma \left[ \sin (\alpha - \pi/4) \sinh \gamma \cos 2 \lambda \right. \]
\[ \left. - \cos (\alpha - \pi/4) \cosh \gamma \sin 2 \lambda \right] = 0, \]
(16)
using the Newton-Raphson technique.

Then $G$ is determined from
\[ G = R \cos \gamma + D \left[ \cos (\gamma - \pi/4) \sinh 4 \lambda \right. \]
\[ + \sin (\gamma - \pi/4) \sin 4 \lambda \]
\[ - 2 E \left[ \cos (\alpha - \pi/4) \sinh 2 \lambda \cos 2 \lambda \right. \]
\[ \left. + \sin (\alpha - \pi/4) \cos 2 \lambda \sin 2 \lambda \right] \times (\cosh 4 \lambda - \cos 4 \lambda)^{-1}, \]
(17)
where
\[ R = Q \left( \frac{\kappa_s \rho \cos \alpha}{\kappa_s \rho \cos \gamma} \right)^{\frac{1}{4}}, \]
(18)
\[ E = \frac{\cos \gamma}{\cos \alpha}, \]
(19)
\[ D = \frac{\kappa_s R^2}{2 Q K}, \]
(20)
Moreover, $C$, a wholly real number in (13), is given by
\[ C = -\frac{\left[ \kappa_s R^2 \sin \gamma - (\sigma/\rho) \kappa_s Q^2 \sin \alpha \right]}{4 K Q} \]
(21)
and the complex coefficient $A$ may be written
\[ A = -\frac{\left[ \kappa_s R^2 e^{i \gamma} + (\sigma/\rho) \kappa_s Q^2 e^{i \alpha} \right]}{2 K Q \cosh \gamma}. \]
(22)

Substituting (17), (21) and (22) in (13) allows the determination of vertical profiles of $u$ and $v$ as a function of wind speed and wind direction with respect to the coastline, eddy viscosity, total water depth, latitude, and surface and bottom friction coefficients. As Jeffreys pointed out, $\cos \alpha$ and $\cos \gamma$ must have the same sign, i.e., alongshore components of wind
and bottom drift are in the same direction. When winds are from a direction between $\pi/2$ and $3\pi/2$, the sign of $\gamma$ in (16) must be changed accordingly. The only poorly understood variable in this development is the eddy viscosity, which I calculated from the expression for a well-mixed surface layer (Neumann and Pierson, 1966):

\[ K = 0.1825 \times 10^{-4} Q \rho^{-1}, \tag{23} \]

which, for wind speeds $Q < 7$ m s$^{-1}$, is in good agreement with values shown in Munk and Anderson (1948).

5. Simplified theory

Neglect of the Coriolis parameters in the initial equations (5) and (6) uncouples these equations and allows straightforward solutions for both $u$ and $v$. Eq. (8) reduces to $\partial^2 u/\partial z^2 = 0$, implying that $u$ is linear with $z$, analogous to turbulent Couette flow. Taking $z$ positive up from the bottom, the solution for $u$ is given by

\[ u = -\frac{1}{K} z + u_b, \tag{24} \]

where $u_b$, the speed near the bottom, is given by the quadratic bottom stress condition

\[ \tau_{zb} = \frac{1}{K} u_b. \tag{25} \]

The alongshore current speed is then easily calculated from (24) with knowledge of only the wind velocity and water depth and using $\tau_{zb} = 2 \times 10^{-3} \sigma (Q \cos \alpha)$, $K$ given by (23), and $k_b \approx 3 \times 10^{-3}$ but dependent on bottom conditions (see Sternberg, 1972). It is amply clear from (24) that the use of a "no-slip" bottom boundary condition would give unrealistically low results for the current speed because in shallow water $u_b$ is typically much greater than the first term on the right side of (24).

Inspection of Table 1 indicates that the calculation of the alongshore current from the simple theory (24) is a good approximation to the complete theory only when the wind angle to the coast is less than about 45°.

In the y or coastal normal direction (9) becomes

\[ \frac{\partial v}{\partial y} = -\frac{g}{\partial y}, \tag{26} \]

which implies a parabolic velocity profile $v = ax^2 + bx + c$, where $a$, $b$, $c$ must be determined by boundary conditions. Using the surface stress condition $\partial v/\partial z |_{y=0} = 2ad + b = \tau_{ye}/K$, the bottom stress condition $\partial v/\partial z |_{y=b} = b = \tau_{yb}/K = k_b \rho g b^2/\kappa$, and the condition of zero net volume flux normal to the coast

\[ \int_0^d v dy = ad^2/3 + bd + c = 0 \]

yields

\[ \frac{1}{2d/K} (\tau_{ye} - \tau_{yb}) z^2 + \frac{\tau_{yb}}{4d/K} z - \frac{d}{2d/K} (\tau_{ye} + 2\tau_{yb}), \tag{27} \]

where $\tau_{ye}$ is the wind stress component normal to the coast, $d$ the total water depth, and $\tau_{yb}$ can be determined explicitly by setting $v = \tau_b$ when $z = 0$ in (27) and solving the resulting quadratic equation in $v_b$. The solution to the quadratic equation gives

\[ v_b = \frac{-3K + (9K^2 - 2k_b \rho g^2 \tau_{ye})^{1/2}}{2k_b \rho g d}. \tag{28} \]

Under reasonable conditions calculations show that $v_b$ is on the order of 0.01 m s$^{-1}$ and reverses near mid-depth so that the flow, from the simplified theory, is indicated to be quasi-parallel to the coast. Since a negative sign in front of the second term of the numerator in (28) gives physically unrealistic speeds (~1 m s$^{-1}$) under moderate winds, this solution can be discarded.

6. Discussion

Calculation of the velocity vectors with depth from the constant eddy viscosity theory (13) under conditions nominal for the present study predict that the current direction should indeed be quasi-parallel to the coast and nearly independent of wind direction and wind speed, as the observations have consistently shown. The absolute speed, however, is strongly dependent on wind direction, being maximum with a parallel-to-the-coast wind and decreasing with increasing wind angle (with approximately a $\cos \alpha$ dependency) to essentially zero values under a normal-to-the-coast wind. Note that with the $\cos \alpha$ dropoff the current speed is reduced by only 50% after the wind has turned a full 75°.

The predictions of (13) under the known wind velocity and water depth are also plotted on Figs. 11–13, the Lake Superior data (unstratified case). The
Fig. 14. Isotachs of alongshore current speeds predicted by the constant eddy viscosity theory [(13)] compared to data from the Arctic coast (dots) [Wiseman et al., 1972]; from Lake Superior (asterisks) [Saylor, 1966]; and from the Florida Gulf Coast (squares) [this paper]. The number labels are the corresponding speeds in $10^{-2}$ m s$^{-1}$.

The magnitude and shape of the speed profiles are clearly in good agreement with the theory; with this constant eddy viscosity theory, the vertical gradient of the speed is rather small under all conditions. The predicted and observed direction profiles similarly agree that the water motion is directed slightly onshore (1°–4°) in the upper ~40% of the water column and slightly offshore (1°–2°) in the underlying waters. Note especially in Fig. 13 that the wind is blowing at 10 m s$^{-1}$ at an angle of 73° to the coast, and yet the water is moving within a few degrees of being parallel to the shoreline. Speeds are considerably reduced below the empirical expectation of 3–5% of the wind speed because a significant amount of the onshore component of the wind stress is used to maintain a slope in the water surface (setup) represented by the pressure gradient term $G$ in (13). Surface slopes on the order of $10^{-3}$ predicted by the theory seem quite reasonable in terms of existing knowledge (Von Arx, 1965).

Numerical calculations were made from (13) to assess the effect of varying the wind velocity, water depth, eddy viscosity, latitude (Coriolis effects), and bottom and surface drag coefficients ($\kappa_b$ and $\kappa_s$). The possible effect of latitude changes on coastal currents appeared to be very interesting. The calculations show, however, that as latitude decreases and the contribution to the momentum balance by the Coriolis forces similarly decreases, the pressure gradient force (surface slope) adjusts itself such that there is no significant modification of the structure of either the current speed or direction.

Varying the surface and bottom friction coefficients $\kappa_s$ and $\kappa_b$, respectively, within reasonable limits (see Sternberg, 1972) produces only minor changes in the current speeds as compared to those caused by changes in wind speed and direction. Similarly, increases in depth are predicted by (13) to produce relatively small changes in current speeds; e.g., under a 10 m s$^{-1}$ wind at 30° to the coastal normal, current speeds increase by only 10% as the depth increases from 6 to 15 m. Whether the wind is onshore or offshore determines only the direction of the $v$ component of the flow (offshore winds produce a slight offshore direction in the surface layer); the speed and essential alongshore character of the flow remain unchanged.

It is encouraging to note that under the water depths and wind velocities under discussion the speed of the current is surprisingly free of influence from changes in the eddy viscosity $K$, a distinct advantage considering our poor knowledge of $K$, an effect also noted by Forstall (1974). At a wind speed of 8 m s$^{-1}$ blowing onshore at an angle of 70° to the coast, calculations [(13)] show that successive changes in the eddy viscosity from 0.01 to 0.03 to 0.05 to 0.07 m² s$^{-1}$ produce current speeds in the surface layer of 0.15, 0.133, 0.129 and 0.127 m s$^{-1}$, respectively. Inasmuch as $K=0.03$ m² s$^{-1}$ is a nominal value for the eddy viscosity at this wind speed, uncertainties in the cur-
rent speed resulting from the estimation of the eddy viscosity amount to at most 11%.

The effect of wind speed and wind angle to the coastline on the alongshore current speed is shown in Fig. 14. The contours of current speed on this figure are calculated from the constant eddy viscosity theory [(13)] and represent the upper 2 m in waters 6–12 m deep. Also plotted on the figure are observed points from data presented in this paper, drogue observations taken in Lake Superior by Saylor (1966), and drogue observations reported by Wiseman et al. (1973) from the coastal waters of the Arctic Ocean. Wiseman et al. (1973) also present data showing that very weak stratification prevailed during the periods of the drogue observations. Wiseman's data are of special interest in that, of his total of 14 observations, 10 were such that the nearshore currents were directed within 15° or less of the shoreline under winds which were offshore 50% of the time, in good agreement with our theoretical expectations. The three sets of data offer basic corroboration of the alongshore current speed dependence on wind speed and direction to the coastline. The standard deviation of the errors (observed minus predicted speeds) is 0.045 m s\(^{-1}\), with 10 of 14 observations plotted on Fig. 14 showing the errors to be negative, i.e., the theory consistently overestimates the speed by 0.04 m s\(^{-1}\).

If the wind is within ±10° of the coastal normal, a quasi-static equilibrium is predicted in which speeds are reduced to less than 0.03 m s\(^{-1}\) throughout the vertical with a slow motion directed onshore in the upper layer and a slow motion directed offshore in the lower layer. A possible realization of this condition in the field was presented in a preliminary report (Sonu et al., 1973, Fig. 8a).

Bretschneider (1967) has also presented a theory on wind-driven currents which considered the vertically integrated equations of motion with essentially the same boundary conditions used by Jeffreys, i.e., surface and bottom stress laws and the condition of no net volume flux normal to the coast. This approach is restricted to predicting the vertically averaged alongshore speed only, all information on \(v\) (water speed normal to the coast) and the vertical structure of \(u\) is suppressed. Bretschneider's steady-state solution (in my notation)

\[
\frac{Q}{4\sqrt{k/\theta}} \left( \frac{\cos \theta}{15} \right)^{1/2}
\]

predicts a family of curves for the alongshore current speed on a wind speed, wind angle diagram very similar to the one on Fig. 14, except that (29) overpredicts Jeffreys' by 0.05–0.1 m s\(^{-1}\). It is further encouraging to note that Bretschneider also finds current speed only weakly dependent on water depth \((Q/\theta)\) and explicitly determines a \(\cos \theta\) dependency of current speed on wind direction. In Eq. (29) \(k\) is a surface stress coefficient, nominally \(3 \times 10^{-4}\), \(n\) is Manning's friction coefficient, nominally 0.025, and \(d\) must be specified in feet.

Comparisons between the data original with this paper (Figs. 4–10), taken under conditions of moderate vertical density stratification, and the constant eddy viscosity theory [(13)] show the theory to be clearly unsatisfactory, although accurate in magnitude to within a factor of 2. Eq. (13) does not, for example, predict the three-layered flow persistent in the Florida data, and it systematically underestimates the near-surface speeds and overestimates the near-bottom speeds.

To determine whether the density gradient could be responsible for the observed effects, the coupled differential equations

\[
\frac{\partial}{\partial x} \left( \frac{K_{\mu}}{\sigma_{\mu}} \right) = -f v
\]

\[
\frac{\partial}{\partial z} \left( \frac{K_{\eta}}{\sigma_{\eta}} \right) = -g + f u
\]

were solved numerically with a Runge-Kutta technique which requires specification of the surface speeds \(u\) and \(v\) and their vertical gradients \((\partial u/\partial z)_{z}=\tau_{x}/K\) and \((\partial v/\partial z)_{z}=\tau_{x}/K\), as well as the functional form of the vertical dependence of the eddy viscosity. The baroclinic pressure gradient term

\[
g \int \omega dz = -\rho \frac{\partial p}{\partial y}
\]

can be estimated from observed salinity cross sections normal to the coast. Using a maximum change of 2% over a kilometer and with \(\Delta z=5 \text{ m}\) this term is calculated at \(~4 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}\), an order of magnitude or more smaller than values expected from the barotropic term \(g(\partial \tau/\partial y)\) in (31), and hence can be safely neglected in the present context.

To this point I have used a power law for \(K\) as a function of \(z\), i.e.,

\[
K = K_0 \left( \frac{z+\epsilon}{d+\epsilon} \right)^n
\]

where \(K_0\) is given by (23) and \(\epsilon\), following Fjeldstad (1929), is a very small number. The exponent \(n\) is adjustable and Fjeldstad's data suggest \(n=0.75\) as a nominal value. The constant in (31) is proportional to the surface slope and was determined by tuning the solution such that the continuity condition (12) was satisfied. Bowden (1962) has reviewed the interrelationship between the vertical profile of the eddy viscosity and the local density gradient via the Richardson number. Although the exact details are still the subject of research (Bowden and Gilligan,
1971), it is clear that the eddy viscosity is greatly affected by the suppression of vertical turbulence and mixing associated with the local density gradients.

The preliminary results are very encouraging in that the numerical solutions (Fig. 15) do indicate that a three-layered velocity profile should develop in the stratified case: a surface layer 1–2 m thick moving slightly onshore at angles of 1°–3° to the coast, a thicker mid-depth layer moving offshore at similar small angles to the coast, and then, abruptly, in the lowest few meters, a sharp turning of the flow in the onshore direction. In some of the calculations this onshore turning approached 30°. Comparison of Figs. 7–9 (the moderately stratified Florida data of this study) and the theoretical predictions of the variable eddy viscosity theory (Fig. 15) indicates that, although exact quantitative agreement is yet to be obtained, the consideration of vertical density gradients via the eddy viscosity does reproduce the essential directional characteristics of the observed velocity fields.

7. Summary and conclusions

Detailed observations by precision theodolite of the movement of drogues within 800 m of a long, straight, sandy shoreline indicate that the direction of currents driven by local winds is predominantly alongshore and that there is little dependence on wind speed or direction. Under moderate conditions, current speed is strongly controlled by the wind angle to the shoreline and to a lesser degree by wind speed. In the moderately stratified waters off the Florida Gulf Coast a subtle three-layered pattern was persistent in the coastal normal component of the velocity profile such that onshore motion was present in a near-surface and a near-bottom layer, whereas intermediate depths were characterized by offshore motion.

In unstratified coastal waters of Lake Superior (Saylor, 1966), similar drogue experiments show a dominant coastal parallel flow despite large wind angles to the shore. The lack of stratification in the Lake Superior data apparently results in a two-layered velocity profile in which onshore components are in the upper layer and offshore components are in the lower layer.

Theoretical solutions (Jeffreys, 1923) to the equations of motion, including the Coriolis term but assuming a constant eddy viscosity, successfully predict the basic alongshore nature of the flow and agree quantitatively with nearshore current velocity observations by Saylor (1966) and by Wiseman et al. (1973) along the Arctic coast, which were taken under a variety of wind speeds and wind angles to the coast. The numerical solution of the differential equations, including a depth-dependent eddy viscosity, accounts for the three-layered flow pattern observed in the moderately stratified (Florida) data.

A few observations remain in the literature, notably one in Sonu (1960) and two in Wiseman et al. (1973), where strong winds nearly perpendicular to the coast
set up current fields significantly different from those expected from the results discussed above, i.e., well-developed \( (0.1-0.2 \text{ m s}^{-1}) \) onshore-offshore speed components. Murray (1970, 1971) also observed strong offshore-directed bottom currents near the coast under near-hurricane winds. These apparently anomalous observations, which obviously have an important bearing on onshore-offshore transport processes, should be explained with improved knowledge of the spatial variations in the eddy viscosity and the inclusion of the nonlinear field accelerations. Numerical work by W. Waldrop (personal communication) with these goals in mind is presently underway.

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