On the Vorticity Balance over Steep Slopes: Kuroshio Intrusions Northeast of Taiwan

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(Manuscript received 14 November 2019, in final form 16 April 2020)

ABSTRACT

The circulation of the Kuroshio northeast of Taiwan is characterized by a large anticyclonic loop of surface intrusion and strong upwelling at the shelfbreak. To study the mechanisms of Kuroshio intrusions, the vorticity balance is examined using a high-resolution nested numerical model. In the 2D depth-averaged vorticity equation, the advection of geostrophic potential vorticity (APV) term and the joint effect of baroclinicity and relief (JEBAR) term are dominant. On the other hand, in the 2D depth-integrated vorticity equation, the main balance is between nonlinear advection and bottom pressure torque. It is shown that JEBAR and APV tend to compensate, and their difference is comparable to bottom pressure torque. Perhaps most significantly, a general framework is provided for examination of vorticity balance over steep slopes through a full 3D depth-dependent vorticity equation. The 3D analysis reveals a well-defined bottom boundary layer over the shelfbreak, about 40 m deep and capped by the vertical velocity maximum. In the upper frictionless layer from the surface to about 100 m, the primary balance is between nonlinear advection and horizontal divergence. In the lower frictional layer, viscous stress is balanced by nonlinear advection and horizontal divergence. The bottom pressure torque, which corresponds to the depth-integrated viscous effect, is a proxy for viscous stress divergence at the bottom. The importance of nonlinear advection is further demonstrated in a sensitivity experiment by removing advective terms from momentum equations. Without nonlinear advection, the bottom pressure torque becomes trivial, the boundary layer vanishes, and the on-shelf intrusion is considerably weakened.

1. Introduction

The Kuroshio is a western boundary current in the North Pacific. It flows northward east of Taiwan, continues east-northeastward along the continental slope in East China Sea (ECS), and finally turns eastward off the south coast of Japan (Nitani 1972). (Fig. 1) The Kuroshio water interacts with shelf waters in ECS, of which the on-shelf intrusion northeast of Taiwan is a well-known phenomenon. The Kuroshio intrusion strongly influences the structure and variation of circulations in the China Seas (Guo et al. 2006; Yang et al.

2012). The associated upwelling also significantly promotes the ecosystem of the China Seas (Su et al. 1994; Tang et al. 2000; Lian et al. 2016). Tang et al. (2000) summarized the historical hydrographic and current observations northeast of Taiwan. They noted a seasonal migration that the Kuroshio core tended to move offshore in summer and onshore in winter. Yang et al. (2012) found that the bottom intrusion sometimes extended as far as the Zhejiang coast. Vélez-Belchí et al. (2013) depicted two types of on-shelf intrusions, large and small, based on the drifter and altimetry observations. They suggested that the large Kuroshio intrusions were often triggered by impinging cyclonic eddies from the western Pacific.

Numerical models have been used to simulate the Kuroshio intrusion and its seasonal variations. Guo et al. (2006) simulated the ECS circulation using a

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FIG. 1. (a) Geography of the northwestern Pacific Ocean and (b) topography of the study region. The black box shown in (a) is the study region. The 100-, 200-, 400-, 600-, 800-, 1000-, and 2000-m isobaths are marked in (b). The topography data are from ETOPO2 (National Geophysical Data Center 2006; <https://www.ngdc.noaa.gov/mgg/global/etopo2.html>).

triple-nested Princeton Ocean Model (POM) model. Wu et al. (2008) simulated the Kuroshio-induced upwelling in connection with a cyclonic flow pattern northeast of Taiwan. Yang et al. (2011, 2012) reproduced the bottom intrusion. More recently, Liu et al. (2014) simulated the circulation northeast of Taiwan with a high-resolution double-nested Regional Ocean Model System (ROMS). Their model results indicated a clear seasonal variation. In the winter season, the on-shelf intrusion is much stronger, with a large portion of the Kuroshio water flowing across the shelfbreak and onto the continent shelf. They noted that the surface intrusions follow two main routes: a large anticyclonic loop at the northern end of Taiwan and a straight northward path onto the ECS shelf.

The conservation of the potential vorticity (PV) is a fundamental dynamic property in describing the ocean circulation. For a homogeneous (shallow water) layer, the PV conservation can be written as (Gill 1982, chapter 7)

$$\frac{D}{Dt} \left(\frac{f + \zeta}{h} \right) = 0, \quad (1)$$

where $D/Dt = \partial/\partial t + \mathbf{u} \cdot \nabla$ is the total derivative, $\mathbf{u} = (u, v)$ is the horizontal velocity vector, $Q = (f + \zeta)/h$ is the PV, ζ is the vertical component of relative vorticity, f is the Coriolis parameter, and h is a water layer thickness.

In a barotropic ocean, h is the total water depth. Equation (1) states that the local change of relative vorticity can be induced by nonlinear vorticity advection, the β effect, and vortex stretching. The β effect is fundamental to the large-scale ocean circulation. The nonlinear advection is significant in an inertial jet, while the vortex stretching becomes dominant over steep slopes. We note that the RHS of Eq. (1) in general includes the forcing/dissipation due to viscous stress (Jackson et al. 2006).

The vorticity dynamics has often been used to examine the Kuroshio on-shelf intrusion. Most previous model studies were based on the 2D depth-averaged vorticity balance derived from depth-averaged momentum equations (Liang and Su 1994; Guo et al. 2006; Oey et al. 2010; Wang and Oey 2016). Guo et al. (2006) calculated the depth-averaged vorticity balance along the 200 m isobath. They found that the primary balance is between advection of geostrophic potential vorticity (APV) and the joint effect of baroclinicity and relief (JEBAR) term. Using an idealized model experiment, Oey et al. (2010) obtained a similar conclusion of 2D vorticity balance between APV and JEBAR. Moreover, they suggested that the winter intrusion is due to an enhanced JEBAR associated with winter cooling. Wang and Oey (2016) found further support based on their analysis of a comprehensive regional Pacific Ocean Model.

In previous studies of the interactions between the Kuroshio and steep bottom slopes, PV conservation is

limited to the 2D depth-averaged vorticity balance. In this study, we formulate a full 3D vorticity balance in a terrain-following (sigma) coordinate model system. The vorticity analysis is applied to model results of Liu et al. (2014) of the Kuroshio intrusion northeast of Taiwan. We also conduct a sensitivity experiment to demonstrate the importance of nonlinear effects in the on-shelf intrusion. The remaining sections are arranged as follows. In section 2, the model setup is introduced. In section 3, the circulation and the momentum balance from the model results are presented. Section 4 shows the 2D and 3D vorticity equations in their applications to the Kuroshio intrusion. In section 5, a sensitivity experiment is conducted by removing nonlinear advective terms in the momentum equations. Discussion and conclusions are given in section 6. The derivation and numerical procedure of the 3D vorticity equation are given in appendixes A and B, respectively.

2. Model setup

The numerical model used in this study is the Regional Ocean Modeling System (ROMS). ROMS is a free-surface, terrain-following, primitive equation ocean model (Shchepetkin and McWilliams 2003, 2005). The present model setup is the same as in Liu et al. (2014). We run the model with two nested grids of different grid spacing. The outer domain (hereafter, NEST1), 110°–138°E, 15°–41°N, has a grid spacing of 1/18°, while the inner domain, 116°–126°E, 22.5°–28°N, has a grid spacing of 1/54° (hereafter, NEST2). The vertical s -coordinate layers are set to 32 levels. The internal time step of NEST1 is 200 s, while the internal time step of NEST2 is 50 s. The wind stress used in both NEST1 and NEST2 is from the QuikSCAT daily wind field. The surface heat and freshwater fluxes are based on a seasonal climatology from the Comprehensive Ocean–Atmospheric Data Set (COADS) (da Silva et al. 1994). The horizontal viscosity is set to zero. The complete model setup and validation can be found in Liu et al. (2014).

The model was spun up for 5 years using the annually repeated surface forcing and open boundary conditions of the year 2000. Then, the model was run for another 9 years. The model results for the seasonal variation of the Kuroshio and its onshore intrusion have been previously verified with altimetry observations and historical hydrographic data. In particular, it was shown that the month of February was representative of the winter season. For consistency, we use the month of February 2001 for the vorticity analysis; the time period is typical of the February climatology in Liu et al. (2014). In ROMS, each term of the momentum equations (both

2D and 3D) is accumulated continuously at every time step. Both the state variables (i.e., velocity, temperature, salinity) and the diagnostic variables (i.e., each term in the momentum equations) are accumulated, and then averaged over the specified time period. The fact that the diagnostic variables are continuously stored is essential for an accurate vorticity analysis.

3. Circulation pattern and momentum balance

a. Circulation pattern of the Kuroshio on-shelf intrusion northeast of Taiwan

Liu et al. (2014) have clearly demonstrated the Kuroshio on-shelf intrusion northeast of Taiwan, using their model's February mean temperature and flow distributions. The Kuroshio generally follows the shelfbreak (Fig. 2). Upon leaving the northeast corner of Taiwan, the current encounters a sharp bend of the continental slope, and part of the Kuroshio surface water runs straight across the shelfbreak. The surface plume makes a large anticyclonic (eastward) loop over the outer shelf, before rejoining the core of the Kuroshio farther downstream. At 100 m, the flow pattern is similar to that at the surface. The Kuroshio temperature front runs across shelfbreak, indicating intrusions of warmer subsurface water. At 200 m, the core of the current is located farther off the slope, relative to the core at the surface, indicating a tilted velocity axis with depths. The frontal isotherms vanish near the shelfbreak.

The vertical velocity is closely related to the horizontal circulation pattern, where the strong vertical velocity coincides with the on-shelf intrusion of the Kuroshio (Fig. 3). The upwelling is most evident at the shelfbreak near the 200-m isobath. The large positive vertical velocity, with maximum at about 100 m (also see Fig. 7), is concentrated at where the core of the Kuroshio encounters the shelfbreak. In contrast, the vertical velocity is small over the broad shelf and in the open ocean.

b. Momentum balance

The depth-averaged momentum balance is calculated from the model results. The horizontal momentum equations in Cartesian coordinates are

$$\underbrace{\frac{\partial u}{\partial t}}_{\text{I}} + \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}_{\text{II}} + \underbrace{-fv}_{\text{III}} + \underbrace{g \frac{\partial \eta}{\partial x} + \int_z^\eta \frac{\partial b}{\partial x} dz'}_{\text{IV}} - \underbrace{\left[\frac{\partial}{\partial z} \left(K_M \frac{du}{dz} \right) + \mathcal{D}^x \right]}_{\text{V}} = 0, \quad (2a)$$

FIG. 2. Mean temperatures (color; °C) and velocity vectors (black arrows) in February at (a) 0, (b) 100, and (c) 200 m. Gray curves are the 100-, 200-, 500-, and 1000-m isobaths.

$$\underbrace{\frac{\partial v}{\partial t}}_I + \underbrace{u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}}_{II} + \underbrace{fu}_{III} + \underbrace{g \frac{\partial \eta}{\partial y} + \int_z \frac{\partial b}{\partial y} dz'}_{IV} - \underbrace{\left[\frac{\partial}{\partial z} \left(K_M \frac{dv}{dz} \right) + \mathcal{D}^y \right]}_V = 0, \quad (2b)$$

where (u, v, w) are the velocity components in x, y , and z directions; η is the surface elevation; $b = gp/\rho_0$ is the buoyancy forcing; ρ is the density; ρ_0 is the reference density; f is the Coriolis coefficient; g is the gravity acceleration; K_M is the vertical eddy viscosity coefficient; and $(\mathcal{D}^x, \mathcal{D}^y)$ are the horizontal viscosity terms.

In Eqs. (2a) and (2b), the terms are referred to as term I, acceleration (accel); term II, advection (adv); term III, Coriolis force (cor); term IV, horizontal pressure gradient (prsgd); and term V, viscosity (visc). The momentum equations are dominated by the geostrophic balance, such that cor and prsgd are an order of magnitude larger than all other terms (figure not shown). Following Gan and Allen (2005), it is convenient to introduce an ageostrophic pressure gradient term, $\text{apg} = \text{cor} + \text{prsgd}$. The horizontal velocity can be decomposed into the geostrophic and ageostrophic velocity components; $\mathbf{u} = \mathbf{u}_g + \mathbf{u}_a$, then, $\text{apg} = f\mathbf{k} \times \mathbf{u}_a$.

FIG. 3. Mean vertical velocity distributions (m s⁻¹) in February at (a) 50, (b) 100, and (c) 200 m. Black curves are the 100-, 200-, 500-, and 1000-m isobaths. The black box marks the region in Figs. 7 and 8 for vertical profiles of vertical velocity and vorticity balance.

The depth-averaged momentum equations can be written as

$$\underbrace{\frac{1}{D} \frac{\partial \bar{u} D}{\partial t}}_{\text{I}} = - \underbrace{\frac{1}{D} \left(\frac{\partial \bar{u} \bar{u} D}{\partial x} + \frac{\partial \bar{v} \bar{u} D}{\partial y} \right)}_{\text{II}} \underbrace{+ f \bar{v}}_{\text{III}} - \underbrace{\left(g \frac{\partial \eta}{\partial x} + \frac{1}{D} \int_{-H}^{\eta} \int_z^{\eta} \frac{\partial b}{\partial x} dz' dz \right)}_{\text{IV}} \underbrace{+ \frac{\tau_x^s}{D}}_{\text{V}} - \underbrace{\frac{\tau_b^x}{D}}_{\text{VI}} + \underbrace{(\bar{\mathcal{D}})^x}_{\text{VII}}, \quad (3a)$$

$$\underbrace{\frac{1}{D} \frac{\partial \bar{v} D}{\partial t}}_{\text{I}} = - \underbrace{\frac{1}{D} \left(\frac{\partial \bar{u} \bar{v} D}{\partial x} + \frac{\partial \bar{v} \bar{v} D}{\partial y} \right)}_{\text{II}} \underbrace{- f \bar{u}}_{\text{III}} - \underbrace{\left(g \frac{\partial \eta}{\partial y} + \frac{1}{D} \int_{-H}^{\eta} \int_z^{\eta} \frac{\partial b}{\partial y} dz' dz \right)}_{\text{IV}} \underbrace{+ \frac{\tau_y^s}{D}}_{\text{V}} - \underbrace{\frac{\tau_b^y}{D}}_{\text{VI}} + \underbrace{(\bar{\mathcal{D}})^y}_{\text{VII}}, \quad (3b)$$

where $D = H + \eta$ is the total water depth, H is the undisturbed water depth, $(\bar{u}, \bar{v})$ are the depth-averaged horizontal velocity components, (τ_x^s, τ_y^s) are the surface stress components, (τ_b^x, τ_b^y) are the bottom stress components, and $[(\bar{\mathcal{D}})^x, (\bar{\mathcal{D}})^y]$ are the depth-averaged horizontal viscosity components. Since $|\eta| \ll H$ and $\partial \eta / \partial t$ are negligible in the transport equation, $\partial(\bar{u} D) / \partial x + \partial(\bar{v} D) / \partial y$, we do not distinguish between D and H for the rest of the text. In Eqs. (3a) and (3b), the terms are the depth averages of term I, acceleration; term II, advection; term III, Coriolis force; term IV, horizontal pressure gradient; term V, surface stress; term VI, bottom stress; and term VII, horizontal viscosity. The ageostrophic pressure gradient term is similarly defined as Coriolis (term III) + pressure gradient (term IV).

Figure 4 shows horizontal distributions of terms in the depth-averaged momentum equation. All the terms of momentum equations are directly obtained from the model diagnostics. Therefore, the momentum budget is perfectly conserved (within the round off error) with no residual. The acceleration, term I, is an order of magnitude smaller than other terms. In the subsequent analysis, the steady state is assumed. The horizontal viscosity can also be neglected. The Coriolis and pressure gradient are not shown, as the geostrophic currents are nondivergent, $\partial \bar{u}_g / \partial x + \partial \bar{v}_g / \partial y = 0$. Over the outer shelf, the ageostrophic pressure gradient is generally eastward, corresponding to a sharp turning of the

surface plume toward the east. An eastward ageostrophic current, that is, an on-shelf intrusion. It is mainly balanced by an opposing advection term. The advection term, in addition, has a strong northward component, which is partially balanced by bottom stress. In the inner shelf (water depth < 100 m), the balance is the classical surface Ekman relation between ageostrophic pressure gradient, corresponding to a northwestward Ekman transport, and southwestward surface wind stress. In the depth-integrated momentum equation, the stress term τ/D is important only in shallow depths. Also, the advection term includes the contributions from both the mean and eddy motions. The eddy motions are mainly associated with short-term, wind-driven velocity fluctuations. Their contribution is generally small by our estimation (figure not shown). Since our model diagnostics are for the total advection terms, for consistency, only the total field is examined.

4. The vorticity balance

a. Vorticity balance from depth-averaged momentum equations

Following Mertz and Wright (1992) and Oey et al. (2014), the depth-averaged vorticity equation in Cartesian coordinate can be derived via cross differentiation of depth-averaged momentum equations, Eqs. (3a) and (3b) (The derivation is described in appendix A). The acceleration terms in momentum equations are negligible, and consequently, the tendency term in vorticity equation is also very small. This is verified that the tendency term is an order of magnitude smaller than other terms. Thus, only the steady-state balance is considered. The vertical component of the depth-averaged vorticity equation is

$$\underbrace{\mathbf{U} \cdot \nabla \left(\frac{f}{D} \right)}_{\text{I}} + \underbrace{\mathbf{U} \cdot \nabla \left(\frac{\bar{\zeta}}{D} \right)}_{\text{II}} = \underbrace{J(\chi, D^{-1})}_{\text{III}} + \underbrace{\nabla \times \left(\frac{\tau_s}{D} \right)}_{\text{IV}} - \underbrace{\nabla \times \left(\frac{\tau_b}{D} \right)}_{\text{V}}, \quad (4)$$

where $\bar{\zeta} = \partial \bar{v} / \partial x - \partial \bar{u} / \partial y$ is the vertical component of relative vorticity of the depth-averaged flow, $\chi = \int_{-H}^{\eta} z b dz'$ is the potential energy anomaly, $J(\chi, D^{-1}) = (\partial \chi / \partial x)(\partial D^{-1} / \partial y) - (\partial D^{-1} / \partial x)(\partial \chi / \partial y)$, and $\mathbf{U} = D \bar{\mathbf{u}}$. Terms in Eq. (4) are term I, advection of geostrophic PV [adv(bPV)]; term II, advection of relative PV [adv(rPV)]; term III, the JEBAR term; term IV, curl of depth-distributed wind stress [curl(ssr)]; and term V, curl of depth-distributed bottom stress [curl(bstr)].

FIG. 4. Horizontal distributions of terms in the depth-averaged momentum equation (m s^{-2}): (a) ageostrophic pressure gradient, (b) advection, (c) surface stress, and (d) bottom stress. Gray curves are the 100-, 200-, 500-, and 1000-m isobaths.

Figure 5 shows horizontal distributions of terms in the depth-averaged vorticity equation. The vorticity balance is mainly contributed by $\text{adv}(\text{bPV})$, JEBAR, and $\text{adv}(\text{rPV})$, while $\text{curl}(\text{sstr})$ and $\text{curl}(\text{bstr})$ are much smaller. The $\text{adv}(\text{bPV})$ term, advection of geostrophic PV (APV), is the largest term (Fig. 5a). A positive $\text{adv}(\text{bPV})$ corresponds to the on-shelf advection, and is mainly balanced by the JEBAR term of a comparable spatial structure (Fig. 5c). JEBAR thus appears to be a primary “force” that drives a cross-shelf barotropic flow. The nonlinear advection $\text{adv}(\text{rPV})$ is secondary and is balanced by $\text{JEBAR} - \text{adv}(\text{bPV})$ (Fig. 5f). It is worth noting that the total depth-averaged vorticity budget (including the tendency term) is highly accurate; the residual is $O(10^{-16}) \text{ s}^{-2}$.

The depth-integrated vorticity equation can be derived by taking the curl of depth-integrated momentum equations (appendix A). The β effect is ignored on this spatial scale; it is straightforward to add βV (V is the northward volume transport) into the vorticity balance. The depth-integrated vorticity equation is

$$\nabla \times (D \overline{\text{adv}}) = \nabla \times (\tau_s - \tau_b) + \frac{1}{\rho_0} J(p_b, D), \quad (5)$$

where $\overline{\text{adv}}$ is the depth-averaged advection term [term II in Eqs. (3a) and (3b)], p_b is the bottom pressure, and $J(p_b, D) = (\partial p_b / \partial x)(\partial D / \partial y) - (\partial D / \partial x)(\partial p_b / \partial y)$. The term on the LHS of Eq. (5) represents the depth-integrated nonlinear vorticity advection. The first term on the RHS of Eq. (5) is the torque of the difference between surface and bottom stresses. The last term of Eq. (5) is the bottom pressure torque (Mertz and Wright 1992; Hughes 2000). Over a sloping bottom, the normal pressure force has a horizontal component, the form drag $p_b \nabla D$. The bottom pressure torque is simply the curl of form drag. Figure 6 shows horizontal distributions of terms in the depth-integrated vorticity equation (divided by D to transform into the depth-averaged form). The stress term is small, and the balance in Eq. (5) is mainly between nonlinear advection and bottom pressure torque. This presents apparent paradox. The nonlinear advection is dominant in the depth-integrated vorticity equation, whereas it appears to be only secondary in the depth-averaged vorticity equation. The total depth-integrated vorticity budget (including the tendency term) is highly accurate; the residual is $O(10^{-15}) \text{ s}^{-2}$.

FIG. 5. Horizontal distributions of terms in the depth-averaged vorticity equation (s^{-2}): (a) advection of background PV, (b) advection of relative PV, (c) JEBAR, (d) curl of depth-distributed surface stress, (e) curl of depth-distributed bottom stress, and (f) difference between JEBAR and background PV advection. Black curves in each panel are the 100-, 200-, 500-, and 1000-m isobaths.

The bottom pressure torque and the JEBAR term are related (Mertz and Wright 1992):

$$\frac{1}{\rho_o} J(p_b, D) = f \mathbf{u}_{gb} \cdot \nabla D = DJ(\chi, D^{-1}) + f \bar{\mathbf{u}}_g \cdot \nabla D, \quad (6)$$

where $\bar{\mathbf{u}}_g$ and $\mathbf{u}_{gb}$ are geostrophic components of the depth-averaged velocity and bottom velocity, respectively. From Eq. (6), the bottom pressure torque can be interpreted as topographic vortex stretching caused by the bottom geostrophic flow. The bottom pressure

FIG. 6. Horizontal distributions of terms in the depth-integrated vorticity equation (s^{-2}): (a) the advection term, (b) torque of surface minus bottom stress, and (c) bottom pressure torque. Each term in Eq. (5) is divided by the total water depth. Black curves in each panel are the 100-, 200-, 500-, and 1000-m isobaths.

FIG. 7. Vertical profiles of the area-averaged vertical velocity in water depths of (a) 140, (b) 180, and (c) 220 m. The analysis box is shown in Fig. 3b.

torque includes both a baroclinic component, JEBAR (multiplied by D), and a barotropic component $f\bar{\mathbf{u}}_g \cdot \nabla D$. As indicated from the momentum balance, the Kuroshio is basically geostrophic. Thus, the depth-averaged velocity is expected to be approximately geostrophic, or $|\bar{\mathbf{u}}_g| \sim |\bar{\mathbf{u}}|$. This is confirmed by the fact that JEBAR $-\text{adv}(\text{bPV})$ and bottom pressure torque (divided by D) are almost indistinguishable (Fig. 5f versus Fig. 6c).

b. Vorticity balance from depth-dependent momentum equations

The three-dimensional vorticity equation is obtained via cross differentiation of depth-dependent momentum equations, Eqs. (2a) and (2b), in Cartesian coordinate:

$$\underbrace{\left(\frac{\partial \text{adv}^y}{\partial x} - \frac{\partial \text{adv}^x}{\partial y}\right)}_{\text{I}} - \underbrace{f \frac{\partial w}{\partial z}}_{\text{II}} = \underbrace{\left(\frac{\partial \text{visc}^y}{\partial x} - \frac{\partial \text{visc}^x}{\partial y}\right)}_{\text{III}}. \quad (7)$$

The notations adv and visc with superscripts x and y stand for the three-dimensional advection and viscosity terms, respectively. Term I on the LHS of Eq. (7) is the nonlinear term, while term II is the divergence or vortex stretching. The nonlinear term includes horizontal and vertical advective of relative vorticity and a tilting term (Holton 2004). Term III on the RHS is the dissipation term. In Eq. (7), the cross differentiation is obtained by calculating horizontal derivatives along the z coordinate. The corresponding numerical procedure in a terrain-following s coordinate is described in appendix B.

Figure 7 shows profiles of the area-averaged vertical velocity in regions where the Kuroshio encounters the shelfbreak (marked in Fig. 3b). The profiles are calculated at three selected water depths, 140, 180, and 220 m, respectively. For each water depth, a grid cell is counted in the ensemble if at least one of its four vertices is greater than the target depth and one of its four vertices is smaller than the target depth. The three vertical velocity profiles have a similar structure, increasing from the surface to about 120–140-m depth, and then decreasing gradually to the bottom. The vertical velocity is large, about $0.5 \times 10^{-3} \text{ m s}^{-1}$. For comparison, the surface Ekman pumping, $\nabla \times \boldsymbol{\tau}_s / f$, is about $0.2 \times 10^{-5} \text{ m s}^{-1}$.

Figure 8 shows vertical profiles of the 3D vorticity balance. The divergence is positive in the upper layer, and negative in the lower layer. The layer interface thus corresponds to the position of the maximum vertical velocity. The interface is approximately constant at about 120–140-m depth. In the upper layer, the vorticity balance is mainly between nonlinear and positive divergence, while dissipation is much smaller. In the lower water column, dissipation is the dominant term, and is balanced by nonlinear and negative divergence. Alternatively, the layer interface can be defined as the top of the bottom boundary layer; though, there is little difference in practice. For all three water depths, the thickness of the bottom boundary layer is the same, about 40 m. It is worth noting that each term in Eq. (7) is calculated by the model diagnostics following the method described in appendix B. The 3D numerical computation is more challenging, compared to 2D cases. Nevertheless, the result is quite satisfactory.

FIG. 8. Vertical profiles of the area-averaged 3D vorticity balance in water depths of (a) 140, (b) 180, and (c) 220 m: nonlinear, divergence, and dissipation. The analysis box is shown in Fig. 3b.

The residual of the total vorticity budget (including the tendency term) is $O(10^{-11})\text{ s}^{-2}$, orders of magnitude smaller than other terms.

For a specific layer k in the model, integrating Eq. (7) vertically from level k to the surface, and dividing each term by D , the layer-integrated vorticity equation from depth-dependent momentum equations can be written as

$$\begin{aligned} \frac{1}{D} \sum_{\sigma=k}^N \left[\left(\frac{\partial \text{adv}^y}{\partial x} - \frac{\partial \text{adv}^x}{\partial y} \right) H_z \right] + \frac{f}{D} w_k \\ = \frac{1}{D} \sum_{\sigma=k}^N \left[\left(\frac{\partial \text{visc}^y}{\partial x} - \frac{\partial \text{visc}^x}{\partial y} \right) H_z \right], \end{aligned} \quad (8)$$

where w_k is the vertical velocity at level k , $H_z = \partial z / \partial \sigma$ is the vertical grid thickness in ROMS. The first term on the LHS is the vertical integration of *nonlinear* terms, while the second term is vortex stretching. The term on the RHS is the vertical integration of *dissipation* terms. A similar equation can be derived for the lower layer, with vortex stretching replaced by $f(w_b - w_k)/D$. Figure 9 shows layer-integrated vorticity balances in the upper layer from the surface to 100 m, and in the lower layer from 100 m to the bottom. In the upper layer, the main balance is between vortex stretching and nonlinear advection, while dissipation is generally small. This indicates that the Kuroshio intrusion, which is mostly confined to the upper 100 m, is mainly caused by the nonlinear (inertial) effect. The Kuroshio tends to “overshoot” across the ECS slope, accompanied by intense upwelling at the shelfbreak. In the lower layer, the layer-integrated dissipation is dominant, and is balanced

by nonlinear advection and horizontal divergence. The striking contrast in the vorticity balance between upper and low layers is missing in the 2D vorticity equation. It is worth noting that the depth integration of 3D vorticity equation is fully consistent with the depth-integrated vorticity equation. In other words, the sum of upper- and lower-layer vorticity balances (Fig. 9) is exactly the same as the depth-integrated vorticity balance (Fig. 6).

The bottom pressure torque of the depth-integrated vorticity budget [Eq. (5)] is not present in the 3D vorticity balance [Eq. (7)]. Appendix A shows how bottom pressure torque arises from the depth integration of the 3D vorticity balance, following Hughes and de Cuevas (2001). The bottom velocity, $w_b = -\mathbf{u}_b \cdot \nabla D$, can be divided into the geostrophic and ageostrophic components. The vortex stretching associated with bottom geostrophic divergence is the same as bottom pressure torque [Eq. (A11a)]. The vortex stretching associated with bottom ageostrophic divergence is due to viscous and nonlinear terms [Eq. (A11b)]. The geostrophic and ageostrophic bottom divergence are nearly compensated that the total bottom vortex stretching $f w_b / D$ is very small (Fig. 10). This indicates that bottom pressure torque is generated by the ageostrophic vortex stretching; the latter is dominated by viscous terms. The bottom viscous stress divergence corresponds to upwelling induced by an upslope bottom Ekman current.

5. No-advection sensitivity experiment

From examination of momentum and vorticity balances, it appears that nonlinear advection plays a major

FIG. 9. Terms in the layer-averaged vorticity balance (s^{-2}): (a),(d) divergence, (b),(e) nonlinear, and (c),(f) dissipation. (top) Integration over the upper 100 m, and (bottom) integration from 100 m to the bottom. Each term is divided by the total water depth. Black curves in each panel are the 100-, 200-, 500-, and 1000-m isobaths.

role in Kuroshio on-shelf intrusion northeast of Taiwan. To further test the importance of advection, a sensitivity experiment, the NO-ADV, was conducted by removing advective terms from momentum equations in NEST2, while keeping same open boundary conditions interpolated from NEST1. The same one-month period

(February 2001) is used in the analysis. In the NO-ADV, an explicit horizontal viscosity is added to maintain numerical stability.

Figure 11 shows horizontal distributions of monthly-mean temperatures and velocities from the NO-ADV sensitivity experiment. After leaving the northeast

FIG. 10. Horizontal distributions of terms in the depth-integrated vorticity equation (s^{-2}): (a) bottom vortex stretching of ageostrophic current, (b) bottom vortex stretching of geostrophic current, and (c) total bottom vortex stretching. Each term is divided by the total water depth. Black curves in each panel are the 100-, 200-, 500-, and 1000-m isobaths.

FIG. 11. Mean temperatures (color; $^{\circ}\text{C}$) and velocity vectors (black arrows) in February from the NO-ADV experiment at (a) 0, (b) 100, and (c) 200 m. Gray curves are the 100-, 200-, 500-, and 1000-m isobaths.

corner of Taiwan, the Kuroshio makes a sharp, almost 90° turn following the orientation of the continental slope, while its core stays offshore of the 200-m isobath. This is in striking contrast to the base case in which the core of the Kuroshio continues in the northeastward direction across the shelfbreak (Fig. 2). The altered flow pattern is also clearly reflected in the isotherm distribution. At 100 m, the isotherms are approximately parallel to the isobaths in the NO-ADV experiment, but they are stretched across the shelfbreak in the base case. From both temperature and velocity distributions, it is clear that the on-shelf intrusion is much weaker in the NO-ADV experiment. The vertical velocity is also much smaller (not shown). We note that in the NO-ADV

experiment the surface currents tend to branch onto the shelf along the Taiwan coast. The surface branch follows the 100-m isobath with little cross-isobath motion.

The depth-averaged and depth-integrated vorticity balances are calculated from the NO-ADV sensitivity experiment (Fig. 12). With advective terms removed, the depth-averaged vorticity balance, Eq. (4), is completely dominated by $\text{adv}(\text{bPV})$ and JEBAR. The surface and bottom stresses are small, and horizontal viscosity is significant only near the coast. The JEBAR term is about twice as large as in the base case. Its spatial pattern, however, is controlled by small-scale topographic features: JEBAR is negative approaching (in the direction of the Kuroshio Current) a canyon and is

FIG. 12. Horizontal distributions of terms in the depth-averaged vorticity equation (s^{-2}) from the NO-ADV experiment: (a) advection of background PV, and (b) JEBAR, and (c) bottom pressure torque divided by the total water depth. Black curves in each panel are the 100-, 200-, 500-, and 1000-m isobaths.

positive leaving a canyon. For the depth-integrated vorticity balance, Eq. (5), without advection, bottom pressure torque is trivial, an order of magnitude smaller than JEBAR or $\text{adv}(\text{bPV})$. Indeed, the bottom pressure torque is comparable to $\text{JEBAR} - \text{adv}(\text{bPV})$. The 3D vorticity balance indicates a weak vertical velocity with no meaningful bottom boundary layer (not shown).

6. Discussion and conclusions

In this study, the vorticity balance for the Kuroshio on-shelf intrusion northeast of Taiwan is examined using a high-resolution nested numerical model. The depth-averaged vorticity equation is formed from depth-averaged momentum equations. This approach, which gives rise to the JEBAR term, is similar to previous model studies, for example, Guo et al. (2006), Oey et al. (2010), and Wang and Oey (2016). The primary balance is between advection of geostrophic potential vorticity (APV) and the JEBAR term. The depth-integrated vorticity balance is also examined. This approach, on the other hand, indicates a primary balance between horizontal advection and bottom pressure torque. To reconcile the difference between these two approaches, it is shown that $\text{JEBAR} - \text{APV}$ is in fact very similar to bottom pressure torque. Whether it is useful to look into the JEBAR and APV separately is probably a matter of taste (Yeager 2015). In the absence of nonlinear advection, bottom pressure torque is trivial, and the on-shelf intrusion is absent. This is dramatized in the NO-ADV experiment in which JEBAR and $\text{adv}(\text{bPV})$ are perfectly compensated.

The 3D vorticity equation is formed from depth-dependent momentum equations. A numerical procedure is presented for cross differentiation of horizontal momentum equations *directly* from the diagnostics of a terrain-following s -coordinate model such as ROMS. To our knowledge, this study is the first time a 3D vorticity balance is examined in coastal oceans of steep slopes. The vorticity profiles indicate a basic two-layered structure, a frictionless upper layer where nonlinear advection is balanced by horizontal divergence, and a dissipative lower layer where vertical viscosity is balanced by nonlinear advection and horizontal divergence. As the 3D vorticity equation is consistent with the depth-integrated vorticity equation, it can be seen that bottom pressure torque mainly corresponds to the depth-integrated viscous effect. Previously, Gan et al. (2016) have considered a domain- and depth-integrated vorticity balance of a three-layer circulation in the South China Sea. Their layered approach is consistent with our study in emphasizing the role of bottom pressure torque. On the other hand, as there is no direct correspondence

between 3D vorticity equation and depth-averaged vorticity equation, great care must be exercised in interpretation of the JEBAR term.

In the traditional Ekman theory, the depth-integrated viscous effect appears as the curl of the bottom stress, the first term on the RHS of Eq. (A7c). In a sloping bottom, the viscous stress divergence, the second term on the RHS of Eq. (A7c), is dominant, and the associated vertical motion is an order of magnitude greater than the traditional Ekman pumping. Moreover, as the divergence term has a simple (linear) vertical structure, nonlinear advection appears to be critical in determining the structure of the bottom boundary layer (Fig. 8). The importance of nonlinear advection is also highlighted in the NO-ADV experiment.

Hughes (2000) and Hughes and de Cuevas (2001) have argued that the western boundary currents are fundamentally inviscid in realistic oceans with sloping sidewalls. This could be easily understood from the depth-integrated vorticity balance. In a narrow boundary current, the vorticity balance must be between βV (V is the northward volume transport) and bottom pressure torque, allowing for modification by nonlinear advection. However, this raises an apparent paradox, as friction is fundamental in the classical western boundary current theory (Jackson et al. 2006). It is customary to interpret bottom pressure torque in terms of the (inviscid) geostrophic flow at the bottom. This is misleading because the bottom geostrophic flow is the consequence of a frictional bottom boundary layer. Since the total bottom vortex stretching is very small (the no-slip bottom boundary condition), geostrophic and ageostrophic vortex stretching are almost equivalent (of an opposite sign). Thus, the bottom pressure torque is perhaps best described as a proxy of the viscous (ageostrophic) stress divergence over sloping bottom. In other words, friction is also fundamental in realistic ocean basins with sloping sidewalls. The ageostrophic vortex stretching vanishes only if there is no friction at the bottom (the free-slip boundary condition).

Acknowledgments. This work is supported by the Natural Science Foundation of China (41730535), the National Key Research and Development Plan of China (2016YFC1401603), the Scientific Research Fund of the Second Institute of Oceanography, MNR (JG1711), the National Programme on Global Change and Air-Sea Interaction (GASI-IPOVAI-04), and the Natural Science Foundation of China (41690120, 41690121, 41621064). The QuikSCAT data are produced by Remote Sensing Systems and sponsored by the NASA Ocean Vector Winds Science Team. Data are available at <http://www.remss.com>.

APPENDIX A

Derivation of the 2D and 3D Vorticity Balances

It is useful to summarize the three different forms of the vorticity balance and their derivations. We assume a constant f plane and neglect horizontal viscosity terms. It is straightforward to include the β effect, the classical Sverdrup balance.

The steady state depth-averaged momentum equations are

$$\frac{1}{D} \left(\frac{\partial \bar{u} \bar{u} D}{\partial x} + \frac{\partial \bar{v} \bar{u} D}{\partial y} \right) - f \bar{v} + g \frac{\partial \eta}{\partial x} + \frac{1}{D} \int_{-H}^{\eta} \int_z^{\eta} \frac{\partial b}{\partial x} dz' dz = \frac{\tau_s^x}{D} - \frac{\tau_b^x}{D}, \quad (\text{A1a})$$

$$\frac{1}{D} \left(\frac{\partial \bar{u} \bar{v} D}{\partial x} + \frac{\partial \bar{v} \bar{v} D}{\partial y} \right) + f \bar{u} + g \frac{\partial \eta}{\partial y} + \frac{1}{D} \int_{-H}^{\eta} \int_z^{\eta} \frac{\partial b}{\partial y} dz' dz = \frac{\tau_s^y}{D} - \frac{\tau_b^y}{D}. \quad (\text{A1b})$$

The depth-averaged 2D vorticity equation is obtained by cross differentiating Eqs. (A1a) and (A1b):

$$\underbrace{\mathbf{U} \cdot \nabla \left(\frac{f}{D} \right)}_{\text{I}} + \underbrace{\mathbf{U} \cdot \nabla \left(\frac{\bar{\zeta}}{D} \right)}_{\text{II}} = \underbrace{J(\chi, D^{-1})}_{\text{III}} + \underbrace{\nabla \times \left(\frac{\boldsymbol{\tau}_s}{D} \right)}_{\text{IV}} - \underbrace{\nabla \times \left(\frac{\boldsymbol{\tau}_b}{D} \right)}_{\text{V}}, \quad (\text{A2})$$

where $\chi = \int_{-H}^{\eta} z b dz'$ is the potential energy anomaly. The first term on the RHS of (A2) is the joint effect of baroclinicity and relief (JEBAR) term. By its definition, the JEBAR term can also be expressed as $-f/D(\bar{u}_g - \mathbf{u}_{gb}) \nabla D$, where $\bar{u}_g$ is the depth-averaged geostrophic velocity and $\mathbf{u}_{gb}$ is the bottom geostrophic velocity (Mertz and Wright 1992).

Multiplying the water depth D to Eqs. (A1a) and (A1b), the steady state depth-integrated momentum equations are

$$\left(\frac{\partial \bar{u} \bar{u} D}{\partial x} + \frac{\partial \bar{u} \bar{v} D}{\partial y} \right) - f \bar{v} D = - \frac{\partial}{\partial x} \int_{-D}^0 p dz + \frac{\partial D}{\partial x} p_b + \tau_s^x - \tau_b^x, \quad (\text{A3a})$$

$$\left(\frac{\partial \bar{v} \bar{u} D}{\partial x} + \frac{\partial \bar{v} \bar{v} D}{\partial y} \right) + f \bar{u} D = - \frac{\partial}{\partial y} \int_{-D}^0 p dz + \frac{\partial D}{\partial y} p_b + \tau_s^y - \tau_b^y, \quad (\text{A3b})$$

where the subscript b represents variables evaluated at the bottom.

Cross differentiating Eqs. (A3a) and (A3b), the depth-integrated vorticity equation (the barotropic vorticity equation) is

$$\left\{ \frac{\partial[(\text{adv})^y D]}{\partial x} - \frac{\partial[(\text{adv})^x D]}{\partial y} \right\} = \left[\frac{\partial(\tau_s^y - \tau_b^y)}{\partial x} - \frac{\partial(\tau_s^x - \tau_b^x)}{\partial y} \right] + \frac{1}{\rho_0} J(p_b, D), \quad (\text{A4a})$$

or in vector form,

$$\nabla \times (D \text{adv}) = \nabla \times (\boldsymbol{\tau}_s - \boldsymbol{\tau}_b) + \frac{1}{\rho_0} J(p_b, D), \quad (\text{A4b})$$

where adv stands for the x and y components of the advective terms in Eqs. (A3a) and (A3b):

$$\begin{aligned} \text{adv} &= [(\text{adv})^x, (\text{adv})^y] \\ &= \left(\frac{\partial \bar{u} \bar{u} D}{\partial x} + \frac{\partial \bar{u} \bar{v} D}{\partial y}, \frac{\partial \bar{v} \bar{u} D}{\partial x} + \frac{\partial \bar{v} \bar{v} D}{\partial y} \right). \end{aligned}$$

The last term in Eq. (A4b) is the bottom pressure torque (Mertz and Wright 1992; Hughes 2000; Jackson et al. 2006; Yeager 2015).

The steady state 3D momentum equations are

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v + g \frac{\partial \eta}{\partial x} + \int_z^{\eta} \frac{\partial b}{\partial x} dz' - \frac{\partial}{\partial z} \left(K_M \frac{du}{dz} \right) &= 0, \end{aligned} \quad (\text{A5a})$$

$$\begin{aligned} u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u + g \frac{\partial \eta}{\partial y} + \int_z^{\eta} \frac{\partial b}{\partial y} dz' - \frac{\partial}{\partial z} \left(K_M \frac{dv}{dz} \right) &= 0. \end{aligned} \quad (\text{A5b})$$

Taking curl of the 3D momentum equations, the 3D depth-dependent vorticity equation is

$$\underbrace{\left(\frac{\partial \text{adv}^y}{\partial x} - \frac{\partial \text{adv}^x}{\partial y} \right)}_{\text{I}} - \underbrace{f \frac{\partial w}{\partial z}}_{\text{II}} = \underbrace{\left(\frac{\partial \text{visc}^y}{\partial x} - \frac{\partial \text{visc}^x}{\partial y} \right)}_{\text{III}}, \quad (\text{A6a})$$

or in vector form,

$$\underbrace{\nabla \times (\text{adv})}_{\text{I}} - \underbrace{f \frac{\partial w}{\partial z}}_{\text{II}} = \underbrace{\nabla \times (\text{visc})}_{\text{III}}, \quad (\text{A6b})$$

where

$$\begin{aligned} \text{adv} &= (\text{adv}^x, \text{adv}^y) \\ &= \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}, u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right), \end{aligned}$$

$$\mathbf{visc} = (\text{visc}^x, \text{visc}^y) = \left[\frac{\partial}{\partial z} \left(K_M \frac{du}{dz} \right), \frac{\partial}{\partial z} \left(K_M \frac{dv}{dz} \right) \right].$$

Following Hughes and de Cuevas (2001), the depth integration of each term in Eq. (A6b) gives

$$\int_{-D}^0 (\text{I}) dz = \nabla \times (D \overline{\mathbf{adv}}) - \nabla D \times \mathbf{adv}_b, \quad (\text{A7a})$$

$$\int_{-D}^0 (\text{II}) dz = fw_b = -f\mathbf{u}_b \cdot \nabla D, \quad (\text{A7b})$$

$$\int_{-D}^0 (\text{III}) dz = \nabla \times (\boldsymbol{\tau}_s - \boldsymbol{\tau}_b) - \nabla D \times \mathbf{visc}_b. \quad (\text{A7c})$$

Combining Eqs. (A7a)–(A7c), the depth-integrated three-dimensional vorticity equation is

$$\nabla \times (D \overline{\mathbf{adv}}) - f\mathbf{u}_b \cdot \nabla D = \nabla \times (\boldsymbol{\tau}_s - \boldsymbol{\tau}_b) - \nabla D \times \mathbf{visc}_b + \nabla D \times \mathbf{adv}_b. \quad (\text{A8})$$

The steady-state momentum equation at the bottom is

$$\mathbf{adv}_b + f\mathbf{k} \times \mathbf{u}_b = -\frac{1}{\rho_0} \nabla p_b + \mathbf{visc}_b. \quad (\text{A9})$$

We can further divide the bottom velocity into the geostrophic component $\mathbf{u}_{gb}$ and ageostrophic component $\mathbf{u}_{ab}$ so that $\mathbf{u}_b = \mathbf{u}_{gb} + \mathbf{u}_{ab}$. The geostrophic part of Eq. (A9) is

$$f\mathbf{k} \times \mathbf{u}_{gb} = -\frac{1}{\rho_0} \nabla p_b. \quad (\text{A10a})$$

The ageostrophic part of Eq. (A9) is

$$\mathbf{adv}_b + f\mathbf{k} \times \mathbf{u}_{ab} = \mathbf{visc}_b. \quad (\text{A10b})$$

The bottom vortex stretching induced by bottom geostrophic flow is obtained by $\nabla D \times$ Eq. (A10a):

$$f\mathbf{u}_{gb} \cdot \nabla D = \frac{1}{\rho_0} J(p_b, D). \quad (\text{A11a})$$

In other words, the bottom pressure torque represents the geostrophic component of the bottom vortex stretching. Similarly, the ageostrophic component of the bottom vortex stretching is obtained by $\nabla D \times$ Eq. (A10b):

$$f\mathbf{u}_{ab} \cdot \nabla D = \nabla D \times \mathbf{visc}_b - \nabla D \times \mathbf{adv}_b. \quad (\text{A11b})$$

The first term on the RHS of Eq. (A11b) represents the viscous stress divergence at the bottom. Substituting Eqs. (A11a) and (A11b) into Eq. (A8), the barotropic vorticity equation Eq. (A4b) is fully recovered. Obviously,

the depth integration of 3D vorticity equation is consistent with the 2D vorticity balance. It is worth noting that $\mathbf{u}_{gb}$ and $\mathbf{u}_{ab}$ are coupled through appropriate bottom boundary conditions, for example, a no-slip condition. In other words, the bottom pressure torque is determined dynamically by the bottom ageostrophic divergence, Eqs. (A11a) and (A11b).

APPENDIX B

Calculation of the 3D Vorticity Equation in ROMS

In ROMS, the calculations of the depth-averaged vorticity equation Eq. (A2) and the depth-integrated vorticity equation Eq. (A4) are straightforward since all the variables in these equations are independent of the transformation between different coordinates. The calculation of the 3D vorticity equation Eqs. (A6a) and (A6b) needs more demonstration.

The transformation of horizontal differentiation of a variable A from z coordinate (x, y, z) to s coordinate (x^*, y^*, σ) is (taking the differentiation on the x direction as an example)

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial x^*} - \frac{1}{H_z} \frac{\partial z}{\partial x^*} \frac{\partial A}{\partial \sigma}, \quad (\text{B1})$$

where $H_z = \partial z / \partial \sigma$ is the vertical grid thickness. The depth-integrated horizontal differentiation is

$$\int_{-D}^0 \frac{\partial A}{\partial x} dz = \int_1^N \frac{\partial A}{\partial x} H_z d\sigma = \int_1^N \left(\frac{\partial A}{\partial x^*} H_z - \frac{\partial z}{\partial x^*} \frac{\partial A}{\partial \sigma} \right) d\sigma, \quad (\text{B2})$$

where $\sigma = 1, 2, \dots, N$ is the number of the vertical s coordinate from the bottom to the surface. The first term inside the integral in Eq. (B2) is

$$\text{I}_k = \frac{\partial A}{\partial x^*} H_z = \frac{\partial (AH_z)}{\partial x^*} - A \frac{\partial H_z}{\partial x^*}. \quad (\text{B3a})$$

Then, the depth-integrated form of I_k is

$$\begin{aligned} \sum_{k=1}^N \text{I}_k &= \frac{\partial}{\partial x^*} \int_1^N (AH_z) d\sigma - \int_1^N A \frac{\partial H_z}{\partial x^*} d\sigma \\ &= \frac{\partial (D\bar{A})}{\partial x^*} - \int_1^N A \frac{\partial H_z}{\partial x^*} d\sigma, \end{aligned} \quad (\text{B3b})$$

where $\bar{A}$ is the depth-averaged A .

The second term inside the integral of Eq. (B2) is

$$\text{II}_k = \frac{\partial z}{\partial x^*} \frac{\partial A}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left(\frac{\partial z}{\partial x^*} A \right) - A \frac{\partial H_z}{\partial x^*}. \quad (\text{B4a})$$

Similarly, the depth-integrated form of Π_k is

$$\begin{aligned}\sum_{k=1}^N \Pi_k &= \int_1^N \frac{\partial}{\partial \sigma} \left(\frac{\partial z}{\partial x^*} A \right) d\sigma - \int_1^N A \frac{\partial H_z}{\partial x^*} d\sigma \\ &= \frac{\partial D}{\partial x^*} A_1 - \int_1^N A \frac{\partial H_z}{\partial x^*} d\sigma,\end{aligned}\quad (\text{B4b})$$

where A_1 is A at the bottom ($\sigma = 1$). After cancelling the second terms in Eqs. (B3b) and (B4b), Eq. (B2) is

$$\frac{1}{D} \sum_{k=1}^N (\mathbf{I}_k - \Pi_k) = \frac{1}{D} \frac{\partial(D\bar{A})}{\partial x^*} - \frac{1}{D} \frac{\partial D}{\partial x^*} A_1. \quad (\text{B5})$$

The transformation of the depth-integrated horizontal differentiation on y direction takes the same form as Eq. (B5). Then, the depth-integrated vertical component of $\nabla \times \mathbf{A} = \text{curl}(\mathbf{A}^x, \mathbf{A}^y)$ in s coordinates is

$$\int_{-D}^0 \nabla \times \mathbf{A} dz = \nabla^* \times (D\bar{\mathbf{A}}) - \nabla^* D \times \mathbf{A}_{\sigma=1}, \quad (\text{B6})$$

where the operator ∇^* is the horizontal differentiation along s -coordinate surfaces [$\nabla^* = (\partial/\partial x^*, \partial/\partial y^*)$]. Equation (B6) is the numerical equivalent of Eqs. (A7a)–(A7c). For example, if the vector $\mathbf{A}$ stands for vertical viscosity terms **visc** in the momentum equation, the depth-integrated of $\nabla \times \mathbf{visc}$ in s coordinate is

$$\int_{-D}^0 (\text{III}) dz = \nabla^* \times (\boldsymbol{\tau}_s - \boldsymbol{\tau}_b) - \nabla^* D \times \mathbf{visc}_{\sigma=1}, \quad (\text{B7})$$

where the horizontal differentiation of 2D variables along the s coordinate is identical to that along the z coordinate in Eqs. (A7a) and (A7c). From Eq. (B7), the depth-integrated three-dimensional vorticity equation [Eq. (8)] in s coordinates can be calculated.

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