Instability Processes in Simulated Finite-Width Ocean Fronts

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(Manuscript received 3 February 2020, in final form 2 July 2020)

ABSTRACT

A simple, isolated front is modeled using a turbulence resolving, large-eddy simulation (LES) to examine the generation of instabilities and inertial oscillations by surface fluxes. Both surface cooling and surface wind stress are considered. Coherent roll instabilities with 200–300-m horizontal scale form rapidly within the front after the onset of surface forcing. With weak surface cooling and no wind, the roll axis aligns with the front, yielding results that are equivalent to previous constant gradient symmetric instability cases. After ~ 1 day, the symmetric modes transform into baroclinic mixed modes with an off-axis orientation. Traditional baroclinic instability develops by day 2 and thereafter dominates the overall circulation. Addition of destabilizing wind forcing produces a similar behavior, but with off-axis symmetric-Ekman shear modes at the onset of instability. In all cases, imbalance of the geostrophic shear by vertical mixing leads to an inertial oscillation in the frontal currents. Analysis of the energy budget indicates an exchange between kinetic energy linked to the inertial currents and potential energy associated with restratification as the front oscillates in response to the vertically sheared inertial current. Inertial kinetic energy decreases from enhanced mixed layer turbulence dissipation and vertical propagation of inertial wave energy into the pycnocline.

1. Introduction

Ocean fronts are defined as regions of rapid lateral density change relative to the background ocean. Surface fronts often extend downward along constant density surfaces and merge with the subsurface pycnocline, providing a connection between the interior ocean and atmosphere. Because of this connection, circulations associated with fronts are able to transport heat, momentum, and tracers between the surface mixed layer and water beneath the ocean boundary layer, frequently leading to concentration of nutrients and high productivity (Niller and Reynolds 1984; Woods 1988; Taylor and Ferrari 2011; Thomas et al. 2016). The same density structure that defines the front can also lead to dynamic instabilities that generate strong vertical circulations either through eddy mixing via symmetric, baroclinic (e.g., Haine and Marshall 1998), and Ekman instabilities (Skyltingstad et al. 2017), or from secondary circulations generated by unbalanced geostrophic shear

and Ekman flow (McWilliams 2017). The net result of these instabilities is restratification of the ocean surface boundary layer (e.g., Fox-Kemper et al. 2008; Thomas et al. 2008).

Recent studies have examined aspects of frontal instabilities using both a constant gradient approach (e.g., Taylor and Ferrari 2010) representing frontal instability embedded within a broad frontal zone, and finite-width frontal systems that can support secondary circulations associated with the isolated geostrophic jet structure (e.g., McWilliams et al. 2015) and baroclinic instability. Using the constant gradient approach with essentially a two-dimensional framework, Taylor and Ferrari (2010) demonstrated that shear associated with the geostrophic current (thermal wind) can provide energy for symmetric instability, generating rapid mixing in fronts forced by either destabilizing winds (in the same direction as the surface geostrophic current) or surface cooling. Skyltingstad et al. (2017) extended this work by increasing the downstream domain size and found mixed-mode symmetric-Ekman shear instabilities that were oriented at an angle to the front. Their simulations also found that abrupt mixing of fronts could generate inertial currents from mixing of the geostrophic shear and generation of unbalanced flow, similar to theory from Tandon and Garrett (1994). Using a constant

Denotes content that is immediately available upon publication as open access.

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DOI: 10.1175/JPO-D-20-0030.1

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background gradient simulation, [Stamper and Taylor \(2016\)](#) also found mixed mode instabilities that evolved from linear symmetric modes in an Eady model representation. Over time, restratification by the symmetric and mixed modes increased the hydrodynamic stability leading to traditional baroclinic instability consistent with theory developed in [Stone \(1966\)](#).

Research on finite width fronts has mostly focused on the formation of submesoscale mixed layer instabilities (e.g., [Barth 1989](#); [Samelson 1993](#); [Samelson and Chapman 1995](#); [Haine and Marshall 1998](#); [Boccaletti et al. 2007](#)) produced by baroclinic instability, and frontogenesis in frontal systems associated with filaments ([McWilliams et al. 2015](#)). A key concept in the latter work centers on the effects of geostrophic shear on the Ekman boundary layer, which can result in a flow balance in frontal regions between the background pressure gradient, Coriolis term, and vertical mixing of momentum. The flow is essentially the geostrophic thermal wind shear modified by vertical turbulent momentum flux ([Stommel 1960](#); [Cronin and Kessler 2009](#); [Gula et al. 2014](#)). In finite width fronts, differences in the Ekman flow balance will lead to convergent and divergent flow at the top and base of the front, and frontogenesis/frontolysis. For example, for a mixed layer front as depicted in [Fig. 1](#), the balanced geostrophic flow is southward at the surface and decreases with depth along with the horizontal pressure gradient generated by the density field (i.e., the thermal wind gradient). Away from the front, the fluid is at rest. Vertical mixing by either wind or surface cooling, will decrease the surface current and Coriolis term, creating a new balanced flow that is oriented toward the east at the surface and west at the mixed layer base in the frontal zone.

Regions of convergence and divergence resulting from this modified geostrophic balance lead to a vertical secondary circulation with downwelling flow on the cold side of the front and upwelling on the warm side of the front. The resulting frontogenesis can produce rapid strengthening of surface fronts and an overall downscale flux of energy ([Sullivan and McWilliams 2018](#)). [Sullivan and McWilliams \(2018\)](#) find that for an $O(1)$ km cold filament, frontogenesis is arrested by turbulence enhanced by the local shear instabilities and wave induced Langmuir circulation. In a more general case, [Crowe and Taylor \(2018, 2019a\)](#) show that frontal evolution for fixed viscosity and diffusion coefficients without wind forcing exhibits a brief phase of frontogenesis, followed by lateral spreading of the front induced by shear dispersion. Their results suggest that the mixing responsible for cross-frontal currents effectively prevents frontogenesis; however, they do not account for the added momentum from surface winds or the action of resolved turbulent circulations from convection.

FIG. 1. Initial temperature ($^{\circ}\text{C}$) and meridional velocity fields.

Neither of these studies has examined how an isolated, finite width front evolves with external wind and surface buoyancy flux forcing. In the filament case examined by [Sullivan and McWilliams \(2018\)](#), the two fronts on either side of the filament are arrested within 24 h of the front initialization. Circulations associated with the filament cause interactions between both fronts that would not occur for an isolated system. [Crowe and Taylor \(2018\)](#) address the problem of an isolated front, but apply a constant eddy viscosity and diffusion with fixed initial stratification and no external forcing. To extend these studies, we model a finite-width front using a large-eddy simulation (LES) model with surface cooling and wind stress. Our goal is to examine how instability processes, such as symmetric and baroclinic instability, interact with ocean turbulence and what effects these processes have on frontal spreading, circulation, and overall propagation. Our simulations follow the progression of instability processes first modeled in [Haine and Marshall \(1998\)](#) for deep convective mixed layers, but for a shallow mixed layer (70 m) with resolved turbulence modeled via LES in place of constant viscosity. In addition, we consider destabilizing wind forcing as studied in, for example, [Thomas and Lee \(2005\)](#) and [Taylor and Ferrari \(2010\)](#) and observed by [D'Asaro et al. \(2011\)](#).

Our experiments also examine the generation of inertial currents by vertical mixing from either surface cooling or from wind and wave generated shear and convective instability. Generation of inertial currents is expected during geostrophic adjustment for a vertically oriented, unbalanced, constant gradient front as demonstrated in [Tandon and Garrett \(1994\)](#). The constant gradient case examined in [Skylvingstad et al. \(2017\)](#) did not account for frontal zone margins where we expect a discontinuity in the inertial currents and possible propagation or dispersion of inertial wave energy (e.g., [Thomas 2017](#)).

The paper begins with a description of the model and experiments in [section 2](#). Results are presented in [section 3](#) for three cases; two with surface cooling and a third case with surface wind stress but no surface

cooling. Our main goal in these experiments is to perturb the balanced geostrophic current without greatly modifying the frontal strength through direct forcing of turbulence by the surface fluxes. [Section 4](#) focuses on analysis of the frontal inertial current generation, turbulence, and energy budgets, followed by conclusions in [section 5](#).

2. Model description and analysis methods

a. Model and experimental design

We use the LES model described in [Skyltingstad and Samelson \(2012\)](#). The model is configured as a channel with horizontal dimensions of 14.4 km $\times$ 6.0 km,

extending 120 m in the vertical. Grid spacing of 3 m is used in all directions. Lateral boundaries are periodic, with rigid lid conditions applied on the top and bottom. Initial conditions are prescribed with a mixed layer extending from the surface to 70-m depth and a constant vertical temperature gradient in the pycnocline with $N^2 = (g/\rho)(\partial\rho/\partial z) = 1.96 \times 10^{-4} \text{ s}^{-2}$, where ρ is density and $g = 9.81 \text{ m s}^{-2}$ is gravitational acceleration. We note that this initial condition differs from that of [Taylor and Ferrari \(2010\)](#) and [Stamper and Taylor \(2016\)](#) in that the surface boundary layer is uniformly mixed rather than stratified. A finite width front is imposed using a zonal background temperature gradient defined using

$$\begin{cases} \frac{dT}{dx} = 0.5 \frac{dT_b}{dx} \tanh\left[\left(x - \frac{x_{\max}}{3}\right)/D\right] + 0.5 \frac{dT_b}{dx}, & \text{for } x < x_{\max}/2, \\ \frac{dT}{dx} = 0.5 \frac{dT_b}{dx} - \frac{dT_b}{dx} \tanh\left[\left(x - \frac{2x_{\max}}{3}\right)/D\right], & \text{for } x > x_{\max}/2, \end{cases} \quad (1)$$

with $x_{\max} = 14.4 \text{ km}$, $D = 150 \text{ m}$, and dT_b/dx representing a constant temperature gradient scale for the front. Model velocities are initialized in thermal wind balance with the imposed thermal gradient,

$$\frac{\partial V_g}{\partial z} = \frac{g\alpha}{f} \frac{dT}{dx} = \frac{1}{f} \frac{db}{dx}, \quad (2)$$

where $\alpha = 2 \times 10^{-4}$ is the approximate thermal expansion coefficient for seawater, b is the buoyancy, and $f = 1 \times 10^{-4} \text{ s}^{-1}$ is the Coriolis parameter. This initialization is similar to the front structure defined in [Crowe and Taylor \(2018\)](#), however, in our case the temperature gradient in the center of the front is nearly constant, with large changes in gradient confined to the edges of the front. [Figure 1](#) provides cross sections of the initial state for our basic experimental setup with a prescribed $dT_b/dx = 0.1^\circ \text{C km}^{-1}$. For this frontal strength, the lateral buoyancy gradient is

$$M^2 = \frac{db}{dx} \approx 2 \times 10^{-7} \text{ s}^{-2}.$$

Three primary cases are simulated using the model. In the first two cases, denoted C10 and C100, the model is forced with surface cooling of -10 and -100 W m^{-2} , respectively, imposed at $t = 0$ and maintained for all $t > 0$. These cases duplicate conditions similar to experiments from [Stamper and Taylor \(2016\)](#), but for a finite-width front rather than effectively an infinite frontal zone. The third case, denoted WS05, imposes a

wind stress aligned with the front so that the Ekman transport is destabilizing (so-called “down front” wind direction). Wind stress magnitude is increased linearly from 0 to -0.05 N m^{-2} over the first 12 h of the simulation. Direct buoyancy flux for case WS05 is set to zero. Beginning the simulations with an immediate imposed heat flux along with an existing mixed layer disrupts the initialized balanced flow similar to an ocean storm event. In all three cases, one effect of these surface fluxes is the destruction of potential vorticity (PV) defined as

$$q = (f\hat{\mathbf{k}} + \nabla \times \mathbf{v}) \cdot \nabla b, \quad (3)$$

where $\mathbf{v}$ is the three-dimensional velocity vector and $\hat{\mathbf{k}}$ is the vertical unit vector. A region of negative PV or $PV \approx 0$ is a necessary condition for the formation of symmetric instability ([Thomas and Lee 2005](#); [Taylor and Ferrari 2010](#)). For fluid in geostrophic balance with uniform buoyancy gradients, $q = fN^2 - M^4/f$, indicating that our boundary layer initial condition in the frontal zone is immediately unstable to symmetric instability ([Taylor and Ferrari 2010](#)).

Both surface cooling and down front wind stress generate turbulence by destabilizing the density gradient and therefore can act to decrease restratification by frontal instabilities. [Mahadevan et al. \(2010\)](#) introduced a “restratification ratio” comparing the strength of restratification, as parameterized by [Fox-Kemper et al. \(2008\)](#), with surface buoyancy flux,

$$R_{\text{MLI}} = \frac{B_o f}{M^4 H^2}, \quad (4)$$

where B_o is the surface buoyancy flux and H is the mixed layer depth. Here we use the notation from Taylor (2016) who apply this scaling with a phytoplankton growth LES model for a front undergoing baroclinic mixed layer instabilities (referenced as MLIs). Mahadevan et al. (2010) found that $R_{MLI} < 0.06$ generated active restratification by mixed layer instabilities whereas $R_{MLI} > 0.06$ suppressed mixed layer instabilities and restratification. For the cases C10 and C100 conducted here, the value of $R_{MLI} = 0.0024$ and 0.024 . We calculate R_{MLI} for case WS05 by substituting the Ekman buoyancy flux, $B_{Ek} = \tau^y M^2 / (\rho_o f)$, yielding $R_{MLI} = 0.05$. These estimates are all smaller than 0.06 suggesting that buoyancy and shear production of turbulence will not dominate over mixed layer instabilities.

b. Analysis methods

Ageostrophic mixed layer frontal instabilities can be divided into waves varying primarily in the cross front direction or “symmetric” modes, and waves varying along the front or “baroclinic” modes (Stone 1966). Kelvin–Helmholtz instabilities are also possible, but are not the primary interest here. Stamper and Taylor (2016) find that in their direct numerical simulation with a constant gradient, the Richardson number,

$$Ri = \frac{N^2}{\left(\frac{\partial u}{\partial z}\right)^2},$$

determines which mode is dominant. For $0.25 < Ri < 1.0$, symmetric instabilities are the main mode at the beginning of the simulation, but transition to mixed modes as Ri increases from restratification. Skillingstad et al. (2017), using LES, note a similar behavior for cases with no wind but surface buoyancy flux. One objective of this study is to determine how surface wind and heat flux control the type of instability and if the progression from symmetric to baroclinic modes modeled in Stamper and Taylor (2016) is produced in a forced LES case.

We are also interested in understanding how secondary circulations develop around fronts in response to inertial currents and direct wind forcing. In cases with destabilizing wind stress, Ekman transport on the cold side of the front will interact with the combined Ekman/geostrophic flow of the front and can develop near surface convergence. Circulations generated by this interaction could lead to further frontogenesis (e.g., McWilliams 2017) and vertical downwelling/upwelling. Generation of inertial currents within the front will also produce regions of convergence and divergence along the edge of the front, possibly generating vertically radiating internal waves below the front.

Analysis of the turbulence kinetic energy (TKE) and momentum budget for the frontal region is conducted using the meridional average flow as the background mean current following Skillingstad and Samelson (2012). Turbulent velocities are defined by subtracting the meridional averaged velocity from each grid point value,

$$u'_i(x, y, z) = u(x, y, z) - \frac{1}{y_{\max}} \sum_{y=0}^{y=y_{\max}} u(x, y, z) \Delta y.$$

Using this decomposition, the horizontally averaged TKE budget equation is defined as

$$\begin{aligned} \frac{\overline{\partial \text{TKE}}}{\partial t} = & \overline{u'_i \frac{\partial u_i}{\partial t}} = -\overline{u'_i u_j \frac{\partial u_i}{\partial x_j}} - \overline{u'_i \frac{\partial p}{\partial x_i}} - \overline{u'_i \delta_{i3} g \frac{\rho'}{\rho_o}} \\ & - \overline{u'_i \frac{\partial (\overline{u''_i u''_j})}{\partial x_j}}, \end{aligned} \quad (5)$$

where $\text{TKE} = 1/2 \overline{u'^2_i}$, $\overline{u''_i u''_j} = -K_m \partial u_i / \partial x_j$, K_m is the subgrid-scale eddy viscosity, p is pressure, and the over-bar denotes horizontal averaging. Terms in (5) are, in order, TKE storage, energy flux divergence (consisting of a vertical transport term and shear production term), pressure work, buoyancy production, and dissipation. The energy flux divergence represents both the shear production and the vertical transport of turbulence. We note that with this flow decomposition, traditional baroclinic wave development is part of the TKE budget.

3. Simulation results

a. Surface buoyancy flux, no wind stress

Our first experiment, case C10, examines how -10 W m^{-2} of cooling without wind forcing disrupts the balanced front initial condition (Fig. 2). Weak instabilities appear in the simulation at hour 18 as shown by the small velocity fluctuations that are angled across the front. By hour 24, these disturbances have strengthened and are mostly aligned with the flow suggesting symmetric instability. This result qualitatively agrees with the DNS experiments presented in Stamper and Taylor (2016). By hour 50, the symmetric mode has weakened and the flow begins to develop baroclinic instability with disorganized wave growth throughout the front, similar to the baroclinic mixed mode discussed in Stamper and Taylor (2016). Baroclinic wave development reaches across the front at hour 72 with the formation of a domain-scale wave feature in surface temperature and strong frontogenesis along the wave front.

Zonal–time cross sections of the surface temperature and cross-front u velocity component provide another view of the front evolution (Fig. 3). Initially, cooling

FIG. 2. (left) Surface temperature and (right) v component current at (a) hour 18, (b) hour 24, (c) hour 50, and (d) hour 72 for case C10.

generates vertical mixing that disrupts the geostrophic balance of the thermal wind circulation. Consequently, an inertial response develops with surface acceleration toward lower pressure on the cold side of the front. The inertial signal is maintained until roughly hour 48 when lateral acceleration from baroclinic instability begins to dominate the flow. Further growth of the baroclinic wave leads to large surface flows at hour 72. Overall movement of the surface front is toward colder water, signifying restratification of the system from an upright mixed layer front to a sloped frontal surface with increased vertical stability.

Cross-section plots of temperature and Ri (Fig. 4) follow the predicted behavior described above. At hour 24 (Fig. 4a), when symmetric instability appears in the

velocity field, the value of Ri within the front is on average greater than 0, but less than 1. Temperature surfaces are almost vertical at this time and circulations associated with symmetric instability appear upright because of the nearly vertical front. At hour 50 (Fig. 4b), restratification is clearly shown by the tilt of temperature contours that develops as symmetric instability transitions to baroclinic instability and Ri gradually increases in the frontal region to a value greater than 1. This tilt increases by hour 72 (Fig. 4c) and we note frontogenesis near the surface between $x = 8$ and 10 km from the developing baroclinic wave system (see also Fig. 2d).

Surface cooling in case C10 generates turbulence through buoyancy production and acts to decrease

FIG. 3. Zonal-time plots of surface (a) temperature and (b) u velocity component for case C10 from $y = 600$ m.

potential vorticity to near zero, which drives the formation of symmetric instability (Taylor and Ferrari 2010) for select periods of time when Ri is between 0.25 and 1. The buoyancy forcing, alone, is a relatively weak source of turbulence as shown by the very small velocity variations in the fluid away from the frontal region (Figs. 2 and 3; see also analysis of the turbulence kinetic energy budget in section 4b). Increasing the flux to -100 W m^{-2} in case C100 produces a more significant source of turbulence, which tends to reduce the vertical strength of frontal instabilities but increase the intensity of shear associated with the inertial current. Plots of the surface temperature and v component of velocity for case C100 in Fig. 5 indicate reduced strength in symmetric modes compared with case C10. Space-time plots from case C100 (Fig. 6) indicate a stronger more persistent inertial response compared with case C10. The onset time of baroclinic instability is somewhat later in case C100 than case C10, which is consistent with the greater influence of turbulent mixing and larger value of R_{MLI} . Stronger buoyancy forced mixing in case C100 causes a greater imbalance between the pressure gradient and Coriolis term, leading to the more significant inertial oscillation. In addition, the increased buoyancy forced mixing maintains a lower Ri , delaying the onset of baroclinic instability in comparison with case C10. Overall cooling from increased surface heat flux in case C100 is evident in the temperature field on the warm side of the front (Figs. 5, 6, this effect is not shown by the color scale on the cold side of the front).

FIG. 4. Zonal-depth cross sections of the average Ri (shaded) and temperature (contours, $^{\circ}\text{C}$) from hour (a) 24, (b) 50, and (c) 72 for case C10.

b. Wind stress, no surface buoyancy flux

Surface wind stress provides another means of destabilizing frontal zones by increasing the boundary layer shear through added surface momentum flux (Thomas and Lee 2005; Taylor and Ferrari 2010; Thomas et al. 2013; Skillingstad et al. 2017). This has also been viewed as an Ekman buoyancy flux (e.g., B_{Ek}) or advection of dense water across the frontal system by surface Ekman transport, e.g., Thomas and Lee (2005), but the turbulence source is shear production, not buoyancy production. In the third experiment, WS05, a southward surface wind stress of 0.05 N m^{-2} is imposed causing increased geostrophic shear production equal to

$$SP_{geo} = -u_*^2 \frac{\partial V}{\partial z},$$

where $u_* = \sqrt{\tau/\rho}$ and τ is the wind stress. This wind forcing has a dramatic effect on both the movement of the front and generation of embedded instabilities (Fig. 7). These instabilities are mixed symmetric-Ekman shear modes, similar to numerical and linear analysis

FIG. 5. As in Fig. 2, but for case C100.

results presented in [Skyltingstad et al. \(2017\)](#), with structure in both the alongfront and cross-front direction. Examination of the velocity field over the full simulation does not indicate a strictly symmetric mode as is more evident in cases C10 and C100; circulations always have a significant meridional (alongfront) wave component. We note that the symmetric-Ekman shear mixed mode differs from the baroclinic mixed mode analyzed in [Stamper and Taylor \(2016\)](#), which draw energy entirely from the baroclinic front. This case also generates an inertial oscillation as shown by a zonal–time cross section (Fig. 8), but with a more turbulent structure than in cases C10 and C100, suggesting more vigorous Ekman mixed mode instabilities and wind generated turbulence. Stronger instabilities may also be related to R_{MLI} being about a factor of 2 larger for case

WS05 in comparison with C100. By hour 50 traditional baroclinic instability again develops, leading to a frontal-scale eddy circulation similar to the cooling forced cases.

One clear difference between the buoyancy forced and wind forced cases is the movement of the front in response to Ekman transport. With buoyancy forcing, vertical mixing generates inertial oscillations in the front, but the net movement of the two water masses is minor. The front in case WS05, however, propagates westward across the domain with a speed of about 0.01 m s^{-1} . Water surrounding the front exhibits a vertical current profile that is essentially in Ekman balance, with westward surface currents that are stronger than the frontal translation velocity. Boundary layer velocities in the interior of the front are reversed in direction

FIG. 6. As in Fig. 3, but for case C100.

relative to the surrounding ocean boundary layer because of the geostrophic shear and the modified Ekman velocity profile that develops in response to unbalanced flow. The difference in velocities generates horizontal divergence and convergence along the trailing and leading edges of the front consistent with that described in Niller and Reynolds (1984), Samelson and de Szoek (1988), Allen et al. (1995), and McWilliams et al. (2015).

Average vertical cross sections shown in Fig. 9 demonstrate the circulation that is generated by the wind interaction with the frontal geostrophic shear. Outside of the front, the vertical shear exhibits an approximate Ekman spiral, with maximum velocities at the surface directed roughly 45° to the right of the wind stress vector. Currents in the front have a more complex profile with the geostrophic shear dominating the v component. Frontal u velocity is strongly affected by vertical mixing from instabilities, with weak positive velocity near the surface and a broad region of negative velocity extending to about 60 m, in contrast to the much shallower Ekman velocity profile outside the front. The overall frontal circulation is anchored by a downward current along the cold side of the front and an upward current on the warm side of the front (Fig. 9c). The divergence of the vertical u momentum flux, defined as $-\partial(u'w')/\partial z$, (Fig. 9d) is in agreement with the zonal velocity structure, showing positive acceleration near the surface and negative acceleration near the base of the frontal mixed layer. This pattern of acceleration from frontal instabilities directly leads to restratification of the front.

The overall frontal circulation has similarities to the simplified frontal system described in Crowe and Taylor (2018, 2019a), which consisted of a turbulent thermal wind balance with a depth-dependent turbulent viscosity. In their study, the balanced flow leads to a cross front circulation with enhanced vertical velocity at the edges of the front, similar to the structure in Fig. 9. The front in their case weakens through lateral spreading produced by shear dispersion by the cross-front circulation. In our simulations, the front displays only marginal spreading before the baroclinic modes dominate the lateral exchange.

4. Within front dynamics

a. Inertial currents

Inertial currents are generated in all of the mixed layer frontal simulations when the initially balanced, near-surface geostrophic shear is disrupted by vertical momentum transport. Observed inertial oscillations often are attributed to short (less than 1 inertial period) episodic wind events that deposit momentum in the ocean mixed layer. Wind forced inertial currents do not exhibit strong shear in the mixed layer and initially decay rapidly with depth below the mixed layer. We note a similar, faint wind-forced inertial response in case WS05 away from the frontal region in Fig. 8b. Wind forced inertial currents can take days to propagate into the pycnocline below the mixed layer, and typically cover broad areas that scale with the size of atmospheric storm systems. In contrast, frontal forced inertial currents develop with strong shear across the depth of the mixed layer from the disruption of geostrophic balance and are confined to the frontal region. Frontal inertial currents are also short lived and appear to dissipate from vertical propagation into the pycnocline, vertical mixing, and the onset of baroclinic instability.

We take a closer look at the frontal generated inertial currents in case C10, which has weak forcing and no wind stress that might complicate our analysis. To isolate the effects of the inertial current, quantities are averaged in the meridional direction as described in the methods section for computing turbulence perturbation parameters, and the geostrophic velocity is subtracted from the flow. We designate these velocity components with a subscript in. Snapshots of the averaged v_{in} and u_{in} component velocities are presented along with contours of potential temperature in Fig. 10, covering the time period from initial current formation through a full cycle of the oscillation. In the first panel, u_{in} has advected the front toward the east and is beginning to reverse direction, while v_{in} is at minimum, which when added to the

FIG. 7. As in Fig. 2, but for case WS05.

geostrophic velocity, generates a supergeostrophic flow (the terms supergeostrophic and subgeostrophic for atmospheric low-level jets are defined on page 110 of [Markowski and Richardson 2010](#)). In the second panel, the westward velocity is advecting the front back toward the initial vertical orientation and the v_{in} component is reduced to near zero. At hour 23, the front is nearly vertical and the v_{in} component opposes the geostrophic flow, generating subgeostrophic conditions. The cycle finishes with u_{in} restratifying the flow by advecting the surface front eastward.

The time evolution of the inertial current system is further revealed by concentrating on the flow characteristics averaged only over the frontal region as shown in Fig. 11. Depth-time plots of the velocity components show how the inertial current forms in response to

vertical mixing exchanging water between the top and bottom of the mixed layer over the first 36 h of the simulation. Reduction of the geostrophic shear leads to a couplet of positive and negative v_{in} , and acceleration of u_{in} by the background pressure gradient associated with the front. The inertial system begins to decay and lose energy through vertical radiation and background turbulence dissipation around hour 40, with greatly reduced velocities and the appearance of a downward propagating wave in the upper pycnocline. A depth-time plot of u_{in} away from the frontal zone clearly shows the formation and propagation of the inertial gravity wave (Fig. 12). The slanted character of the velocity field is consistent with an upward propagating internal wave phase (downward group velocity) with vertical phase speed of about $5.6 \times 10^{-4} \text{ m s}^{-1}$. Assuming a vertical

FIG. 8. As in Fig. 3, but for case WS05.

wavelength of ~ 30 m, this phase speed yields a frequency of $1.2 \times 10^{-4} \text{ s}^{-1}$, or slightly higher than the inertial frequency.

A key term in the momentum budget responsible for the formation of the inertial current system is the vertical flux divergence (defined above) for the u and v velocity components. These two terms (Figs. 13a,b) represent the vertical transport of momentum by turbulence and, in the case of the v component, act to disrupt the initial geostrophic balance. At hour 6, the flux divergence of the v component opposes the geostrophic shear (see Fig. 1) by increasing the velocity at the surface and decreasing the velocity at the mixed layer base. The imbalance reduces the Coriolis term in the u momentum equation, causing acceleration of u and vertical transport that acts to reinforce the inertial current during the first 36 h. Momentum transport is modulated by the movement of the front and vertical stratification, with maximum values occurring when the front is vertically aligned at hours 6, 24, and ~ 38 , suggesting that much of the transport is produced by symmetric and mixed mode instabilities active at these times (e.g., see Fig. 2b). After hour 48, the growth of baroclinic instability drives large acceleration of the cross frontal u velocity responsible for further restratification of the frontal region.

The growth of baroclinic instability coincides with the decay in inertial current energy as shown by the kinetic energy, $\text{KE} = 1/2(u_{in}^2 + v_{in}^2)$, in Fig. 13c. We also note that vertical transport of momentum when the front is nearly vertical tends to reduce the kinetic energy

FIG. 9. Meridional and time averaged (a) u , (b) v , and (c) w component currents along with (d) divergence of $u'w'$ from case WS05. Time average is from every hour between 18 and 24.

contained in the inertial currents, possibly because of increased turbulence dissipation. This is supported by a plot of the subgrid-scale dissipation rate in Fig. 12d, which indicates high dissipation during vertical mixing events at hours 6 and 24. Dissipation also increases rapidly, especially near the surface, as baroclinic instability intensifies after hour 48.

Wind-driven inertial currents in the ocean surface mixed layer are often long lasting, persisting for many days before propagating vertically into the ocean interior. Here, the inertial currents decay at a rapid rate, suggesting that the limited scale and dynamic instability of the front quickly remove energy from inertial currents. The ultimate fate of this energy is either dissipation through turbulence or rapid vertical propagation in

FIG. 12. Meridionally averaged u_i (shaded) and temperature ($^{\circ}\text{C}$; contours) at $x = 12$ km as a function of depth and time for case C10.

b. Turbulence energy budget

Symmetric and mixed mode symmetric-Ekman shear instabilities typically gain energy from the geostrophic, wind-driven, and ageostrophic shear while baroclinic instability extracts potential energy via buoyancy production. Using the TKE budget (5), we can assess the importance of the shear and baroclinic terms as shown in Fig. 14 for case C10. We note that about 3/4 of the model domain is outside of the frontal region and has only buoyancy production as a source of TKE. Therefore, the relative importance of the buoyancy term is magnified in the domain average plots. At hour 24, shear production as indicated by the positive flux divergence term is similar in magnitude to the buoyancy production in the upper 10 m of the mixed layer, but dominates the TKE budget near the base of the mixed layer. Shear production in this case, which is mostly from geostrophic currents, is the source of energy for the coherent roll

circulations shown in Fig. 2 indicative of symmetric instability. This arrangement changes at hour 50, when baroclinic mixed mode instability is gaining strength. Buoyancy and pressure transport have increased near the surface, and the shear production has been mostly replaced by vertical transport in the divergence term as shown by the negative values near the surface. Further growth of baroclinic modes is indicated at hour 60 when the buoyancy production completely overwhelms shear production, making up the majority of new eddy energy production and TKE dissipation.

Perturbations calculated by removing the meridional mean velocity represent both turbulence and other small-scale processes, such as internal gravity waves. In the mixed layer, most of this energy is contained in turbulence. However, below the mixed layer, perturbation quantities are more likely to represent internal gravity waves produced by interaction of the inertial current with the upper pycnocline, similar to the mechanism discussed in Shakespeare and Taylor (2016). Growth of these waves are indicated in Fig. 14 by the maximum in pressure transport term near 70-m depth. This production term is consistent with the formation and propagation of the near-inertial wave shown in Figs. 11 and 12, which develops after hour 48. Turbulence dissipation is weak in the pycnocline because of stronger vertical stratification and because the wave systems have horizontal scales that are relatively large (e.g., about the frontal width) and are not strongly affected by the subgrid-scale eddy diffusion.

FIG. 13. Average vertical flux divergence of (a) u component and (b) v component velocities for case C10. (c) The kinetic energy of the inertial current; (d) the total subgrid dissipation of energy. All averages except for the subgrid dissipation are taken between $x = 5400$ and 9000 m.

FIG. 14. Horizontally averaged terms from the case C10 TKE budget for hour (a) 24, (b) 50, and (c) 60.

c. Inertial current energy budget

The inertial current system modeled here can be described using the reduced equation set

$$\begin{aligned}\frac{\partial u}{\partial t} &= f(v - v_g), \\ \frac{\partial v}{\partial t} &= -fu,\end{aligned}\quad (6)$$

where the horizontal pressure gradient term is expressed as the geostrophic v_g velocity component and we assume the flow does not have horizontal gradients and vertical velocity is negligible. Multiplying each equation by the component velocity, u and v , yields the total inertial kinetic energy equation,

$$\frac{\partial \text{KE}}{\partial t} = -fv_g u. \quad (7)$$

where $\text{KE} = 1/2(u^2 + v^2)$. We can also define an equation for the time rate of change of background potential energy, $\text{PE} = \int_{-z}^0 \rho g dz$, as

$$\frac{\partial \text{PE}}{\partial t} = -u \frac{\partial \text{PE}}{\partial x} = -ug \int_{-z}^0 \frac{\partial \rho}{\partial x} dz. \quad (8)$$

Our assumption here is that PE is completely defined by the background density structure and there is no density variation in the alongfront direction, y . Substituting the thermal wind equation (2) into (8) yields

$$\frac{\partial \text{PE}}{\partial t} = fu \int_{-z}^0 \frac{\partial v_g}{\partial z} dz. \quad (9)$$

Integrating (7) and (9) over a depth z gives the average energy budget equations,

$$\frac{\partial \overline{\text{KE}}}{\partial t} = -f \overline{u} \overline{v_g} z, \quad \frac{\partial \overline{\text{PE}}}{\partial t} = f \overline{u} \overline{v_g} z, \quad (10)$$

where the overbar indicates the average over the model depth. The equivalence of these two budget equations indicates a transfer between the inertial current KE and PE following the rotation of the current and differential advection of the frontal density gradient.

Plots of depth averaged energy budget terms in Fig. 15 demonstrate the transfer between PE and KE implied by (10) for case C10. As shown in Fig. 10, the initial unbalanced flow produced by vertical mixing of the geostrophic shear leads to a positive u velocity component near the surface and negative u component near the mixed layer base in the frontal zone. Advection of the background temperature field by this current temporarily restratifies the water column in the front, effectively decreasing the potential energy of the system at hour ~ 15 . The v component at this time has accelerated

FIG. 15. Vertically averaged terms from the total energy budget described in the text. Also shown is the averaged TKE associated with the growth of baroclinic instability after hour 48.

near the surface to a speed that is greater than the initial geostrophic velocity, which is reflected in the positive KE perturbation shown in Fig. 15 at hour ~ 15 . Rotation of the flow removes the stratification as the near surface front moves westward and the pycnocline surface orientation reverts to the original vertical direction at hour ~ 23 . Inertial KE is a minimum at this time and the total energy has returned to values similar to the initial condition. Growth of baroclinic instability after 48 h is indicated by the increasing value of TKE shown in Fig. 15. We also note that the average PE decreases as TKE increases and the inertial oscillation energy gradually decays.

5. Conclusions

Ocean fronts have taken the center stage in recent research on small-scale mixing and transport in the ocean. Here we examine a simple, isolated front using a turbulence resolving model to better understand the interaction between baroclinicity and surface fluxes in generating both vertical transport and horizontal spreading of fronts. Both surface cooling and surface wind stress are considered. In general, without wind stress we find that frontal instabilities follow a life cycle similar to Stamper and Taylor (2016), beginning with 200–300-m lateral scale coherent roll structures that are aligned with the front. With weak surface cooling and no wind, the alignment is nearly perfect, yielding results that are equivalent to symmetric instability cases described in Haine and Marshall (1998) and Taylor and Ferrari (2010). Over time, the symmetric modes transform into baroclinic modes with an off-axis orientation, referred to as baroclinic mixed modes in Stamper and Taylor (2016). After ~ 48 h, traditional baroclinic instability develops along the front and dominates the overall circulation. Off-axis modes are also predicted for baroclinic instability modified by a constant viscosity as shown through linear analysis and simulations in Crowe and Taylor (2019b). However, the angle of unstable modes in their study were nearly cross frontal, like traditional baroclinic modes, in contrast with our results where the angle is usually less than 45° .

Cases with destabilizing winds follow a similar pathway starting with small-scale roll instabilities that eventually are overwhelmed by frontal-scale baroclinic waves. However, unlike the cases with only surface buoyancy flux, our wind-forced experiment does not generate an initial, linear symmetric mode that transforms to the mixed baroclinic mode. Instead, off-axis coherent structures form, consistent with mixed symmetric-Ekman shear mode instabilities described in Skillingstad et al. (2017), and dominate the flow structure until traditional

baroclinic instability is initiated. The combination of geostrophic shear associated with the front and wind-forced shear in the Ekman layer lead to the mixed symmetric-Ekman shear instability (Skillingstad et al. 2017).

The lack of a pure symmetric mode is also noted in the cold filament cases studied by Sullivan and McWilliams (2018). They found no indication of symmetric circulations with destabilizing wind forcing; however, off-axis coherent structures are evident in the surface v component velocity shown in their Figs. 18c and 18d that are consistent with mixed symmetric-Ekman shear mode instabilities. They note that these off-axis structures are not present on the other side of the filament, where Ekman transport stabilizes the vertical temperature gradient. In our simulations, both mixed symmetric-Ekman shear modes and symmetric instabilities were only produced when the front was nearly vertical with Ri less than 1.0.

This research suggest that cases where pure symmetric instability acts as the primary mechanism for restratification are only active when cooling (and convection) is the main source of mixing, and shear is dominated by the geostrophic current. For example, in cases of deep convection (~ 500 m) with strong surface cooling, as examined in Haine and Marshall (1998), shear produced by wind forcing is relatively weak compared with geostrophic shear. In addition, it is important for modeling frontal systems to use a domain with sufficient alongfront dimension. If the alongfront distance is too short, then the simulation domain will force symmetric two-dimensional processes and miss mixed mode instabilities.

Sullivan and McWilliams (2018) note that in their filament experiment, the fronts did not undergo baroclinic instability over the time period of the simulation because of strong turbulence suppression. Our results also indicate that vertical mixing from turbulence and symmetric and mixed mode instabilities delay the onset of baroclinic instability by maintaining a low mixed layer stability. Eventually the action of these instabilities lead to restratification and baroclinic development. The small scale of the filament and short simulation period in Sullivan and McWilliams's experiments likely prevent growth of baroclinic modes.

The imbalance of the geostrophic shear forced by both turbulence and frontal instabilities generates inertial oscillations, agreeing with the simplified model presented in Tandon and Garrett (1994). In their study, time-dependent solutions with inertial oscillations yield an average $Ri = 0.5$, consistent with our simulation results when oscillations are active. Inertial oscillations have also been reported in conjunction with frontal instabilities, for example, Thomas et al. (2016) note the possible role of inertial shear in the energetics of

symmetric instability with the Gulf Stream front. Similarly, Farrar (2003) mentions the combined role of geostrophic shear and inertial oscillations in boundary layer mixing. More recently, Yu et al. (2019) present evidence of symmetric instability for a transient front in which inertial shear was also a significant feature of the analysis. In both of these cases, the inertial shear was assumed to be wind forced, but could also have resulted from mixing of the thermal wind shear by destabilizing winds and weak buoyancy forcing. In general, fronts are likely regions for strong interaction between inertial currents and balance flows as reviewed by Thomas (2017). In particular, the sloping isopycnals within fronts produce both wave reflection and absorption promoting exchange of inertial wave energy with the background flow similar to cases presented here.

All of the experiments conducted in this work suggest that frontal instabilities evolve over periods of days for 1–10-km-scale fronts. Vertical transport from symmetric and mixed mode instabilities, while important, is only active for a small period of time before much stronger, classic baroclinic instability dominates the flow. An important question is if special parameterizations are necessary for representing these normally subgrid-scale processes in ocean circulation models given their rather brief life cycle and their source from possibly resolved vertical current shear.

Acknowledgments. Funding for this research was provided by the Office of Naval Research (ONR N00014-18-1-2083) and the National Science Foundation (OCE-1435407). We thank two anonymous reviewers for comments and suggestions that greatly helped to improve this manuscript.

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