Energetics of Seamount Wakes. Part II: Wave Fluxes

B. PERFECT

Mechanical Engineering, University of Washington, Seattle, Washington

N. KUMAR

Civil and Environmental Engineering, University of Washington, Seattle, Washington

J. J. RILEY

Mechanical Engineering, University of Washington, Seattle, Washington

(Manuscript received 1 May 2019, in final form 30 December 2019)

ABSTRACT

Seamounts are thought to facilitate ocean mixing through unsteady wake processes, and through the generation of internal waves, which propagate away from the seamount and later break. The relative importance of these processes is examined for idealized, isolated seamounts (with characteristic width D and height H) in uniform barotropic flow U . A range of Coriolis parameters f and buoyancy frequencies N are used such that a broad parameter space of low Froude numbers (U/NH) and low Rossby numbers (U/fD) is considered. Results indicate that eddy processes energetically dominate the internal wave energy flux in this range of parameter space. The internal wave field is specifically examined and partitioned into steady lee waves and unsteady, wake-generated waves. It is found that the lee wave energy flux cannot be explained by existing analytical theories. A lee wave model by Smith is then extended into the low-Froude-number regime and the effect of rotation is included. While strongly stratified experiments have previously indicated that only the top U/N of an obstacle generates internal waves, the effect of rotation appears to modify this wavemaking height. Once the U/N height is revised to account for rotation, the lee wave energy flux can be reasonably accurately reproduced by the extended Smith model.

1. Introduction

A vital yet difficult challenge in ocean modeling for climate applications is obtaining an accurate representation of the diapycnal mixing (MacKinnon et al. 2017). While the ocean interiors tend to produce relatively little mixing (e.g., Munk and Wunsch 1998), the boundaries can contain extremely energetic processes that may be a primary driver for basin-scale diapycnal mixing (Nash et al. 2007). In particular, the interaction of abrupt bathymetry such as seamounts with ocean tides and currents can produce internal waves, mesoscale eddies, turbulence, and ultimately mixing. Because there is such a wide range of interactions between ocean currents and bathymetry, however, a

comprehensive understanding of topographically induced mixing phenomena is lacking. Traditionally, bathymetry-induced mixing has been tied to the internal wave flux associated with the ocean bathymetry. Parameterizations for topographic internal wave flux have generally been confined to 2D ridges (e.g., St. Laurent and Garrett 2002; Llewellyn Smith and Young 2003; St. Laurent et al. 2003; Klymak et al. 2010). The underlying 2D theory for these parameterizations cannot necessarily be extended to 3D obstacles because the flows can go around, in addition to over, 3D obstacles; Zhang et al. (2017) finds, for example, that 2D theory overestimates internal wave flux for a Gaussian seamount.

In addition to the shortcomings of applying 2D wave theory to a 3D problem, recent research has suggested that seamount wakes and islands may also play an important role in the process of extracting energy from a the incident flow field (e.g., Dong et al. 2007; Chen et al. 2015; Perfect et al. 2018; Srinivasan et al. 2019; MacKinnon et al. 2019; Zeiden et al. 2019; Perfect et al. 2020, hereafter Part I).

Denotes content that is immediately available upon publication as open access.

Corresponding author: B. Perfect, bperfect@uw.edu

DOI: 10.1175/JPO-D-19-0104.1

© 2020 American Meteorological Society. For information regarding reuse of this content and general copyright information, consult the AMS Copyright Policy (www.ametsoc.org/PUBSReuseLicenses).

A motivating issue, then, is the relative importance of the internal wave field generated by the seamount compared to the hydrodynamic wake in extracting energy from the ambient flow and ultimately causing diapycnal mixing. It is expected that the answer may vary dramatically based on a number of factors, some of which will be addressed in this paper. Chief among them are the Froude and the Rossby numbers, which we define as

$$\text{Fr} \equiv \frac{U}{NH}, \quad \text{Ro} \equiv \frac{U}{fD}, \quad (1)$$

where D and H are the characteristic width and height of the seamount, U is the barotropic velocity, f is the f -plane Coriolis parameter, and N is the uniform buoyancy frequency. Part I performed a series of numerical simulations with varying Froude and Rossby number and computed the wake energy extracted by a seamount from the ambient flow. In conjunction with the present paper, this permits a direct comparison between the energies in the wake and in the internal waves generated by a seamount.

To properly assess the internal waves produced in the simulations, we expand upon classical internal wave theory and apply it to the flow regimes found in the simulations. When stratified fluid flows past an obstacle, the flow over that obstacle creates perturbations that generate internal gravity waves. The 2D case has been addressed for a wide variety of flows and obstacle shapes (e.g., Bell 1975a; Vallis 2006). The present case deals with 3D bathymetry, which invites the flow to move predominantly around, rather than over the obstacle; this effect is especially apparent at low Froude numbers (Brighton 1978). If the fluid is sufficiently stably stratified, fluid near the base of the obstacle in the oncoming flow can lack the kinetic energy necessary to overcome the potential energy barrier associated with ascending toward the top of the seamount, which is at a lower density (Sheppard 1956; Drazin 1961). The flow at lower levels then *must* go around the seamount. The transition from a bulk characterization of the flow going over versus around the seamount has been experimentally determined to occur at a critical Froude number around 0.4 (Hunt and Snyder 1980). However, experimental values for the critical Froude number can vary considerably with topographic shape (Boyer et al. 1987; Vosper et al. 1999). It should be noted as well that a potential energy argument (also known as Sheppard's criterion) suggests a critical Froude number of around 1. For Froude numbers significantly below this critical value, experiments suggest a bifurcation in the flow behavior (Riley et al. 1976; Brighton 1978).

When flow bifurcation occurs (see Fig. 1), the top $h_c = U/N$ of the topography forms a region where the flow

FIG. 1. In the cap model for flow past an obstacle, a bifurcation occurs in the flow. Fluid elements more than U/N below the summit are directed around the obstacle, and fluid elements above that plane will lift up to contour the bathymetry.

goes over the seamount, hereafter referred to as the “cap.” The height of the cap h_c corresponds to the vertical length scale for which $\text{Fr} = 1$. There is evidence that the plane of separation that delineates the bifurcation between flow over and flow going around is tilted slightly down toward the lee side (Brighton 1978; Dalziel et al. 2011), but this is a higher-order effect with respect to the cap height. The flow over the cap is associated with the generation of internal lee waves (Voisin 2007). Below the cap, the flow is directed around, in the form of a stratified turbulent wake, generally with vortex shedding (Lin and Pao 1979; Hunt and Snyder 1980; Vosper et al. 1999). Together, the assumptions that the lee waves are generated entirely by the top h_c of the seamount, and the flow more than h_c below the summit moves horizontally and forms a stratified wake, are referred to here as the cap model. The cap model is presented visually in Fig. 1. A similar concept known as the lowest overtopping streamline (LOTS) is used to determine the portion of the bathymetry responsible for internal waves within the context of blocked ridge flow (e.g., Arbic et al. 2019). However, here the flow is not blocked, but rather forms a separated wake, and we consider the 3D nature of the wave generation. When the wake contains significant vertical perturbations, it can generate “random” internal gravity waves (e.g., Brandt and Rottier 2015; Meunier et al. 2018). While the overall internal wave field can contain signals from both lee and wake-generated waves, they are considered to be dynamically separate (Gilreath and Brandt 1985; Riley and Lelong 2000). In the context of a steady flow past a seamount, the different types of internal gravity waves may be distinguished by the steady (lee) and unsteady (wake) components.

In the following sections of this paper, we will discuss the series of numerical simulations upon which this analysis is based (section 2), and then compare the

relative importance of the lee waves, the wake-generated waves, and the hydrodynamic wake itself (section 3). The remainder of the paper is focused on obtaining an accurate model for the lee waves. Section 4 summarizes several analytical models for lee wave flux (e.g., Bell 1975a; Smith 1980; Voisin 2007) and extends them to the low Froude number and rotating (when possible) regime using the cap assumption. The performance of these models is compared against the lee wave energy flux in the simulations, and the model performance is discussed in section 5. In section 6, we assess the validity of the cap model with respect to the simulations, and then relax some of the assumptions of the cap model. We demonstrate that with some information about the isopycnal displacements from a simulation, lee wave energy flux estimates are significantly improved. Finally, concluding remarks are in section 7.

2. Seamount simulations

The forthcoming analysis is based on the results of 18 simulations carried out using the Regional Ocean Modeling System (ROMS) framework (Shchepetkin and McWilliams 2005). The model configuration is explained in full detail in Part I and Perfect et al. (2018). These simulations all contain the same bathymetry, but differ in the planetary vorticity f and background stratification N , as noted in Fig. 2. The bathymetry consists of a flat bottom, with an isolated Gaussian seamount of height 3500 m and full width at half maximum (FWHM) of 23.4 km. The water is 5000 m deep. The height and FWHM are labeled H and D , respectively. The seamount is subjected to a uniform eastward flow of $U = 0.1 \text{ m s}^{-1}$, and the flow is kept in geostrophic balance with a tilted free surface. A quadratic drag law is used to include the effect of bottom drag; no other forcing was considered.

The 18 simulations depict the development of a von Kármán vortex street in the lee of the seamount. The shed vortices exhibit different structures based on the Burger number of the flow (Perfect et al. 2018),

$$\text{Bu} \equiv \left(\frac{\text{Ro}}{\text{Fr}} \right)^2 = \left(\frac{NH}{fD} \right)^2, \quad (2)$$

where H and D describe the height and FWHM of the seamount, and the Froude and Rossby numbers are as defined in Eq. (1). The Burger number is also found to control the eddy energy extracted from the mean flow (Part I). Indeed, most of the energetic quantities that do not involve turbulent scales are found to approximately be functions of the Burger number.

The simulation parameters were chosen to span a range of rotation and stratification conditions that might

FIG. 2. Schematic of the ROMS model setup. A Gaussian seamount with height H and full width at half maximum D is subjected to a uniform inlet flow with spatially uniform N and f . A von Kármán vortex street appears in the wake, indicated by the blue and red eddies with opposite vorticity.

be encountered by a seamount (see Fig. 3 and Fig. 2; Part I). The β -plane effects were neglected, and a spatially uniform background buoyancy frequency was used. The Coriolis parameter was set to either 2×10^{-5} , 4×10^{-5} , or $8 \times 10^{-5} \text{ s}^{-1}$. Six approximately logarithmically spaced values ranging from 2×10^{-4} to $2 \times 10^{-3} \text{ s}^{-1}$ were used for the buoyancy frequency. Put in terms of the Froude and Rossby number, this parameter space corresponds to $0.014 < \text{Fr} < 0.14$ and $0.053 < \text{Ro} < 0.21$. Therefore, $0.14 < \text{Bu} < 221$. A critical Burger number separates a regime with rotationally dominated, vertically aligned vortices (circles with a blue background in Fig. 3) from the regime with stratification-dominated, vertically decoupled vortices (triangles with a red background in Fig. 3). While this transition corresponds to significant changes in the eddy structure and wake energetics, it is not clear whether it is significant for the internal wave generation.

It is important to mention that, although all the simulations are carried out with the same ambient current speed of 0.1 m s^{-1} and seamount height of 3500 m, because of dynamic similarity the results are much more general than for these specific conditions. Assuming that the modeling assumptions are consistent, then two cases with the same Rossby and Froude numbers but possibly differing U , H , f , and N will be dynamically similar, so that the solution from one can be directly obtained from the solution to the other. This topic is addressed more fully in Part I of this study.

3. Wave energy flux evaluation

A key result of this study is that the rate of energy flux due to internal waves can be directly compared to the

FIG. 3. The 18 simulations used for this study are plotted by Froude and Rossby number (lower and left axis, respectively), as well as by buoyancy frequency and Coriolis parameter (top and right axis, respectively). The different shedding regimes are noted on the figure, with the symbols indicating regime associated with each simulation.

generation rate of eddy kinetic energy. This comparison evaluates the extent to which the simulated seamount can be described as a “stirring rod” versus a source of internal waves for this idealized case of an isolated seamount in a steady, barotropic flow with uniform stratification. Note that the flow conditions in the present study emphasize the wake development, and the results are probably not representative of the comparison for the case of a 2D ridge in tidal flow (e.g., St. Laurent and Garrett 2002; Llewellyn Smith and Young 2003; St. Laurent et al. 2003; Klymak et al. 2010). Nonetheless, large, isolated seamounts are common enough in Earth’s oceans that the Gaussian seamount of the present study is a reasonable bathymetry for comparing the energetics of the wake and internal waves (Wessel et al. 2010).

We find, in line with Gilreath and Brandt (1985), that the internal wave field consists of a steady (in the frame of the seamount) lee wave pattern and an unsteady, random wave field arising from wake-adjustment processes. The lee waves, particularly at large Rossby number (see Fig. 4a), resemble a Kelvin wake (also reported in Smith 1980). This pattern is visualized in Fig. 4 using horizontal slices of the time-averaged vertical velocity. As the effect of rotation increases (Figs. 4b,c), the lee wave wake pattern becomes increasingly asymmetrical and the lee waves are trapped closer to the seamount. The low-frequency waves that would normally propagate at low angle and form the far-field component of the Kelvin wake are predominantly subinertial, forming evanescent modes that are confined near the topography.

To separately compute the vertical flux of steady lee waves and unsteady (wake-generated) waves, we

decompose the vertical velocity w field using a time average ($\bar{\cdot}$), as done in Part I:

$$w = \bar{w} + \hat{w}. \quad (3)$$

We also consider a decomposition of the density field that consists of a uniform background density ρ_0 and an ambient stratification profile in the vertical $\rho_s(z)$, plus a perturbation density that includes time-averaged $\bar{\rho}$ and unsteady $\hat{\rho}$ components. Then,

$$\rho = \rho_0 + \rho_s(z) + \bar{\rho}(x, y, z) + \hat{\rho}(x, y, z, t). \quad (4)$$

We will drop the arguments from the perturbation terms for the sake of brevity, but it is understood that they vary spatially and that the unsteady component also has a time dependence.

The vertical energy fluxes (per unit area) associated with lee waves and wake waves are computed as

$$F''_{\text{lee}} = \bar{w} \int_{-h}^0 \bar{\rho} g \, dz, \quad F''_{\text{wake}} = \overline{\hat{w} \int_{-h}^0 \hat{\rho} g \, dz}, \quad (5)$$

where the upper limit of integration is the ocean surface. In the latter equation, the unsteady wave energy flux is time averaged. These equations only produce meaningful results above the seamount summit, where the eddy field does not influence the density and velocity fields. A bulk value for the wave energy flux is obtained by integrating the local vertical energy flux over a horizontal plane $\mathcal{A}$ spanning the simulation domain:

$$F_{\text{lee}} = \int_{\mathcal{A}} \bar{w} \int_{-h}^0 \bar{\rho} g \, dz \, d\mathcal{A}, \quad F_{\text{wake}} = \int_{\mathcal{A}} \hat{w} \int_{-h}^0 \hat{\rho} g \, dz \, d\mathcal{A}. \quad (6)$$

FIG. 4. The lee wave Kelvin wake pattern is demonstrated using the steady vertical velocity field $\bar{w}$. Each subfigure plots a horizontal slice of $\bar{w}$ taken $2h_c$ above the summit of the seamount. (a) The wake pattern is largely unaffected by rotation and it propagates for several hundred kilometers downstream. (b),(c) At progressively lower Rossby numbers, the wake becomes asymmetric and is confined near the seamount.

These energy fluxes represent the steady and unsteady components of the vertical component of the pressure work. The lee and wake wave energy fluxes are equivalent to the pressure work on the top boundary of the calculation domain for (respectively) the mean and eddy energy equations in Part I. We find that the vertical decay in the wave energy flux above the seamount is sufficient to prevent wave reflections from impacting the energy flux calculations.

The terms F_{lee} and F_{wake} are compared to the rate of eddy energy generation in Fig. 5. The mean to eddy energy conversion (details for this calculation are provided in Part I) represents the rate at which eddy kinetic energy is extracted from the mean flow and is found to dominate both the lee wave and wake wave energy fluxes in all cases. Part I reports that the mean to eddy conversion initially increases with the Burger number, before reaching a saturation value above a critical Burger number. Because the mean to eddy conversion is several decades larger than the wave energy fluxes, the Burger number dependence is readily discernible only in the right hand side of Fig. 5c, which corresponds to low Burger number. The wake and lee wave energy fluxes do not share the same scaling as do the mean to eddy conversion, but they appear to scale in a mutually similar manner. The nature of this dependence will be discussed in greater detail in later sections.

For $Fr < 1$, the Froude number describes approximately the proportion of the seamount that can generate lee waves. The remaining lower portion primarily produces eddies, which may in turn radiate wake waves. Despite the great majority of the seamount producing unsteady motion, the wake-generated wave energy flux

is lower than the lee wave energy flux in all cases. This finding is consistent with Brandt and Rottier (2015), which finds that, for $Fr < 1$, the energy flux of the wake waves is dominated by the energy flux of lee waves. Physically, the stratification inhibits vertical motion, which is necessary for the generation of gravity waves. Vertical and ageostrophic motions generated by the wake are quickly damped out downstream of the seamount. In a nonrotating regime, Bonneton et al. (1996) and Kadri et al. (1996) describe wake waves that substantially decay within 10 buoyancy periods. For comparison, the advection time for the flow to move past the seamount in the present simulations corresponds to $50 < Nt < 500$. The wake, then, ages very rapidly in its ability to generate waves, especially relative to the time scale of the eddy shedding (i.e., the buoyancy time scale is approximately an hour, whereas the eddy shedding period is several days). Consistent with this, we find that most of the unsteady wave energy flux is concentrated near the seamount where the eddies are forming, and very little is generated in the farther wake. While the wake vortices weaken as they advect downstream, as found by Part I, that weakening is not associated with significant internal wave generation, indicating that vortex weakening is caused by dissipative processes, rather than adjustments that produce internal wave radiation. This runs contrary to the findings of Bonneton et al. (1993, 1996), who demonstrate a strong coupling between coherent wake structures and wake wave radiation. In those laboratory experiments, however, the time scale of the internal waves and that of the wake eddies are of the same order, so that the generation process is more efficient than in our oceanographic case, where the

FIG. 5. Comparison of the power associated with the production of eddies, the lee wave energy flux, and the eddy-generated internal wave energy flux. The comparison is presented for (a) $f = 2 \times 10^{-5}$, (b) $f = 4 \times 10^{-5}$, and (c) $f = 8 \times 10^{-5}$.

time scales are very disparate. Additionally, by the time coherent vortices may be distinguished in the wake, the wake is already quite old, and scaling by Voisin (1991) suggests that nonrotating wake waves would have almost entirely decayed by that point. In our simulations, any wake wave radiation that is coupled to the developed eddies falls below the noise level in the pressure and vertical velocity fields.

4. Lee wave flux modeling

It is desirable to model the energy flux due to lee waves in Fig. 5 using analytical internal wave theory. Parameterizations for topographic internal wave production are important for large-scale ocean simulations, but no such model exists for the case of seamounts in low-Fr flow with rotation. Commonly, topographic lee wave parameterizations are based on the 2D, nonrotating, large-Fr model proposed by Bell (1975a) (see, e.g., Melet et al. 2013b). While the simple model is convenient to use, its restrictive assumptions are violated nearly everywhere, except for certain ocean ridges, and it is not clear how well the model performs outside of its underlying assumptions. Other lee wave models of interest include the effects of arbitrary 3D bathymetry Smith (1980), and low Froude number Voisin (2007), but their underlying assumptions preclude the present topic.

Therefore, we derive a novel lee wave model to compute the lee wave energy flux for 3D seamounts in

low-Fr, rotating flow. This new model is structurally similar to that of Smith (1980), and so the derivation below is abridged to focus on the novel alterations from Smith (1980).

In addition, we propose a nonrotating, low-Fr reformulation of the Bell model, in order to provide a simple extension into the low-Fr regime, and we assess its predictive ability with respect to the lee wave energy flux data in section 3. We also compute lee wave energy flux using Voisin's model, which is the only preexisting analytical model derived with a low-Fr assumption.

The lee wave flux models that we compare to the flux computed from the simulations are described below, with particular focus on the assumptions that are necessary in order to apply them to the current simulations.

a. Voisin model

Voisin (2007) analytically computes the wavemaking drag for an axisymmetric Gaussian seamount in a low-Froude-number uniform flow, in a nonrotating frame. The wavemaking drag is independent of the drag associated with flow separation. We therefore relate the wavemaking drag to the wave energy flux by multiplying the drag by the uniform flow velocity U , yielding,

$$F_{\text{Voisin}} = \frac{8}{15} \text{Fr}^{3/2} H a \rho_0 U^3, \quad (7)$$

where a is the characteristic horizontal scale of the Gaussian seamount. This model, then, can directly

replicate the geometry and stratification of the present study, but cannot be used to consider rotational effects.

b. A low-Froude-number extension to the bell model

Bell (1975a) develops a classical theory for the wave flux for 2D nonrotating flow over a small obstacle. This work is further developed by Bell (1975b) to include more features such as 3D terrain and rotation, but the simpler Bell (1975a) model has emerged as a widely used model for estimating wave radiation in oceanographic applications (Melet et al. 2013a). Our intent, then, is to apply a popular contemporary model to the present simulations and evaluate the results. In the case of a witch of Agnesi in steady flow, as studied in Bell's paper, the radiated wave energy flux is found to be

$$F = \frac{\rho_0 \pi}{2} U^2 H^2 N l, \quad (8)$$

where we have added an implied length scale l in the cross-stream direction for dimensional consistency. To extend this expression to a low-Froude-number regime, the topographic height H should be replaced by the cap height U/N . Bell's argument that the wave flux for a witch of Agnesi is approximately equivalent to that of a Gaussian is extended to the Gaussian cap, without additional justification (though we later find that this assumption is reasonable). Equation (8), with the cap height substitution, yields

$$F = \frac{\rho_0 \pi}{2} \frac{U^4}{N} l. \quad (9)$$

If we take l to be the cross-stream extent of the cap, further simplification is possible. For a Gaussian defined by $h = H e^{-y^2/a^2}$, the horizontal extent of the cap is

$$l = a \sqrt{-\log(1 - \text{Fr})} \approx a \sqrt{\text{Fr}}, \quad (10)$$

where in the last step we have taken a first-order expansion about $\text{Fr} = 0$. The wave flux, then, can be modeled as

$$F_{\text{Bell}} = \frac{\pi}{2} \text{Fr}^{3/2} H a \rho_0 U^3. \quad (11)$$

The low-Fr Bell model differs from the Voisin estimate, Eq. (7), by a constant, but the Froude dependency and dynamical representation are identical. This is an important result that establishes the feasibility of a low-Fr extension to the Bell model and demonstrates parallels between two lee wave models with very different derivations. The first-order expansion in Eq. (10) also yields the same result for a witch of Agnesi (a common

topographical form in analytical studies; e.g., Smith 1980), though the higher-order representations are different.

c. A low-Froude-number 3D model with rotation

Smith (1980) models the isopycnal displacement field of a 3D witch of Agnesi in nonrotating, high-Froude number flow, and analytically explores the behavior of this field in asymptotic conditions. However, the underlying method is generally valid for all bathymetry if $\text{Fr} \geq 1$. We may extend this model to a low-Froude regime by using the cap model. Rather than using the actual bathymetry of the idealized seamount, we treat the surface of the flow bifurcation (as in Fig. 1) as a virtual bottom. Under the assumptions of the cap model, no waves may be generated from flow below the bifurcation, so that component of the flow may be neglected. Treating the virtual bottom as a revised bathymetry, the seamount effectively becomes an obstacle with $\text{Fr} = 1$, which is within the range of applicability of Smith's (1980) theory.

For the extension of this lee wave energy flux model to cases with rotation, consider the steady, inviscid Navier–Stokes equations within the Boussinesq approximation for a rotating flow; linearizing these equations about a mean flow U in geostrophic balance gives

$$U u_x - f v = -\frac{1}{\rho_0} p_x, \quad (12)$$

$$U v_x + f u = -\frac{1}{\rho_0} p_y, \quad (13)$$

$$\rho_0 U w_x = -p_z - \bar{\rho} g, \quad (14)$$

$$U \bar{\rho}_x = -w \frac{d\rho_s}{dz}, \quad \text{and} \quad (15)$$

$$u_x + v_y + w_z = 0. \quad (16)$$

Here the subscripts are taken to be derivatives, lower case variables represent steady perturbations, and the density terms are as defined in Eq. (4). We define the isopycnal displacement η in terms of the steady density perturbation $\bar{p}$ as

$$\bar{p} = \frac{\rho_0 N^2}{g} \eta, \quad (17)$$

and then Eq. (15) can be written as

$$w = U \eta_x. \quad (18)$$

To complete the problem, the kinematic condition can be written as

$$w(x, y, 0) = U \frac{\partial}{\partial x} h(x, y), \quad (19)$$

where $z = h(x, y)$ defines the bottom boundary of the cap, which is the upper surface in Fig. 1. The equation for the isopycnal displacement is then found to be (e.g., Kundu et al. 2004)

$$\frac{\partial}{\partial x^2} (\nabla^2 \eta) + \frac{f^2}{U^2} \frac{\partial^2}{\partial z^2} \eta + \frac{N^2}{U^2} \nabla_H^2 \eta = 0. \quad (20)$$

We take the Fourier transform [denoted as $\mathcal{F}\{\cdot\} = \widehat{(\cdot)}$] of this relation to give

$$k^2(k^2 + l^2) \hat{\eta} - k^2 \hat{\eta}_{zz} + \frac{f^2}{U^2} \hat{\eta}_{zz} - \frac{N^2}{U^2} (k^2 + l^2) \hat{\eta} = 0. \quad (21)$$

Note that $\widehat{(\cdot)}$ now denotes the Fourier transform, rather than the eddy scale, which was used earlier. This differential equation permits solutions of the form

$$\hat{\eta}(k, l, z) = \hat{\eta}(k, l, 0) e^{imz}, \quad (22)$$

where

$$m^2 = \frac{(k^2 + l^2)(N^2 - U^2 k^2)}{U^2 k^2 - f^2}. \quad (23)$$

Under the hydrostatic approximation (i.e., $Uk \ll N$) this reduces to

$$m^2 = \frac{(k^2 + l^2)N^2}{U^2 k^2 - f^2}, \quad (24)$$

which is the dispersion relationship for hydrostatic inertial internal gravity waves.

The hydrostatic approximation is invoked for two reasons. First, the simulations to which this wave flux estimate will be compared are also hydrostatic. Second, we find that relaxing the hydrostatic approximation does not significantly affect the forthcoming results, and yet results in a less computationally expensive model when compared to the nonhydrostatic version. For a nonhydrostatic simulation, it may be desirable to use the nonhydrostatic dispersion relation.

Using the kinematic boundary condition [Eq. (19)], the flow at $z = 0$ should contour the bathymetry. In the low-Fr extension, the surface of flow bifurcation (n.b., Fig. 1) is the contour of interest that the flow follows, and this should be used in place of the bathymetry. For a flow contour described by $h(x, y)$, we can say that, $\hat{\eta}(k, l, 0) = \hat{h}(k, l)$. The isopycnal displacement can then be written as the inverse transform of the product of the wavenumber transform of $h(x, y)$ and the vertical wave propagation:

$$\eta(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{h}(k, l) e^{imz} e^{-i(kx+ly)} dk dl. \quad (25)$$

Equation (25) may be evaluated numerically. From η , the internal wave flux may be evaluated using linear theory; for a hydrostatic flow, Eqs. (14) and (18) become, respectively,

$$p(x, y, z) = \int_z^{\infty} \rho_0 N^2 \eta(x, y, z') dz', \quad (26)$$

$$w(x, y, z) = U \eta_x(x, y, z). \quad (27)$$

In Eq. (26), the integral computes the effect of all isopycnal displacements above the fluid element of interest for an infinitely deep ocean. In practice, the displacements decay with height and the pressure field is sufficiently converged by a height of $50h_c$. The internal wave flux is computed as

$$F_{\text{Smith}} = \iint_{\mathcal{A}} p w dA = \iint_{\mathcal{A}} \mathcal{F}^{-1} \{ ik \hat{h}(k, l) e^{imz} \} \times \int_z^{\infty} \rho_0 N^2 \mathcal{F}^{-1} \{ \hat{h}(k, l, 0) e^{imz} \} dz' dA, \quad (28)$$

where $\mathcal{A}$ is taken to be a surface that encloses the causality envelope of the topography (i.e., all wave radiation from the mountain must cross $\mathcal{A}$).

5. Model performance

We evaluate the performance of three nonrotating lee wave flux models, based on Bell (1975a) [see Eq. (11), referred to as Bell hereafter], Voisin (2007) [see Eq. (7), referred to as Voisin hereafter], and Smith (1980) [see Eq. (28), referred to as nonrotating Smith hereafter]. In the nonrotating Smith model, the Coriolis parameter is set to zero in Eq. (24). A fourth model is also based on Smith's model, but includes a nonzero Coriolis parameter, and is referred to as the rotating Smith model. The lee wave energy fluxes predicted by these models are plotted alongside the simulation data in Fig. 6.

The nonrotating models (the straight red lines in Fig. 6) do not depend on the Rossby number, and so they appear identical in each frame. The Bell and Voisin estimates share the same $\text{Fr}^{3/2}$ scaling and differ only by a constant multiplicative factor, as explained in sections 4a and 4b. The Voisin model more closely matches the data, which is to be expected because Voisin explicitly addresses a Gaussian in strongly stratified flow, whereas the Bell model contains significant assumptions in extending the model to a 3D, low-Froude-number regime. We observe that the nonrotating Smith model also obeys a $\text{Fr}^{3/2}$ scaling, though

FIG. 6. Comparison of several lee wave models (plotted as lines) against lee wave energy flux data (plotted as black asterisks). The three panels indicate the three Coriolis parameters used, as in Fig. 5. The straight red lines represent the nonrotating models: Voisin (dashed), Bell (dot-dash), and Smith (solid). The curved solid blue line is the rotating Smith model.

this cannot be readily derived from Smith's formulation. Together, the nonrotating models capture the broad trends of the internal wave energy flux at low and moderate rotation (Figs. 6a,b), but it is clear that the $Fr^{3/2}$ scaling is incorrect. The simulation data suggest that, in agreement with Brandt and Rottier (2015), the wave energy flux scales approximately with Fr^2 . As rotation becomes strong (Fig. 6c), the Froude dependence of the simulation data changes dramatically, which is not captured by the nonrotating models.

The rotating Smith model (the solid blue lines in the panels of Fig. 6) is nearly identical to the nonrotating Smith model when the Rossby number is relatively large (Figs. 6a,b). At low Rossby number, the rotating Smith model diverges significantly from its nonrotating counterpart and much more closely follows the simulation data. In Fig. 6b, the addition of rotation to the Smith model causes the model to adjust away from the simulation data in the high-Fr range. The rightmost simulation data point may be an outlier, for reasons discussed in section 6.

The small values for the wave energy flux in Fig. 6c can be justified as a consequence of the dispersion relation for linear internal gravity waves. A substantial proportion of the waves excited by the topography become evanescent modes at low Rossby number. These modes, which were free to propagate at higher Ro, are now confined to the seamount, as shown in Fig. 4. For a nonhydrostatic application, the range of

propagating frequencies is $f < \omega < N$ (while we do not invoke the upper bound restriction in our analysis, we do not find significant energy at frequencies above N , and so this range effectively encapsulates the range of frequencies seen). As f and N approach each other, as in the right side of Fig. 6c, the allowable band of frequencies becomes quite narrow. In the most extreme case of the present study, the internal wave band is restricted to $8 \times 10^{-5} \text{ s}^{-1} < \omega < 2 \times 10^{-4} \text{ s}^{-1}$.

6. Cap height model relaxation

In the previous sections we have assumed, uncritically, that the interaction of the wavemaking flow and the seamount can be perfectly described as flow over the cap, the top $h_c = U/N$ of the seamount. While this framework is generally supported by potential energy arguments and experimental observations, our data suggest that the cap model is inadequate, especially under the effect of rotation. As suggested by Fig. 4, as the Rossby number decreases, the simulated lee waves become increasingly asymmetric. The Smith models, which are the only models that compute the structure of the lee waves, cannot reproduce the observed asymmetry with a cap model. Additionally, we find that the cap heights in the simulations can be described to first order by h_c but contain significant deviations that require a correction to the cap height.

In this section, we investigate the inadequacies of the cap model through the use of isopycnal surfaces and the

FIG. 7. Normalized isopycnal deflection maps 1500 m below the seamount summit. The direction of flow in each image is from top to bottom. Images are arranged in the same pattern as the data points in Fig. 3.

cap heights computed from the simulations. We then relax the assumption that $h_c = U/N$ and modify the rotating Smith model to use a cap height correction factor. This modification to the Smith model takes two forms: the cap height correction factor can be taken directly from the simulation or, more generally, an empirical fit to the simulation data can prescribe a correction factor based on the rotation and stratification. The assumption that the waves are generated by a kinematic boundary condition on the theoretical seamount cap is also relaxed. In this case, the Fourier transform of the cap $[\hat{h}(k, l)]$ in the rotating Smith model is replaced with the Fourier transform of an isopycnal surface. This correction, referred to as the isopycnal Smith model, permits the asymmetries observed in the simulation to be reproduced in the model.

a. Isopycnal displacements

A key assumption of the cap model is that at vertical levels more than h_c below the summit, the flow is confined to horizontal planes. We test this assumption by calculating, for every simulation, the vertical perturbation of an isopycnal that is well below the cap. We use the isopycnal corresponding to $\rho_s(-3000\text{ m})$, which is 1500 m below the seamount summit in the incoming flow. These isopycnal displacement maps are normalized by h_c , and then plotted in Fig. 7. For strong stratification and weak rotation, the isopycnal displacement is negligible. The bottom right corner of Fig. 7, however, indicates isopycnal displacements exceeding h_c ,

in clear violation of the cap model. The large isopycnal displacements are caused by adjustment processes associated with the turning flow. The length scale associated with these displacements suggests that they are large relative to U/f , and therefore correspond to subinertial motion. While these structures violate the cap model by producing significant isopycnal displacements below the cap, it is not clear whether this violation impacts, to a significant degree, the lee wave field and, in particular, the internal wave energy flux. The fluid flowing over the deflected isopycnals would likely not produce propagating waves because the associated motion would tend to be subinertial. The simulation results support this interpretation; there is no evidence that propagating waves are generated at the seamount flanks.

Figure 8 also plots normalized isopycnal displacements, but at a vertical level that is approximately at the seamount summit. Unlike in Fig. 7, significant displacements on the order of h_c exist for all of the simulations. The cap model predicts that isopycnal displacements of h_c should be observed at the seamount summit, and Fig. 8 supports this prediction, qualitatively. In each plot in Fig. 8, the upstream side of the seamount summit (just above the center mark in each image) contains an isopycnal elevation where the flow moves over the summit. Other structures commonly appear in some simulations, such as an isopycnal trough in the lee of the summit, a meridional tilt, and wavelike oscillations. These structures have significantly smaller length scales than those in Fig. 7

FIG. 8. As in Fig. 8, but isopycnal deflections are taken at the seamount summit.

and may contribute to the lee waves. This possibility is addressed in section 6b.

We can refine the observation in Fig. 8 that the summit isopycnal displacement is on the order of h_c by directly calculating the actual cap height from the simulation. The actual cap height is defined as the vertical excursion from the ambient density profile, $\rho_0 + \rho_s(z)$, that is required for the densest parcel of water that is found at the summit of the seamount. For a given maximum density at the seamount summit, $\rho_{\max}$, the actual cap height is

$$h_{\text{actual}} = \frac{[\rho_0 + \rho_s(z) - \rho_{\max}]g}{\rho_0 N^2}. \quad (29)$$

We find, in Fig. 9a, that the peak isopycnal displacement scales only roughly as N^{-1} , as predicted by $h_c \equiv U/N$. When the computed isopycnal displacement is normalized by h_c , deviations from the theoretical cap height become even more apparent (see Fig. 9b). The actual cap height differs from the U/N scaling by as much as $\pm 40\%$. The abscissa in Fig. 9b is

$$\lambda \equiv \frac{\text{Ro}}{\text{Bu}}, \quad (30)$$

which is defined by Perret et al. (2006) to be a scaling parameter for interface deviation in a rotating, two-layer flow past an obstacle. This quantity also appears as the ratio of the horizontal to vertical contributions to the Ertel potential vorticity (Pedlosky 2003) and was found

to play a role in eddy processes for flow past an island (Dong et al. 2007). The scaling parameter λ is a natural choice to represent the normalized isopycnal displacement, and in Fig. 9b we suggest a λ -based scaling that explains the deviation from the cap model. The modified cap height h_c^* is defined to be

$$h_c^* = h_c \times 1.42 \times \lambda^{0.15}. \quad (31)$$

That is, we can represent the actual cap height as the theoretical cap height, multiplied by a correction factor that is a function of λ . The modified cap height h_c^* can also be considered as a least squares fit to represent the h_{actual} data.

b. Revised wave energy flux estimates

The violations of the cap model, detailed above, can be accounted for by modifying the rotating Smith model. It is not immediately clear, however, whether the inaccuracies of the lee wave energy flux models in Fig. 6 are caused by the inadequacies of the cap model. To address this, we implement three modifications to the rotating Smith model and assess their ability to improve the lee wave energy flux estimates. The modifications, in order of increasing complexity and required simulation data, are as follows:

- 1) The surface of flow bifurcation is modified by using the empirical cap height from Eq. (31) in place of h_c . The revised $h(x, y)$ is then used in the rotating

FIG. 9. (a) The maximum isopycnal deflection at the summit is plotted against the buoyancy frequency. (b) Once normalized by the predicted cap height, the summit isopycnal deflection is plotted vs λ , a dimensionless parameter associated with isopycnal displacement.

Smith model. This method is referred to as the cap height fit Smith.

- 2) The cap height is computed directly from the simulation data using Eq. (29). This is then used to create a revised $h(x, y)$ to be input into the rotating Smith model. This method is referred to as the cap height corrected Smith.
- 3) The isopycnal surface that just barely reaches the seamount summit may be treated as the surface of flow bifurcation. This surface is treated as $h(x, y)$ for the rotating Smith model. The surface may therefore be asymmetric, as expected from Fig. 4. This method is referred to as the isopycnal Smith.

In Fig. 10, the Smith model modifications are plotted alongside the lee wave energy flux data. The simulation data are plotted as black stars. The cap height fit Smith and the cap height corrected Smith models are plotted as blue lines and blue circles, respectively. Finally, the wave fluxes obtained from the summit isopycnals are plotted as red squares. The cap height fit tracks the simulation data quite well and represents a relatively robust fit with respect to the cap height corrected model. The most striking aspect of this comparison is that the slope of the modeled wave flux with respect to Fr now matches the simulation data in Figs. 10a and 10b. Note that the uncorrected models all exhibited a $Fr^{3/2}$ dependence, which underpredicted the slope of the simulation data. The cap height fit model permits the lee wave energy flux to be predicted continuously in the Froude–Rossby space, as shown in Fig. 11. The isopycnal Smith model overestimates the energy flux in Figs. 10a and 10b, although the

Froude dependence is still correctly captured. The rightmost data point in Fig. 10b seems to diverge from the trend set by the rest of the points, as noted in section 5. This behavior is captured by the isopycnal Smith model, but none of the other models. This suggests that the large measured flux is due to the isopycnal structure, rather than actual cap height. In Fig. 10c, the isopycnal Smith model nearly perfectly captures the lee wave flux.

An additional benefit of using the measured isopycnal with the rotating Smith model is that it permits the observed asymmetries in the isopycnal to be included in the calculated structure of the lee wave. Figure 12 plots three typical examples of the lee wave flux at $3h_c$ above the seamount. The simulation output, the rotating Smith with a basic cap model, and the rotating Smith with the isopycnal displacement input are all shown. While both models approximately reproduce the observed lee wave pattern when the wake is symmetric, the isopycnal input must be used to capture the full structure of the wake when strong rotation causes asymmetries.

7. Conclusions

Through a series of numerical simulations, we have shown that, for an isolated seamount in uniform flow, the rate that eddy energy is extracted from the mean flow significantly exceeds the rate of internal wave energy flux. These results are robust across a broad range in stratification and rotation values within the low-Froude-number regime. Caution should be taken, however, in extrapolating the present study to a realistic

FIG. 10. The lee wave energy flux is the same as in Fig. 6, but now the modified Smith models are plotted alongside the simulation data. The cap height corrected Smith and cap height fit Smith models are indicated by blue circles and a solid blue line, respectively. The isopycnal Smith model is plotted using red squares.

seamount. Tidal forcing was not considered, which could enhance internal wave generation relative to eddy energy generation (Buijsman et al. 2014). Additionally, it is difficult to determine a single Froude number to describe any given seamount, because stratification and currents may have complex spatial and temporal variation, as well as considerable uncertainty in the abyssal ocean (King et al. 2012; Timko et al. 2013). But, in general, slow abyssal currents and the prevalence of large seamounts (e.g., Wessel et al. 2010) suggest that a significant number of seamounts could be classified as relatively low Froude number. We therefore highlight the possibility that some seamounts may exhibit the wake-dominated energetics seen in this study, particularly large seamounts in regions with minimal tidal forcing and strong stratification.

We show that lee wave energy flux models such as Bell (1975a) and Smith (1980) can be extended to the low-Froude-number regime by considering the tendency for the flow to bifurcate and only flow over the top h_c of the seamount. Both of these models, once modified, predict the same functional form in the low-Froude-number limit as the explicitly low-Froude-number lee wave flux model by Voisin (2007). However, these models are inadequate to describe the lee wave fluxes seen in our simulations because they do not account for rotation. Once the Smith model is extended to include rotation, the model predictions are able to capture the reduced lee wave energy flux that occurs at low Ro . Further model improvements

are obtained by correcting the cap height based on simulation data, and by replacing the surface of flow bifurcation with the simulation isopycnal that just reaches the seamount summit.

The classical view that the flow is confined to horizontal planes below the cap appears to be violated by the observed isopycnal displacement, particularly when the Rossby number is below $O(0.1)$ (Brighton 1978; Dalziel et al. 2011). The isopycnal deflections raise interesting questions about flow patterns around 3D topography in a rotating, stratified regime. In conjunction with work such as Dalziel et al. (2011),

FIG. 11. Predicted lee wave energy flux, as a function of the Froude and Rossby number, based on the cap height correction in Eq. (31).

FIG. 12. (a)–(c) A typical case with weak rotation. The wave flux is plotted $3h_c$ above the seamount summit for the simulation data, the rotating Smith model, and the isopycnal Smith model. (d)–(f) As in (a)–(c), but for a case with strong rotation and an asymmetric wake.

which looked at isopycnal deflections in the purely stratified case, there appear to be potentially important deviations from the cap height model that merit further study.

In purely stratified flow past 3D obstacles, the cap height is defined by $h_c = U/N$ and the wave energy flux scales with $Fr^{3/2}$. We find that, with rotation, the cap height also depends on the parameter λ , and that this new dependency is able to explain the discrepancy between the $Fr^{3/2}$ scaling and the simulation data (as in Fig. 10a). The empirical cap height relationship, Eq. (31), does not reduce to U/N as $f \rightarrow 0$, so clearly further work is needed to explore how cap height varies in the transition from nonrotating to rotating flow and to unify the two regimes.

Acknowledgments. This work was facilitated through the use of advanced computational, storage, and networking infrastructure provided by the Hyak supercomputer system and funded by the STF at the University of Washington. B. Perfect would like to thank B. Blakeley for productive conversation. N. Kumar acknowledges support from the Office of Naval Research, Littoral and Geosciences Division (Grant N00014-17-1-2890). J. J. Riley acknowledges support from PACCAR. The numerical code used for this work is open source

and located at myroms.org. The postprocessed dataset and model settings used to produce the above results are available through the Open Science Framework at osf.io/ntscf. Raw model simulations are archived and will be provided upon request by the corresponding author.

REFERENCES

- Arbic, B. K., O. B. Fringer, J. M. Klymak, F. T. Mayer, D. S. Trossman, and P. Zhu, 2019: Connecting process models of topographic wave drag to global eddying general circulation models. *Oceanography*, **32**, 146–155, <https://doi.org/10.5670/oceanog.2019.420>.
- Bell, T. H., 1975a: Lee waves in stratified flows with simple harmonic time dependence. *J. Fluid Mech.*, **67**, 705–722, <https://doi.org/10.1017/S0022112075000560>.
- , 1975b: Topographically generated internal waves in the open ocean. *J. Geophys. Res.*, **80**, 320–327, <https://doi.org/10.1029/JC080i003p00320>.
- Bonneton, P., J.-M. Chomaz, and E. J. Hopfinger, 1993: Internal waves produced by the turbulent wake of a sphere moving horizontally in a stratified fluid. *J. Fluid Mech.*, **254**, 23–40, <https://doi.org/10.1017/s0022112093002010>.
- , —, E. Hopfinger, and M. Perrier, 1996: The structure of the turbulent wake and the random internal wave field generated by a moving sphere in a stratified fluid. *Dyn. Atmos. Oceans*, **23**, 299–308, [https://doi.org/10.1016/0377-0265\(95\)00435-1](https://doi.org/10.1016/0377-0265(95)00435-1).
- Boyer, D. L., P. A. Davies, W. R. Holland, F. Biolley, and H. Honji, 1987: Stratified rotating flow over and around isolated

- three-dimensional topography. *Philos. Trans. Roy. Soc. London*, **322A**, 213–241, <https://doi.org/10.1098/rsta.1987.0049>.
- Brandt, A., and J. R. Rottier, 2015: The internal wavefield generated by a towed sphere at low Froude number. *J. Fluid Mech.*, **769**, 103–129, <https://doi.org/10.1017/jfm.2015.96>.
- Brighton, P. W. M., 1978: Strongly stratified flow past three dimensional obstacles. *Quart. J. Roy. Meteor. Soc.*, **104**, 289–307, <https://doi.org/10.1002/qj.49710444005>.
- Buijsman, M. C., and Coauthors, 2014: Three-dimensional double-ridge internal tide resonance in Luzon Strait. *J. Phys. Oceanogr.*, **44**, 850–869, <https://doi.org/10.1175/JPO-D-13-024.1>.
- Chen, G., and Coauthors, 2015: Observed deep energetic eddies by seamount wake. *Sci. Rep.*, **5**, 17416, <https://doi.org/10.1038/srep17416>.
- Dalziel, S. B., M. D. Patterson, C. P. Caulfield, and S. Le Brun, 2011: The structure of low-Froude-number lee waves over an isolated obstacle. *J. Fluid Mech.*, **689**, 3–31, <https://doi.org/10.1017/jfm.2011.384>.
- Dong, C., J. C. McWilliams, and A. F. Shchepetkin, 2007: Island wakes in deep water. *J. Phys. Oceanogr.*, **37**, 962–981, <https://doi.org/10.1175/JPO3047.1>.
- Drazin, P., 1961: On the steady flow of a fluid of variable density past an obstacle. *Tellus*, **13**, 239–251, <https://doi.org/10.3402/tellusa.v13i2.9451>.
- Gilreath, H., and A. Brandt, 1985: Experiments on the generation of internal waves in a stratified fluid. *AIAA J.*, **23**, 693–700, <https://doi.org/10.2514/3.8972>.
- Hunt, J. C. R., and W. H. Snyder, 1980: Experiments on stably and neutrally stratified flow over a model three-dimensional hill. *J. Fluid Mech.*, **96**, 671–704, <https://doi.org/10.1017/S0022112080002303>.
- Kadri, Y., P. Bonneton, J.-M. Chomaz, and M. Perrier, 1996: Stratified flow over three-dimensional topography. *Dyn. Atmos. Oceans*, **23**, 321–334, [https://doi.org/10.1016/0377-0265\(95\)00433-5](https://doi.org/10.1016/0377-0265(95)00433-5).
- King, B., M. Stone, H. P. Zhang, T. Gerkema, M. Marder, R. B. Scott, and H. L. Swinney, 2012: Buoyancy frequency profiles and internal semidiurnal tide turning depths in the oceans. *J. Geophys. Res. Oceans*, **117**, C04008, <https://doi.org/10.1029/2011JC007681>.
- Klymak, J. M., S. Legg, and R. Pinkel, 2010: A simple parameterization of turbulent tidal mixing near supercritical topography. *J. Phys. Oceanogr.*, **40**, 2059–2074, <https://doi.org/10.1175/2010JPO4396.1>.
- Kundu, P. J., I. M. Cohen, and D. R. Dowling, 2004: *Fluid Mechanics*. 3rd ed. Elsevier, 759 pp.
- Lin, J. T., and Y. H. Pao, 1979: Wakes in stratified fluids. *Annu. Rev. Fluid Mech.*, **11**, 317–338, <https://doi.org/10.1146/annurev.fl.11.010179.001533>.
- Llewellyn Smith, S. G., and W. R. Young, 2003: Tidal conversion at a very steep ridge. *J. Fluid Mech.*, **495**, 175–191, <https://doi.org/10.1017/S0022112003006098>.
- MacKinnon, J. A., and Coauthors, 2017: Climate process team on internal wave-driven ocean mixing. *Bull. Amer. Meteor. Soc.*, **98**, 2429–2454, <https://doi.org/10.1175/BAMS-D-16-0030.1>.
- , M. H. Alford, G. Voet, K. L. Zeiden, T. M. S. Johnston, M. Siegelman, S. Merrifield, and M. Merrifield, 2019: Eddy wake generation from broadband currents near Palau. *J. Geophys. Res. Oceans*, **124**, 4891–4903, <https://doi.org/10.1029/2019JC014945>.
- Melet, A. V., R. W. Hallberg, S. Legg, and K. L. Polzin, 2013a: Sensitivity of the ocean state to the vertical distribution of internal-tide-driven mixing. *J. Phys. Oceanogr.*, **43**, 602–615, <https://doi.org/10.1175/JPO-D-12-055.1>.
- , M. Nikurashin, C. Muller, S. Falahat, J. Nycander, P. G. Timko, B. K. Arbic, and J. A. Goff, 2013b: Internal tide generation by abyssal hills using analytical theory. *J. Geophys. Res. Oceans*, **118**, 6303–6318, <https://doi.org/10.1002/2013JC009212>.
- Meunier, P., S. Le Dizès, L. Redekopp, and G. R. Spedding, 2018: Internal waves generated by a stratified wake: Experiment and theory. *J. Fluid Mech.*, **846**, 752–788, <https://doi.org/10.1017/jfm.2018.278>.
- Munk, W., and C. Wunsch, 1998: Abyssal recipes II. *Deep-Sea Res. I*, **45**, 1977–2010, [https://doi.org/10.1016/S0967-0637\(98\)00070-3](https://doi.org/10.1016/S0967-0637(98)00070-3).
- Nash, J. D., M. H. Alford, E. Kunze, K. Martini, and S. Kelly, 2007: Hotspots of deep ocean mixing on the Oregon continental slope. *Geophys. Res. Lett.*, **34**, L01605, <https://doi.org/10.1029/2006GL028170>.
- Pedlosky, J., 2003: WKB theory for internal gravity waves. *Waves in the Ocean and Atmosphere*, Springer, 75–89.
- Perfect, B., N. Kumar, and J. Riley, 2018: Vortex structures in the wake of an idealized seamount in rotating, stratified flow. *Geophys. Res. Lett.*, **45**, 9098–9105, <https://doi.org/10.1029/2018GL078703>.
- , —, and —, 2020: Energetic of seamount wakes. Part I: Energy exchange. *J. Phys. Oceanogr.*, **50**, 1365–1382, <https://doi.org/10.1175/JPO-D-19-0105.1>.
- Perret, G., A. Stegner, M. Farge, and T. Pichon, 2006: Cyclone-anticyclone asymmetry of large-scale wakes in the laboratory. *Phys. Fluids*, **18**, 036603, <https://doi.org/10.1063/1.2179387>.
- Riley, J., and M. P. Lelong, 2000: Fluid motions in the presence of strong stable stratification. *Annu. Rev. Fluid Mech.*, **32**, 613–657, <https://doi.org/10.1146/annurev.fluid.32.1.613>.
- , H. Liu, and F. Geller, 1976: A numerical and experimental study of stably stratified flow around complex terrain. Environmental Protection Agency Tech. Rep. EPA/600/4-76/021, 42 pp.
- Shchepetkin, A. F., and J. C. McWilliams, 2005: The Regional Ocean Modeling System (ROMS): A split-explicit, free-surface, topography following coordinates ocean model. *Ocean Modell.*, **9**, 347–404, <https://doi.org/10.1016/j.ocemod.2004.08.002>.
- Sheppard, P., 1956: Airflow over mountains. *Quart. J. Roy. Meteor. Soc.*, **82**, 528–529, <https://doi.org/10.1002/qj.49708235418>.
- Smith, R. B., 1980: Linear theory of stratified hydrostatic flow past an isolated mountain. *Tellus*, **32**, 348–364, <https://doi.org/10.3402/tellusa.v32i4.10590>.
- Srinivasan, K., J. C. McWilliams, M. J. Molemaker, and R. Barkan, 2019: Submesoscale vortical wakes in the lee of topography. *J. Phys. Oceanogr.*, **49**, 1949–1971, <https://doi.org/10.1175/JPO-D-18-0042.1>.
- St. Laurent, L., and C. Garrett, 2002: The role of internal tides in mixing the deep ocean. *J. Phys. Oceanogr.*, **32**, 2882–2899, [https://doi.org/10.1175/1520-0485\(2002\)032<2882:TROIIT>2.0.CO;2](https://doi.org/10.1175/1520-0485(2002)032<2882:TROIIT>2.0.CO;2).
- , S. Stringer, C. Garrett, and D. Perrault-Joncas, 2003: The generation of internal tides at abrupt topography. *Deep-Sea Res. I*, **50**, 987–1003, [https://doi.org/10.1016/S0967-0637\(03\)00096-7](https://doi.org/10.1016/S0967-0637(03)00096-7).
- Timko, P. G., B. K. Arbic, J. G. Richman, R. B. Scott, E. J. Metzger, and A. J. Wallcraft, 2013: Skill testing a three-dimensional global tide model to historical current meter records. *J. Geophys. Res. Oceans*, **118**, 6914–6933, <https://doi.org/10.1002/2013JC009071>.
- Vallis, G. K., 2006: *Atmospheric and Oceanic Fluid Dynamics*. Cambridge University Press, 745 pp.

- Voisin, B., 1991: Internal wave generation in uniformly stratified fluids. Part I. Green's function and point sources. *J. Fluid Mech.*, **231**, 439–480, <https://doi.org/10.1017/S0022112091003464>.
- , 2007: Lee waves from a sphere in a stratified flow. *J. Fluid Mech.*, **574**, 273–315, <https://doi.org/10.1017/S0022112006004095>.
- Vosper, S. B., I. P. Castro, W. H. Snyder, and S. D. Mobbs, 1999: Experimental studies of strongly stratified flow past three-dimensional orography. *J. Fluid Mech.*, **390**, 223–249, <https://doi.org/10.1017/S0022112099005133>.
- Wessel, P., D. Sandwell, and S.-S. Kim, 2010: The global seamount census. *Oceanography*, **23**, 24–33, <https://doi.org/10.5670/oceanog.2010.60>.
- Zeiden, K. L., D. L. Rudnick, and J. A. MacKinnon, 2019: Glider observations of a mesoscale oceanic island wake. *J. Phys. Oceanogr.*, **49**, 2217–2235, <https://doi.org/10.1175/JPO-D-18-0233.1>.
- Zhang, L., M. C. Buijsman, E. Comino, and H. L. Swinney, 2017: Internal wave generation by tidal flow over periodically and randomly distributed seamounts. *J. Geophys. Res. Oceans*, **122**, 5063–5074, <https://doi.org/10.1002/2017JC012884>.