Parametric Subharmonic Instability of the Semidiurnal Internal Tides at the East China Sea Shelf Slope

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ABSTRACT

On the basis of measurements from an observing mooring system, the observational evidence of parametric subharmonic instability (PSI) that transfers energy from semidiurnal internal tides (ITs) to the subharmonic waves at the East China Sea continental shelf slope is presented for the first time. Although the mooring station is very close to the energetic semidiurnal IT generation site, about 76% of the observed shear variance is contained in the near-inertial band, which is found to have comparable upward- and downward-propagating energy components. Bispectra and bicoherence estimates further confirm the occurrence of PSI transferring energy from the low-mode M2 ITs (vertical wavelength of ~1000 m) to high-mode subharmonic waves (vertical wavelength of ~200 m). The calculated energy transfer rate $\mathcal{g}$ reveals an averaged net value of $\mathcal{g} = 5 \times 10^{-9}$ W kg$^{-1}$. Strong temporal variation of $\mathcal{g}$ is found that is not exactly in phase with the semidiurnal energy flux. After looking into the local vorticity fields, it is strongly suggested that the varying background relative vorticity associated with the evolving Kuroshio has modified the efficiency of PSI at the mooring location through changing the local effective inertial frequency.

1. Introduction

Internal tides (ITs) primarily generated by barotropic tidal flow over topography supply an important energy source for small-scale mixing in the ocean. Although the global pattern of the generation of ITs is relatively well quantified (e.g., Simmons et al. 2004; Niwa and Hibiya 2011, 2014), where and how the ITs lose their energy still remains an open question. One widely accepted hypothesis is that the ocean mixing is sustained by a class of resonant wave–wave interactions transferring energy from large-scale internal waves to smaller scales (McComas and Bretherton 1977; Müller et al. 1986). Parametric subharmonic instability (PSI), which transfers energy from a large-scale “parent wave” to a pair of small-scale “daughter waves” with nearly half the frequency of the parent wave, is one of the most important interactions identified. Because of the scale separation between the parent and daughter waves, PSI is an efficient way in the downscale energy transfer.

In theory, the PSI works most efficiently at the critical latitude (for M2 IT: 28.8°N/S) where the group velocity of the subharmonic wave vanishes, allowing their exponential buildup. Using a series of two-dimensional numerical experiments, Hibiya et al. (1998, 2002) showed that the downscale cascading of low-mode semidiurnal baroclinic tidal energy was clearly latitude dependent. MacKinnon and Winters (2005) predicted a catastrophic energy loss from M2 IT to M1 subharmonic waves at the M1 PSI critical latitude (28.8°N, where \( \omega_r = \omega_{M1} \)). The latitudinal dependence of PSI as theoretically and numerically predicted was further verified by both finescale and microstructure observations carried out over a wide area in the North Pacific (Hibiya and Nagasawa 2004; Hibiya et al. 2007).

Motivated by these results, more observations have been carried out to examine the role of PSI in decaying the ITs and driving small-scale mixing in the ocean. On the basis of the continuous velocity measurements,
statistically significant phase locking between the IT and upward- and downward-propagating subharmonic waves, which is an intrinsic feature of PSI, has been documented both at (MacKinnon et al. 2013) and below (Carter and Gregg 2006; Xie et al. 2011; Sun and Pinkel 2013) the critical latitude. The calculated energy transfer rates from the semidiurnal ITs to subharmonic waves suggested that PSI could account for a small but significant fraction of the local turbulent dissipation showing the important role of PSI in determining the mixing pattern (Sun and Pinkel 2013). Using mooring observations at the critical latitude north of Hawaii, MacKinnon et al. (2013) also found that the calculated PSI energy transfer rates were of roughly the same order as the local dissipation rates. However, the observed PSI did not show a significant drain of energy from the IT, which is totally different from the previously modeled “catastrophic energy loss from M$_2$ IT” at the critical latitude (MacKinnon and Winters 2005).

Recently, it has been found that some realistic oceanic background conditions being ignored in most previous idealized simulations, such as the presence of background geostrophic flow (Richet et al. 2017; Yang et al. 2018; Dong et al. 2019) and nonuniform stratification (Hazewinkel and Winters 2011; Onuki and Hibiya 2018), can significantly influence the PSI behavior. Richet et al. (2017) showed that the Doppler effect of the mean current could shift the frequencies of primary ITs, hence, further changed the position of PSI critical latitudes. Yang et al. (2018) also found that the existence of background geostrophic flow could significantly modify the behavior of PSI in a different way from Richet et al. (2017). It is numerically predicted that the relative vorticity induced by the spatially varying background geostrophic flow can change the PSI efficiency by changing the local effective Coriolis frequency $f_{\text{eff}}$. This is related to the fact that the propagation of PSI-generated subharmonic wave is controlled by $f_{\text{eff}}$ rather than the local Coriolis frequency $f$. More recently, using 3D simulations, Dong et al. (2019) achieved similar results to Yang et al. (2018), that the presence of the background geostrophic flow can significantly change the PSI efficiency.

The East China Sea (ECS) lies at the northwest of the Pacific Ocean. The Kuroshio western boundary current enters northeast of Taiwan extending northeastward along the shelf slope. The ECS is one of the most energetic M$_2$ barotropic-to-baroclinic conversion sites among the world oceans (Niwa and Hibiya 2014). Based on the regional three-dimensional numerical model study, Niwa and Hibiya (2004) reported that the rate of conversion of the M$_2$ barotropic to baroclinic energy integrated over the ECS shelf slope and Ryukyu Island chain reached 4.1 and 14.9 GW, respectively.

In contrast to the strong M$_2$ IT conversion and propagation, there is still no study documenting the role of PSI in cascading IT energy on the ECS shelf slope. The southern half of the ECS lies equatorward of 28.8°N; the PSI can thus potentially transfer energy from M$_2$ to M$_1$ without violating the internal wave theory that $f \leq \omega \leq N$. On the basis of observations from a mooring station located at the shelf slope of the ECS, northeast of the island of Taiwan, this study aims to document the occurrence and properties of PSI at the ECS shelf slope for the first time. The potential influence of Kuroshio on the PSI efficiency is discussed. Section 2 describes the basic observational methods and results. Section 3 explores the properties of semidiurnal ITs and near-inertial waves (NIWs). Evidence of PSI is shown in section 4. The potential role of Kuroshio on the temporal variability of PSI is discussed in section 5. Section 6 summarizes the results.

2. Instruments and data

2a. Observation methods

Two-month (27 August–28 October 2013) time series of horizontal velocity, temperature, salinity, and pressure were obtained from a mooring station (25°25.013’N, 122°44.878’E) northeast of the island of Taiwan (Fig. 1b). The station has a water depth of about 750 m. Two model 75K acoustic Doppler current profilers (ADCPs), measuring the vertical profiles of horizontal velocity, were deployed at the middle of the water column with one looking upward and the other looking downward. The ADCP pings every 10 s with a temporal resolution of 5 min and a vertical resolution of 8 m. Valid velocity measurements covered a depth range from ~50 to ~710 m. Because of the separation between the upward- and downward-looking ADCP records, there is a gap of velocity measurements at the depth range between ~437 and ~463 m. This is supplemented by linearized interpolation in the following analysis with the related influence almost invisible in velocity plots (Figs. 2a,b).

The mooring chain was equipped with 24 temperature sensors (SBE56) and 7 conductivity–temperature–depth sensors (CTD; SBE37). Information of the corrected sensor depths and temporal resolution are presented in Table 1. Taking account of the coarse resolution of salinity measurements in the vertical direction, only the velocity and temperature data are utilized in this study.

In the presence of energetic ITs, the induced vertical displacements of isopycnals can usually heave the layers of velocity shear, particularly that of the high-vertical-wavenumber motions (Sherman 1989). This can result in Doppler-shifted spectral peaks (e.g., at $\omega f + M_2$ for the
NIW). To minimize the advective distortion effect, semi-Lagrangian reference frame (in contrast to the Eulerian frame) is used here. Transformations from the Eulerian to semi-Lagrangian reference frame are conducted by reference measurements to isotherms. A set of isotherms are selected with constant mean spacing (approximately 8 m), and the shears along each isotherm are next calculated by linear interpolation. Furthermore, internal waves follow curved ray paths in the stratified ocean and their amplitudes vary with

![Fig. 1. (a) Bottom bathymetry surrounding the island of Taiwan (from the ETOPO1 database). (b) Inset of the outlined square in (a), showing the location of the mooring station, denoted by the red triangle. The red arrow denotes the observed depth-integrated semi-diurnal baroclinic energy flux at the mooring station. Corresponding field observational mooring results of Lien et al. (2013) are indicated as orange arrows.](image)

![Fig. 2. Depth–time maps of the (a) zonal velocity, (b) meridional velocity, (c) temperature, and (d) isotherm displacements.](image)
depth. However, to provide the PSI energy transfer rate estimates in meaningful physical units (section 4), the quantities analyzed here are not Wentzel–Kramers–Brillouin (WKB) stretched and scaled (Leaman and Sanford 1975).

b. Depth–time structure

The unfiltered depth–time maps of the horizontal velocity in the Eulerian coordinate show clear signatures of Kuroshio and semidiurnal tides (Figs. 2a,b). The Kuroshio appears as a strong northeastward current (>1 m s\(^{-1}\)) reaching 300-m depth from 8 September to 14 October. Apparent spring–neap cycles of semidiurnal tides are evident especially in the meridional velocity component. Contours of temperature and isotherm displacement also show semidiurnal variations that indicate the baroclinic structure of tides (Figs. 2c,d).

Typhoon Fitow passed near the mooring station on 6 October, which induced strong vertical entrainment influencing the entire water column. The isotherms are lifted up for more than 50 m, driven by the strong Ekman-pumped upwelling event along the typhoon path (Price et al. 1994).

c. Spectral estimates

The vertical wavenumber–frequency spectra of horizontal velocity are calculated next to look at the energy content as a function of wavenumber and frequency. The horizontal baroclinic kinetic energy spectra show significant peaks at the near-inertial/diurnal (~1 cpd) and semidiurnal (~2 cpd) bands (Fig. 3a). The M\(_2\) IT has the strongest peak, which is characterized by the lowest vertical wavenumber (~0.002 cpm). In contrast, the NIWs have a much larger vertical wavenumber that could contain contributions from a variety of waves: the waves at the inertial frequency (\(f\) at 27.9 h), diurnal ITs (K\(_1\) at 23.93 h and O\(_1\) at 25.82 h) and the subharmonic waves of semidiurnal ITs (M\(_1\) at 24.84 h and S\(_1\) at 24 h). These waves are not clearly distinguished from each other in the spectra.

As a result of the vertical-scale separation between the NIWs and semidiurnal ITs, the shear variance is mostly concentrated at the near-inertial band (Fig. 3b).

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**Table 1. Mooring measurement sensor depths and the corresponding temporal resolution.**

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Corrected sensor depths (m)</th>
<th>Temporal resolution (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward-looking 75K ADCP</td>
<td>445</td>
<td>5</td>
</tr>
<tr>
<td>Downward-looking 75K ADCP</td>
<td>455</td>
<td>5</td>
</tr>
<tr>
<td>SBE56</td>
<td>155, 165, 174, 184, 204, 213, 223, 233, 252, 262, 272, 282, 313, 372, 400, 459, 490, 521, 572, 592, 617, 642, 666, and 691</td>
<td>1</td>
</tr>
<tr>
<td>SBE37</td>
<td>145, 194, 243, 292, 344, 427, and 552</td>
<td>1</td>
</tr>
</tbody>
</table>

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**Fig. 3.** Vertical wavenumber–frequency spectra of (a) horizontal baroclinic velocity and (b) vertical shear of horizontal velocity in the semi-Lagrangian coordinate. For reference, the frequencies of the important wave components are indicated by vertical dashed lines.
About 76% of the observed shear variance lies within the frequency range of 0.8–1.5 $v_f$. The velocity shear is usually used as an important parameter in parameterizing the mixing intensity (e.g., Gregg 1989); therefore, the physical process underlying the high-wavenumber NIWs may be of crucial importance for the turbulent mixing at the ECS shelf slope.

3. Observed wave properties

Properties of the semidiurnal ITs and NIWs are discussed separately in the present section. We extract the wave fields by a bandpass filter in frequency space. Possible connections between the NIWs and semidiurnal ITs are suggested at the end of the section.

a. Semidiurnal internal tides

The energy flux $F$ is a useful quantity to identify the energy sources, propagation direction, and energy sinks of ITs:

$$F = p' \mathbf{u}' ,$$

where $p$ and $\mathbf{u} = (u, v)$ are the internal wave-induced hydrostatic pressure perturbation and horizontal velocity vector, respectively. The prime denotes a baroclinic disturbance bandpass filtered in the semidiurnal range (0.8–1.3$w_M$ as applied here). The perturbation pressure was estimated by integrating the full-depth profiles of the density perturbation $\rho'$ using the hydrostatic equation (Nash et al. 2005).

Figure 4 shows that although the calculated semidiurnal energy flux is highly variable it directs to the southeast most of the time with a clear mode-1 signature. The depth-integrated energy flux has an average value of 6.3 kW m$^{-1}$. Based on measurements from several floats’ trajectory and five mooring stations around the North Mien-Hua Canyon, Lien et al. (2013) observed similar southeastward- or eastward-propagating semidiurnal ITs emitting from the edge of the canyon (as shown in Fig. 1b). Their observed depth-integrated energy flux varied in a range of 3.0–10.7 kW m$^{-1}$. By performing numerical simulations, multiple barotropic-to-baroclinic energy conversion sites were found on the continental slope, shelf break and around North Mien-Hua Canyon. After the semidiurnal ITs are generated, they mostly propagate seaward in a low-mode structure (Lien et al. 2013). The observed semidiurnal ITs here thus coincide well with the previous observationally and numerically predicted results.

b. Near-inertial waves

The NIWs have relatively high mode structures as can be identified from the bandpass-filtered (0.8–1.5$\omega$) zonal velocity (Fig. 5a). Wind disturbance is usually expected to
be an important generation source of the NIWs, for which the generated NIWs have a predominant downward energy propagation from the sea surface (Garrett 2001). The observed zonal current, however, does not show clear vertical phase propagation. On the basis of the theory that different energy-propagation direction is associated with different signs of frequency and wavenumber in the Fourier space, being related to the wave polarization theory (Leaman and Sanford 1975; Lien et al. 2013), the zonal velocity is next decomposed into upward- and downward-propagating directions. Figures 5b and 5c show that the NIWs have comparable downward- and upward-energy-propagating components, which strongly suggests that the wind generation should not contribute much to the observed NIWs.

We note that even when Typhoon Fitow influences the mooring station the upward phase propagation (downward energy propagation) only dominates the upper layer (<250 m; 6–10 October) (Fig. 5a). Most of the NIWs are still characterized by the “checkerboard” structure. Moreover, the NIWs already exist before the approach of the typhoon on 2 October, and interestingly, appear first at the middepth (300–500 m). This also indicates that the observed NIWs are not wind generated.

We next calculate the depth-integrated horizontal kinetic energy within the near-inertial and semidiurnal bands following

$$E = \int_{h_1}^{h_2} (u'^2 + v'^2) \rho_0/2 \, dz,$$

where \(\rho_0 = 1024 \text{ kg m}^{-3}\) is the reference density; \(h_1\) and \(h_2\) are the lower and upper integration limits. Because the ITs are generated by the interaction between barotropic tide and topography, the baroclinic velocities are usually strongly related to the barotropic velocity that forces them. However, the temporal variation of NIWs does not appear to follow the local diurnal forcing \((K_1 + O_1)\) predicted by the TOPEX/Poseidon global tidal model (TPXO; Egbert and Erofeeva 2002) (Figs. 6a,c). This is different from the semidiurnal ITs, which show close temporal variability with the barotropic tidal amplitudes (Figs. 6b,c). Moreover, with similar generation mechanisms of the interaction between barotropic tide and topography, diurnal and semidiurnal ITs are usually characterized by a similar vertical structure.
However, the NIWs observed here have strikingly vertical high-mode structures—a situation that is clearly different from the vertical structure of the semidiurnal ITs.

In contrast, the temporal variability of the depth-integrated horizontal kinetic energy of NIWs seems to be strongly linked to that of the semidiurnal ITs (Figs. 6a,b). They show a close variation trend with their energy peaks generally coinciding with each other. One exception occurs around 27 September when the near-inertial energy peak corresponds to relatively low semidiurnal energy level. During this period, the upward-energy-propagating NIWs dominate over the downward energy component, which may be associated with the bottom-generation or -reflection of NIWs.

From observations at the Hawaiian Ridge, Rainville and Pinkel (2006) documented the occurrence of a similar phenomenon that the diurnal baroclinic energy flux varied in accord with the fortnightly cycle of the barotropic semidiurnal, rather than the diurnal, tide. PSI was further identified as a candidate transferring energy from the low-mode semidiurnal IT to higher-mode subharmonic internal waves (e.g., M2 to M1), supplying the observed diurnal baroclinic energy flux. It is strongly suggested that PSI may have occurred at the mooring site here and played an important role in transporting energy from semidiurnal ITs to the subharmonic waves, inducing the comparable upward and downward near-inertial energy.

4. Evidence of PSI

a. Analysis methods

As one kind of triad interactions, PSI requires a constant phase difference between the interacting waves. One useful method to estimate the phase locking between waves with different frequencies and wavenumbers is the bispectrum, and the normalized version, the bicoherence (Kim and Powers 1979). As an example, bispectrum in frequency domain can be expressed as

\[
B(\omega_1, \omega_2) = E \left[ \hat{X}_{\omega_1} \hat{Y}_{\omega_2} \hat{Z}_{\omega_1+\omega_2} \right] = E \left[ |\hat{X}_{\omega_1}| |\hat{Y}_{\omega_2}| |\hat{Z}_{\omega_1+\omega_2}| e^{-i(\theta_1+\theta_2+\theta_3)} \right],
\]

(3)

where \(\hat{X}_{\omega}, \hat{Y}_{\omega},\) and \(\hat{Z}_{\omega}\) are the Fourier coefficients of variables \(x, y,\) and \(z\) in frequency space. The magnitude of the bispectrum depends on both of the magnitudes (\(|\hat{X}_{\omega_1}|, |\hat{Y}_{\omega_2}|,\) and \(|\hat{Z}_{\omega_1+\omega_2}|\)) and relative phases (\(\theta_1, \theta_2,\) and \(\theta_3\)).
and \( \theta_3 \) of the respective Fourier coefficients. The bicoherence follows

\[
b^2(\omega_1, \omega_2) = \frac{|B(\omega_1, \omega_2)|^2}{E[\hat{X}_{\omega_1}^2]E[\hat{Y}_{\omega_1}^2]E[\hat{Z}_{\omega_1+\omega_2}^2]}. \tag{4}\]

With the influence of wave amplitude eliminated, bicoherence measures the phase locking between the interacting triads.

PSI theory predicts the energy transfer from the low-mode semidiurnal ITs to high-mode subharmonic waves, which could represent the first step in cascading the low-mode internal wave energy toward the small-scale mixing. An accurate estimation of the PSI-induced energy transfer rate is important for evaluating the role of PSI in cascading the IT and generating turbulent mixing. After separating the variables into background components (semidiurnal ITs) and fluctuating components (subharmonic waves), the energy transfer rate \( g \) follows

\[
g = -\frac{\partial U}{\partial x} u \frac{\partial V}{\partial y} - \frac{\partial U}{\partial y} v \frac{\partial V}{\partial x} - \frac{\partial V}{\partial x} u \frac{\partial V}{\partial y} \tag{5}\]

where \( U \) and \( V \) represent the background tidal flow; \( u \) and \( v \) represent the fluctuating subharmonic waves. This formula has been widely used in numerical studies in diagnosing the energy balance of ITs when PSI occurs (e.g., MacKinnon and Winters 2005; Hazewinkel and Winters 2011; Onuki and Hibiya 2015). However, the horizontal divergence terms included in Eq. (5) are very difficult to be calculated on the basis of oceanic observations.

Recently, as an attempt to calculate the PSI energy transfer in the real ocean, Sun and Pinkel (2013) and MacKinnon et al. (2013) have developed Eq. (5) by transforming the horizontal divergence terms according to the internal wave polarization relation theory. Several assumptions have been further applied during the derivation process with the details omitted here. In the present study, \( g \) is calculated following the methods developed by Sun and Pinkel (2013)

\[
g \approx u_{1z} u_{2z} \eta \left( \frac{-m_1 \omega}{m_1 m_2} \right), \tag{6}\]

where \( u_{1z} \) and \( u_{2z} \) represent the subharmonic shear that bandpassed in a frequency range of (0.8–1.5)\( \omega_1 \); \( \eta \) is the semidiurnal (0.8–1.3\( \omega_{M2} \)) vertical displacement; \( m_1 \) is the vertical wavenumber of the interacting semidiurnal IT; \( m_1 \) and \( m_2 \) are the vertical wavenumbers of the two interacting subharmonic waves. The values of \( m_1, m_2 \), and \( m_3 \) can be obtained in the vertical wavenumber bi-coherence estimates.

b. Estimates of the bispectra and bicoherence

To better focus on the wave–wave interactions associated with PSI, we perform a prefilter to the bispectral methods following Sun and Pinkel (2013). The inputs have referred the variables that appear at the PSI energy transfer equations [Eq. (6)], that is, velocity shears with upward energy propagation, velocity shears with downward energy propagation and the vertical isotherm displacements. The bispectrum thus becomes

\[
B(\omega_1, \omega_2) = E\left[ -U_1(\omega_1) U_2(\omega_2) \eta\eta^*(\omega_1 + \omega_2) \right], \tag{7}\]

where \( U_1 \) and \( U_2 \) represent the velocity shears in the complex form associated with the upward and downward energy propagation, respectively.

The sign of \( g \) represents a fixed direction of energy transfer, which depends on \( B(\omega_1, \omega_2) \) and \( m_1 \omega/(m_1 m_2) \) (Sun and Pinkel 2013). The PSI-generated subharmonic waves have nearly opposite vertical wavenumbers, making \( m_1 m_2 < 0 \). A close examination of the semidiurnal ITs reveals a predominant downward phase propagation indicating that \( \omega m_1 < 0 \) (the figure is omitted). Thus, a positive real part of \( B(\omega_1, \omega_2) \) is associated with positive PSI energy transfers.

Data are divided into half-overlapping 7.5-day windows when calculating the frequency bispectra. The real part of the frequency bispectrum shows a large positive peak located at the third quadrant \( (\omega_1 < 0 \text{ and } \omega_2 < 0) \) that represents clockwise rotation with time, a common feature for the NIWs in the Northern Hemisphere (Fig. 7a). The largest value is located at \( (\omega_1, \omega_2, \omega_3) = (-\omega_{M1}, -\omega_{M1}, \omega_{M2}) \) instead of the cross point of \( (\omega_1, \omega_2) = (-\omega_r, -\omega_r) \). This indicates the potential significant interaction between the \( M_2 \) IT and two \( M_1 \) subharmonic waves.

The normalized form, bicoherence, allows an estimation of the significance level of the interaction. The significance level for the bicoherence depends on the degrees of freedom \( n_{\text{dof}} \) and are given as 90% \( = (4.6/n_{\text{dof}})^{1/2} \), 95% \( = (6/n_{\text{dof}})^{1/2} \), 99% \( = (9.2/n_{\text{dof}})^{1/2} \) (Elgar and Guza 1988). The \( n_{\text{dof}} \) depends on the number of wave groups sampled. Here, we suggest the independent time and vertical scales to be 2 days and 200 m, respectively, which is close to the values employed in previous studies (Carter and Gregg 2006; MacKinnon et al. 2013). This gives the 95% confidence threshold of 0.19. The calculated bicoherence in frequency plane has a maximum value of 0.23, thus indicating a confident phase locking between the interacting triads (Fig. 7b).

Similarly, the bispectra analysis as a function of vertical wavenumber is also calculated so as to examine the vertical scales of the interacting triads. In analogy
to Eq. (7), the bispectrum in vertical wavenumber space follows

\[ B(m_1, m_2) = E \left[ -\tilde{U}_1^*(m_1) \tilde{U}_2^*(m_2) \tilde{\eta}^*(m_1 + m_2) \right]. \]  

(8)

No overlapping 428-m windows (the velocity measurement coverage in the vertical direction) have been applied during the vertical-wavenumber bispectra calculation. The fact that the large values concentrated along the line \( m_1 + m_2 \approx 0 \) means that both \( m_1 \) and \( m_2 \) must be much larger than the sum of them \( (m_1 + m_2) \) (Figs. 7c,d). This indicates the scale separation between the \( M_2 \) parent wave and the subharmonic daughter waves as predicted by the PSI theory. The bispectra (Fig. 7c) has also shown some negative peaks; however, none of these negative peaks is statistically significant (Fig. 7d). The largest bicoherence value locates at \( (m_1, m_2, m_3) = (0.0049, 0.0059, -0.001 \text{ cpd}) \) indicating a vertical wavelength of \( \sim 1000 \text{ m} \) for the \( M_2 \) IT and \( \sim 200 \text{ m} \) for the \( M_1 \) subharmonic waves. We note that the estimated vertical wavelength of the \( M_2 \) IT is slightly larger than the local water depth. This is related to the processing algorithm used here (Matlab “fft”), which pads the input with trailing zeros to the length next higher power of 2 from the current data length. The frequency and vertical-wavenumber bispectral estimates together support statistically confident PSI triads transferring energy from the low-mode \( M_2 \) IT to high-mode \( M_1 \) subharmonic waves.

c. The PSI energy transfer

The energy transfer rate \( g \) from the semidiurnal ITs to subharmonic waves is next calculated. As revealed in Eq. (6), \( g \) is determined not only by the subharmonic shear and semidiurnal vertical displacement but also the vertical wavenumber of the interacting triads. An accurate estimation of \( g \) needs to get the information of the vertical wavenumber of the interacting triads. As shown in Fig. 6, we separate the observation period into four stages (stages 1–4), each roughly covering a fortnight. Bispectral estimates are carried out within every single stage. Table 2 summarizes the vertical wavenumber of the predominant interacting triads at four stages (Figures are omitted). The interacting parent wave has a constant...
low vertical wavenumber \((m_3 \approx -1 \times 10^{-3} \text{ cpm})\); however, the wavenumber of the daughter waves \((m_1 \text{ and } m_2)\) varies among the four stages. The identified daughter waves during stages 2 and 3 have a smaller vertical wavenumber than those during stages 1 and 4. Here, the estimated vertical wavenumber of the interacting triads during each stage are used in calculating \(g\), rather than applying the fixed values.

Figure 8a shows the depth–time map of \(g\) with the depth-averaged results \(<g>\) shown in the lower panel. The overall positive value confirms the constant energy transfer from semidiurnal ITs to subharmonic waves. A net \(<g>\) of \(5 \times 10^{-9} \text{ W kg}^{-1}\) is found after averaging all the measurements. This is comparable to the previous estimates at the Kaena Ridge, Hawaii (averaging at \(2 \times 10^{-9} \text{ W kg}^{-1}\); Sun and Pinkel 2013) and that north of the Hawaii Ridge near the M2 critical latitude of \(29^\circ\text{N}\) (orders of \(10^{-9} \text{ W kg}^{-1}\); MacKinnon et al. 2013).

Note that despite \(g\) being overall positive, negative values also occur (Fig. 8a). This does not necessarily indicate an inverse energy transfer from subharmonic waves to semidiurnal ITs. The observed semidiurnal ITs and subharmonic waves have a constant phase locking within the PSI prevailing environment that has induced the overall positive \(g\) observed here. However, if there is any process other than PSI that generates energetic waves within the frequency range of the subharmonic waves, it can break the phase locking in Eq. (6). For example, Typhoon Fitow has induced strong wind-generated NIWs at depth 150–270 m during 6–10 October. These waves have no phase correlation with the local semidiurnal ITs, which has induced strong blue and red patches in \(g\) at the upper layer during that time (Fig. 8a). With enough waves included, the averaged \(g\) should be equal to zero.

One of the most interesting phenomena in the observed \(g\) is that it has experienced strong temporal variation. Stages 2 and 3 are characterized by the most energetic \(<g>\), which reaches \(6 \times 10^{-9}\) and \(9 \times 10^{-9} \text{ W kg}^{-1}\), respectively. With the semidiurnal ITs acting as the parent wave here, it is natural to infer that the semidiurnal ITs should have positive effects on the PSI energy transfer. Strong baroclinic energy flux \(F_{\text{semi}}\) is considered to induce intense \(g\). Based on continuous velocity measurements at the PSI critical latitude, MacKinnon et al. (2013) showed that the observed three pulses of energetic PSI energy transfer occur during the spring tides. It is shown here that over the
first three stages \( \sigma \) grows with the intensification of \( F_{\text{semi}} \), which agrees well with the inference (as summarized in Table 2). However, one exception occurs during stage 4 when the largest \( F_{\text{semi}} \) (10.7 kW m\(^{-1}\)) has induced the least efficient PSI ((\( \sigma \) = 1 \times 10^{-9} \text{ W kg}^{-1})). This disproportionate relation is very different from that occurring during the first three stages. It is conjectured that there may exist other factors, in addition to the semidiurnal energy flux, that have modulated the behavior of PSI. This question is explored in the next section.

5. Discussion

PSI is expected to act most efficiently at the critical latitude, with an abrupt switch off at higher latitudes and gradual relaxation toward lower latitudes. This is related to the fact that the frequency of the generated M\(_2\) subharmonic waves deviates from the local Coriolis frequency \( f \) with distance south of the critical latitude. This will change the group velocity of the generated subharmonic waves, modifying the PSI efficiency.

By carrying out numerical experiments, one of the most important conclusions revealed by Yang et al. (2018) is that the existence of relative vorticity \( \zeta \) can significantly modify the behavior of PSI by changing \( f_{\text{eff}} \). At latitudes equatorward of the critical latitude, the PSI efficiency can significantly increase if there is positive \( \zeta \) shifting \( f_{\text{eff}} \) closer to the critical frequency (e.g., \( \omega_{\text{M41}} \) for M\(_2\) IT). This is equivalent to shifting to higher latitude and vice versa. Based on this theoretical work, it is predicted that once there is spatially varying background geostrophic flow generating \( \zeta \) in the real ocean, the PSI efficiency may be modified accordingly. As one of the two strongest western boundary currents, Kuroshio flows along the ECS shelf slope and has influenced our mooring station. Previous transects velocity observations showed that Kuroshio front tends to induce a “wall” of positive and negative vorticity on the left and right side to the flow direction, respectively (Rainville and Pinkel 2004).

Here, the effective Coriolis frequency at our mooring station is calculated following (Kunze 1985)

\[
f_{\text{eff}} = f + \frac{\zeta}{2} = f + \frac{1}{2} \left( \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right),
\]

where \( U \) and \( V \) are the absolute surface geostrophic velocities obtained from the Copernicus Marine and Environment Monitoring Service (CMEMS). Figure 9f shows that the calculated \( f_{\text{eff}} \) at the mooring site varies significantly with time. A typical trend of \( f_{\text{eff}} > f \) and \( f_{\text{eff}} < f \) appears during the first three stages and the fourth stage, respectively. The shift of \( f_{\text{eff}} \) is associated with the variation of Kuroshio. The mooring station is located at the left-to-middle side of the Kuroshio flow direction during the first three stages, which are characterized by positive \( \zeta \) and larger \( f_{\text{eff}} \) (Figs. 9a–c).

Although the calculated \( f_{\text{eff}} \) is based on surface geostrophic velocities, it can be regarded as a good indicator of the geostrophic vorticity below the surface.

To conceptualize the effect of \( \zeta \), we plot the latitude corresponding to the effective inertial frequency at the right axis of Fig. 9f, which is named as the “effective latitude.” Our observation site is located to the south of the critical latitude. The effective latitude is shifted northward (closer to the critical latitude) during the first three stages and southward during stage 4. The estimated PSI energy transfer in section 4 has shown that PSI was more efficient in transferring energy during the first three stages than that during stage 4 (as summarized in Table 2). This shift of PSI efficiency thus follows the variation of \( f_{\text{eff}} \) well consistent with the scenario described by Yang et al. (2018).

The observational results here together with the previous numerical predictions (Yang et al. 2018) all suggest that in the realistic ocean where geostrophic flows and mesoscale eddies prevail, the ITs cascading by PSI and the associated diapycnal mixing may depend a lot on the background vorticity field. However, we should note that due to the limited length of time of the observation, the vorticity effect on PSI intensity as suggested occurring at the mooring station is a fairly reasonable inference, rather than a robust conclusion. The mooring observations here only cover four spring–neap cycles, which is not a long enough time to isolate the influence of Kuroshio. Observations that last for longer periods in the future may provide further insight. As predicted in Yang et al. (2018), with the presence of strong negative vorticity, PSI can even occur poleward of the critical latitude, which is unlikely in the classical theory. At the northeast of ECS, there is a Tokara strait which is mainly located poleward of the M\(_1\) critical latitude and is characterized by both intense Kuroshio and strong semidiurnal ITs (Wang et al. 2018). Possible occurrence of PSI at the Tokara strait is expected, which awaits further exploration.

From the two months of measurements, the mean semidiurnal depth-integrated tidal energy flux is estimated to be 6.1 kW m\(^{-1}\) at our mooring site. Integrating the PSI energy transfer rate \( \sigma \) vertically yields the areal density of about 2.6 \times 10^{-3} \text{ W m}^{-2}. On the basis of these estimates, about 4% of the semidiurnal energy flux can be transferred to the subharmonic waves in 100 km of distance. This PSI-induced attenuation rate of the IT is a little bit lower than that revealed in previous studies.
[~12% in Alford et al. (2007) and ~7.5% in Sun and Pinkel (2013)]. Because of the absence of local dissipation measurements, it is difficult to evaluate the role of PSI in the local mixing field. On the basis of turbulence profiling measurements at ~28.5°N on the ECS break, Matsuno et al. (2005) observed that the dissipation varied between 10^{-10} and 10^{-8} W kg^{-1}. The calculated net $g$ of 5 × 10^{-9} W kg^{-1} here is close to the dissipation range, which may imply a significant role of PSI in local mixing. The possible influence of Kuroshio on the PSI behavior is of particular interest in the future, which may dramatically change the PSI acting efficiency. Nevertheless, the occurrence of PSI and the associated energy transfer have been clearly depicted in the present study.

6. Conclusions

One robust conclusion of this work is that PSI is working at the observation site, which is documented for the first time in the ECS region. The observed semidiurnal ITs contribute most to the wave activities at the mooring station. The associated energy flux appears mostly in the first-mode structure with an averaged depth-integrated value of 6.3 kW m^{-2} directing to the southeast. However, about 76% of the shear variance is contained in the near-inertial band because of vertical-high-mode structure. Apart from the surface wind-driven generation mechanism, these NIWs are found to have comparable upward- and downward-energy-propagating components.

Frequency and vertical wavenumber bispectral estimates show statistical phase locking between low-mode $M_2$ IT and the high-mode $M_2$ subharmonic waves. This is the key feature as expected when PSI occurs. The calculated PSI energy transfer further reveals a net rate of $5 \times 10^{-9} W kg^{-1}$ from the semidiurnal ITs to the subharmonic waves. This accounts for a 4% loss of semidiurnal energy flux over 100 km of distance, which is not an intense cascade. However, if all the PSI transferred energy dissipates locally, we expect that it may account for a significant fraction of the local mixing.
One of the most interesting results observed here is the strong temporal variation of PSI, which cannot be fully interpreted by the variation of local semidiurnal energy flux. The variation of background vorticity field associated with the changing Kuroshio pattern is further suggested to be the other important modulating factor, besides the semidiurnal energy flux. Our mooring station is located equatorward of the critical latitude. Observations show higher PSI efficiency during the first three stages when positive $\xi$ has shifted $f_{\text{eff}}$ to a larger value closer to the critical frequency ($f_{\text{eff}} = \omega_{M1}$). This work thus may be regarded as the first observational evidence supporting the theory of the effect of $\xi$ on modulating the PSI efficiency as revealed by Yang et al. (2018), although the length of the time of the measurements here is too short to yield this as a robust conclusion. We expect more investigations of this topic in the future from longer observations at a location with the presence of both strong ITs and background geostrophic currents.

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