Internal Tide Nonstationarity and Wave–Mesoscale Interactions in the Tasman Sea

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ABSTRACT

Internal tides, generated by barotropic tides flowing over rough topography, are a primary source of energy into the internal wave field. As internal tides propagate away from generation sites, they can dephase from the equilibrium tide, becoming nonstationary. Here, we examine how low-frequency quasigeostrophic background flows scatter and dephase internal tides in the Tasman Sea. We demonstrate that a semi-idealized internal tide model [the Coupled-Mode Shallow Water model (CSW)] must include two background flow effects to replicate the in situ internal tide energy fluxes observed during the Tasmanian Internal Tide Beam Experiment (TBeam). The first effect is internal tide advection by the background flow, which strongly depends on the spatial scale of the background flow and is largest at the smaller scales resolved in the background flow model (i.e., 50–400 km). Internal tide advection is also shown to scatter internal tides from vertical mode-1 to mode-2 at a rate of about 1 mW m^{-2} . The second effect is internal tide refraction due to background flow perturbations to the mode-1 eigenspeed. This effect primarily dephases the internal tide, attenuating stationary energy at a rate of up to 5 mW m^{-2} . Detailed analysis of the stationary internal tide momentum and energy balances indicate that background flow effects on the stationary internal tide can be accurately parameterized using an eddy diffusivity derived from a 1D random walk model. In summary, the results identify an efficient way to model the stationary internal tide and quantify its loss of stationarity.

KEYWORDS: Ocean; Australia; Baroclinic flows; Eddies; Internal waves; Shallow-water equations

1. Introduction

Internal tides propagate at the interfaces of density layers and are the primary energy source of the global internal wave field (Munk and Cartwright 1966; Egbert and Ray 2000; Wunsch and Ferrari 2004; Waterhouse et al. 2014). While internal tides are generated over rough topography, they rarely dissipate locally. Instead they often propagate for thousands of kilometers, distributing global internal wave energy (Shriver et al. 2012; Zhao et al. 2016; Buijsman et al. 2016). Techniques such as harmonic analysis and plane-wave fitting have produced global maps of internal tide sea surface displacement from satellite altimetry (Ray and Mitchum

1997; Zhao et al. 2016; Zaron 2015). While these techniques accurately estimate stationary tides (i.e., the signal component phase-locked with the equilibrium tide), they are largely unable to estimate nonstationary tides, or tides whose phases wander with respect to the equilibrium tide (Ray and Zaron 2011; Shriver et al. 2014; Zaron 2017; Nelson et al. 2019). Interactions between internal tides and slowly evolving (i.e., quasigeostrophic) background flows can dephase internal tides from the equilibrium tide, and the estimated global distribution of nonstationary tides suggests this process is important. Mesoscale eddies can alter the propagation of low-mode internal waves (Rainville and Pinkel 2006; Park and Watts 2006; Dunphy and Lamb 2014; Ponte et al. 2015; Dunphy et al. 2017; Duda et al. 2018) and semidiurnal tides are largely nonstationary along the equator and in western boundary currents (Buijsman et al. 2017; Zaron 2017; Savage et al. 2017a). While several studies have examined the temporal variability

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of tidal nonstationarity (Zaron and Egbert 2014; Buijsman et al. 2017) and the local correlation between tidal nonstationarity and the strength of the mesoscale field (Pickering et al. 2015), little is known about the dominant horizontal scales of the background flows in these interactions, or whether these interactions merely redirect/refract the internal tides, or actively dissipate and/or scatter them to higher modes. Quantifying such interactions is difficult because wave–background flow interactions are hard to observe and computationally expensive to model over large spatial and temporal scales. Here, we find that tidal nonstationarity is also dependent on the horizontal scale of the mesoscale field, and not only on its strength.

In the Tasman Sea, Macquarie Ridge (the underwater extension of the South Island, New Zealand) generates an internal tidal beam that propagates northwest to the continental slope of Tasmania. This beam has a large stationary and weak nonstationary component (Waterhouse et al. 2018). It is approximately unidirectional because Macquarie Ridge is the only major source of internal tides in the Tasman sea, and the basin is relatively shielded from remotely generated internal tides (Zhao et al. 2018). From January to March of 2015, the Tasman Tidal Dissipation Experiment (TTIDE) observed the open-ocean propagation of the internal tide beam and its dissipation and reflection on the Tasman slope (Pinkel et al. 2015). Earlier in the experiment, Johnston et al. (2015) observed a standing internal tide off the Tasmanian coast using gliders. The standing wave was caused by internal tide reflection at the continental slope, which was simulated in detail by Klymak et al. (2016).

The Tasmanian Internal Tide Beam Experiment (TBeam) was conducted in tangent to TTIDE, and aimed to study propagation of the M_2 energy fluxes between the generation site and the Tasman Shelf (Waterhouse et al. 2018). Figure 8 of Waterhouse et al. (2018) shows the energy fluxes computed from moored observations from the TBeam experiment (recreated here in Fig. 5). Though nonstationary semidiurnal energy flux at the TBeam mooring was approximately 25% of total semidiurnal internal tidal energy flux, the nonstationary internal tide caused large short-term variability in both the magnitude and heading of the tidal energy fluxes. While the heading of the stationary tidal energy fluxes is relatively steady, directed from the generation site toward the Tasman Shelf (ranging between $\sim 140^\circ$ and 160° due to the spring–neap cycle), the heading of the total tidal energy fluxes varies dramatically throughout the experiment. This suggests that even in a region with a weak nonstationary tide, the nonstationary tide can still significantly alter the tidal energy flux.

The Coupled-Mode Shallow Water model (CSW) (Kelly et al. 2016) is an internal tide process model that has been used to understand nonstationary tides. In the Gulf Stream, Kelly et al. (2016) used CSW to show that a strong western boundary current such as the Gulf Stream can refract a propagating internal tide wave. Here, we use CSW to study the effect of mesoscale eddies on the semidiurnal internal tide in the Tasman Sea, particularly in the region of the TBeam experiment. An important feature of CSW is that it can be run with and without a background flow; this allows us to quantify the effect of this background flow on the propagation of the internal tide.

The remainder of the paper includes a short review of tidal analysis methods (section 2), descriptions of the TBeam mooring data (Waterhouse et al. 2018) and modeling methods (section 3). Section 4 provides a comparison of CSW with in situ observations and altimetry. Section 5 analyzes which background flow effects dominate internal tide nonstationarity in the Tasman Sea by quantifying the separate effects of wave advection, scattering, and refraction by the background flow, and contextualizes this in terms of previous studies, (e.g., Zaron and Egbert 2014; Buijsman et al. 2017). This section also aims to determine how internal tide propagation depends on the horizontal scale of the background flow. Section 6 focuses on stationary internal tide dynamics and develops parameterizations for background flow effects. Section 7 summarizes the results.

2. Tidal analyses

Before analyzing observations and numerical simulations, it is useful to review the concepts of orthogonal modes and tidal stationarity.

a. Vertical modes

Horizontal velocity, $\mathbf{u}(\mathbf{x}, z, t) = [u(\mathbf{x}, z, t), v(\mathbf{x}, z, t)]$, and pressure $p(\mathbf{x}, z, t)$, can be written as a sum of orthogonal vertical modes

$$\mathbf{u}(\mathbf{x}, z, t) = \sum_{n=0}^{\infty} \mathbf{u}_n(\mathbf{x}, t) \phi_n(z), \quad (1a)$$

$$p(\mathbf{x}, z, t) = \sum_{n=0}^{\infty} p_n(\mathbf{x}, t) \phi_n(z), \quad (1b)$$

where $\mathbf{x} = [x, y]$, z , and t are horizontal, vertical, and time coordinates, $\mathbf{u}_n(\mathbf{x}, t)$ and $p_n(\mathbf{x}, t)$ are the velocity and pressure modal amplitudes, n is the vertical mode number, and $\phi_n(z)$ is the vertical structure function (i.e., the mode) computed by solving the Sturm–Liouville problem

$$\partial_z \left(\frac{1}{N_0^2} \phi_{nz} \right) + \frac{1}{c_n^2} \phi_n = 0, \quad \text{with} \quad \phi_{nz}(0) = \phi_{nz}(-H) = 0, \quad (2a)$$

and the orthogonality condition

$$\frac{1}{H} \int_{-H}^0 \phi_m \phi_n dz = \delta_{mn}, \quad (3)$$

where z subscripts indicate partial derivatives, $N_0(\mathbf{x}, z)$ is the buoyancy frequency, $c_n(\mathbf{x})$ are the eigenspeeds, $H(\mathbf{x})$ is depth, and δ_{mn} is the Kronecker delta (e.g., Kundu 2004). In practice, N_0^2 is computed from an observed density profile and the system is solved using a finite-difference or a spectral method (Kelly 2016; Early et al. 2020). The vertical velocity modes, Φ_n , are defined in appendix and related to the horizontal velocity modes via $\Phi_{nz} = \phi_n$.

Exploiting the orthogonality of the modes, the vertical-mode amplitudes are

$$\mathbf{u}_n(\mathbf{x}, t) = \frac{1}{H} \int_{-H}^0 \phi_n(z) \mathbf{u}(\mathbf{x}, z, t) dz, \quad (4a)$$

$$p_n(\mathbf{x}, t) = \frac{1}{H} \int_{-H}^0 \phi_n(z) p(\mathbf{x}, z, t) dz. \quad (4b)$$

b. Stationary and nonstationary tides

To separate tidal and nontidal variability, the time series of the modal amplitudes, $\mathbf{u}_n(\mathbf{x}, t)$ and $p_n(\mathbf{x}, t)$, are bandpassed around the M_2 tidal frequency $\omega_{M_2} \pm 0.4$ cpd, retaining both the stationary semidiurnal constituents and the phase-shifted internal tides. After bandpassing, least squares fits to tidal harmonics determine the M_2 and S_2 constituents. The sum of the M_2 and S_2 time series is the stationary tide¹

$$\mathbf{u}_n^s = \mathbf{u}_n^{M_2} + \mathbf{u}_n^{S_2}, \quad (5a)$$

$$p_n^s = p_n^{M_2} + p_n^{S_2}. \quad (5b)$$

The nonstationary tide is the difference between the bandpassed and stationary tidal time series

$$\mathbf{u}_n^r = \mathbf{u}_n - \mathbf{u}_n^s, \quad (6a)$$

$$p_n^r = p_n - p_n^s. \quad (6b)$$

¹ The stationary tide can alternatively be defined from ensemble averages of tidal constituents over many short time windows or using the response method, but the results are nearly identical (Munk and Cartwright 1966; Kelly et al. 2015).

Because variance in the nonstationary tide increases with record length (Nash et al. 2012; Ansong et al. 2015), nonstationary tides calculated over only a few months are an underestimate of the multidecadal nonstationary tide. Because stationary tides are determined by least squares regression, they satisfy the “normal equation” (see Wunsch 2006) and the stationary and nonstationary signals are orthogonal with respect to time averaging, e.g.,

$$\overline{\mathbf{u}_n^r p_n^r} = 0, \quad (7)$$

where the overbar indicates a time average.

3. Observations and model

a. TBeam

A full water-column mooring was deployed from the Research Vessel (R/V) *Revelle* at 44.5°S, 153°E on 10 January and recovered on 28 February 2015. While this study only takes advantage of the mooring data, full details of the observations collected during the TBeam campaign (including shipboard measurements from the R/V *Falkor* from 16 January through 13 February 2015) can be found in Waterhouse et al. (2018). On the mooring, pressure measurements were collected every minute using six Sea-Bird SBE37s and an RBR-Concerto at 37, 285, 493, 975, 1680, 2707, and 4726 m, with temperature observations made at 31 other depths along the mooring. The velocity observations were collected using upward-facing 300- and 75-kHz ADCPs at 38- and 748-m depths, sampling every 10 min. The direct correlation between temperature and density observed from the CTD measurements made at eight stations discussed in Waterhouse et al. (2018) made for a straightforward conversion from the high vertical resolution temperature observations to density. Mooring knock-down events were accounted for by interpolating all velocity and density observations to a vertical coordinate system. The first two pressure modal amplitudes (p_1 and p_2) and velocity amplitudes ($\mathbf{u}_1$ and $\mathbf{u}_2$) were computed by direct projection of the density profiles, respectively (see Waterhouse et al. 2018). Due to the vertical resolution of the observations, only the lowest two modes could be reliably computed, and are thus the only modes analyzed here and in Waterhouse et al. (2018).

b. Coupled-Mode Shallow Water model

CSW solves the evolution equations for the vertical-mode amplitudes of tidal transport $\mathbf{U}_n(\mathbf{x}, t) = H\mathbf{u}_n$ and pressure $p_n(\mathbf{x}, t)$ using

$$\begin{aligned} \mathbf{U}_{nt} + \sum_{m=0}^{\infty} (\bar{\mathbf{u}}_{mn} \cdot \nabla) \mathbf{U}_m + f \hat{\mathbf{k}} \times \mathbf{U}_n = -H \nabla p_n \\ - \sum_{m=0}^{\infty} H \mathbf{T}_{mn} p_m, \end{aligned} \quad (8a)$$

$$\begin{aligned} \frac{H p_{nt}}{c_n^2} + \frac{H}{c_n^2} \sum_{m=0}^{\infty} \bar{\mathbf{u}}_{mn} \cdot \nabla p_m + \frac{\delta c_n^2}{c_n^2} \nabla \cdot \mathbf{U}_n = -\nabla \cdot \mathbf{U}_n \\ + \sum_{m=0}^{\infty} \mathbf{T}_{nm} \cdot \mathbf{U}_m, \end{aligned} \quad (8b)$$

where t subscripts indicate time derivatives and f is the inertial frequency (see [appendix](#)). Equations (8a) and (8b) are analogous to the shallow-water equations describing the evolution of horizontal momentum and surface displacement, respectively. The equations for $n > 0$ are forced by prescribing surface-tide velocities $\mathbf{U}_0$ computed a priori (forcing terms involving p_0 are negligible; see [Kelly 2016](#)). The terms with summations couple modes and produce scattering. The $\mathbf{T}_{mn}$ terms are the topographic coupling coefficients, and $\bar{\mathbf{u}}_{mn}$ are the background flow coupling coefficients [i.e., projections of the depth-dependent quasigeostrophic current $\bar{\mathbf{u}}(\mathbf{x}, z, t)$]

$$\mathbf{T}_{mn}(\mathbf{x}) = \frac{1}{H} \int_{-H}^0 \phi_n \nabla \phi_m dz \quad (\text{m}^{-1}), \quad (9a)$$

$$\bar{\mathbf{u}}_{mn}(\mathbf{x}, t) = \frac{1}{H} \int_{-H}^0 \bar{\mathbf{u}}(\mathbf{x}, z, t) \phi_m \phi_n dz \quad (\text{m s}^{-1}), \quad (9b)$$

$$\delta c_n^2(\mathbf{x}, t) = \frac{1}{H} \int_{-H}^0 \delta N^2(\mathbf{x}, z, t) \Phi_n \Phi_n dz \quad (\text{m}^2 \text{s}^{-2}). \quad (9c)$$

The term $\mathbf{T}_{mn}$ is zero where the bottom is flat and stratification is horizontally uniform (i.e., where $\nabla \phi = 0$); $\bar{\mathbf{u}}_{mn}$ is diagonal where quasigeostrophic currents are barotropic, and dense where geostrophic currents are baroclinic. The term δc_n^2 (derived in [appendix](#)) accounts for changes in propagation (i.e., refraction) by evolving stratification, and does not couple modes.

CSW is numerically implemented on a spherical $1/20^\circ$ C-grid using a finite-volume formulation and eight vertical modes. Test simulations (not shown) established that the open-ocean mode-1 and mode-2 solutions, which we focus on here, are insensitive to increased resolution. Higher resolution improves the accuracy of coastal tides and topographic scattering, but those aspects have been examined using other models (D. Brazhnikov and H. Simmons 2017, personal communication; [Klymak et al. 2016](#)). Transport and pressure evolve using a

staggered second-order Adams–Bashforth time-stepping algorithm. The vertical structure functions (ϕ_n and Φ_n) and eigenspeeds (c_n) are computed once at each location using the spectral method of [Kelly \(2016\)](#) with the 21-yr mean stratification (1994–2015) from the Hybrid Coordinate Ocean Model (HYCOM; [Chassignet et al. 2009](#)), and global bathymetry ([Smith and Sandwell 1997](#)). The (static) topographic coupling coefficients are computed using centered differences and numerical integration. Prescribed TPXO surface tide transports ($\mathbf{U}_0$) force the model through the topographic coupling terms, allowing us to utilize both highly accurate surface tides and long time steps (as permitted by the Courant–Friedrichs–Lewy condition; [Kundu 2004](#)). To eliminate some complicated standing waves, we limit surface-tide forcing to the Macquarie Ridge and neglect the (weak) generation sites in the northern part of the domain, allowing us to focus on the fate of the internal tide that radiates from Macquarie Ridge.

CSW requires a dissipation scheme for numerical stability. The exact form and implementation of the scheme is somewhat arbitrary and solely designed to have minimal impact on the dynamics of interest. To this end, we stabilize the simulations with a weak horizontal numerical viscosity, $\nu_N = 27.5 \text{ m}^2 \text{s}^{-1}$ ([Bryan et al. 1975](#)), and a flow-relaxation region (sponge) along the domain boundary ([Lavelle and Thacker 2008](#)) and in shallow locations with poor “wave resolution” (see [Adcroft et al. 1999](#)). We do not analyze the dynamics of our numerical viscosity, but we report viscous dissipation simply to show that it is an order of magnitude weaker than the advective terms in most of the model domain ([section 5a](#)).

The time-dependent background flow terms, $\bar{\mathbf{u}}_{mn}$ and δc_n^2 , are updated once per day by interpolating snapshots from a global $1/12^\circ$ nontidal HYCOM simulation (experiment 91.1; [HYCOM.org](#); [Chassignet et al. 2009](#)) to the $1/20^\circ$ CSW grid. The power spectral density of surface relative vorticity in the horizontal plane ($\bar{\zeta}/f$, where $\bar{\zeta} = \overline{dv/dx} - \overline{du/dy}$) shown in [Fig. 1](#) shows a peak near $\lambda = 200 \text{ km}$. Temporally averaged surface vorticity and eddy kinetic energy (EKE) highlight a region in the southern part of the study region, associated with the Antarctic Circumpolar Current (ACC). There is also a region of strong relative vorticity and EKE along the eastern coast of southern Australia, associated with the East Australian Current (EAC). However, in the interior of the basin, where the internal tidal beam has been observed, vorticity and EKE are relatively weak and small scale.

The model equations (8) in CSW are slightly different than those derived by [Kelly et al. \(2016\)](#). The new equations (i) invoke the horizontal geometric approximation and (ii) rely on a time-scale separation rather than an amplitude separation (see [appendix](#) for the new derivation

FIG. 1. (a) Background flow surface kinetic energy computed from HYCOM and averaged from 1 Jan to 1 Mar 2015. (b) Background flow surface relative vorticity (ζ/f) computed from HYCOM and averaged over the same time period. (c) Wavenumber spectral density (computed as in [Savage et al. 2017b](#)) of surface kinetic energy and relative vorticity computed from daily snapshots of HYCOM. Colored rectangles show the various background flow scales examined in [section 5b](#); red shows scales of 600–800 km, purple shows scales of 400–600 km, blue shows scales of 200–400 km, and yellow shows scales of 50–200 km.

and a complete list of simplifying assumptions). The term “geometric approximation” is borrowed from optics and means that the propagation medium changes slowly over many wavelengths.² Here, the horizontal geometric approximation omits terms that include explicit horizontal derivatives of the background flow. Because the background flow is in geostrophic balance to first order (see [appendix](#)), this approximation also neglects terms that explicitly include vertical shear, which would need to be balanced by (neglected) horizontal buoyancy gradients. This approximation does not require the background flow be spatially uniform. In fact, vertical and horizontal

background flow variability are critical for our central results. The geometric approximation is formally invalid where quasigeostrophic eddies and internal tides have comparable horizontal length scales, but we employ it everywhere because it stabilizes the (linearized) dynamical system. Including the explicit terms in the momentum equations that involve background flow strain would permit exponentially growing waves that quickly destabilize the numerical model (e.g., see the dispersion relations derived by [Kunze 1985](#); [Zaron and Egbert 2014](#)). Nonlinear terms could be included to cap this exponential growth, but these terms would greatly complicate the model, negating CSW’s greatest strengths: simplicity and numerical efficiency.

Scaling the mode-1 internal tide dispersion relation supports our use of the horizontal geometric approximation. [Zaron and Egbert \(2014\)](#) derived a formula for the relative changes in mode-1 phase speed:

² This is a similar constraint to the WKB approximation, which assumes the amplitude and wavenumber of the solution changes slowly over many wavelengths ([Bühler 2009](#)).

FIG. 2. Mean amplitudes during 2015 of nondimensional changes in mode-1 phase speed $\delta c_p/c_p$ due to $1/12^\circ$ HYCOM background flow (a) stratification, (b) advection, and (c) horizontal shear, as defined by (10), which was derived by Zaron and Egbert (2014). The color bar has a $\log_{10}$ scale.

$$\frac{\delta c_p}{c_p} \approx \frac{1}{2} \frac{|\delta c_1^2|}{c_1^2} + \sqrt{1 - f^2/\omega^2} \frac{|\bar{\mathbf{u}}_{11}|}{c_1} + \frac{1}{2} \frac{f^2}{\omega^2} \frac{\bar{\xi}_{11}}{f}, \quad (10)$$

where the terms on the right-hand side represent the contributions of background flow stratification, advection, and vertical vorticity, respectively. While deriving (10), Zaron and Egbert (2014) neglected vertical variability in the background flow to avoid modal coupling, thus the only nonzero term in the background flow summation is in $\bar{\mathbf{u}}_{11}$ [see (8a)]. The root-mean-square (rms) magnitude of each term in (10) was computed using (9) with daily snapshots of $1/12^\circ$ HYCOM output during all of 2015. The annual mean terms were an order of magnitude smaller than the RMS values because the background flow is primarily associated with unsteady eddies. The HYCOM analysis indicates that phase speed changes in the Tasman Sea, like the North Pacific (Zaron and Egbert 2014), are dominated by stratification and advection, while shear effects are an order of magnitude smaller (Fig. 2). These results indicate that the horizontal geometric approximation (i.e., neglecting shear) does not significantly degrade predictions of internal tide propagation through the HYCOM mesoscale in this region.

4. Comparison of CSW to altimetry and moored observations

a. M_2 amplitude and phase from CSW

CSW simulated M_2 internal tides interacting with HYCOM background flows during all of 2015 (and one month of spinup). The simulation reveals a distinct mode-1 beam, with a surface displacement of 1 cm, propagating from Macquarie Ridge toward Tasmania (Fig. 3). The

stationary component of surface displacement agrees with satellite observations (Fig. 4). Specifically, the model and data have statistically equivalent amplitude, phase, direction, and wavelength, when both datasets are fit to a plane wave using data subsampled at the eight stations occupied during TBeam (Waterhouse et al. 2018). The model and satellite data also agree well with in situ observations (cf. Fig. 4 here with Fig. 6 in Waterhouse et al. 2018).

b. Mode-1 and mode-2 tidal energy fluxes

A comparison of the mode-1 tidal energy fluxes computed from both the observations and model demonstrates the internal tide variability due to a background field (Fig. 5). The observational fluxes (black lines) are recalculated as in Fig. 8 of Waterhouse et al. (2018). The dashed black line represents the stationary tidal energy fluxes calculated from observations, and the solid black line represents the total tidal energy fluxes. The beating between the M_2 and S_2 tidal constituents creates the fortnightly oscillation (i.e., the spring–neap cycle) evident in both the stationary and nonstationary energy fluxes. Though the magnitude of the nonstationary tidal flux is small compared to the stationary flux, most of the variability in both the magnitude and direction of the total tidal energy flux is due to the nonstationary tide (Waterhouse et al. 2018). For comparison with the observations, tidal energy fluxes computed from CSW simulations run with (yellow lines) and without (purple lines) background flow are also shown in Fig. 5. For the model run without background flow, the magnitude of the total tidal energy flux is nearly identical to the magnitude of the stationary tidal energy flux. For the model run with background flow, there is a considerably larger difference between the stationary and total tidal energy flux amplitudes. This is shown more

FIG. 3. Internal tide beam in the Tasman Sea. Contours show the (a) depth of the Tasman Sea (km) and (b) amplitude (cm) and (c) phase ($^{\circ}$) of the stationary M_2 mode-1 sea surface height from the CSW model. TBeam observations from January to March 2015 include eight CTD-LADCP stations (white diamonds) and one full-depth mooring (red circle). The internal tide generation site is along Macquarie Ridge, south of New Zealand (NZ) in (a). Mainland Australia (AUS) is noted in (a). Black contours indicate the 1000-m isobath.

clearly by examining the ratio of total tidal energy flux amplitude to stationary tidal energy flux amplitude, plotted in the bottom panel of Fig. 6. In the CSW simulation run with no background flow, the ratio of total to stationary tidal energy flux amplitude is approximately equal to one for the entire time series, whereas for the TBeam mooring observations as well as the CSW simulation forced by a background flow, the ratio of total to stationary tidal energy flux amplitude varies over time. In the TBeam mooring observations, this ratio ranges from less than 0.5 to greater than 2, whereas this ratio tends to be closer to one for the CSW simulation (ranging from approximately 0.6 to 1.5). This demonstrates that not only are the TBeam energy fluxes larger than those calculated in CSW, but also that CSW is not fully capturing the fraction of total tidal energy that is nonstationary.

Though the velocities from CSW compared well with the observations (not shown), the pressure estimated in CSW is approximately half of the pressure measured from the TBeam mooring. As internal tide beams are composed of interfering or superimposed plane waves, they are sensitive to the details of phasing, direction, and generation. While the CSW phasing and sea surface height amplitude described here agrees well with altimetry in general (Fig. 4), small errors in generation location could cause large differences between the model and in situ observations in some locations. This difference in pressure causes the model to underestimate the energy fluxes compared to the mooring data. While the magnitude of the energy fluxes calculated from both simulations is about half of the magnitude of the fluxes calculated from the TBeam mooring, the ratio of stationary tidal energy

FIG. 5. (top) Amplitude (kW m^{-1}) and (middle) heading ($^{\circ}$) of the semidiurnal mode-1 energy flux. (bottom) Ratio of total to stationary tidal energy flux amplitudes. Observations from the TBeam mooring (Waterhouse et al. 2018) include the total (solid black) and stationary (dashed black) energy flux amplitude and heading. Modeled energy flux from CSW include those with background flow (total, solid yellow; stationary, dashed) and without background flow (total, solid purple; stationary, dashed purple). The CSW results without background flow are nearly identical, rendering the purple dashed line indistinguishable from the purple solid line.

CSW, such as interactions with other propagating internal waves. The heading of the total observed tidal energy fluxes varies over 160° over the observation period, which has been attributed to possible variations occurring at the generation site (Waterhouse et al. 2018).

Amplitude of the mode-1 and mode-2 energy fluxes computed from CSW with and without background flow, plotted in Fig. 6, shows that the mode-1 energy is being scattered into higher modes by the background flow (seen also in section 5a). Note that, in Fig. 6, the mode-2 energy fluxes are multiplied by 10, to be more easily compared with the mode-1 energy fluxes. When the background flow is added to the model, mode-2 internal tide energy flux magnitude increases by an average of $\sim 50\%$, implying that the mode-2 internal tides are more sensitive to background flow than mode 1, consistent with previous studies (Rainville and Pinkel 2006; Zaron and Egbert 2014). Although the magnitude of the mode-

2 energy fluxes increases by $\sim 50\%$ with the addition of background flow, the average mode-1 energy flux magnitude decreases by 20%. This simultaneous loss of mode-1 energy and gain of mode-2 energy implies that a fraction of the additional mode-2 energy can be attributed to intermodal scattering by the background flow.

As shown in (Rainville and Pinkel 2006), the mode-2 internal tide is more susceptible to interactions with background flows, and therefore has a larger fraction of nonstationary variance. This can be seen easily in comparing mode-1 and mode-2 ratios of total to stationary internal tide energy flux amplitude, as plotted in the bottom panel of Fig. 6. The total-to-stationary energy flux ratio is considerably larger in mode-2 than mode-1. Additionally, the mode-2 ratio is almost exclusively greater than one, demonstrating that the background flow typically energizes mode-2, likely from intermodal scattering as discussed in the previous

FIG. 6. (top) Amplitude (kW m^{-1}) of the mode-1 and mode-2 semidiurnal energy flux from CSW. Mode-2 energy flux amplitudes are multiplied by 10 for scale. Dashed lines are stationary internal tide components, as in Fig. 5. (bottom) Ratio of total to stationary tidal energy flux amplitudes.

paragraph (and in detail in section 5a). The ratios also hint at differences in the causes of mode-1 and mode-2 nonstationarity. While the mode-1 energy flux amplitudes without a background flow were almost indistinguishable from the stationary energy flux amplitudes (Figs. 5, 6), there are clear differences between the total mode-2 energy flux amplitudes without background flow and the stationary mode-2 energy flux amplitudes. This difference suggests that, unlike the mode-1 tidal nonstationarity, which is primarily caused by the additional background flow, the nonstationarity of

the mode-2 tide is partially dependent on other factors, like subtle changes in stratification over sloping bathymetry.

5. Analysis of internal tide advection by a low-frequency background flow

a. Internal tide energy balance

The steady-state mode- n energy balance is derived by computing $(8a) \cdot \mathbf{U}_n/H + (8b)p_n$ and time averaging

$$\begin{aligned} & \sum_{m=0}^{\infty} \left\{ \underbrace{(\bar{\mathbf{u}}_{mn} \cdot \nabla) \mathbf{U}_m \cdot \frac{\mathbf{U}_n}{H} + \bar{\mathbf{u}}_{mn} \cdot \nabla p_m \frac{H p_n}{c_n^2}}_{A_{mn}} \right\} + \underbrace{\frac{\delta c_n^2}{c_n^2} p_n \nabla \cdot \mathbf{U}_n}_{R_n} \\ &= -\nabla \cdot \underbrace{\overline{\mathbf{U}_n p_n}}_{\mathbf{F}_n} + \sum_{m=0}^{\infty} \underbrace{(\overline{\mathbf{U}_m p_n} \cdot \mathbf{T}_{nm} - \overline{\mathbf{U}_n p_m} \cdot \mathbf{T}_{mn})}_{C_{mn}}, \end{aligned} \quad (11)$$

where A_{mn} is the “advection” term and R_n is the “refraction” term due to wave-background flow interactions. The term $\mathbf{F}_n$ is the energy flux, and C_{mn} is topographic intermodal scattering. When the background flow is zero, the left-hand side is zero and energy flux divergence equals topographic scattering.

Averaged over the full year of 2015, the first-order mode-1 energy balance is between energy flux divergence

and topographic internal tide generation C_{01} , which can reach 100 mW m^{-2} (not shown). The advection term ΣA_{m1} , intermodal topographic scattering ΣC_{m1} , and dissipation are also large near the Macquarie Ridge, because local internal tide energy is enhanced by topographic generation. In the center of the basin, internal tide energy is smaller, and weak energy-flux convergence is balanced by the advection term and intermodal scattering, which

FIG. 7. The total M_2 mode-1 energy balance, averaged over 2015, is between (a) energy flux convergence (corrected for internal tide generation), (b) topographic scattering, (c) viscous dissipation, (d) background flow advection, (e) background flow refraction, and (f) a residual (this includes the energy tendency, numerical error, and dissipation by boundary sponges). Red indicates mode-1 energy loss in every panel except (b).

can reach 5 mW m^{-2} (Fig. 7). Dissipation (by the background horizontal viscosity) and the refraction term R_1 are negligible. Errors in the energy balance occur over rough topography and along the boundary sponge, where radiating waves are artificially dissipated. Within the model domain, 57% of mode-1 generation is scattered to higher modes by rough topography, but most of this scattering occurs in shallow water near Macquarie Ridge. In the open ocean, (defined as regions deeper than 3000 m), only 13% of mode-1 generation is scattered to higher modes by rough topography. In addition, 8% of mode-1 generation is scattered to higher modes by the background flow (9% occurring below 3000 m), 1% is dissipated by numerical viscosity and the remaining 34% is absorbed along the boundaries by the sponge. Overall, the energy balance analysis indicates that CSW background flow terms are weaker than topographic internal tide generation, but are significant and quantifiable in the center of the basin.

The advection term in Eq. (11) can be conveniently separated into symmetric and antisymmetric parts. The symmetric part $A_{S,mn} = (A_{mn} + A_{nm})/2$ can be written as a flux divergence (provided the background flow is horizontally nondivergent), which has an area integral of zero, via the divergence theorem, provided $\bar{\mathbf{u}}_{mn} = 0$ on the boundaries. The antisymmetric part $A_{A,mn} = (A_{mn} - A_{nm})/2$ describes intermodal scattering by the background flow, because energy transfer from mode-1 to mode-2 is equal and opposite to energy transfer from mode-2 to mode-1, etc. The self-interaction (diagonal) terms are zero. For the mode-1 M_2 tide, $\Sigma A_{S,m1}$ is equally positive and negative (i.e., it simply pushes energy around), while $\Sigma A_{A,m1}$ is persistently $1\text{--}2 \text{ mW m}^{-2}$, indicating a net sink of mode-1 energy (Fig. 8). While this energy loss is small (8% of internal tide generation within the model domain), it is indistinguishable from open-ocean turbulent dissipation in the Tasman Sea ($1\text{--}2 \text{ mW m}^{-2}$; Waterhouse et al. 2018) and previous

FIG. 8. Mode-1 energy advection by the background flow A_{mn} can be separated into (a) a symmetric component $A_{S,mn}$ that laterally transports energy and (b) an asymmetric component $A_{A,mn}$ that scatters energy between vertical modes. Red indicates mode-1 energy loss.

numerical estimates of intermodal scattering by the Gulf Stream ($0\text{--}2\text{ mW m}^{-2}$; Kelly et al. 2016).

b. Dependence of internal tide advection on horizontal scale of the background flow

The effects of the flow in modifying the distribution of the internal tide energy are quantified by A_{mn} and R_n in Eq. (11). For mode-1 and mode-2, background flow effects were largest near the generation site (south of New Zealand) and in the southern part of the basin, where the ACC is energetic. In general, advection in mode-1

(Fig. 9a) was larger than that in mode-2 (Fig. 9b). This is to be expected, as there is generally much more energy in mode-1 internal tides than in mode-2 internal tides. However, as was discussed in terms of the energy flux in the section 4b, the fraction of mode-2 energy advection is larger compared to the mode-1 energy advection than the fraction of mode-2 tidal energy to mode-1 tidal energy. This suggests once again that the background flow has a larger overall effect on mode-2 than on mode-1.

To further investigate the advection of the internal tide by the background flow, we analyzed the spatial

FIG. 9. Average advection, $A_n = \sum_m A_{mn}$ (mW m^{-2}), in (a) mode-1 and (b) mode-2.

FIG. 10. As in Fig. 9, but for CSW simulations run with spatially filtered background flow fields. The top row is mode-1, while the bottom row is mode-2. The columns (from left to right) show A_n associated with background flow scales ranging from 50 to 200 km, from 200 to 400 km, from 400 to 600 km, and from 600 to 800 km.

dependence of the wave-energy advection on the horizontal scale of the background flow. To do this, we spatially bandpassed the HYCOM background flow velocities prior to including them in the internal tide model. Four background flow scale ranges were chosen for analysis: a 50–200-km range, a 200–400-km range, a 400–600-km range, and a 600–800-km range; these ranges (shown as shaded regions of different colors in Fig. 1c) separated the eddy field captured by HYCOM from the general, large scale background flow. The wave-energy advection calculated from CSW using the various horizontal scalings are shown in Fig. 10. The simulations run with background flow dynamics ranging in size from 50 to 400 km contributed to the advection shown in Fig. 9, the largest contribution coming from the horizontal scales between 200 and 400 km. Additionally, background flows of this scale have a peak in the spectral density of surface vertical vorticity, but not in EKE (Fig. 1), suggesting that while background flow dynamics with horizontal scales between 200 and 400 km are not the most energetic in HYCOM, they have a larger amplitude vertical vorticity (dependent on the horizontal scale of the background flow) than larger scale flows. This suggests that the wave-energy advection of the internal tide may be more dependent on the horizontal scale of the background flow, and less associated with

the kinetic energy. Supplementing what was found in Buijsman et al. (2017), which showed that the temporal variability of the vertically sheared flows has important implications for tidal nonstationarity, here we show that spatial variability of the background flow is also important in tidal nonstationarity.

6. Parameterizing the energy lost from the stationary internal tide

One can separate stationary and nonstationary internal tides by post processing CSW simulations with wave-background flow interactions. However this method relies on an accurate time-dependent background flow, long model integrations, and extensive postprocessing, all of which can be computationally expensive. Background flow interactions are also difficult to observe from in situ measurements, because terms such as A_{mn} and R_n require impractically high spatial and temporal resolution. Here, we pursue an efficient method of predicting the stationary tide, and the energy lost from it, without knowledge of the instantaneous background flow (or A_{mn} and R_n). The method isolates the evolution equations for the stationary tide and uses a closure scheme to mimic background flow effects with an eddy viscosity and/or diffusion. With this parameterization, the CSW

FIG. 11. The stationary M_2 mode-1 energy balance, computed from the 2015 CSW simulation, is between (a) energy flux convergence (corrected for internal tide generation), (b) background flow advection, and (c) background flow refraction. Red indicates stationary mode-1 energy loss.

model reaches steady state after about 50 tidal cycles, allowing one to predict the stationary tide from short simulations and only a statistical description of the background flow.

Stationary tide equations

Computing the stationary component of each term in Eq. (8) yields the stationary CSW equations

$$\mathbf{U}_{nt}^s + f\hat{\mathbf{k}} \times \mathbf{U}_n^s = -H\nabla p_n^s - H \sum_{m=0}^{\infty} p_m^s \mathbf{T}_{mn} - \sum_{m=0}^{\infty} [(\bar{\mathbf{u}}_{mn} \cdot \nabla) \mathbf{U}_m]^s, \quad (12a)$$

$$\frac{Hp_{nt}^s}{c_n^2} = -\nabla \cdot \mathbf{U}_n^s + \sum_{m=0}^{\infty} \mathbf{U}_m^s \cdot \mathbf{T}_{nm} - \frac{1}{c_n^2} [\delta c_n^2 \nabla \cdot \mathbf{U}_n]^s - \frac{H}{c_n^2} \sum_{m=0}^{\infty} [\bar{\mathbf{u}}_{mn} \cdot \nabla p_m]^s, \quad (12b)$$

where quantities in the square brackets depend on both the stationary and nonstationary flow. These terms describe wave advection and refraction in the evolving quasigeostrophic flow.

The stationary mode- n energy balance is

$$\begin{aligned} & \sum_{m=0}^{\infty} \left\{ \underbrace{[(\bar{\mathbf{u}}_{mn} \cdot \nabla) \mathbf{U}_m]^s \cdot \frac{\mathbf{U}_n^s}{H} + [\bar{\mathbf{u}}_{mn} \cdot \nabla p_m]^s \frac{Hp_n^s}{c_n^2}}_{A_{mn}^s} \right\} + \underbrace{\frac{p_n^s}{c_n^2} [\delta c_n^2 \nabla \cdot \mathbf{U}_n]^s}_{R_n^s} \\ &= -\nabla \cdot \underbrace{\overline{\mathbf{U}_n^s p_n^s}}_{\mathbf{F}_n^s} + \sum_{m=0}^{\infty} \underbrace{(\overline{\mathbf{U}_m^s p_n^s} \cdot \mathbf{T}_{nm} - \overline{\mathbf{U}_n^s p_m^s} \cdot \mathbf{T}_{mn})}_{C_{mn}^s}, \end{aligned} \quad (13)$$

where the left-hand side includes energy loss due to background flow advection and refraction, respectively, and the right-hand side includes energy flux divergence and topographic effects (generation and scattering). Note that time averaged cross terms (e.g., $\overline{\mathbf{U}_n^s p_n^s}$) are zero because the stationary signals are orthogonal to the residuals with respect to time averaging, a feature of least squares fits, ensemble averages, and the response

method. Also note that a nonstationary tide only develops if the background flow is time dependent.

The open-ocean energy balance of the stationary mode-1 tide during the 2015 CSW simulation (with time-varying HYCOM background flow) is between energy flux convergence and decay by background flow interactions ($\sum A_{m1}^s$ and R_1^s , which dissipate 28% of the stationary internal tide; Fig. 11). Unlike the total mode-1

tide, the stationary tide decays rapidly in the center of the basin (through loss of coherence) and less than 6% of the stationary internal tide power is lost at the boundary sponges. In addition, R_1^s plays a major role in attenuating (decohering) the stationary tide, while R_1 produced negligible changes in the energy of the total tide (cf. Figs. 7, 11). This result indicates that, over long distances, minor changes in mesoscale stratification strongly modulate the phase (i.e., travel time) of the internal tide, without significantly changing its amplitude. Moreover, because R_1 is negligible, the substantial loss of stationary energy through R_1^s is compensated by an equivalent gain in nonstationary energy.

Since the terms on the left-hand side of Eq. (13) slowly remove energy from the stationary tide, we parameterize the terms in square brackets in Eq. (12) as diffusion (see also Kafiabad et al. 2019). Thus, we close the equations for the stationary tide in the presence of a background flow with the ansatz:

$$-\sum_{m=0}^{\infty} [(\bar{\mathbf{u}}_{mn} \cdot \nabla) \mathbf{U}_m^s] \approx \nu \nabla^2 \mathbf{U}_n^s, \quad (14a)$$

$$-\frac{1}{c_n^2} [\delta c_n^2 \nabla \cdot \mathbf{U}_n^s] - \frac{H}{c_n^2} \sum_{m=0}^{\infty} [\bar{\mathbf{u}}_{mn} \cdot \nabla p_m^s] \approx \kappa \frac{H}{c_n^2} \nabla^2 p_n^s, \quad (14b)$$

where ν is an eddy viscosity and κ is an eddy diffusivity. Total stationary energy loss is then approximately

$$\underbrace{-\nu \frac{\mathbf{U}_n^s}{H} \cdot \nabla^2 \mathbf{U}_n^s - \kappa \frac{H p_n^s}{c_n^2} \nabla^2 p_n^s}_{K_n} \approx \sum_{m=0}^{\infty} A_{mn}^s + R_n^s. \quad (15)$$

One can propose numerous parameterizations for ν and κ with varying complexity (including formulas where viscosity is represented by a tensor), but we only examine two basic parameterizations:

- 1) $\nu = \Gamma \sqrt{E} l$ and $\kappa = 0$, where E is mean surface eddy kinetic energy (EKE) and l is the observed eddy diameter (Klocker and Abernathey 2014). We approximate l as the mode-1 internal Rossby radius $\lambda_1 = c_1/f$ plus 40 km, which crudely approximates observed eddy diameters (Klocker and Abernathey 2014). The term $\Gamma = 0.075$ is a (tunable) spatially uniform constant (sometimes called a “mixing efficiency”) that was chosen to maximize agreement between explicit mode-1 energy transfer ($\sum A_{m1}^s + R_1^s$) and the parameterized transfer (K_1) in our simulations.
- 2) $\nu = 0$ and $\kappa = \text{Var}[\delta c_1]/(2l/c_1)$, which is equivalent to the diffusion coefficient for a one-dimensional random walk, $\kappa = \delta x^2/(2\Delta t)$ (Kundu 2004). As a wave propagates through a stochastic eddy field, each eddy

produces a displacement along the mean ray path of $\delta x = \pm \text{Std}[\delta c] \Delta t$ over the time it takes the wave to traverse the eddy, $\Delta t = l/c_1$ (where l is the eddy diameter defined above). Note that δc_1^2 is defined in Eq. (9c) so that it can be positive or negative.

We estimated viscosity, diffusivity, and their associated energy transfers K_n a posteriori using the stationary transport and pressure from the 2015 CSW simulations with HYCOM background flow. The parameterized viscosity in method 1 (Fig. 12a) is two orders of magnitude larger than the numerical viscosity used to stabilize the model (which is $\nu_N = 27 \text{ m}^2 \text{ s}^{-1}$, see section 3b). But, the parameterized viscosity is weaker than the diffusivity in method 2 (Figs. 12a,b) because the waves have more kinetic than potential energy, a ratio that is a function of latitude. Both parameterizations of mode-1 energy loss (K_1) are qualitatively similar to the exact energy loss ($\sum A_{m1}^s + R_1^s$), indicating that both parameterizations are reasonable (cf. Figs. 11, 12). The variances explained by each parameterization (on a 1° grid, to discount inevitable small-scale discrepancies) were $r^2 = 0.39$ and $r^2 = 0.76$ for methods 1 and 2, respectively.

Background flow parameterization 2, based on a random walk model, explained the majority of stationary mode-1 energy loss in the 2015 CSW simulation, even without a tunable mixing efficiency. To further evaluate the skill of this parameterization, we ran a short (60 tidal cycles) CSW simulation using this κ parameterization and no background flow. This simulation accurately reproduced the stationary mode-1 tide from the year-long simulation ($r^2 = 0.82$ on the native $1/20^\circ$ grid), and had differences in sea surface amplitude and phase that were only less than 5 mm and $\pm 20^\circ$, respectively (Fig. 13). This agreement suggests that the κ parameterization is an efficient way to incorporate background flow interactions in global simulations of the stationary internal tide.

7. Summary

The analyses confirm the importance of wave-background flow interactions for predicting low-mode internal tides in the Tasman Sea. A semi-idealized model, which includes internal tide advection and refraction by the background flow, captures the character, but not details, of the nonstationary tide observed during the TBeam experiment. Energy balance analyses indicate that both advection and refraction by the background flow attenuate the stationary tides at a combined rate of $\mathcal{O}(1-10) \text{ mW m}^{-2}$. Additional analyses indicate that (i) advection of the internal tide is heavily dependent on the spatial scale of the background eddy field, (ii) mode-2

FIG. 12. (a) Eddy viscosity (see [Klocker and Abernathy 2014](#)) is slightly weaker than (b) eddy diffusion computed from a random walk model. (c),(d) Both background flow energy terms qualitatively resemble stationary energy flux convergence (see [Fig. 11a](#)). The coefficient of determination r^2 is reported for K_1 and $\Sigma A_{m,1} + R_1$ on a 1° grid.

tides are more sensitive to background flow dynamics than mode-1 tides (see also [Rainville and Pinkel 2006](#)), and (iii) advection by the background flow scatters mode-1 energy to mode-2 at a rate of $\mathcal{O}(1) \text{ mW m}^{-2}$ (see also [Dunphy and Lamb 2014](#); [Kelly et al. 2016](#)).

Nonstationary internal tides account for an appreciable fraction of sea surface height variability in some regions ([Savage et al. 2017a](#)). These signals are likely to contaminate satellite observations of low-frequency submesoscale dynamics ([Savage et al. 2017b](#)) that will be collected during the upcoming Surface Water and Ocean Topography satellite altimetry mission (SWOT; [Fu et al. 2012](#)). The results here have some implications for predicting stationary and nonstationary internal tides:

First, in the open ocean, the stationary mode-1 tide primarily attenuates by dephasing with the tidal potential, not by topographic scattering, intermodal scattering by a time-invariant background flow, or turbulent dissipation. Attenuation by dephasing can be accurately parameterized by an eddy diffusion based on local background flow statistics.

Second, internal tide advection by the background flow increases at horizontal scales smaller than 400 km, suggesting accurate, high resolution, general circulation models are necessary for useful predictions of the nonstationary internal tide. However, background flow strain and vorticity also increase at smaller horizontal scales, so wave-background flow dynamics that were omitted here by the geometric approximation may become important (e.g., [Polzin 2010](#)).

FIG. 13. The stationary M_2 mode-1 surface displacement (a) amplitude and (d) phase from the 2015 CSW simulation are well approximated by a shorter simulation with (b),(e) background flow effects parameterized by κ . (c),(f) The differences between the simulations are small, as quantified by the coefficient of determination, $r^2 = 0.82$.

Last, the dominant sink of the nonstationary mode-1 tide remains largely unknown. In the Tasman Sea, we found that intermodal scattering by rough topography (13% below 3000 m) and the background flow (8%) were weak, and a large fraction of generated mode-1 energy (34%) radiates to the domain boundaries. It is possible that additional dynamics primarily dissipate the mode-1 tide, such as wave–wave interactions (e.g., MacKinnon et al. 2013; Eden and Olbers 2014).

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APPENDIX

Derivation of the CSW Equations

The Boussinesq, hydrostatic, f -plane, inviscid equations of motion in Cartesian coordinates are

$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} + w\mathbf{u}_z + f\hat{\mathbf{k}} \times \mathbf{u} = -\nabla p, \quad (\text{A1a})$$

$$0 = -p_z + b, \quad (\text{A1b})$$

$$b_t + \mathbf{u} \cdot \nabla b + wb_z + wN_0^2 = 0, \quad (\text{A1c})$$

$$\nabla \cdot \mathbf{u} + w_z = 0. \quad (\text{A1d})$$

We examine the asymptotic behavior of the system for the small parameter (Wagner et al. 2017)

$$\varepsilon \equiv \frac{U}{fL} \sim \frac{|(\mathbf{u} \cdot \nabla)\mathbf{u}|}{|f\mathbf{u}|} \ll 1, \quad (\text{A2})$$

which is the Rossby number when U and L correspond to quasigeostrophic flow and a measure of nonlinearity when U corresponds to a wave with frequency $\omega \sim f$ and wavenumber $k \sim L^{-1}$ (i.e., $c = \omega/k \sim fL$ is the phase speed, so $U/c \sim \varepsilon$). Waves oscillate at time scales $\tilde{t} \sim f^{-1}$, so quasigeostrophic motions with time scales $\tau \sim (\varepsilon f)^{-1} = LU^{-1}$ evolve slowly, and partial derivatives can be scaled

$$\partial_t \rightarrow \partial_{\tilde{t}} + \varepsilon \partial_{\tau}. \quad (\text{A3})$$

Similarly, if waves and quasigeostrophic motions have horizontal length scales $\tilde{x} \sim L$ then structures with large (regional) length scales $\chi \sim L\varepsilon^{-1}$ have small gradients

$$\nabla \rightarrow \nabla_{\tilde{x}} + \varepsilon \nabla_{\chi}. \quad (\text{A4})$$

The leading-order equations are

$$\mathbf{u}_{0\tilde{t}} + f\hat{\mathbf{k}} \times \mathbf{u}_0 = -\nabla_{\tilde{x}} p_0, \quad (\text{A5a})$$

$$0 = -p_{0z} + b_0, \quad (\text{A5b})$$

$$b_{0\tilde{t}} + w_0 N_0^2 = 0, \quad (\text{A5c})$$

$$\nabla_{\tilde{x}} \cdot \mathbf{u}_0 + w_{0z} = 0, \quad (\text{A5d})$$

and the $\mathcal{O}(\varepsilon)$ equations are

$$\begin{aligned} \mathbf{u}_{0\tau} + \mathbf{u}_{1\tilde{t}} + (\mathbf{u}_0 \cdot \nabla_{\tilde{x}})\mathbf{u}_0 + w_0 \mathbf{u}_{0z} + f\hat{\mathbf{k}} \times \mathbf{u}_1 = -\nabla_{\chi} p_0 \\ - \nabla_{\tilde{x}} p_1, \end{aligned} \quad (\text{A6a})$$

$$0 = -p_{1z} + b_1, \quad (\text{A6b})$$

$$b_{0\tau} + b_{1\tilde{t}} + \mathbf{u}_0 \cdot \nabla_{\tilde{x}} b_0 + w_0 b_{0z} + w_1 N_0^2 = 0, \quad (\text{A6c})$$

$$\nabla_{\chi} \cdot \mathbf{u}_0 + \nabla_{\tilde{x}} \cdot \mathbf{u}_1 + w_{1z} = 0. \quad (\text{A6d})$$

a. Leading-order background flow

Each field comprises a background flow and wave component, e.g., $u = \bar{u} + u'$ respectively. The average of Eq. (A5) over fast time (denoted by an overbar) yields a geostrophic flow

$$f\hat{\mathbf{k}} \times \bar{\mathbf{u}}_0 = -\nabla_{\tilde{x}} \bar{p}_0, \quad (\text{A7a})$$

$$0 = -\bar{p}_{0z} + \bar{b}_0, \quad (\text{A7b})$$

$$\bar{w}_0 = 0, \quad (\text{A7c})$$

$$\nabla_{\tilde{x}} \cdot \bar{\mathbf{u}}_0 = 0, \quad (\text{A7d})$$

i.e., a steady horizontally nondivergent flow in thermal wind balance. A key result is that the leading-order background flow is independent of large-scale spatial gradients (i.e., $\bar{\mathbf{u}}_0$ is independent of $\nabla_{\chi} \bar{p}_0$), so that one can include regional stratification without the corresponding geostrophic currents. Wagner et al. (2017) note that the next-order background flow evolves according to the standard quasigeostrophic equation and is unaffected by wave-background flow interactions.

b. Wave evolution

The leading-order wave equations describe linear waves. They arise by subtracting Eq. (A7) from Eq. (A5),

$$\mathbf{u}'_{0\tilde{t}} + f\hat{\mathbf{k}} \times \mathbf{u}'_0 = -\nabla_{\tilde{x}} p'_0, \quad (\text{A8a})$$

$$0 = -p'_{0z} + b'_0, \quad (\text{A8b})$$

$$b'_{0\tilde{t}} + w'_0 N_0^2 = 0, \quad (\text{A8c})$$

$$\nabla_{\tilde{x}} \cdot \mathbf{u}'_0 + w'_{0z} = 0. \quad (\text{A8d})$$

The $\mathcal{O}(\varepsilon)$ wave equations are

$$\begin{aligned} \mathbf{u}'_{0\tau} + \mathbf{u}'_{1\tilde{t}} + (\bar{\mathbf{u}}_0 \cdot \nabla_{\tilde{x}})\mathbf{u}'_0 + (\mathbf{u}'_0 \cdot \nabla_{\tilde{x}})\bar{\mathbf{u}}_0 + w'_0 \bar{\mathbf{u}}_{0z} \\ + (\mathbf{u}'_0 \cdot \nabla_{\tilde{x}})\mathbf{u}'_0 + w'_0 \mathbf{u}'_{0z} + f\hat{\mathbf{k}} \times \mathbf{u}'_1 = -\nabla_{\tilde{x}} p'_1, \end{aligned} \quad (\text{A9a})$$

$$0 = -p'_{1z} + b'_1, \quad (\text{A9b})$$

$$\begin{aligned} b'_{0\tau} + b'_{1\tilde{t}} + \bar{\mathbf{u}}_0 \cdot \nabla_{\tilde{x}} b'_0 + \mathbf{u}'_0 \cdot \nabla_{\tilde{x}} \bar{b}_0 + w'_0 \bar{b}_{0z} \\ + \mathbf{u}'_0 \cdot \nabla_{\tilde{x}} b'_0 + w'_0 b'_{0z} + w'_1 N_0^2 = 0, \end{aligned} \quad (\text{A9c})$$

$$\nabla_{\tilde{x}} \cdot \mathbf{u}'_1 + w'_{1z} = 0, \quad (\text{A9d})$$

where spatial gradients in the waves are ignored at scales much greater than a wavelength (e.g., $\nabla_{\chi} p'_0 \approx 0$). The wave-wave advective terms [e.g., $(\mathbf{u}'_0 \cdot \nabla_{\tilde{x}})\mathbf{u}'_0$] only force an Eulerian mean flow and second-harmonic wave field,

so they can be ignored when these features are not of interest.

Only the leading-order background velocity fields are present in the advective terms, so waves only feel the geostrophic component of the background flow. One might suspect that the advective terms simplify after substituting the leading-order wave balances Eq. (A8) and thermal wind relations Eq. (A7), but we have not found this to be the case. In general, waves exchange energy with the geostrophic flow, even though the background flow evolves independent of waves.

c. Projection of wave equations on to modes

Recombining Eqs. (A8) and (A9) without the wave-wave terms, and making the horizontal geometric approximation (see section 3b) yields

$$\mathbf{u}'_t + (\bar{\mathbf{u}} \cdot \nabla) \mathbf{u}' + f \hat{\mathbf{k}} \times \mathbf{u}' = -\nabla p', \quad (\text{A10a})$$

$$0 = -p'_z + b', \quad (\text{A10b})$$

$$b'_t + \bar{\mathbf{u}} \cdot \nabla b' + w'(N_0^2 + \bar{b}_z) = 0, \quad (\text{A10c})$$

$$\nabla \cdot \mathbf{u}' + w'_z = 0. \quad (\text{A10d})$$

Projecting Eq. (A10) onto vertical modes yields the CSW equations, Eqs. (8) and (9), where primes have been dropped to lighten notation and $\bar{b}_z$ has been renamed $\delta N^2(\mathbf{x}, z, t)$.

The vertical velocity modes Φ_n are necessary for deriving modal projections, and they satisfy the eigenvalue problem

$$\Phi_{nzz} + \frac{N_0^2}{c_n^2} \Phi_n = 0 \quad \text{with} \quad \Phi_n(0) = \Phi_n(-H) = 0, \quad (\text{A11})$$

and orthogonality condition

$$\frac{1}{H} \int_{-H}^0 \frac{N_0^2}{c_n^2} \Phi_m \Phi_n dz = \delta_{mn}. \quad (\text{A12})$$

The equation for δc_n^2 , (9c), arises from perturbing the eigenvalue problem (A11):

$$(\Phi_{mzz} + \delta \Phi_{mzz}) + \frac{N_0^2 + \delta N^2}{c_m^2 + \delta c_m^2} (\Phi_m + \delta \Phi_m) = 0, \quad (\text{A13})$$

where δN^2 is a small observed variation in buoyancy frequency and δc^2 and $\delta \Phi_m$ are small variations that must be determined. The lowest-order balance in Eq. (A13) is Eq. (A11). The next-order balance is

$$\frac{\delta c_m^2}{c_m^2} \Phi_{mzz} + \frac{\delta N^2}{c_m^2} \Phi_m + \left[\delta \Phi_{mzz} + \frac{N_0^2}{c_m^2} \delta \Phi_m \right] = 0, \quad (\text{A14})$$

where the quantity in square brackets is zero because $\delta \Phi_m$ can be expanded in terms of Φ_m and is zero at the boundaries. Proof that the bracketed quantity is zero follows from projecting Eq. (A14) onto Φ_n , integrating by parts twice, substituting Eq. (A11), and using Eq. (A12). The projection of the nonbracketed quantities onto Φ_n provides the formula for δc_n^2 ,

$$\frac{1}{H} \int_{-H}^0 (\delta c_m^2 \Phi_{mzz} \Phi_n + \delta N^2 \Phi_m \Phi_n) dz = 0, \quad (\text{A15a})$$

$$\delta c_m^2 \frac{1}{H} \int_{-H}^0 \frac{N_0^2}{c_m^2} \Phi_m \Phi_n dz = \frac{1}{H} \int_{-H}^0 \delta N^2 \Phi_m \Phi_n dz, \quad (\text{A15b})$$

where Eq. (A11) has been used. Applying the orthogonality condition to Eq. (A15b) yields Eq. (9c).

d. Summary of approximations

In addition to the starting assumptions of hydrostatic, Boussinesq, f -plane, inviscid dynamics, the derivation of the CSW equations required the following:

- 1) The Rossby number is small and waves are linear, as defined by $\varepsilon \ll 1$. This also ensures quasigeostrophic perturbations to N^2 are small.^{A1}
- 2) The quasigeostrophic time scale $\tau \sim (\varepsilon f)^{-1}$ is much longer than wave time scale $\tilde{t} \sim f^{-1}$. This is the key to separating quasigeostrophic and wave dynamics.
- 3) The spatial scale of the regional stratification $\chi \sim L\varepsilon^{-1}$ is much larger than the spatial scale of the quasigeostrophic flow $\tilde{x} \sim L$. This is the key to including regionally variable stratification.
- 4) The spatial scale of the quasigeostrophic flow is much larger than the wavelength of the internal tide (i.e., the horizontal geometric approximation; see section 3b).
- 5) Variable topography can be ignored in the background flow terms and background flows can be ignored in the topographic scattering terms. We do not formally justify the omission of simultaneous background flow and topographic effects, but topographic effects tend to dominate in localized regions over abrupt features, while background flow effects tend to accumulate slowly as internal waves propagate thousands of kilometers through the open ocean. Thus, to some extent, topographic and background flow effects are separable by context.

^{A1} From the buoyancy equation $\bar{b}_{0r} \sim N^2 \bar{w}_1$, so $\bar{b}_{0z} \sim N^2 U/(fL) = \varepsilon N^2$, where $\bar{w}_{1z} \sim \varepsilon U/L$ from continuity.

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