Energetics of Seamount Wakes. Part I: Energy Exchange

B. PERFECT

Mechanical Engineering, University of Washington, Seattle, Washington

N. KUMAR

Civil and Environmental Engineering, University of Washington, Seattle, Washington

J. J. RILEY

Mechanical Engineering, University of Washington, Seattle, Washington

(Manuscript received 1 May 2019, in final form 30 December 2019)

ABSTRACT

Seamounts have been theorized to act as “stirring rods,” converting barotropic flow into an unsteady wake, turbulence, and diapycnal mixing. The energetics of these processes are not well understood, but they may have implications for basin-scale mixing calculations. This study presents the results of a series of simulations for idealized seamounts in steady barotropic flow, with varying degrees of stratification and rotation. The kinetic energy within each simulation domain is decomposed into the mean kinetic energy, unsteady eddy energy, and turbulent kinetic energy; evolution equations are derived for each. Within the evolution equations, energy exchange terms arise, which relate the various forms of kinetic energy and potential energy. Key exchange terms, such as the rate at which the mean flow is converted into eddy energy, are compared across the Froude–Rossby parameter space. It is shown that the conversion terms associated with mesoscale motions are a function of the Burger number, which is consistent with a quasi-geostrophic flow regime. Conversely, conversion terms associated with turbulent processes scale with the product of the Froude and Rossby number. The amount of energy extracted from the mean flows suggests that wake effects may be significant for the parameter range and model assumptions studied. These results suggest that some seamounts may indeed act as oceanic stirring rods.

1. Introduction

Adequately modeling the effects of mixing processes in the ocean is vital for the study of climate (e.g., [MacKinnon et al. 2017](#)). The physics that drive ocean mixing are often quite subtle, despite accounting for a significant fraction of the global ocean–atmosphere energy budget. Ever since [Munk \(1966\)](#) remarked about the “missing” mixing in the ocean energy budget, the oceanographic community has been working toward identifying physical processes that contribute to the observed basin-averaged diapycnal diffusivity ([Ferrari et al. 2016](#)). One such process is the effect of bathymetry, particularly seamounts. Previous studies have highlighted the ability of seamounts

to extract energy from the barotropic flow in the form of internal gravity waves ([Nikurashin and Ferrari 2009](#); [Klymak et al. 2010](#)). The global topography-induced internal wave energy flux is estimated to be in the range of 200–750 GW, which is of a similar order of magnitude as the input of wind energy into the ocean ([Nikurashin and Ferrari 2011](#); [Wright et al. 2014](#)).

It has also been hypothesized that seamounts might act as “stirring rods” and contribute to water column mixing ([Munk and Wunsch 1998](#)). Yet, only recently there has been increased scientific interest in determining the wake dynamics associated with flow patterns past islands and seamounts, and associated energy extraction from the flow field (e.g., [Dong et al. 2007](#); [Gula et al. 2016](#); [Dong et al. 2018](#); [Zeiden et al. 2019](#); [MacKinnon et al. 2019](#)). Rare measurements of seamount wakes in the South China Sea reveal deep energetic eddies with a baroclinic structure ([Chen et al. 2015](#)). Long-term glider measurements of flow past an

Denotes content that is immediately available upon publication as open access.

Corresponding author: B. Perfect, bperfect@uw.edu

DOI: 10.1175/JPO-D-19-0105.1

© 2020 American Meteorological Society. For information regarding reuse of this content and general copyright information, consult the [AMS Copyright Policy](#) (www.ametsoc.org/PUBSReuseLicenses).

island in the North Pacific Ocean delineate energetic, ageostrophic events on the time scales from weeks to months and have been categorized using the Rossby number (Zeiden et al. 2019). Synoptic measurements of these island wakes indicate complex three-dimensional stratified flow (MacKinnon et al. 2019). Observations and numerical simulations of stratified flow past sloping headlands also suggest complex flow structure (McCabe et al. 2006; Canals et al. 2009; Callendar et al. 2011), while field studies aimed at mapping diapycnal diffusivity note enhanced mixing associated with topography and sloping boundaries (e.g., Toole et al. 1997; Naveira Garabato et al. 2011; Kunze et al. 2012). Yet, a characterization of the wake dynamics of seamounts in terms of varying stratification and Coriolis parameter is not well understood.

Wake dynamics due to interaction between stratified flow and three-dimensional obstacles have also been considered in laboratory studies. For example, Brighton (1978) and Vosper et al. (1999) report vigorous vortex shedding for 3D obstacles in strongly stratified flow in a laboratory, a result with obvious parallels to a large seamount in an ocean current. These two studies also served as experimental confirmation that when the obstacle's height significantly exceeds the vertical length scale set by the stratification (i.e., U/N , where U and N are the ambient current speed and buoyancy frequency, respectively), the flow predominantly moves around rather than over, and is mostly confined to horizontal planes (e.g., Riley et al. 1976). It is this tendency for the flow to contour around the obstacle that is responsible for the flow separation that enables eddy generation. These studies do not include Coriolis acceleration, however, which is a critical aspect of the seamount wake problem. It is possible to simulate flow past a seamount with both stratification and rotation using a rotating table (e.g., Boyer and Chen 1987; Codiga 1993), however experiments become increasingly difficult to conduct as the stratification and rotation increase. A state-of-the-art 10-m rotating table was recently used to evaluate the internal waves produced by an isolated seamount in an unprecedented parameter space of low Froude and Rossby numbers (Raja 2018). In this study, vortex shedding was also observed, but only treated qualitatively because the focus of the study was on internal wave generation.

Numerical modeling offers an opportunity for a quantitative investigation into seamount wakes; regional models have proven to be effective methods of reproducing observed flow patterns near real topography (Beckmann and Haidvogel 1997; Coutis and Middleton 2002; Dong and McWilliams 2007; Boutov et al. 2010). Traditionally, these models have been used within the context of understanding

the hydrodynamics of a specific site, but they have the potential to offer more insight into the underlying physics when used with idealized settings. Idealized regional models for flow past a mountain have been popular within the atmospheric sciences community, though typically the effect of stratification is much weaker than in the ocean and the Coriolis acceleration is usually neglected (e.g., Smolarkiewicz and Rotunno 1989; Schär and Smith 1993; Schär and Durran 1997; Epifanio and Rotunno 2005). Notably, Bauer et al. (2000) performed a series of idealized simulations, much in the same way as the present paper, except the parameters of interest were the Froude number and the aspect ratio of the mountain.

In a study of flow past a sloping headland, Canals et al. (2009) motivate a need to explore how flow past headlands and seamounts varies within the Froude–Rossby number parameter space. Zeiden et al. (2019) also stress working with a framework consisting of nondimensional parameters in order to facilitate comparisons between field observations and idealized numerical studies. Recently, Perfect et al. (2018) performed a series of simulations of an idealized seamount, demonstrating that vortex shedding patterns are strongly dependent on the Froude and Rossby number of the ambient flow. A similar set of wake regimes can be seen in simulations by Srinivasan et al. (2019). While Srinivasan et al. (2019) explores a similar Froude number regime, the Rossby number is held constant and the seamount is much less steep and shorter relative to the bottom boundary layer height.

In this study, simulations from Perfect et al. (2018) are further explored to address the broader question of whether seamounts can act as stirring rods, with a focus on Froude and Rossby number dependence. Particularly, we create a framework for evaluating the energy dynamics of flow past seamounts. Of particular importance is how oceanography-relevant terms such as the diapycnal flux, the rate of eddy generation, and the internal wave flux might be parameterized. Similar approaches based on using a set of energy equations to trace the conversion of energy between different forms have been used for realistic simulations in the Pacific Northwest (MacCready et al. 2009). The parameter space of study is motivated by the range of Froude and Rossby numbers that might be seen by a large seamount in an energetic abyssal current, but as we find in section 2c, this space is broadly applicable through dynamic similarity to a wide range of seamounts.

This article is organized as follows. In section 2, we describe the numerical model and the series of simulations that were conducted. We also offer a brief description of the simulation output, which is described in

further detail in Perfect et al. (2018). In section 3, we derive a formalism in order to separately describe the energetics associated with the mean flow, the unsteady eddy-scale flow, and the Reynolds-averaged turbulence. The resulting equations yield a set of energy exchange terms that represent important physical processes within the flow. This framework is then applied in sections 4a and 4b, which separately describe the various energy reservoirs, and the rate of energy transfer between them. We identify the terms that determine the leading-order dynamics within the domain and analyze their behavior within the Froude–Rossby parameter space in section 5. Implications for ocean mixing are discussed in section 6, followed by concluding remarks. Analysis of the flux of internal wave energy in these simulations, and comparison with the wake energetics, is reserved for a companion paper (Perfect et al. 2020).

2. Numerical model

a. Model configuration

The analysis in this paper is based on a series of simulations conducted using an idealized domain within the Regional Ocean Modeling System (ROMS) framework (Shchepetkin and McWilliams 2005). ROMS solves the 3D primitive equations, using the Boussinesq and hydrostatic approximations. Evolution equations for the turbulent kinetic energy k (TKE) and dissipation rate ϵ are simulated using the k – ϵ realization of the generic length scale closure (Umlauf and Burchard 2003; Warner et al. 2005). The idealized domain consists of an axisymmetric 3500-m Gaussian mountain with a full width at half maximum (FWHM) of 23.4 km surrounded by a flat bottom, as represented in Fig. 1. These dimensions are approximately modeled after those of Fieberling guyot (Brink 1995). The dimensions of the domain are 180 km in the streamwise direction, 120 km in the cross-stream direction, and with a maximum depth of 5000 m. It is discretized with 1/3-km horizontal resolution and 80 vertical terrain-following levels, such that the effective grid size is $540 \times 360 \times 80$. The vertical grid stretching function is chosen to enhance resolution at the bottom and middle of the water column, where the most important dynamics occur.

The boundary layer is modeled using a quadratic drag law with a constant drag coefficient of 2×10^{-3} , which is consistent with the best practice for regional models established by Arbic and Scott (2008). Turbulence closure is achieved via the k – ϵ realization of the generic length scale model (Warner et al. 2005; Umlauf and Burchard 2003), but the minimum (specific) turbulent kinetic energy k is reduced from the model default of

FIG. 1. Schematic of the ROMS model setup. A Gaussian seamount is subjected to a uniform inlet flow with spatially uniform N and f .

7.6×10^{-6} to $10^{-10} \text{ m}^2 \text{ s}^{-2}$. This modification is vital for resolving the TKE trends and also limits the background erosion of the stratification to act on a much longer time scale than the residence time of the flow in the domain. To achieve model stability, a small horizontal pseudo-viscosity ν_T was added; the seamount Reynolds number based on this viscosity was approximately 2000. The viscous term thus acts as a subgrid-scale term that absorbs the horizontal, downscale transfer of energy and stabilizes the numerical scheme. Sensitivity studies with respect to grid size, drag coefficient, and viscosity were carried out to ensure that the results were robust. Isopycnal mixing is produced by horizontal advection and by the explicit viscosity and implicit (numerical) viscosity in the numerical scheme; shear-produced diapycnal mixing is governed by the turbulence model (Buijsman et al. 2012) and is our primary focus for mixing in this study.

The model is forced with a uniform eastward flow at the western boundary of $U = 0.1 \text{ m s}^{-1}$. The northern and southern boundaries use open-type boundary conditions, intended to radiate out disturbances. The eastern boundary uses a sponge layer to produce a nearly uniform outflow of $U = 0.1 \text{ m s}^{-1}$. The model uses f -plane rotation with magnitude f , and a linear density profile, such that a single value of N characterizes the background stratification. The vertically uniform buoyancy frequency and velocity profiles are mostly characteristic of the Southern Ocean, but many abyssal seamounts in other parts of the ocean will also experience nearly uniform N and U (e.g., King et al. 2012; Balwada et al. 2016). Common features such as surface intensification of currents and a thermocline are

excluded; while these confounding factors will certainly interact with near-surface seamounts and islands, our aim is to explore the types of mechanisms in seamount wakes rather than the exact features. The free surface boundary conditions enforce a north–south tilt that maintains a geostrophic balance with eastward flow. No other atmospheric or boundary forcing was considered. Each simulation was spun up for approximately 60 days, and then run for an additional 80 days.

b. Model parameterization

The model can be parameterized with six quantities: the seamount height H and FWHM D , the Coriolis parameter f , the Brunt–Väisälä frequency N , the inlet velocity U , and the pseudoviscosity ν_T . A natural set of dimensionless groups to describe this model are the Reynolds number ($Re \equiv UD/\nu_T$), the Froude number ($Fr \equiv U/NH$), the Rossby number ($Ro \equiv U/fD$), the seamount aspect ratio (H/D), and the depth ratio (of the seamount height to the water depth). The depth ratio has been demonstrated to be a controlling parameter in determining the vertical extent of the eddies (Boyer and Chen 1987; Srinivasan et al. 2019); here it is maintained at a constant value of 0.6, which tends to limit the formation of Taylor column structures that exceed the height of the seamount. The Reynolds number is approximately 2000, which, in line with Dong et al. (2007), may be assumed to faithfully allow the representation of the eddy processes in a Reynolds-number-independent manner. The Froude and Rossby numbers are manipulated by changing the Brunt–Väisälä frequency and Coriolis parameter, respectively. Eighteen simulations spanning a range of $2 \times 10^{-4} < N < 2 \times 10^{-3} \text{ s}^{-1}$ and $2 \times 10^{-5} < f < 8 \times 10^{-5} \text{ s}^{-1}$ ($0.014 < Fr < 0.14$ and $0.053 < Ro < 0.21$) were conducted. The set of simulations can be visualized in Fig. 3, and the results are briefly explained in section 2d.

c. Relevance to the ocean

Site-specific studies of seamounts and island wakes demonstrate that flow patterns can be highly variable and sensitive to the spatial and temporal fluctuations of velocity and density fields (Caldeira and Sangrà 2012; Zeiden et al. 2019; MacKinnon et al. 2019). In considering the vast majority of seamounts for which high-resolution data are not known, let alone reliable bathymetry, tailored studies are not practical. Instead, it is useful to consider simple parameterizations, as in this study, to provide first order estimates of the wake energetics based on parameters that can be readily obtained (i.e., the Froude and Rossby number).

A database of nearly 25 000 seamounts by Kim and Wessel (2011) provides a representative sample of

FIG. 2. A seamount database from Kim and Wessel (2011) was combined with HYCOM simulated mean climatology (2008–10) to produce estimates of bulk Froude and Rossby numbers for each seamount. A heat map of seamounts in Froude–Rossby number space for which $U > 2 \text{ cm s}^{-1}$ is presented here, with the present area of study overlaid (black rectangle). Through dynamic similarity, the current simulations can be used to analyze eddy structures at a range of ocean-relevant values of the Froude and Rossby number.

global seamounts, including their locations and approximate sizes. We combine this with velocity, temperature, and salinity fields from HYCOM climatology in order to produce estimates for a bulk Froude and Rossby number associated with each seamount. Figure 2 presents a mapping of this database to Froude–Rossby space, with the parameter space of the present study overlaid as a black box. Because abyssal currents can be arbitrarily small (and therefore the Froude and Rossby number can also be arbitrarily small), we have only included seamounts for which $U > 2 \text{ cm s}^{-1}$. Figure 2 paints a picture of a diverse parameter space, even while distilling the complexities of any particular seamount into bulk parameters.

While the current study only considers one seamount height and width, and one current speed, the results are much more general than for these specific conditions. Through dynamic similarity, the Froude and Rossby number dependence should be independent of the seamount size and current speed, as long as the key modeling assumptions are consistent. Under these assumptions, the effects of seamount size and current speed would be governed by scaling laws dictated by the energy rate of mean flow incident upon the seamount.

The modeling assumptions necessary for dynamic similarity are that (i) the flow structure is Reynolds number independent, (ii) the effect of the ocean surface is not important, and (iii) the effect of the bottom boundary layer is the same for the cases considered.

FIG. 3. Different shedding regimes are observed, determined by the Burger number of the associated simulation. At sufficiently high Bu , vortices become vertically incoherent, resulting in the “decoupled” shedding regime (top left). Vertically coherent vortex shedding (bottom right) is characteristic of wakes in other vortex shedding literature.

Condition i was convincingly demonstrated by [Dong et al. \(2007\)](#) by considering seamount flows with varying Re . Condition ii requires that the seamount height is no more than about 60% of the depth of the ocean, as described above in [section 2b](#). Finally, condition iii requires that the seamount height dominates the size of the bottom boundary layer, ensuring that the large-scale flow structure is not significantly changed by the presence of a boundary layer. Under these constraints, our findings within the Froude–Rossby parameter space can be applied to seamounts with varying f , N , U , D , and H .

d. Description of simulation results

In [Perfect et al. \(2018\)](#), vortex shedding (see [Fig. 3](#)) was observed in the lee of the seamount in each of the simulations. The vortex structures primarily fell under one of two regimes: tall, vertically coherent eddies that spanned the entire height of the seamount, and vertically incoherent eddies that behaved like isolated layers of flow past a cylinder. The transition between coherent and incoherent eddies is governed approximately by a critical Burger number [$Bu \equiv (Ro/Fr)^2 = (NH/fD)^2$] that we label as Bu_{crit} . The critical Burger number appears to be between 2 and 12, accounting for simulation data and theoretical arguments. For sufficiently strong stratification (low Fr) the vertical alignment produced by the rotation is overcome by the density stratification and the vortices become decoupled. Physically, the regime shift is related to the relative size of the baroclinic

radius of deformation ($R_d \equiv NH/\pi f$; [Gill 1982](#)) and the seamount diameter. Eddies are locked into a horizontal length scale governed by R_d , until R_d exceeds the seamount size, at which point the eddies behave similarly to a 2D von Kármán wake based on the local seamount diameter. These conclusions are supported by high-resolution modeling of an equatorial island (i.e., a high- Bu environment) showing strongly three-dimensional eddy patterns at length scales much smaller than the baroclinic deformation radius ([Simmons et al. 2019](#)).

3. Equations of motion

The present simulations have a uniform inflow of kinetic energy, which is then transformed, by interaction with the bathymetry, into eddies, internal waves, turbulence, and related potential energy. To trace the pathways between these forms of energy, we derive a set of energy equations that include exchange terms between these different forms. These relationships are presented schematically in [Fig. 4](#).

a. Definition of velocity decomposition

In the output of the ROMS simulations the flow is divided between the spatially and temporally resolved motions and the smaller-scale, higher-frequency turbulence. The unresolved turbulence is represented in ROMS by the evolution equations for k and ϵ and associated modeling. The resolved motions may be further divided into steady and unsteady components, using a

FIG. 4. Qualitative view of the different energy transfer terms. The mean, eddy, and turbulent kinetic energy each have exchange terms with each other. Meanwhile, each scale of kinetic energy interacts with potential energy and dissipation. The mean flow can produce steady lee waves that radiate out of the domain, and the unsteady wake may also produce internal wave radiation.

time average over the entire simulation. The ensemble average imposed by the turbulence model, and the simulation time average naturally divide the flow into three components: the time mean flow, unsteady but resolved motions such as eddies and internal waves, and the turbulence. We take advantage of this natural scale separation to analyze our simulations using a three-tiered velocity decomposition. Such a decomposition has been previously been utilized by Phillips (1966) in a study of turbulence in a periodic surface gravity wave field and by Hussain and Reynolds (1970) in the study of a periodic wave in a turbulent shear flow. Here, we extend these previous three-tiered decompositions by deriving the appropriate equations for geophysical flow and further developing the terms that arise.

Kang et al. (2016) studied the time mean and Reynolds-averaged unsteady components of a ROMS simulation; we include the terms studied here, but also consider dynamically important terms that arise from the three-tiered equations. These additional terms represent the effects of the turbulence, and significantly, its interaction with the resolved scales. While the turbulence itself is unresolved, it still has a leading order effect on the dynamics, and its effects can be estimated from the Reynolds-averaged quantities that are computed by the RANS model.

The three-tiered decomposition utilized here is defined by two types of averages with the following notation: $\langle(\cdot)\rangle$, the traditional Reynolds average used in turbulence modeling that is associated with the simulation time step, and $\overline{(\cdot)}$, a time average over a time much greater than that of the large-scale unsteady, eddying motions. In the context of modeled flow past a seamount, take u_i to be the total velocity of the flow, $U_i = \langle u_i \rangle$ to represent the Reynolds-averaged velocity,

that is the output of the ROMS code that includes a RANS model, and u'_i to represent the turbulent velocity that is averaged out by the Reynolds averaging. Note that u'_i is not computed by ROMS, but its averaged kinetic energy $k = 1/2 \langle u'_i u'_i \rangle$ is computed by ROMS. Then $u_i = U_i + u'_i$. The term U_i may be further decomposed into a time average over many Strouhal periods $\overline{U}_i$, which is time independent, and the unsteady, eddy-scale motion $\hat{u}_i$, such that $U_i = \overline{U}_i + \hat{u}_i$. The motion at these three scales must sum to the total velocity u_i , so

$$u_i = U_i + u'_i = \overline{U}_i + \hat{u}_i + u'_i. \quad (1)$$

The same decomposition holds for all field variables, including the reduced pressure, $\phi = p/\rho_0$ and the buoyancy $b = g[(\rho - \rho_0)/\rho_0]$ (where ρ_0 is the fixed Boussinesq density):

$$b = B + b' = \overline{B} + \hat{b} + b', \quad \phi = \Phi + \phi' = \overline{\Phi} + \hat{\phi} + \phi'. \quad (2)$$

b. Derivation of the momentum equations

Here we apply the triple decomposition to obtain equations for $\overline{U}_i$, $\hat{u}_i$, and u'_i , which will then be used to derive the energy equations for the time-averaged flow, for the unsteady motions related to the wake, and for the turbulence. We start with the Navier–Stokes equations subject to the Boussinesq approximation for a rotating, stratified fluid, represented as

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial \phi}{\partial x_i} + 2\nu \frac{\partial s_{ij}}{\partial x_j} - 2\epsilon_{ijk} \Omega_j u_k - b\delta_{i3}, \quad (3)$$

where $2\mathbf{\Omega} \times \mathbf{U} = 2\epsilon_{kij}\Omega_j u_k$ is the Coriolis force. We also identify the strain rate $s_{ij} \equiv 1/2[(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)]$.

The equations for U_i and u'_i are first obtained from Reynolds averaging Eq. (3):

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial \Phi}{\partial x_i} + 2\nu_T \frac{\partial S_{ij}}{\partial x_j} - 2\epsilon_{ijk}\Omega_j U_k - B\delta_{i3} - \frac{\partial R_{ij}}{\partial x_j}, \quad (4)$$

$$\underbrace{\frac{\partial u'_i}{\partial t} + U_j \frac{\partial u'_i}{\partial x_j}}_{5,6} = -\frac{\partial}{\partial x_j} \left(\underbrace{\phi' \delta_{ij}}_7 + \underbrace{u'_i u'_j}_7 - 2\nu s'_{ij} - R_{ij} \right) - \underbrace{u'_j \frac{\partial U_i}{\partial x_j}}_{8,9} - 2\epsilon_{ijk}\Omega_j u'_k - b'\delta_{i3}, \quad (5)$$

where $R_{ij} = \langle u'_i u'_j \rangle$ is the Reynolds stress. Note at this point ν has been replaced by ν_T in Eq. (4) for U_i , as discussed in section 2b. Equation (4) is then split into the steady and the unsteady, eddy components in the following way. The equation for $\bar{U}_i$ can be obtained by time averaging Eq. (4), giving

$$\underbrace{\frac{\partial \bar{U}_i}{\partial t}}_{=0} + \bar{U}_j \underbrace{\frac{\partial \bar{U}_i}{\partial x_j}}_1 = -\frac{\partial}{\partial x_j} (\bar{\Phi} \delta_{ij} - 2\nu_T \bar{S}_{ij} + \bar{R}_{ij} + \bar{T}_{ij}) - 2\epsilon_{ijk}\Omega_j \bar{U}_k - \bar{B} \delta_{i3}. \quad (6)$$

Here it has been convenient to define the eddy component of the Reynolds stress as $T_{ij} = \hat{u}_i \hat{u}_j$, and then decompose both R_{ij} and T_{ij} in terms of their time-averaged and unsteady components as

$$R_{ij} = \bar{R}_{ij} + \hat{R}_{ij}, \quad T_{ij} = \bar{T}_{ij} + \hat{T}_{ij}. \quad (7)$$

Finally, writing Eq. (4) in terms of the decomposition $U_i = \bar{U}_i + \hat{u}_i$, and then subtracting Eq. (6) gives the equation for $\hat{u}_i$:

$$\underbrace{\frac{\partial \hat{u}_i}{\partial t} + \bar{U}_j \frac{\partial \hat{u}_i}{\partial x_j}}_2 = -\frac{\partial}{\partial x_j} \left(\underbrace{\hat{u}_i \hat{u}_j}_3 + \hat{\phi} \delta_{ij} - 2\nu_T \hat{s}_{ij} + \hat{R}_{ij} - \bar{T}_{ij} \right) - \underbrace{\hat{u}_j \frac{\partial \bar{U}_i}{\partial x_j}}_4 - 2\epsilon_{ijk}\Omega_j \hat{u}_k - \hat{b} \delta_{i3} \quad (8)$$

Note that the original Eq. (3) can be recovered by summing the above three equations. The decompositions

in b , ϕ , s_{ij} , and the time and rotation terms are recovered trivially. The nine advective acceleration terms originating from the triple decomposition have been indicated with underbrace numbering and, when summed together, they are equal to the original advection term $u_j(\partial u_i/\partial x_j)$. The Reynolds stress terms cancel, using Eq. (7). Therefore, this trio of equations represents the momentum equation, decomposed into equations for the three components of motion that we are considering.

c. Derivation of the energy equations

To obtain the energy equations for these three components of the motion, we multiply the equation in $\bar{U}_i$ with $\bar{U}_i$, the equation in $\hat{u}_i$ with $\hat{u}_i$, and the equation in u'_i with u'_i . Then, to obtain the overall energy exchanges, the equations are volume integrated and time averaged, producing a balance between surface flux terms and volumetric energy conversion terms. These conservation equations are as follows:

Mean flow kinetic energy (MKE = $1/2\bar{U}_i\bar{U}_i$):

$$\int_S \left(\frac{1}{2} \bar{U}_j \bar{U}_i^2 + \bar{\Phi} \bar{U}_j - 2\nu_T \bar{U}_i \bar{S}_{ij} + \bar{U}_i \bar{R}_{ij} + \bar{U}_i \bar{T}_{ij} \right) n_j dS \\ = \int_V \left(\underbrace{\bar{R}_{ij} \frac{\partial \bar{U}_i}{\partial x_j}}_{-MT} + \underbrace{\bar{T}_{ij} \frac{\partial \bar{U}_i}{\partial x_j}}_{-ME} - \underbrace{2\nu_T \bar{S}_{ij} \bar{S}_{ij}}_{MV} - \underbrace{\bar{B} \bar{U}_3}_{MB} \right) dV. \quad (9)$$

Eddy flow kinetic energy (EKE = $1/2\hat{u}_i\hat{u}_i$):

$$\int_S \left(\frac{1}{2} \bar{U}_j \hat{u}_i^2 + \hat{\phi} \hat{u}_j - 2\nu_T \hat{u}_i \hat{s}_{ij} + \hat{u}_i \hat{R}_{ij} \right) n_j dS \\ = \int_V \left(\underbrace{-\bar{T}_{ij} \frac{\partial \bar{U}_i}{\partial x_j}}_{-ME} + \underbrace{\hat{R}_{ij} \frac{\partial \hat{u}_i}{\partial x_j}}_{-ET} - \underbrace{2\nu_T \hat{s}_{ij} \hat{s}_{ij}}_{EV} - \underbrace{\hat{b} \hat{u}_3}_{EB} \right) dV. \quad (10)$$

Turbulent kinetic energy (TKE = $1/2\langle u'_i u'_i \rangle$):

$$\int_S \left(\bar{U}_j k + \langle \phi' u'_j \rangle - 2\nu \langle u'_i s'_{ij} \rangle \langle u'_i s'_{ij} \rangle + \frac{1}{2} \langle u'_j u'^2 \rangle \right) n_j dS \\ = \int_V \left(\underbrace{-\bar{R}_{ij} \frac{\partial \bar{U}_i}{\partial x_j}}_{-MT} - \underbrace{\hat{R}_{ij} \frac{\partial \hat{u}_i}{\partial x_j}}_{-ET} - \underbrace{2\nu \langle s'_{ij} s'_{ij} \rangle}_{TV} - \underbrace{\langle b' u'_3 \rangle}_{TB} \right) dV. \quad (11)$$

Here the viscous and pseudoviscous terms are evaluated as, for example (see [Kundu et al. 2004](#)),

$$2\nu u'_i \frac{\partial s'_{ij}}{\partial x_j} = 2\nu \frac{\partial(u'_i s'_{ij})}{\partial x_j} - 2\nu s'_{ij} s'_{ij}, \quad (12)$$

where $\epsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$ is the kinetic energy dissipation rate.

On the right-hand sides of Eqs. (9)–(11), every term represents either viscous or pseudoviscous dissipation, buoyant exchange with potential energy, or an exchange with one of the other energy components. Each term is labeled according to the forms of energy that are being exchanged: M is MKE, E is EKE, T is TKE, V is (real or pseudo) viscous dissipation, and B is potential energy (buoyancy). MB, for example, describes the rate of energy conversion from the mean flow into potential energy. ME, MT, and ET each appear twice in the above equations; because they represent exchange processes among the different scales of kinetic energy, they appear as conjugate pairs that do not alter the total kinetic energy. All of the labeled terms are active throughout the domain, and their role is sketched in [Fig. 4](#). The left-hand-side terms are accounted as energy fluxes through and rates of work at the domain boundaries. The first term on the left-hand side of each of the above equations pertains to the advective flux of kinetic energy at the scale of interest. The second term pertains to pressure work and internal wave radiation. The third term is the viscous or pseudoviscous work being done at the boundary of the domain. The fourth term in [Eq. \(11\)](#) is flow of turbulent kinetic energy through the domain surfaces due to the turbulence. The remaining left-hand-side terms do not have well-established physical interpretations. In general, the net right-hand-side conversions should be mostly balanced by the pressure work and the flux of kinetic energy, but the large open boundaries and deep domain results in very large uncertainties in the pressure work, preventing a proper budget closure (see also [MacCready et al. 2009](#)). The volumetric conversion terms, however, do not suffer from such uncertainty; several terms such as ME, MB, and EB have been accurately computed from ROMS in other energy conversion studies ([Dong et al. 2007](#); [Kang and Curchitser 2015](#)).

Note that in Eqs. (9) and (10) the model pseudoviscosity has been used. At these scales of motion, MV and EV represent the loss of energy into the subgrid scale. Mathematically, these terms remove energy from the simulation, though physically they represent a combination of the downscale energy cascade, plus losses to molecular dissipation. Note that the terms in Eqs. (9)–(11) given by Reynolds averaging, which represent averaging over the fluctuating turbulent motions, are

modeled in ROMS from the k - ϵ generic length scale closure. In this sense, the turbulent conversion terms have modeled representations that are accurate to the same degree as the simulations for which they are computed.

4. Results

The decompositions defined by Eqs. (1) and (2) are applied to the field variables in the output of the 18 simulations outlined in [Fig. 3](#). Then, the energy conversion terms in Eqs. (9)–(11) are computed. To mitigate any spurious boundary effects, the volume over which the conversion terms are analyzed sits fully within the horizontal boundaries of the computational domain. The outer 10 km (30 cells) of the sides (north and south in the model), 13.3 km (40 cells) of the inlet, and 33.3 km (100 cells) of the outlet are excluded. The seamount itself, as well as any upstream effects of the seamount, are included in the analysis domain. Below we present the time-averaged, volume-integrated energy reservoir and energy conversion results for the array of simulations outlined in [Fig. 3](#). We refrain from nondimensionalizing our results, as we conclude that it is easier to understand them in dimensional form. The results can be easily nondimensionalized, however, using the characteristic velocity and length scales from [section 2b](#).

a. Reservoir term evaluation

Neglecting thermal energy produced by dissipation, the energy within the domain exists as either kinetic or potential energy. The kinetic energy may be further partitioned using Eqs. (9)–(11) into the MKE, EKE, and TKE. The potential energy quantity of interest is the available potential energy (APE); this quantity is not decomposed like the kinetic energy but treated as a single value. The kinetic energies are computed as volume-integrated time series:

$$\text{MKE}(t) = \int_V \rho_0 \frac{1}{2} \overline{U_i(\mathbf{x}, t)} \overline{U_i(\mathbf{x}, t)} dV, \quad (13)$$

$$\text{EKE}(t) = \int_V \rho_0 \frac{1}{2} \hat{u}_i(\mathbf{x}, t) \hat{u}_i(\mathbf{x}, t) dV, \quad \text{and} \quad (14)$$

$$\text{TKE}(t) = \int_V \rho_0 [k(\mathbf{x}, t)] dV. \quad (15)$$

Because the specific turbulent kinetic energy k is directly output from ROMS, we volume integrate this quantity directly rather than the turbulent velocities themselves, which are not computed. The APE is computed via the sorting method established by [Winters et al. \(1995\)](#). It should be noted that, because the analysis domain contains an inflow and outflow, the time series

FIG. 5. Energy reservoirs are divided into mean kinetic energy, eddy kinetic energy, turbulent kinetic energy, and available potential energy. Each quantity is represented as a boxplot of the results of the 18 simulations.

for the APE does not evolve as for a closed system. Fluctuations in APE may be due to baroclinic processes and irreversible mixing, but also boundary fluxes.

Each time series for an energy reservoir tends to oscillate about a mean, which is reported as a bulk value for each simulation. In Fig. 5 the range of bulk reservoir values over all simulations are compiled in the form of a boxplot for each energy reservoir. Each boxplot, then, contains a dataset of 18 time-averaged reservoir values. The middle two quartiles of the data fall within the box, whose central line is the median, and the nonoutlier extremes are indicated by the bars above and below the box. Outliers are displayed as individual red markers. Because the majority of the flow within the domain is unperturbed by the seamount, most of the energy exists as MKE, and there is very little intersimulation variability for this reservoir. The EKE and APE contain similar amounts of energy, approaching 10% of the MKE, with the EKE containing somewhat more. The TKE is several orders of magnitude below the MKE. The energy-containing submesoscale motions are largely resolved by the simulations, and thus do not need to be included in the turbulence modeling. We note that there is approximately a decade of variability in the APE and EKE between simulations, and the TKE contains nearly two decades of variability. These intersimulation differences will be discussed in detail in section 5a.

b. Conversion term evaluation

The volumetric terms labeled in Eqs. (9)–(11) represent rates of conversion between different forms of energy. The exchange terms (ME, MT, ET) represent the

transmission of energy between the mean flow, the eddies, and the turbulence; the buoyancy terms (MB, EB, TB) represent the rate of production of potential energy through buoyancy; MV and EV represent energy losses from the mean flow and the eddies into the sub-grid scale; and TV represents losses from turbulence into heat due to turbulent dissipation. These terms each exhibit complex spatial and temporal patterns, which might be related to the emergence of various fluid instabilities (as studied by Dong et al. 2007; Srinivasan et al. 2019), but such a treatment is beyond the scope of this work. For the current study, we restrict our analysis to the volume-integrated, time-averaged conversion terms, and assess the aggregate spatial distribution between the bottom boundary layer (BBL) and the free stream. The intent here is to relate the bulk energetic quantities of interest to the underlying control parameters and establish useful relationships for future parameterization. It is necessary to separate out the BBL because for many terms [e.g., TV, in agreement with MacCready et al. (2009)] nearly all of the conversion is concentrated in the BBL, and these processes are not significantly affected by the presence of the seamount. Each of the terms are therefore split into two components: the BBL contribution, which is defined to be the bottom 100 m of the domain, and the free stream contribution, comprising the remainder of the domain.

A summary of the conversion terms is presented in Fig. 6. Each boxplot represents the distribution of the time-averaged conversion for all simulations, as was done with Fig. 5. Each panel contains the volumetric conversion terms that appear in the MKE (Fig. 6a), EKE (Fig. 6b), and TKE (Fig. 6c) equations. Terms are presumed to be net sinks in their respective kinetic energy equation. Terms that act as sources of energy are indicated with a negative on the abscissa label. The exchange terms each appear twice, once as a source, and again as a sink in the conjugate scale of motion. The dominant term in both the mean and eddy equations is ME, representing the massive energy transfer from the mean flow into unsteady vortices. The TKE equation is dominated by the BBL partition of TV, which is numerically consistent with the formulation of MacCready et al. (2009), which finds that $TV \approx \rho_0 C_D |u|^3$. The eddy losses to the subgrid scale and to buoyancy (EV and EB) are both significant terms, which will also be discussed in greater detail in section 5b.

5. Energetics scaling

It is desirable to evaluate how the energetics terms discussed in the previous section depend on rotation and stratification. Of particular importance, in addition to

FIG. 6. The magnitudes of the various energy conversion terms are presented as boxplots of the average values from the 18 simulations. Each conversion term is split into the component that occurs in the boundary layer and the free stream component.

the reservoir terms, are the most significant pathways by which energy is removed from the mean flow (ME and MT), and removed from the eddy motions (EV and EB). These latter terms govern the leading-order behavior that describes the stirring and mixing that takes place in the lee of the seamount. While this analysis cannot prescribe a universal parameterization, it can suggest how the effects of rotation and stratification might enter such a parameterization. To our knowledge, this is the first parametric analysis of these terms.

a. Reservoir term scaling

The energy reservoirs for each simulation are plotted in Fig. 7. Symbols represent the time mean of that reservoir for a particular simulation, and the error bars denote the associated standard deviation of the time series. The different symbols/colors indicate groupings of simulations with the same Coriolis parameter. The EKE data, given in Fig. 7a, approximately collapse when plotted against the Burger number, $(NH/fD)^2$. The two strongest stratification cases, for $f = 8 \times 10^{-5} \text{ s}^{-1}$, appear

to be outliers; otherwise, the EKE grows with Bu until it saturates near Bu_{crit} and then remains at a constant maximum value. We believe that this behavior is due to the eddy size scaling observed by Perfect et al. (2018) and Srinivasan et al. (2019); below Bu_{crit} , the eddies generally do not exceed R_d , the baroclinic deformation radius, which scales with $\sqrt{N/f} \propto \sqrt{Bu}$. However, above Bu_{crit} , the eddy length scale is determined by the seamount size, which manifests as an upper bound on eddy size, limiting the growth of EKE for $Bu > Bu_{\text{crit}}$. The dashed line indicates a $Bu^{1/2}$ scaling, which matches well with the data. These findings establish a simple scaling law for the energy contained in the wake of a seamount of a given size and ambient velocity, but perhaps more importantly appear to impose an upper bound on that energy.

Figure 7b plots the APE versus the Burger number. Again, the data approximately collapse, in this instance forming a peak near Bu_{crit} . In its linearized form, the APE scales as the square of the product of the buoyancy frequency and the isopycnal displacement. The isopycnal displacement is governed by both the rotation and the stratification; at very high stratification the isopycnal displacement is suppressed. With strong rotation, vertically coherent eddies appear to limit the growth of displacements. We find that the critical Burger number provides the optimum balance between rotation-dominant and stratification-dominant systems; the point where vortices decouple (see Fig. 3) corresponds to the maximum APE. While the existence of a maximum APE is a straightforward consequence, its value and association with Bu_{crit} are novel results. The falloff in APE at high-Bu scales approximately as $Bu^{-1/2}$, or equivalently, f/N .

The TKE, in contrast with the EKE and APE, scales poorly with Bu, as demonstrated in Fig. 7c. When plotted against $Fr Ro$ as in Fig. 7d, however, the three lines at various rotation values collapse. The collapsed data suggest that the TKE reservoir scales like $(Fr Ro)^{3/2}$. This relationship seems to be robust across most of the parameter space, except possibly for very low Burger number. Unlike the EKE and APE, the TKE scaling does not change significantly near Bu_{crit} . This result is consistent with the idea that both rotation and stratification act to suppress 3D turbulence (Praud et al. 2006), but somewhat surprising given that the eddies that provide the energy input for the turbulence obey a very different scaling. We conclude that the TKE is approximately independent of the eddy structure and appears to have no memory of the geometry of the seamount. Rather, the TKE is determined by the more fundamental properties of geophysical turbulence. An alternate interpretation

FIG. 7. Suggested scalings for the (a) EKE, (b) APE, and (c), (d) TKE. Each marker corresponds to the time mean of a single simulation, with error bars indicating the standard deviation of the time series. Shaded regions indicate a range for the critical Burger number and dashed lines offer possible scaling relationships as indicated in the legends.

of the $Fr Ro$ scaling is possible if the Froude number is redefined in terms of a bulk Richardson number. Consider U/H to be a measure of the bulk shear caused by the seamount. Then,

$$Fr = \frac{U/H}{N} = Ri^{-1/2}, \quad (16)$$

and

$$Fr Ro = Fr^2 \frac{Ro}{Fr} = Ri^{-1} Bu^{1/2}. \quad (17)$$

This relationship establishes a Burger number dependence for the TKE, but also indicates that the TKE depends on the inverse Richardson number. With increased stratification, the bulk Richardson number is increased, which reduces turbulence generated from flow shear.

b. Conversion term scaling

The pathways that describe how the mean flow is converted into unsteady and turbulent motions, ME and MT, are plotted in Fig. 8. The shape and scaling for the ME conversion term (Fig. 8a) is nearly identical to that of the EKE scaling in Fig. 7a. In agreement with Dong et al. (2007), we find that most of the conversion happens within one diameter of the seamount. A significant and novel consequence of this finding is that the upper bound on the EKE is imposed upon the eddies at their time of creation, and not as a result of the rotation and stratification acting on the eddies as they advect.

The MT term (Fig. 8b) shares many of its characteristics with the TKE reservoir (Fig. 7d). The curves for both terms collapse when plotted against the product of the Froude and Rossby number. The suggested scaling line, however, is steeper for the MT conversion $[(Fr Ro)^2]$

FIG. 8. Scaling for (a) the mean to eddy energy conversion and (b) the mean to turbulent energy conversion.

than for the TKE reservoir $[(Fr\ Ro)^{3/2}]$. We attribute this difference to the fact that the TKE equation contains several terms of leading order importance (see Fig. 6) and encompasses the effects of a variety of physical processes. MT, however, is driven purely by shear in the mean flow. The nature of the scaling for MT suggests that the primary driver for this term has no memory of the large-scale wake structure, and instead depends on the properties of rotating, stratified turbulence.

The largest sinks in the eddy kinetic energy equation are the losses to the model pseudoviscosity (EV) and buoyancy (EB). EV (see Fig. 9a) exhibits small error bars, indicating that the EV contains very little temporal variation. The time mean values largely fall in a narrow band around 5×10^6 W; only three data points fall more than 40% away from this value. In general, stronger rotation always produces a lower conversion value, but the dependence on stratification is much more complex, especially for low Bu. No nondimensional scaling was found that could sufficiently collapse the data, and so the Burger number is used as the ordinate. We attribute the lack of a viable scaling to the wide range of physical processes that contribute to this term. The horizontal gradients that contribute to the EV conversion are active both near to the seamount, where a large local Rossby number contributes to centrifugal instability (e.g., Srinivasan et al. 2019), and in the far-field wake, where eddies may decay by instability or directly by viscosity. Furthermore, the viscosity in ROMS, which is a combination of numerical viscosity and an explicit pseudoviscosity that acts as a subgrid-scale viscosity, necessarily models subgrid-scale dynamics. The effect of

this term, then, is likely influenced by the modeled pseudoviscosity, which is different from the real viscosity in an oceanographic context. In sum, we find that this term is highly complex, and likely requires further study in specific regions of the wake, and with higher-fidelity codes.

The buoyancy term, EB (Fig. 9b), contains significantly more temporal variability than the EV term, as indicated by the large error bars. This term may also be referred to as the baroclinic conversion rate, in the sense of eddy energy being generated via instability from the potential energy reservoir (Dong et al. 2007). However, we find that, over the domain, the EB term is positive, representing a net production of potential energy from the EKE reservoir. This term includes all effects of unsteady vertical advection of a density anomaly, encompassing eddy motions as well as internal waves that may or may not leave the analysis domain. The generation of buoyancy tends to happen in bursts, resulting in significant uncertainty in the time series. However, there appears to be a broad peak in the EB conversion that encompasses Bu_{crit} . The chaotic regime associated with the initiation of eddy decoupling forms conditions that are particularly amenable to the generation of potential energy.

6. Discussion

a. Quasigeostrophy

The prevalence of Burger number scalings for quantities such as the EKE, ME conversion, and EB conversion suggests that the eddy processes can be parameterized, at least partially, by the Burger number. It follows that the

FIG. 9. Scaling for (a) the eddy to viscosity conversion and (b) the eddy to potential energy conversion.

Burger number should enter into the leading-order dynamics that govern flow past a seamount. We find that this hypothesis can be satisfied if the flow past the seamount can be considered to be approximately quasigeostrophic (QG). While QG eddies are well studied (e.g., Stegner and Dritschel 2000; Dritschel and De La Torre Juárez 1996), it has not been previously established how the energetics of topographically generated eddies relate to quasigeostrophy. When properly nondimensionalized, the QG equations depend only on the Burger number as the sole nondimensional parameter (Pedlosky 1987, Eq. 6.8.11). This relationship is often obscured through the use of dimensional variables, or by the use of alternate names or representations for the Burger number. The Burger number has variously been expressed as the square of a nondimensional height (Srinivasan et al. 2019), the stratification parameter (Pedlosky 1987), the ratio $(L_R/D)^2$ (e.g., Pedlosky 1987; Perfect et al. 2018), and as other ad hoc symbols.

The question of whether flow past a seamount can be considered to be quasigeostrophic is then important for justifying a Burger-number-based parameterization of the seamount energetics. To assess whether the present simulations are near a QG regime, velocities can be decomposed into balanced (geostrophic) and unbalanced (ageostrophic) motions. The density field in the simulations can be used to compute a geostrophic velocity, and when this velocity is subtracted from the simulation velocity, the residual can be considered to be the ageostrophic component. In doing this, we find that the only significant ageostrophic motions are very near the seamount and, depending on f , there comprise between 5% and 20% of the horizontal flow velocity; therefore, in

general, the overall flow is approximately consistent with QG flow.

The literature shows that a small bulk Froude and Rossby number are neither sufficient nor necessary for establishing quasigeostrophy. MacKinnon et al. (2019) finds significant ageostrophic motions in an island wake that would normally fall into the QG regime, and conversely, Williams et al. (2010) demonstrates that many flow scenarios that exist outside of the traditional requirements for the QG regime may in fact be well described by quasigeostrophy. So while the demonstrated Burger number dependence of quantities such as the mean to eddy conversion fit within QG theory, this is not a foregone conclusion, because these relationships have not previously been demonstrated in relation to flow past topography. To wit, other quantities such as the EV conversion term do not conform to this, and present ongoing questions related to how they might be parameterized. We suggest that EV, for example, may represent a blend of QG and non-QG processes; fluid instabilities that are not relevant to QG theory can also play a role that may only become apparent in certain simulations.

b. Far-field influences

When the eddies advect out of the analysis domain, they still contain considerable energy. Typically 50%–80% of the EKE is unconverted into other forms of energy, and still leaves the domain as unsteady eddy motion, as indicated in Fig. 10a. In Fig. 10a, the streamwise distribution of EKE (i.e., averaged in time, cross stream, and vertical) for each simulation is normalized by its respective total EKE (see Fig. 7a).

FIG. 10. (a) Streamwise EKE distribution, normalized by total EKE, for all simulations. The envelope indicates the minimum and maximum normalized values, and the dashed line is the mean for all simulations. The outline of the seamount is shown below. (b) A typical map of the EKE, which has been averaged in time and in the cross-stream direction.

The dashed line is the mean across all simulations, while the shaded region around it indicates the range of normalized EKE values across all simulations. The streamwise distribution of EKE exhibits a strong degree of similarity among all simulations. This novel result suggests that both regimes of eddy structure are subject to the same rate of decay as they advect downstream. A typical distribution for the EKE, averaged in the cross-stream direction, is given in Fig. 10b. The eddy energy is primarily located toward the bottom of the domain and tends to decay both vertically and in the streamwise direction. Cases with very low Burger number (and a depth ratio approaching unity) may exhibit some EKE well above the seamount summit, though Perfect et al. (2018) did not detect any substantial vorticity above the summit.

The EKE boundary flux term [the first right-hand-side term of Eq. (10)] represents a significant amount of energy whose fate is likely to be converted into internal waves, turbulence, and diapycnal mixing. In the treatment of the energetics of flow past a seamount, the eddy motions that advect out of the domain cannot be neglected. In a realistic ocean, the EKE leaving the domain can later generate turbulence, internal waves, dissipation and mixing, and can also interact with other topography or coherent flow structures.

c. Diapycnal diffusivity

It is often useful to quantify the effect of energetic events in the ocean in terms of the associated

diapycnal diffusivity (e.g., Munk 1966; Waterhouse et al. 2014). The diapycnal diffusivity refers to a diffusion coefficient associated with turbulent mixing of density (Osborn 1980):

$$K_p = \frac{g \langle w' \rho' \rangle}{\rho_0 N^2}, \quad (18)$$

In Fig. 11a, we calculate the time-averaged, volume-averaged diapycnal diffusivity for the present simulations, obtained from the modeled turbulent buoyancy flux. Except for the strongest two stratifications tested, the diapycnal diffusivity scales with Fr Ro , peaking at $3 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$. The largest calculated diapycnal diffusivities are consistent with findings in the vicinity of Fieberling Guyot, a large midlatitude seamount (Kunze and Toole 1997). This significantly exceeds open-ocean estimates of around $10^{-5} \text{ m}^2 \text{ s}^{-1}$ and would constitute one of the extreme mixing events that are believed to keep the basic-averaged diapycnal diffusivity around $10^{-4} \text{ m}^2 \text{ s}^{-1}$ (Munk and Wunsch 1998). This constitutes the first study demonstrating seamount wake mechanisms that could produce contribute to the missing mixing picture.

The lowest calculated diffusivities, however, are well below even open-ocean estimates; this is because the background diffusivity for the simulations was reduced to $10^{-9} \text{ m}^2 \text{ s}^{-1}$, and so a typical background ocean mixing state was omitted. Because only the mixing in the simulation was either associated with the seamount

FIG. 11. (a) Volume-averaged diapycnal diffusivity is plotted vs the product of the Froude and Rossby numbers. (b) The volume-integrated diapycnal flux is plotted vs the Burger number.

wake or effectively molecular, we interpret the cases with very low K_p as contributing effectively no additional mixing over whatever background value would ordinarily be appropriate.

Because the volume over which the diffusivity is averaged is somewhat arbitrary, a better measure of the impact of the seamount that could be better parameterized is the total turbulent buoyancy flux, presented in Fig. 11b. This quantity is identical to the TB term in Eq. (10), when expressed in terms of ROMS output: $\text{TB} = g \int_V -K_p (\partial \bar{p} / \partial z) dV = g \int_V \bar{w}' \bar{p}' dV$. The TB term does not have a simple scaling relationship with any combination of the Froude and Rossby numbers. In general, however, increasing rotation decreases TB. For a constant Rossby number and varying Froude number, TB has a peak at the Froude number for which $\text{Bu} = \text{Bu}_{\text{crit}}$. The two limiting cases of very strong and very weak stratification both correspond to cases with weak diapycnal mixing. In the case of strong stratification, vertical motion is arrested, and so little mixing beyond molecular diffusivity may take place. In the opposite case of stratification approaching zero, vertical motion exists, but the potential energy of that motion tends toward zero. This effect is especially evident in the points for $f = 8 \times 10^{-5} \text{s}^{-1}$ in Fig. 11b, where both weak and strong stratification both yield substantially lower diapycnal diffusivity.

7. Conclusions

Through the use of a set of high-resolution simulations, we have explored the energetic aspects of flow

past a seamount for a variety of rotation and stratification conditions. We find that energetic quantities related to quasigeostrophic motions (i.e., production of eddies, EKE, and unsteady buoyancy production) scale with the Burger number, consistent with theory (Majda 2003; Pedlosky 1987). Furthermore, the nature of this dependency suggests that there are important changes near Bu_{crit} , which is associated with the vertical decoupling of the eddies (Perfect et al. 2018). In contrast, quantities that are more closely related to turbulence and are not considered relevant to the quasigeostrophic regime exhibit a dependence on Fr Ro . These quantities increase with Fr Ro , which is consistent with the idea that turbulence is suppressed by the presence of rotation and stratification.

The above results motivate the need to further investigate seamount wakes. Because seamounts are mostly unresolved in global models, it may be important to include the effect of wakes in topographic parameterizations. The present results suggest that a Burger number-based parameterization could be a useful starting point in this regard and give values for such a parameterization for a range of the Burger number. Several major open questions remain, however. The first, which is the subject of Perfect et al. (2020), pertains to how the energy flux into the seamount wake compares to the energy flux of internal waves for an isolated seamount. This is a necessary comparison in order to justify the inclusion of wake effects in addition to internal wave radiation. Some other remaining questions are how the energetics of seamount wakes are affected by tides, nonuniform currents, nonuniform stratification profiles, and more realistic bathymetry. Recent work such as

Zeiden et al. (2019) has opened the door for in situ measurements of seamount wakes; this study establishes an idealized case for comparison and suggests several potentially important phenomena for future investigation.

The mathematical structure of section 3 could also be applied to another scenario of interest—the case where tides are included. A phase average over the tidal period could be used in place of the time average. The equations of motion would then partition the flow into the unsteady but phase-averaged eddy motion, the motions occurring within each tidal phase, and the turbulence. Such simulations and analyses are possible but require significantly more computational resources and analysis. Because of the relevance of the problem with a steady, nontidal flow, the more complex tidal problem is left for future work.

Acknowledgments. This work was facilitated through the use of advanced computational, storage, and networking infrastructure provided by the Hyak supercomputer system and funded by the STF at the University of Washington. B. Perfect would like to thank B. Blakeley for productive conversation. N. Kumar acknowledges support from the Office of Naval Research, Littoral and Geosciences Division (Grant N00014-17-1-2890). J. J. Riley acknowledges support from PACCAR. The numerical code used for this work is open source and located at myroms.org. The postprocessed dataset and model settings used to produce the above results are available through the Open Science Framework at osf.io/ntscf. Raw model simulations are archived and will be provided upon request by the corresponding author.

REFERENCES

- Arbic, B. K., and R. B. Scott, 2008: On quadratic bottom drag, geostrophic turbulence, and oceanic mesoscale eddies. *J. Phys. Oceanogr.*, **38**, 84–103, <https://doi.org/10.1175/2007JPO3653.1>.
- Balwada, D., K. G. Speer, J. H. LaCasce, W. B. Owens, J. Marshall, and R. Ferrari, 2016: Circulation and stirring in the southeast Pacific Ocean and the Scotia Sea sectors of the Antarctic circumpolar current. *J. Phys. Oceanogr.*, **46**, 2005–2027, <https://doi.org/10.1175/JPO-D-15-0207.1>.
- Bauer, M. H., G. J. Mayr, I. Vergeiner, and H. Pichler, 2000: Strongly nonlinear flow over and around a three-dimensional mountain as a function of the horizontal aspect ratio. *J. Atmos. Sci.*, **57**, 3971–3991, [https://doi.org/10.1175/1520-0469\(2001\)058<3971:SNFOAA>2.0.CO;2](https://doi.org/10.1175/1520-0469(2001)058<3971:SNFOAA>2.0.CO;2).
- Beckmann, A., and D. B. Haidvogel, 1997: A numerical simulation of flow at Fieberling Guyot. *J. Geophys. Res.*, **102**, 5595–5613, <https://doi.org/10.1029/96JC03414>.
- Boutov, D., A. Santos, E. Luís, and J. Videman, 2010: Numerical modeling of vorticity dynamics in oceanic wakes. *V European Conference on Computational Fluid Dynamics ECCOMAS CFD 2010*, J. C. F. Pereira, A. Sequeira, and J. M. C. Pereira, Eds., ECCOMAS, 10 pp., <http://congress2.cimne.com/eccomas/proceedings/cfd2010/papers/01698.pdf>.
- Boyer, D. L., and R.-R. Chen, 1987: Laboratory simulation of mountain effects on large-scale atmospheric motion systems: The Rocky Mountains. *J. Atmos. Sci.*, **44**, 100–123, [https://doi.org/10.1175/1520-0469\(1987\)044<0100:LSOME0>2.0.CO;2](https://doi.org/10.1175/1520-0469(1987)044<0100:LSOME0>2.0.CO;2).
- Brighton, P. W. M., 1978: Strongly stratified flow past three dimensional obstacles. *Quart. J. Roy. Meteor. Soc.*, **104**, 289–307, <https://doi.org/10.1002/qj.49710444005>.
- Brink, K. H., 1995: Tidal and lower frequency currents above Fieberling Guyot. *J. Geophys. Res.*, **100**, 10817, <https://doi.org/10.1029/95JC00998>.
- Buijsman, M. C., S. Legg, and J. Klymak, 2012: Double-ridge internal tide interference and its effect on dissipation in Luzon strait. *J. Phys. Oceanogr.*, **42**, 1337–1356, <https://doi.org/10.1175/JPO-D-11-0210.1>.
- Caldeira, M., and P. Sangrà, 2012: Complex geophysical wake flows Madeira Archipelago case study. *Ocean Dyn.*, **62**, 785–797, <https://doi.org/10.1007/s10236-012-0528-6>.
- Callendar, W., J. M. Klymak, and M. G. G. Foreman, 2011: Tidal generation of large sub-mesoscale eddy dipoles. *Ocean Sci.*, **7**, 487–502, <https://doi.org/10.5194/os-7-487-2011>.
- Canals, M., G. Pawlak, and P. MacCready, 2009: Tilted baroclinic tidal vortices. *J. Phys. Oceanogr.*, **39**, 333–350, <https://doi.org/10.1175/2008JPO3954.1>.
- Chen, G., and Coauthors, 2015: Observed deep energetic eddies by seamount wake. *Sci. Rep.*, **5**, 17416, <https://doi.org/10.1038/srep17416>.
- Codiga, D. L., 1993: Laboratory realizations of stratified seamount-trapped waves. *J. Phys. Oceanogr.*, **23**, 2053–2071, [https://doi.org/10.1175/1520-0485\(1993\)023<2053:LROSST>2.0.CO;2](https://doi.org/10.1175/1520-0485(1993)023<2053:LROSST>2.0.CO;2).
- Coutis, P. F., and J. H. Middleton, 2002: The physical and biological impact of a small island wake in the deep ocean. *Deep-Sea Res. I*, **49**, 1341–1361, [https://doi.org/10.1016/S0967-0637\(02\)00029-8](https://doi.org/10.1016/S0967-0637(02)00029-8).
- Dong, C., and J. C. McWilliams, 2007: A numerical study of island wakes in the southern California Bight. *Cont. Shelf Res.*, **27**, 1233–1248, <https://doi.org/10.1016/j.csr.2007.01.016>.
- , —, and A. F. Shchepetkin, 2007: Island wakes in deep water. *J. Phys. Oceanogr.*, **37**, 962–981, <https://doi.org/10.1175/JPO3047.1>.
- , Y. Cao, and J. C. McWilliams, 2018: Island wakes in shallow water. *Atmos.–Ocean*, **56**, 96–103, <https://doi.org/10.1080/07055900.2018.1448750>.
- Dritschel, D. G., and M. De La Torre Juárez, 1996: The instability and breakdown of tall columnar vortices in a quasi-geostrophic fluid. *J. Fluid Mech.*, **328**, 129–160, <https://doi.org/10.1017/S0022112096008671>.
- Epifanio, C. C., and R. Rotunno, 2005: The dynamics of orographic wake formation in flows with upstream blocking. *J. Atmos. Sci.*, **62**, 3127–3150, <https://doi.org/10.1175/JAS3523.1>.
- Ferrari, R., A. Mashayek, T. J. McDougall, M. Nikurashin, and J.-M. Campin, 2016: Turning ocean mixing upside down. *J. Phys. Oceanogr.*, **46**, 2239–2261, <https://doi.org/10.1175/JPO-D-15-0244.1>.
- Gill, A. E., 1982: *Atmosphere–Ocean Dynamics*. Academic Press, 662 pp.
- Gula, J., M. J. Molemaker, and J. C. McWilliams, 2016: Topographic generation of submesoscale centrifugal instability and energy dissipation. *Nat. Commun.*, **7**, 12811, <https://doi.org/10.1038/ncomms12811>.
- Hussain, A. K. M. F., and W. C. Reynolds, 1970: The mechanics of an organized wave in turbulent shear flow. *J. Fluid Mech.*, **41**, 241–258, <https://doi.org/10.1017/S0022112070000605>.

- Kang, D., and E. N. Curchitser, 2015: Energetics of eddy-mean flow interactions in the Gulf Stream region. *J. Phys. Oceanogr.*, **45**, 1103–1120, <https://doi.org/10.1175/JPO-D-14-0200.1>.
- , —, and A. Rosati, 2016: Seasonal variability of the Gulf Stream kinetic energy. *J. Phys. Oceanogr.*, **46**, 1189–1207, <https://doi.org/10.1175/JPO-D-15-0235.1>.
- Kim, S. S., and P. Wessel, 2011: New global seamount census from altimetry-derived gravity data. *Geophys. J. Int.*, **186**, 615–631, <https://doi.org/10.1111/j.1365-246X.2011.05076.x>.
- King, B., M. Stone, H. P. Zhang, T. Gerkema, M. Marder, R. B. Scott, and H. L. Swinney, 2012: Buoyancy frequency profiles and internal semidiurnal tide turning depths in the oceans. *J. Geophys. Res.*, **117**, C04008, <https://doi.org/10.1029/2011JC007681>.
- Klymak, J. M., S. Legg, and R. Pinkel, 2010: A simple parameterization of turbulent tidal mixing near supercritical topography. *J. Phys. Oceanogr.*, **40**, 2059–2074, <https://doi.org/10.1175/2010JPO4396.1>.
- Kundu, P. J., I. M. Cohen, and D. R. Dowling, 2004: *Fluid Mechanics*. 3rd ed. Elsevier, 759 pp.
- Kunze, E., and J. M. Toole, 1997: Tidally driven vorticity, diurnal shear, and turbulence atop feeblering seamount. *J. Phys. Oceanogr.*, **27**, 2663–2693, [https://doi.org/10.1175/1520-0485\(1997\)027<2663:TDVDSA>2.0.CO;2](https://doi.org/10.1175/1520-0485(1997)027<2663:TDVDSA>2.0.CO;2).
- , C. MacKay, E. E. McPhee-Shaw, K. Morrice, J. B. Girton, and S. R. Terker, 2012: Turbulent mixing and exchange with interior waters on sloping boundaries. *J. Phys. Oceanogr.*, **42**, 910–927, <https://doi.org/10.1175/JPO-D-11-075.1>.
- MacCready, P., N. S. Banas, B. M. Hickey, E. P. Dever, and Y. Liu, 2009: A model study of tide- and wind-induced mixing in the Columbia River Estuary and plume. *Cont. Shelf Res.*, **29**, 278–291, <https://doi.org/10.1016/j.csr.2008.03.015>.
- MacKinnon, J. A., and Coauthors, 2017: Climate process team on internal wave–driven ocean mixing. *Bull. Amer. Meteor. Soc.*, **98**, 2429–2454, <https://doi.org/10.1175/BAMS-D-16-0030.1>.
- , M. H. Alford, G. Voet, K. L. Zeiden, T. M. S. Johnston, M. Siegelman, S. Merrifield, and M. Merrifield, 2019: Eddy wake generation from broadband currents near Palau. *J. Geophys. Res. Oceans*, **124**, 4891–4903, <https://doi.org/10.1029/2019JC014945>.
- Majda, A., 2003: *Introduction to PDEs and Waves for the Atmosphere and Ocean*. Courant Lecture Notes, Vol. 9, American Mathematical Society, 234 pp.
- McCabe, R. M., P. MacCready, G. Pawlak, R. M. McCabe, P. MacCready, and G. Pawlak, 2006: Form drag due to flow separation at a headland. *J. Phys. Oceanogr.*, **36**, 2136–2152, <https://doi.org/10.1175/JPO2966.1>.
- Munk, W., 1966: Abyssal recipes. *Deep-Sea Res.*, **13**, 707–730, [https://doi.org/10.1016/0011-7471\(66\)90602-4](https://doi.org/10.1016/0011-7471(66)90602-4).
- , and C. Wunsch, 1998: Abyssal recipes II: Energetics of tidal and wind mixing. *Deep-Sea Res. I*, **45**, 1977–2010, [https://doi.org/10.1016/S0967-0637\(98\)00070-3](https://doi.org/10.1016/S0967-0637(98)00070-3).
- Naveira Garabato, A. C., R. Ferrari, and K. L. Polzin, 2011: Eddy stirring in the Southern Ocean. *J. Geophys. Res.*, **116**, C09019, <https://doi.org/10.1029/2010JC006818>.
- Nikurashin, M., and R. Ferrari, 2009: Radiation and dissipation of internal waves generated by geostrophic motions impinging on small-scale topography: Application to the Southern Ocean. *J. Phys. Oceanogr.*, **40**, 2025–2042, <https://doi.org/10.1175/2010JPO4315.1>.
- , and —, 2011: Global energy conversion rate from geostrophic flows into internal lee waves in the deep ocean. *Geophys. Res. Lett.*, **38**, L08610, <https://doi.org/10.1029/2011GL046576>.
- Osborn, T. R., 1980: Estimates of the local rate of vertical diffusion from dissipation measurements. *J. Phys. Oceanogr.*, **10**, 83–89, [https://doi.org/10.1175/1520-0485\(1980\)010<0083:EOTLRO>2.0.CO;2](https://doi.org/10.1175/1520-0485(1980)010<0083:EOTLRO>2.0.CO;2).
- Pedlosky, J., 1987: *Geophysical Fluid Dynamics*. Springer, 710 pp.
- Perfect, B., N. Kumar, and J. Riley, 2018: Vortex structures in the wake of an idealized seamount in rotating, stratified flow. *Geophys. Res. Lett.*, **45**, 9098–9105, <https://doi.org/10.1029/2018GL078703>.
- , —, and —, 2020: Energetics of seamount wakes. Part II: Wave fluxes. *J. Phys. Oceanogr.*, **50**, 1383–1398, <https://doi.org/10.1175/JPO-D-19-0104.1>.
- Phillips, O. M., 1966: *The Dynamics of the Upper Ocean*. Cambridge University Press, 269 pp.
- Praud, O., J. Sommeria, and A. M. Fincham, 2006: Decaying grid turbulence in a rotating stratified fluid. *J. Fluid Mech.*, **547**, 389–412, <https://doi.org/10.1017/s0022112005007068>.
- Raja, K., 2018: Internal waves and mean flow in the presence of topography. Ph.D. thesis, Université Grenoble Alpes, 148 pp.
- Riley, J., H. Liu, and F. Geller, 1976: A numerical and experimental study of stably stratified flow around complex terrain. Environmental Protection Agency Tech. Rep. EPA/600/4-76/021, 42 pp.
- Schär, C., and R. B. Smith, 1993: Shallow-water flow past isolated topography. Part I: Vorticity production and wake formation. *J. Atmos. Sci.*, **50**, 1373–1400, [https://doi.org/10.1175/1520-0469\(1993\)050<1373:SWFPIT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1993)050<1373:SWFPIT>2.0.CO;2).
- , and D. R. Durran, 1997: Vortex formation and vortex shedding in continuously stratified flows past isolated topography. *J. Atmos. Sci.*, **54**, 534–554, [https://doi.org/10.1175/1520-0469\(1997\)054<0534:VFAVSI>2.0.CO;2](https://doi.org/10.1175/1520-0469(1997)054<0534:VFAVSI>2.0.CO;2).
- Shechepetkin, A. F., and J. C. McWilliams, 2005: The Regional Oceanic Modeling System (ROMS): A split-explicit, free-surface, topography-following-coordinate oceanic model. *Ocean Modell.*, **9**, 347–404, <https://doi.org/10.1016/j.ocemod.2004.08.002>.
- Simmons, H. L., B. Powell, S. Merrifield, S. Zedler, and P. Colin, 2019: Dynamical downscaling of equatorial flow response to Palau. *Oceanography*, **32**, 84–91, <https://doi.org/10.5670/oceanog.2019.414>.
- Smolarkiewicz, P. K., and R. Rotunno, 1989: Low Froude number flow past three-dimensional obstacles. Part I: Baroclinically generated lee vortices. *J. Atmos. Sci.*, **46**, 1154–1164, [https://doi.org/10.1175/1520-0469\(1989\)046<1154:LFNFPT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1989)046<1154:LFNFPT>2.0.CO;2).
- Srinivasan, K., J. C. McWilliams, M. J. Molemaker, and R. Barkan, 2019: Submesoscale vortical wakes in the lee of topography. *J. Phys. Oceanogr.*, **49**, 1949–1971, <https://doi.org/10.1175/JPO-D-18-0042.1>.
- Stegner, A., and D. G. Dritschel, 2000: A numerical investigation of the stability of isolated shallow water vortices. *J. Phys. Oceanogr.*, **30**, 2562–2573, [https://doi.org/10.1175/1520-0485\(2000\)030<2562:ANIOTS>2.0.CO;2](https://doi.org/10.1175/1520-0485(2000)030<2562:ANIOTS>2.0.CO;2).
- Toole, J. M., R. W. Schmitt, K. L. Polzin, and E. Kunze, 1997: Near-boundary mixing above the flanks of a midlatitude seamount. *J. Geophys. Res.*, **102**, 947–959, <https://doi.org/10.1029/96JC03160>.
- Umlauf, L., and H. Burchard, 2003: A generic length-scale equation for geophysical turbulence models. *J. Mar. Res.*, **61**, 235–265, <https://doi.org/10.1357/002224003322005087>.
- Vosper, S. B., I. P. Castro, W. H. Snyder, and S. D. Mobbs, 1999: Experimental studies of strongly stratified flow past three-dimensional orography. *J. Fluid Mech.*, **390**, 223–249, <https://doi.org/10.1017/S0022112099005133>.

- Warner, J. C., C. R. Sherwood, H. G. Arango, and R. P. Signell, 2005: Performance of four turbulence closure models implemented using a generic length scale method. *Ocean Modell.*, **8**, 81–113, <https://doi.org/10.1016/j.ocemod.2003.12.003>.
- Waterhouse, A. F., and Coauthors, 2014: Global patterns of diapycnal mixing from measurements of the turbulent dissipation rate. *J. Phys. Oceanogr.*, **44**, 1854–1872, <https://doi.org/10.1175/JPO-D-13-0104.1>.
- Williams, P. D., P. L. Read, and T. W. Haine, 2010: Testing the limits of quasi-geostrophic theory: Application to observed laboratory flows outside the quasi-geostrophic regime. *J. Fluid Mech.*, **649**, 187–203, <https://doi.org/10.1017/S0022112009993405>.
- Winters, K. B., P. N. Lombard, J. Riley, and E. A. D’Asaro, 1995: Available potential energy and mixing in density-stratified fluids. *J. Fluid Mech.*, **289**, 115–128, <https://doi.org/10.1017/S002211209500125X>.
- Wright, C. J., R. B. Scott, P. Ailliot, and D. Furnival, 2014: Lee wave generation rates in the deep ocean. *Geophys. Res. Lett.*, **41**, 2434–2440, <https://doi.org/10.1002/2013GL059087>.
- Zeiden, K. L., D. L. Rudnick, and J. A. MacKinnon, 2019: Glider observations of a mesoscale oceanic island wake. *J. Phys. Oceanogr.*, **49**, 2217–2235, <https://doi.org/10.1175/JPO-D-18-0233.1>.