Thorpe Turbulence Scaling in Nighttime Convective Surface Layers in the North Indian Ocean

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ABSTRACT: We use profiles from a Lagrangian float in the north Indian Ocean to explore the usefulness of Thorpe analysis methods to measure vertical scales and dissipation rates in the ocean surface boundary layer. An rms Thorpe length scale $L_T$ and an energy dissipation rate $\epsilon_T$ were computed by resorting the measured density profiles. These are compared to the mixed layer depth (MLD) computed with different density thresholds, the Monin–Obukhov (MO) length $L_{MO}$ computed from the ERA5 reanalysis values of wind stress, and buoyancy flux $B_0$ and dissipation rates $\epsilon$ from historical microstructure data. The Thorpe length scale $L_T$ is found to accurately match MLD for small ($<0.005$ kg m$^{-2}$) density thresholds, but not for larger thresholds, because these do not detect the warm diurnal layers. We use $\xi = L_T/L_{MO}$ to classify the boundary layer turbulence during nighttime convection. In our data, 90% of points from the Bay of Bengal (Arabian Sea) satisfy $\xi < 1$ ($1 < \xi < 10$), indicating that wind forcing is (both wind forcing and convection are) driving the turbulence. Over the measured range of $\xi$, $\epsilon_T$ decreases with decreasing $\xi$, i.e., more wind forcing, while $\epsilon$ increases, clearly showing that $\epsilon/\epsilon_T$ decreases with increasing $\xi$. This is explained by a new scaling for $\xi \ll 1$, $\epsilon_T = 1.15B_0^{0.64} \xi^{-1}$ compared to the historical scaling $\epsilon = 0.64B_0 + 1.76\xi^{-1}$. For $\xi \ll 1$ we expect $\epsilon = \epsilon_T$. Similar calculations may be possible using routine Argo float and ship data, allowing more detailed global measurements of $\epsilon_T$, thereby providing large-scale tests of turbulence scaling in boundary layers.

KEYWORDS: Marine boundary layer; Mixing; Stability; Atmosphere-ocean interaction; Buoyancy; Surface fluxes

1. Introduction

a. The ocean boundary layer

Turbulent mixing in the upper layers of the ocean regulates most of the air–sea dynamical processes, including air–sea flux transfer (Munk and Wunsch 1998; Cisewski et al. 2008), redistribution of energy into the interior of the ocean, and marine ecological functioning (Huang et al. 2017). Turbulence is a three-dimensional process that is chaotic, happens in several intertwined time and spatial scales, and is difficult to accurately measure in the ocean. Hence, results from laboratory turbulence experiments or direct numerical simulations (DNS) or large-eddy simulation (LES) models have often been directly applied to the oceanic surface boundary layer (OSBL) turbulence (Reichl and Hallberg 2018).

Major sources of turbulence in the OSBL are the surface wind stress and the surface buoyancy flux (Shay and Gregg 1984; Lombardo and Gregg 1989, hereinafter LG89). The large variation in shortwave radiation from night to day drives a diurnal cycle in OSBL properties, which can be a major source of variability. In this cycle, the OSBL loses heat to the atmosphere during the night, promoting unstable stratification in the surface layer. The resulting convective overturns convert the potential energy into turbulent kinetic energy (TKE) energizing the OSBL turbulence and deepening the mixed layer. In contrast, the OSBL gains heat from solar insolation during the day, promoting stable stratification in the surface layer. This suppresses turbulence, often restricting it to a shallow wind-forced surface layer on top of the previous night’s mixed layer. The layer between the deep nighttime convective mixing layer and the daytime shallow turbulent layer is called the remnant layer, which is marked by decaying turbulence (see Fig. 1a of Callaghan et al. 2014). This daily cycle driven by buoyancy and wind stress is modified by surface gravity and internal wave breaking and Langmuir circulations (D’Asaro 2014), although there is not yet a firm consensus on the best way to model their role (Qiao et al. 2016; Cai et al. 2017). Overall, despite over 50 years of development (Kraus and Turner 1967) models of this and other boundary layer processes often have significant biases (Zhu et al. 2020; Sallée et al. 2013).

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b. Mixed layer depth

Most models specify boundary layer turbulence by the combination of a length scale and some measure of the rate of energy transformation. Multiple length scales can be defined. The first, and simplest, is the distance from the wall $z$. A second one is the mixed layer depth (MLD), the depth of the quasi-homogeneous layer near the surface with uniform characteristics. A common issue here is that the methods of MLD estimation are often arbitrary, based on the gradient criteria of either temperature or density, and depending on different averaging periods (hourly, daily, or even monthly) of data. The literature is vastly divergent since the quantitative outcome of the analysis varies largely if a different threshold is selected for the MLD estimation.

c. Monin–Obukhov length

A third length scale is provided by Monin–Obukhov (MO) similarity theory (Monin and Obukhov 1954). For a horizontally uniform and stationary flow bounded by a solid wall with stress and heat flux at the wall, MO theory defines the Monin–Obukhov scale

$$L_{MO} = \frac{\mu}{\kappa B_0}$$  \(1\)

for wind stress $\mu$, buoyancy flux $B_0$, von Kármán constant $\kappa = 0.4$, and water density $\rho$. Traditionally, the value of $L_{MO}$ is negative when the buoyancy flux forces unstable stratification, i.e., convection, and positive when it forces stable stratification. Since we will consider only the convective case, $L_{MO}$ will be positive for convection here. The Monin–Obukhov depth $L_{MO}$ quantifies the relative importance of the stress and the buoyancy flux. For $z/L_{MO} \ll 1$, the stress dominates and a classical logarithmic boundary layer is found. For $z/L_{MO} \gg 1$, the buoyancy flux dominates, forming free convection in the unstable case and suppressing the turbulence in the stable case. For intermediate distances, MO theory specifies the scaling of the turbulent and mean quantities. MO scaling accurately describes conditions in the atmospheric boundary layer (Businger et al. 1971) especially under unstable conditions or weakly stable conditions. In the ocean, its usefulness is less well established (LG89; D’Asaro 2014; Zheng et al. 2021) due to the complicating effects of surface wave forcing and radiation absorption which introduce additional length scales. The complications due to penetrative radiation do not occur at night; this analysis thus focuses mostly on nighttime data.

d. Thorpe overturning scale

In this paper, we investigate a fourth length scale for the boundary layer, the Thorpe or overturning scale $L_T$ (Thorpe 1977). In Thorpe’s method, the vertical potential density profiles are reordered into stable monotonic profiles which contain no inversions. The Thorpe displacement $d$, the difference between the displaced water parcel depth of the reordered profile and its actual depth in the observations, measure the size of individual turbulent eddies. The rms Thorpe displacement is thus a turbulent length scale $L_T$. This approach is widely used in the stratified ocean interior where shear instability is the major source of mixing.

Thorpe analyses can also be used to estimate turbulent kinetic energy dissipation rate $\varepsilon$, through the Ozmidov scale (Ozmidov 1965; Dillon 1982),

$$L_O = \left( \frac{\varepsilon}{N^2} \right)^{0.5},$$  \(2\)

where $N$ is the density stratification. When the turbulence is limited by stratification a strong correlation between $L_O$ and $L_T$, with an average relationship of

$$L_O = 0.8L_T$$  \(3\)

is found. Combining Eqs. (2) and (3),

$$\varepsilon_T = 0.64 L_T^2 N^3$$  \(4\)

which provides an estimate of the dissipation rate over the region of the overturn defined by $L_T$. The notation $\varepsilon_T$ is used here to distinguish the dissipation rate estimated from Thorpe methods from its true value. Numerous studies (Dillon 1982; Itsweire 1984; Crawford 1986; Ferron et al. 1998; Smyth and Moum 2000; Smyth et al. 2001; Mater et al. 2015; Ferron et al. 1998) support (3) and (4) on average, although a perfect relationship is not expected due to spatial and temporal variability of the turbulent field. It also is not expected to apply instantaneously, but only to averages over turbulent events. For example, Smyth and Moum (2000) find that $L_T/L_O$ increases monotonically during a mixing event and Rohr et al. (1988) show that $L_T/L_O < 1$ for low Richardson number, when the turbulence grows, and $L_T/L_O > 1$ for high Richardson number when it decays. Scotti (2015), analyzing turbulence resulting from instabilities of gravity waves, suggests that $L_T$ and $L_O$ are linearly correlated for shear instabilities, but that $L_T > L_O$ for convective instabilities.

e. Length scale comparisons

Here, we compare MLD, $L_{MO}$, and $L_T$ in the ocean boundary layer in two ways. First, motivated by previous observations (Brainerd and Gregg 1995; Cisewski et al. 2008; Sutherland et al. 2014) suggesting that $L_T$ is usually a more accurate measure of mixing depth than MLD, we explore the feasibility of using $L_T$ as an alternate metric for MLD. In this comparison, it is important to distinguish between the different interpretations of these depths. MLD measures the depth to which turbulence has recently mixed the boundary layer. The time since mixing depends on the threshold chosen, with larger thresholds implying longer times. Thus, a threshold larger than the typical magnitude of daytime surface density decrease due to solar heating will not capture the diurnal shallowing of the mixed layer, while a smaller threshold will. In contrast, $L_T$ measures the mixing depth on the time scale of a turbulent overturning of the boundary layer, typically a few thousand seconds, or more simply, the local mixing depth at the time of measurement. The Monin–Obukhov depth $L_{MO}$ measures the depth over which surface stress dominates turbulence production compared to surface heating and cooling and is usually computed from averages
over many boundary layer overturn scales. Thus, the relationship between these depends on both the dynamics of the boundary layer and the time scale being considered.

f. Dissipation rate comparisons

1) CLASSICAL THEORY

Second, we explore the usefulness of estimating the boundary layer dissipation rate from (4). Our evaluation builds on previous scaling relationships for \( \epsilon \) in oceanic boundary layers, in particular LG89. LG89 argue that the relative importance of wind and buoyancy forcing is quantified by the ratio \( \xi = H/L_{MO} = \kappa B_{0} H/\mu \epsilon \), where \( H \) is the mixed layer depth. For \( \xi < 1 \), wind forcing dominates turbulence, for \( \xi > 10 \), buoyancy flux dominates and both processes have a comparable contribution to turbulence generation if \( 1 < \xi < 10 \). Since, as we will show, \( L_{T} \) is a good measure of MLD, we will use the ratio \( \xi = L_{T}/L_{MO} \). When convection dominates, \( \xi > 10 \), Shay and Gregg (1986) and LG89 show that \( \epsilon \) is nearly uniform with depth across the mixed layer with a value proportional to the surface buoyancy flux in the same way as free convection in the atmospheric boundary layer:

\[
\epsilon = 0.64 \epsilon_{f}, \quad \epsilon_{f} = B_{0}. \tag{5}
\]

When wind-forcing dominates, the situation is more complex due to the influence of wave breaking and Langmuir forcing, both of which can strongly affect the dynamics of the OSBL. Ignoring these, LG89 start with surface layer scaling with \( B_{0} = 0 \):

\[
\epsilon = \frac{u_{*}^{3}}{k H}, \tag{6}
\]

where \( z \) is the distance from the surface, and average this over the mixed layer depth to predict an average wind-driven dissipation rate \( \epsilon_{z} \), which scales with

\[
\epsilon_{z} = \frac{u_{*}^{3}}{k H} = B_{0} \xi^{-1}. \tag{7}
\]

For mixed conditions, LG89 scale their observations of \( \epsilon \) as a sum

\[
\epsilon = 0.64 \epsilon_{f} + 1.76 \epsilon_{z}, \tag{8a}
\]

\[
\frac{\epsilon}{B_{0}} = 0.64 + 1.76 \xi^{-1}, \tag{8b}
\]

and show that (8) fits their entire dataset under all conditions, albeit with considerable scatter.

2) APPLICATION TO THORPE SCALING IN BOUNDARY LAYERS

Here, we investigate the scaling of \( \epsilon f \) within this framework. We anticipate that the scaling will be different from that appropriate for shear instability in the ocean interior because buoyancy is generating turbulence rather than inhibiting it. Furthermore, \( \epsilon f \) and \( \epsilon \) will scale differently, because while \( \epsilon \) measures all of the kinetic energy dissipation, \( \epsilon f \) only measures that part associated with the collapse of unstable density anomalies. For example, in a homogeneous fluid, \( \epsilon f \) is necessarily zero, while \( \epsilon \) is not.

We follow the notation from Table 1 of LG89, summarizing the results in our Table 1. For \( \epsilon \) in the mixed layer, the scaling parameters are \( B_{0} \), \( H \) for convection and \( u_{*} \), \( H \) for wind forcing, where \( H \) is the mixed layer depth. Following (4), we scale \( \epsilon f \) as \( L_{z}^{2} N^{3} \) where the sorted density profile is scaled as \( N^{2} = \Delta b/H \) and the buoyancy anomaly \( \Delta b = B_{0} T/H \), the input of buoyancy over the time \( T \) that it takes to mix the layer, spread over the layer depth \( H \). Mixing of the boundary layer occurs via the large eddies with a typical velocity of \( w_{*} = (B_{0} H)^{1/3} \) for convection and \( u_{*} \) for wind forcing, so that \( T = H/w_{*} \) and \( T = H/u_{*} \), respectively. For convection, this yields a scaling \( \epsilon f \propto B_{0} \), not surprisingly since this is the only possible dimensional choice. For wind forcing, it yields

\[
\epsilon f \sim H^{1/2} \left( \frac{B_{0}}{u_{*}} \right)^{3/2} \sim B_{0} \xi^{1/2} \tag{9}
\]

and

\[
\frac{\epsilon}{\epsilon f} \sim \xi^{3/2}. \tag{10}
\]

For pure convection \( \epsilon f \) and \( \epsilon \) are both proportional to \( B_{0} \), although not necessarily with the same constant of proportionality. For wind forcing, \( \epsilon \) increases rapidly with increasing wind stress (7), not surprisingly because the forcing increases, while \( \epsilon f \) decreases with wind stress (9) because the increased mixing decreases the density anomaly. Thus, in the convective regime, the ratio \( \epsilon/\epsilon f \) is constant, while in the wind-forced regime (10) it decreases with increasing wind.
The goal of this paper is, therefore, to evaluate the usefulness of using these Thorpe methods in the ocean boundary layer, specifically using $L_T$ to measure turbulence length scales and $c_T$ to measure dissipation rates. In section 2, various datasets used in this study are described. Details of Thorpe scale analysis and the procedures followed for Thorpe reordering are described in sections 3 and 4, respectively. The main results of the paper are described in section 5, followed by a summary and discussion in sections 6 and 7.

2. Data

Fine-scale upper-ocean measurements were taken using a custom-made Lagrangian float (model MLFIII, number MLF75; D’Asaro 2003) in three deployments; two in the Bay of Bengal (10–28 May 2016 and 7–30 July 2018) and one in the Arabian Sea (18 January–5 February 2018). The deployment locations are shown in Fig. 1a. Measurements include upper-ocean temperature, salinity, and current profiles. During the three deployments, the float profiled the upper ocean approximately at 3-h intervals up to around 100-m depths and then moved upward to settle on a predefined isopycnal for several hours to measure velocity before surfacing to transmit the data to the ground station. The float measures potential density with CTDs mounted on both the top and bottom of the float, thereby easily detecting shifts in calibration. This analysis uses only the downward profiles, as the float moves more slowly on the downcast and thus has higher resolution, particularly near the surface. The absolute accuracy of the measurement is estimated from the difference between these, about 0.005 kg m$^{-3}$. The short-term noise, which is more important for the analyses presented here, is estimated from the difference between subsequent measurements in the upper mixed layer where gradients are small. Here, 5% of the measurements differ by less than 0.0001 kg m$^{-3}$, 20% by less than 0.0005 kg m$^{-3}$ and 50% by less than 0.003 kg m$^{-3}$. We take 0.005 kg m$^{-3}$ as an estimate of the single point measurement noise. More details about the Lagrangian float and processing methods are provided in Kumar et al. (2019).

The first MLF75 deployment was done in the Bay of Bengal on 10 May 2016 (BoB16) near the center of a warm core anticyclonic eddy (13.49°N, 83.92°E). It then moved westward and was overrun by Tropical Storm Roanu as described in detail by Kumar et al. (2019). After the storm passage, MLF75 turned north-northeast and was recovered on 28 May 2016.

The second float deployment was in the Arabian Sea on 18 January 2018 (AS18) at 18.45°N, 67.52°E. The float moved in the southwest direction and was recovered on 5 February.
The third deployment was again in the Bay of Bengal on 7 July 2018 (BoB18) near the center of a warm core eddy at 13.05°N, 83.80°E. The float moved in a clockwise direction around the eddy and was recovered on 30 July 2018. In all the cases, the mission parameters, profiling, and parking depths were changed as and when required as per the changing hydrography and local weather conditions.

Air–sea heat fluxes, 10-m wind speed, and surface wind stress were extracted from ERA5 analysis (https://cds.climate.copernicus.eu/cdsapp#!/dataset/reanalysis-era5-single-levels?tab; Hersbach et al. 2020) at 1-hourly intervals along the float track. Data from these products were then extracted along the float tracks and temporally interpolated at float surfacing time intervals. Typical deviations between these fluxes and those from a high-quality surface mooring in the northern Bay of Bengal (Weller et al. 2016) are 20 W m⁻², with somewhat larger values during the summer monsoon and an annual average difference of 8 W m⁻². A recent study by Pokhrel et al. (2020) suggests that ERA5 is the best product for heat fluxes in the Indian Ocean region compared to other available reanalysis and other blended products. Based on this, we conservatively assume errors of 30% for nighttime buoyancy flux in our region (50 W m⁻²) out of typical nighttime values of 150 W m⁻². Similarly, we conservatively assume errors of 20% in wind stress, 10% in u*.

3. Methodology

Thorpe scales were computed from the density profiles measured by the SBE-41 CTD on the bottom of the float. Profiles were measured as the float profiled downward from a communications session on the surface. Measurements were made every 1–2 m for the two Bay of Bengal deployments and approximately every 0.5 m for the Arabian Sea deployment. Temperature and salinity data from the CTD were regridded at 0.25-m vertical resolution, and following Thorpe (1977), Thorpe length scales were estimated by reordering the vertical potential density profiles. Details are as follows:

1) We used a threshold density noise level of \( \delta \rho = 5 \times 10^{-4} \) kg m⁻³ in our calculations (Finnigan et al. 2002; Koch-Larrouy et al. 2015). Any density spike between consecutive vertical levels less than the noise density level was considered as a false overturn due to instrument or random error.

2) Gargett and Garner (2008) proposed an intermediate density profile which was slightly modified by Park et al. (2014). An intermediate profile is built by vertically traversing the original profile from top to bottom, copying its values directly to the intermediate profile as long as successive data values differ from each other by more than the threshold value given above. Differences less than the threshold value are excluded by retaining the preceding original data value in the intermediate profile. A similar profile, sorted from the bottom of the profile to the top is also created. The profile used for analysis is the average of both these profiles.

3) Gargett and Garner (2008) proposed a vertical overturning ratio defined as \( R_o = \min(L^+, L^-, L^{-}) \), where \( L \) is the total vertical extent of an overturn and \( L^+ \) and \( L^- \) are the cumulative extent occupied by positive and negative Thorpe displacements. They suggested a critical \( R_o \) value of 0.2, below which a prospective overturn is suspected to be due to residual salinity spiking. We followed \( R_o = 0.2 \) to identify any false overturning.

4) The overturn size must be larger than the minimum overturn size given by \( L_{\text{min}} = 2\delta \rho N^2 \rho_0 \) (Galbraith and Kelly 1996).

![Figure 2](https://example.com/figure2.png)

**FIG. 2.** (a) Buoyancy flux (blue) and wind stress (red) along the float track during the Bay of Bengal deployment in 2016. (b) Thorpe displacements (in color), Thorpe depth (bold black), and mixed layer depths (MLD) based on different density criteria. The red circles indicate the Monin–Obukhov depth (L_{MO}).
The Thorpe calculations were not done whenever the above four conditions are not met in our data. We also ignored smaller overturns within larger overturns since they are assumed to be perturbations within the same mixing event. Also, any overlapping overturns are joined together vertically.

We calculated buoyancy flux as the sum of heat fluxes and haline contributions by following the global tropical moored array (GTMA) methods (https://www.pmel.noaa.gov/ocs/flux-documentation) as follows:

\[ B = B_h + B_w, \]

where \( B_h = \alpha Q_{\text{net}}/C_p \) is the component due to surface heat flux, \( \alpha \) is the coefficient of thermal expansion of seawater, \( Q_{\text{net}} \) is the net heat flux, \( C_p \) is the specific heat of seawater, and \( B_w = \beta \rho S(P - E) \) is the component due to freshwater flux, where \( \beta \) is the coefficient of haline contraction, \( \rho \) is the water density, \( S \) is the sea surface salinity, \( P \) is the precipitation, and \( E \) is evaporation.

4. Results

a. Forcing and \( T-S \) structure

Figures 1b–d show the temperature–salinity (\( T-S \)) diagrams of the three deployments colored as a function of yearday. The magenta dots represent the \( T-S \) relations at 5-m depth ranges and the magenta line connects these dots as time progresses. For BoB16 (Fig. 1b) MLF75 passes from warmer (31.5°C) and fresher (32.6 psu) surface values at the beginning of the float deployment to a cooler (29.5°C) and saltier (33 psu) regime at the end of the deployment. This transition occurs during the passage of Tropical Storm Roanu, which makes the interpretation of these data particularly complex (Kumar et al. 2019). During BoB18, the \( T-S \) relationship is stable (Fig. 1c). During AS18, the surface \( T-S \) is stable, but the deeper values show two distinct water masses. At the start, the float is in the fresher water mass, but on day 25 it crosses a subsurface front into a weakly stratified water mass about 0.6 psu saltier that splits the shallow pycnocline. As will be shown below, this water mass dramatically shallows the boundary layer compared to the initial value.

The upper panel of Figs. 2, 4, and 6 show the buoyancy forcing (blue lines), and friction velocity (red lines) extracted along the float path. The lower panel of Figs. 2, 4, and 6 show the Thorpe displacement \( d \) (in color), Thorpe length scale \( L_T \) (thick black line), Monin–Obukhov depth \( L_{MO} \), and different estimates of density-based MLDs (0.005, 0.01, 0.05, 0.1, and 0.125 kg m\(^{-3}\) thresholds) as indicated in the figure legend.
Thorpe and Monin–Obukhov scales are only shown for nighttime. Figures 3, 5, and 7 show the average diurnal cycle for selected regions of uniform diurnal behavior for each float deployment.

The three cases show distinct buoyancy and wind forcing. In BoB16 (Fig. 2) strong daytime radiation creates a peak of surface buoyancy flux of more than $5 \times 10^{-7} \text{ m}^2 \text{s}^{-3}$ with small values of $u^*$ before yearday 137. Between yearday 139–141 Tropical Storm Roanu passed over the float with very strong winds ($u^* > 0.02 \text{ m s}^{-1}$) and a peak buoyancy loss of $-6 \times 10^{-7} \text{ m}^2 \text{s}^{-3}$. After the storm, the diurnal cycle in buoyancy flux resumes, but with stronger $u^*$. In BoB18 (Fig. 4) strong, persistent winds ($u^* > 0.015 \text{ m s}^{-1}$) and slightly weaker buoyancy forcing were seen in the first half of the BoB18 record (up to yearday 199) and with the winds decreasing in the second half of the data. In AS18 (Fig. 6) moderate positive buoyancy flux and wind occurred at the beginning and end of the record, with weaker winds and buoyancy flux during the middle period.

b. Depth scales

1) BoB16

The BoB16 record documents the transition from pre-monsoon conditions to a more active period. Three distinct phases of the upper-ocean structure (Fig. 2) are seen. From yearday 131 to 138, the mixed layers are very shallow, less than 10 m. The ocean response to the strong diurnal forcing is present (see Kumar et al. 2019), but it is hard to see in Fig. 2. Nighttime cooling with weak winds results in small values of $L_{MO}$. From yearday 138 to 142, as Tropical Storm Roanu passed over the float, the mixed layers are much deeper with $L_T$ reaching a maximum depth of ~40 m on day 139.5. The $L_{MO}$ is even deeper, reaching 70 m. After the storm, the diurnal forcing returns with stronger winds so that the mixed layers and $L_{MO}$ are both deeper than before the storm. The diurnal cycle during this period (Fig. 3, yeardays 142–148) shows individual nighttime average values of $L_{MO}$ between 30 and 70 m (Fig. 2), but average values of ~50 m (Fig. 3), significantly deeper than the individual 5–15-m depths of $L_T$ or its 20 m composite value (Fig. 3). On average, nighttime values of $L_{MO}$ are about a factor of 3 larger than $L_T$. $L_{MO}$ for small thresholds are similar to $L_T$, while $L_{MO}$ with larger thresholds are similar to $L_{MO}$.

2) BoB18

The BoB18 data (Fig. 4) were taken during the active phase of the summer monsoon. Layer depths, $L_T$ and $L_{MO}$, were deeper than during most of BoB16. A weak diurnal signal seen in the first part of the record (yearday 189–199) is composited in Fig. 5. $L_{MO}$ shoals slightly during daytime in response to the daily warming. Nighttime $L_{MO}$ is deeper than both $L_T$ and the deepest MLD (MLD$_{p=0.125}$). From yearday 200 onward, the wind speed weakens ($u^* < 0.015$) and $L_{MO}$ shoals to a depth between the deepest MLD (MLD$_{p=0.125}$) and $L_T$. Overall, the pattern is similar to that in BoB16. $L_{MO}$ are usually deeper than $L_T$. MLD for small thresholds are similar to $L_T$, while MLD with larger thresholds are similar to $L_{MO}$.

3) AS18

During AS18 (Fig. 6) strong diurnal variations in the upper ocean occur until yearday 26. The composite diurnal cycle (Fig. 7) shows $L_T$, MLD$_{p=0.005}$, and MLD$_{p=0.01}$ shoaling by nearly 40 m from day to night. However, unlike in the BoB
cases, nighttime $L_{MO}$ is typically about half of $LT$. The boundary layer turbulence is thus more dominated by convection during this period than during the BoB cases as discussed in section 5d below. After day 28, the boundary layers are much shallower due to the salty pycnocline eddy and the diurnal cycle is hard to resolve. Nighttime $L_{MO}$ is again deeper than $LT$.

c. Do threshold-based mixed layer depths correspond to Thorpe mixing depths?

One common observation from these three records is that the three largest MLD criteria (0.05, 0.1, and 0.125 kg m$^{-3}$) did not capture the strong upper-ocean diurnal structures, whereas MLDs based on 0.005 and 0.01 kg m$^{-3}$ threshold capture the diurnal structure of the upper ocean quite well. The $d$ (color shading) and $LT$ of the upper ocean also followed a clear diurnal structure as seen in Figs. 2–7. It is also seen that MLD based on 0.005 kg m$^{-3}$ coincide well with $d$. Figure 8 shows the scatter between $LT$ and MLDs computed with different thresholds for both daytime (Fig. 8a) and nighttime (Fig. 8b) conditions. The best comparison was found between $LT$ and MLD$_{p=0.005}$ for daytime (correlation = 0.85, slope = 1.2, and mean bias = 2.7 m) and nighttime conditions (correlation = 0.71, slope = 0.96, and mean bias = 5.8m). For other MLD criteria, the slopes are shallower (correlation < 0.8 and mean bias > 10 m; see Table 2). Thus, in this dataset, $LT$ is an effective and parameter-free method of estimating MLD that corresponds with a threshold method using a very small threshold. This is consistent with previous results as discussed in section 7.

d. How are Ozmidov depths and Thorpe mixing depths related?

Figure 9 plots $LT$ versus Monin–Obukhov depth ($L_{MO}$) for convective conditions, mostly nighttime. As discussed in sections 1b and 1d, the ratio $LT$/$L_{MO}$ can be used to characterize the forcing regimes of the boundary layer. The red and blue lines, marking $LT$/$L_{MO}$ = 10 and $LT$/$L_{MO}$ = 1, respectively, thus define the boundaries of these regimes (LG89, Fig. 9). Data to the left of the red line are convection dominated, data to the right of the blue line are wind dominated, and data between the lines have significant contributions from both.

Overall, more than 90% of the data from the three datasets fall in the $LT$/$L_{MO}$ < 1 regime, suggesting that on average wind
stress drives turbulence in the upper ocean during nighttime conditions. The large black and blue circles, showing the average values for the nighttime equilibrium (0–4 h) from Figs. 3 and 5 for BoB16 and BoB18, respectively, are clearly in this wind-stress-driven regime as is also true during the passage of Tropical Cyclone Ronau (black stars). The AS18 data vary widely but during nighttime equilibrium (large red circle) \( \frac{LT}{L_{MO}} \approx 10 \) and the boundary layer is mostly dominated by both convection and wind forcing. Early in BoB16, both \( LT \) and \( L_{MO} \) are small (black dots, lower left) with their ratio suggesting contributions from both convection and wind forcing. However, \( LT \) is at the smallest possible value given the processing and is thus likely overestimated.

Studying the surface energetics of the mixed layer in the north Indian Ocean, Prasad (2004) concluded that the energy required for effecting similar magnitude mixing in the Bay of Bengal is about 3 times greater than that in the Arabian Sea owing to the strong stratification existing in the Bay. Our results are consistent with these findings. Our Fig. 9 shows that the OSBL mixing in the Bay is largely affected by the surface winds as convective activity is limited in the stratified upper layers. In contrast, weak stratification of the upper layers of the Arabian Sea supports both convective and wind-driven mixing.

5. How does \( \varepsilon_T \) scale?

Figure 10 compares the scaling of \( \varepsilon/B_0 \) and \( \varepsilon_T/B_0 \). Values from LG89 are replotted from LG89’s Fig. 10 as blue dots. Values of \( \varepsilon_T \) are computed from the steady equilibrium region (0–4 h average) from the nighttime composite diurnal cycles of Figs. 3, 5, and 7 and from 2016 yeardays 139.5 to 141 of Fig. 2 for Cyclone Roanu. The nighttime data are identified as “BoB16,” “BoB18,” and “AS18” and the 2016 Roanu data as “Roanu.” For each, average values of \( \varepsilon_T, B_0, \) and \( H \) and were computed and the scaled values plotted in Fig. 10 as large red dots. Uncertainties in the forcing are indicated by the spread of 100 smaller dots computed from random Gaussian variations of \( u^* \) and \( B_0 \) by 10% and 30%, respectively.

The value of \( \varepsilon/B_0 \) for LG89 form a cloud spreading about a factor of 2 around the solid black line (8), LG89’s scaling. For \( \varepsilon_T/L_{MO} > 10 \), the value is nearly constant at about 0.6, as expected for purely convective conditions. For \( \varepsilon_T/L_{MO} < 1 \), \( \varepsilon_T \) decreases, roughly consistent with LG89’s scaling (8b). These data and the black line define three regimes, wind forced for small values \( \varepsilon_T/L_{MO} \approx 1 \), convective regime for large values \( \varepsilon_T/L_{MO} \approx 10 \), and a mixed regime between these, as labeled in the figure.

Dissipation derived from Thorpe analysis, \( \varepsilon_T \), clearly scales differently from the dissipation \( \varepsilon \) despite the large errors in the forcing. In the range of \( \varepsilon_T/L_{MO} \) corresponding to our data, \( \varepsilon \) increases while \( \varepsilon_T \) decreases. This decrease is consistent with that predicted by (9) within the estimated errors (cloud of small dots) as shown by the red line, implying a prediction of

\[
\varepsilon_T = 1.15 B_0 \left( \frac{H}{L_{MO}} \right)^{1/2}
\]

for the wind-forced regime. At the smallest values of \( \varepsilon_T/L_{MO} \), for BoB16, the value of \( \varepsilon_T \) is about 0.8, far below the LG89 scaling and clearly in the wind-forced regime. At the largest values of \( \varepsilon_T/L_{MO} \), for AS18, the value of \( \varepsilon_T \) is close to both (9) and (8), while at higher values, not measured here, we expect...
to apply. This suggests that through the mixed regime and in the convective regime, the value of $\varepsilon_T$ becomes similar to that of $\varepsilon$, as indicated by the red dashed line.

6. Summary

A custom-made Lagrangian float profiled the ocean surface boundary layer (OSBL) hydrographic conditions during three deployments in the Arabian Sea and Bay of Bengal. Using these 70 days of fine-scale float hydrographic data, we explore the usefulness of Thorpe analysis methods to measure vertical scales and dissipation rates in the OSBL, primarily focusing on periods of negative surface buoyancy forcing where the effects of shortwave penetration are negligible. Our main results are as follows:

1) Using the ratio of $L_T$, the Thorpe depth, and $L_{MO}$, the Monin–Obukhov depth, 90% of the data from the three datasets fall in a regime where $L_T/L_{MO} < 1$, placing them in a wind forced mixing regime. All the Bay of Bengal data are primarily wind-driven as convection is limited with the stratified upper ocean while those from the Arabian Sea are driven both by convective forcing and wind stress forcing.

2) The Thorpe depth scale $L_T$ is comparable to the mixed layer depth (MLD) computed using small density criteria ($\delta \rho = 0.005 \text{ kg m}^{-3}$) under both nighttime convective and daytime restratifying conditions although it is somewhat better correlated at night (0.96 versus 0.85). This is consistent with previous studies and delineates the diurnal variations in mixing depth. Since $L_T$ does not require the selection of an arbitrary density criteria, it should be more widely used as a measure of mixing on the time scale of the turbulent overturns.

3) The value of $L_T$ is poorly correlated with MLD computed using large density criteria ($\delta \rho > 0.05 \text{ kg m}^{-3}$ and larger) since these do not capture the significant diurnal variations in active mixing depth, instead measuring the maximum depth of the previous mixing. Thus, $L_T$ is not a good way to measure mixing depths on these longer time scales, unless explicitly measured and averaged over the entire period.

4) The dissipation rate computed from the available potential energy between the sorted and unsorted profiles, $\varepsilon_T$, is generally less than the total dissipation $\varepsilon$ with the difference increasing with decreasing values of $L_T/L_{MO}$, i.e., stronger wind forcing. This is approximately described by a scaling relationship (10) with a rough estimate of the scaling constant given in (11). Given the small amount of data and the large spread in values, this scaling certainly needs

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FIG. 7. Diurnal climatology of (a) buoyancy flux (blue) and friction velocity (red); (b) Thorpe displacement (in color), Thorpe depth, and MLD (in lines); and (c) $\varepsilon_T$ (color) for Arabian Sea data from yearday 18–24 of 2018. The red circles indicate the Monin–Obukhov depth ($L_{MO}$).
further validation, and the scaling constant is uncertain to at least a factor of 2.

5) These results suggest that $\tau_T$ maybe a useful measure of the total turbulent kinetic energy dissipation $\varepsilon$ in the convective regime and, with better quantification of the ratio $\varepsilon/\tau_T$ (11), it could also be useful in the mixed and wind-forced regimes. In either, it may be a useful measure of the potential energy dissipation in the boundary layer.

7. Discussion

Why is $L_T$ an effective way to measure mixed layer depth? Visually, the depth of the ocean mixed layer is often obvious. Often, the transition between the well-mixed region and the strong gradients beneath it is so strong that the exact criteria for selecting the depth make little difference in the depth estimate. However, this is not always the case, and elaborate empirical schemes have been developed (Kara et al. 2000; de Boyer Montégut et al. 2004; Lorbacher et al. 2006) most of which rely on specific dimensional density or temperature thresholds. The Thorpe mixing length avoids these by using the profile itself to set the threshold. An actively mixing “mixed layer” is not homogeneous, but has both a mean density profile, denser at the top for convection and fluctuations within the layer, due to the mixing of denser surface water into the layer (e.g., Fig. 12 of Steffen and D’Asaro 2002). Stable density gradients significantly larger than these fluctuations inhibit the mixing and thus mark the bottom of the actively mixing layer. Thus, the “right” density threshold for determining this depth is the magnitude of the density fluctuations within the mixed layer. The Thorpe sorting algorithm automatically uses this to compute a length scale and thus effectively finds a sensible layer depth.

The analysis presented here uses fluxes from ERA-5, which is globally available, and the Lagrangian float measures temperature and salinity using the same sensors used on Argo floats. Thus, the analysis presented here could be done globally. Two factors may limit such an analysis. First, Argo temperature and salinity are often only reported to a precision of 0.001 psu or 0.001°C, close to the magnitude of the density fluctuations and somewhat higher than the typical noise floor of the sensor (although less than its absolute accuracy). Second, Argo CTD sensors are turned off before they reach the surface, and thus do not measure the region of high gradient and large density anomaly just below the surface. Similarly, hydrographic observations from conventional CTD systems and underway profiling CTD systems provide opportunities for the Thorpe scale analysis for turbulence estimation. But like in Argo float data, the topmost CTD observations are erroneous due to the ship wake. An additional issue with the CTD data is that the ship’s rolling and pitching induce false density overturns and hence erroneous $\varepsilon$ calculations. Ship observations

Table 2. Correlation coefficient, slope, and mean bias of different estimates of mixed layer depth (MLD) vs Thorpe depth for daytime stratified and nighttime convective conditions.

<table>
<thead>
<tr>
<th></th>
<th>Daytime stratified conditions</th>
<th>Nighttime convective conditions</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$r^2$ Slope Bias</td>
<td>$r^2$ Slope bias</td>
</tr>
<tr>
<td>MLD $\rho=0.125$</td>
<td>0.55 1.014 21.9</td>
<td>0.63 1.011 20.6</td>
</tr>
<tr>
<td>MLD $\rho=0.1$</td>
<td>0.57 1.055 20.1</td>
<td>0.65 1.023 18.6</td>
</tr>
<tr>
<td>MLD $\rho=0.05$</td>
<td>0.63 1.150 15.6</td>
<td>0.66 0.995 15.0</td>
</tr>
<tr>
<td>MLD $\rho=0.01$</td>
<td>0.82 1.329 5.6</td>
<td>0.69 0.989 8.8</td>
</tr>
<tr>
<td>MLD $\rho=0.005$</td>
<td>0.85 1.228 2.7</td>
<td>0.71 0.956 5.8</td>
</tr>
</tbody>
</table>

Fig. 8. Scatterplot of Thorpe length with various MLDs with different $\Delta \rho$ conditions for (a) daytime conditions and (b) nighttime conditions.
may, however, benefit from better measurements of the air–sea fluxes. These issues will probably not prevent a useful analysis but may need to be considered.

The analysis presented here assumes Monin–Obukhov scaling of turbulence in the boundary layer and ignores the well-documented deviations of this scaling in data and models due to the effects of surface waves (e.g., Li et al. 2019; D’Asaro 2014; D’Asaro et al. 2014) including clear deviations from MO scaling (Zheng et al. 2021). The scaling and analyses for $\varepsilon_T$ presented here offer a simple way to obtain widespread measurements of boundary layer turbulent properties without using specialized equipment and perhaps thereby test these more modern models of the upper-ocean boundary layer.

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Data availability statement. Processed data from the Lagrangian float are available upon request to the corresponding author. ERA5 data used in this article are available from https://cds.climate.copernicus.eu/cdsapp#!/dataset/reanalysis-era5-single-levels?tab=form.

REFERENCES


