ABSTRACT: Mode-1 internal tides can propagate far away from their generation sites, but how and where their energy is dissipated is not well understood. One example is the semidiurnal internal tide generated south of New Zealand, which propagates over a thousand kilometers before impinging on the continental slope of Tasmania. In situ observations and model results from a recent process-study experiment are used to characterize the spatial and temporal variability of the internal tide on the southeastern Tasman slope, where previous studies have quantified large reflectivity. As expected, a standing wave pattern broadly explains the cross-slope and vertical structure of the observed internal tide. However, model and observations highlight several additional features of the internal tide on the continental slope. The standing wave pattern on the sloping bottom as well as small-scale bathymetric corrugations lead to bottom-enhanced tidal energy. Over the corrugations, larger tidal currents and isopycnal displacements are observed along the trough as opposed to the crest. Despite the long-range propagation of the internal tide, most of the variability in energy density on the slope is accounted by the spring–neap cycle. However, the timing of the semidiurnal spring tides is not consistent with a single remote wave and is instead explained by the complex interference between remote and local tides on the Tasman slope. These observations suggest that identifying the multiple waves in an interference pattern and their interaction with small-scale topography is an important step in modeling internal energy and dissipation.

KEYWORDS: Internal waves; Tides; Continental shelf/slope; In situ oceanic observations

1. Introduction

Surface tides, through the generation and subsequent breaking of internal tides, are thought to provide a significant contribution of the ∼2 TW of power required to maintain deep-ocean diapycnal mixing and the observed abyssal stratification (Munk and Wunsch 1998). In turn, diapycnal mixing plays a key role in both the vertical and horizontal circulations at depth (Lumpkin and Speer 2007; Melet et al. 2016). Thus, a better understanding of the energy pathways that ultimately drive diapycnal mixing is necessary to improve our models of the global circulation, in particular, of the lower meridional overturning circulation (MacKinnon et al. 2017).

In contrast to the high-wavenumber internal tides (de Lavergne et al. 2019; Vic et al. 2019), mode-1 waves propagate away from their generation sites for hundreds to thousands of kilometers and redistribute the energy available for deep-ocean mixing across the ocean basins (Ray and Mitchum 1996; Egbert and Ray 2000; Alford 2003; Zhao et al. 2016; Alford et al. 2019). After long-range propagation, low-mode internal tides may reach continental slopes where several processes associated with tide–topography interactions can play a role in the wave energy budget. de Lavergne et al. (2019) parameterized some of these processes and found that internal-wave breaking on continental slopes can account for 10%–15% of global tidal dissipation.

Understanding the processes that determine the fate of the remote internal tide energy upon impinging on the continental slope is the goal of the Tasman Tidal Dissipation Experiment (T-TIDE; Pinkel et al. 2015). Observations and numerical models have been used to study different aspects of the internal tide generated at Macquarie Ridge, which then propagates ∼1400 km toward Tasmania.

Macquarie Ridge has been shown to be strong a generation site in satellite observations (Egbert and Ray 2000), theoretical (Morozov 1995) and numerical models (Niwa and Hibiya 2001; Simmons et al. 2004). From a regional analysis of altimeter data, Zhao et al. (2018) characterized the spatial variability of internal-tide generation along the ridge and calculated that an M2, mode-1 energy flux of ∼1 GW propagates toward Tasmania (Fig. 1a). In addition to good agreement with the altimetry-based estimate, energy flux from in situ observations in the center of the basin also show a spring–neap cycle, which is consistent with radiation of M2 and S2 internal tides from Macquarie Ridge (Waterhouse et al. 2018). From numerical simulations, Klymak et al. (2016, hereafter K16) found that the
incident wave is diffracted before it arrives on the continental slope, leading to larger (smaller) flux toward southeastern (northeastern) Tasmania (reproduced in Fig. 1b). Coincidentally, the region with the largest incoming energy flux also has highest modeled reflectivity, where ≈80% of the incoming mode-1 internal tide energy flux reflects as mode 1.

Previous observations have been used to characterize the internal tide on the eastern Tasman slope and corroborate the large reflectivity of its southeastern sector. Glider observations by Boettger et al. (2015) indicate the source of the internal tide is primarily remote as opposed to local generation on the continental slope. More extensive glider observations by Johnston et al. (2015) show standing wave features in the cross-slope structure of internal-tide energy, characteristic of high reflectivity. Their reflectivity estimate (90%) is higher than those based on satellite altimetry (60%; Zhao et al. 2018) and numerical simulations (80%; K16). Furthermore, Johnston et al. (2015) identified a northward propagating (approximately along-slope) internal tide, which K16 identified as a slope wave generated by the remote forcing on the slope.

Despite recent observations on eastern Tasmania, the sampling of available glider and satellite observations has precluded a description of the internal tide temporal variability. In particular, altimetry-based energy flux estimates are only calculated from the sea surface height tidal signal that has constant amplitude and phase at any given location (i.e., the stationary signal1). On one hand, if lunar (M₂) and solar (S₂) internal tides have amplitudes and phases that are constant, then the only temporal variability that would be observed is due to their beating (i.e., the spring–neap cycle). On the other hand, processes such as wave–mean flow interactions (e.g., Duda et al. 2018) or variable internal-wave generation (e.g., Zilberman et al. 2011), can create nonstationary variability of the internal tide, which requires more comprehensive ocean models for accurate prediction. Therefore, it is not known whether the stationarity assumption of current internal-tide energy budgets (de Lavergne et al. 2019) is a good approximation for the Tasman slope.

In situ observations (Johnston et al. 2015) have focused on the horizontal variability of the internal tide over O(10–100) km instead of its vertical structure on the continental slope. Regions where the internal tide energy is bottom-enhanced may trigger more tidally driven turbulence near the bottom. These regions may be determined by several processes associated with tide–topography interactions. For example, small-scale topography can lead to wave breaking (Nash et al. 2007) and interactions of the remote internal tide with the local surface tide can change the internal wave generation on the slope (Kelly and Nash 2010). Identifying which processes are relevant is an important step toward improving energy budgets of the internal tide on the continental slope.

In this paper, we present new in situ observations from T-TIDE and, along with a hierarchy of models, further describe the reflection process on the southeastern Tasman slope. In particular, our 2-month-long mooring observations capture the bottom-enhanced tidal energy in the reflective portion of the continental slope and provide an estimate of the temporal variability. In addition to being broadly consistent with a standing wave, our observations show that (i) both the standing wave and small-scale topography lead to bottom-enhanced energy, (ii) the tidal energy is primarily stationary, and (iii) the spring–neap cycle is a function of the complex interference between remote and local tides.

We proceed by describing our measurements, the numerical models we use to compare with observations, and the analysis methods (section 2). We then present model predictions of the standing wave pattern (section 3a), the observations (section 3b) and the overall agreement between them (section 3c). In addition to the large-scale structure of the standing wave, bathymetric
corrugations lead to significant spatial variability of near-bottom energy density at small spatial scales (section 3d). From the temporal variability of internal-tide energetics (section 3e), we show that the internal tide is primarily stationary, but the observed timing of the spring–neap cycle does not match with the prediction from the group velocity across the Tasman Sea. We use simple trigonometric calculations to argue this mismatch is a result of interference between the remote internal tide with the local tide on the slope (section 3f). We conclude with a brief discussion of the incident energy flux (section 4) and the summary of our results (section 5).

2. Methods

a. Observations

The T-TIDE field experiment took place between January and March of 2015. We carried out extensive observations along eastern Tasmania, focusing on two regions on the slope where we deployed 10 moorings and occupied several CTD–lowered acoustic Doppler current profiler (CTD-LADCP) stations and transects.

We will present observations from three moorings and one station in southeastern Tasmania (Fig. 2). In an along-slope averaged sense, the bathymetry has a concave shape, with supercritical topography ($\gamma > 1$, see Fig. 2) in the upper part of the continental slope and subcritical ($\gamma < 1$) further offshore. Three subsurface moorings were deployed on the upper half of the continental slope, where the mean water depth is 1170 m (Table 1) and the slope is locally supercritical. While the moorings were deployed at roughly the same cross-slope location (relative to the width of the continental slope), two moorings (T5 and T6) were deployed in a corrugation trough and the other (M5) on the adjacent crest of the bathymetry. Moorings T5 and T6 were each instrumented with two downward-looking acoustic Doppler current profilers (ADCPs): a 300-kHz RDI Workhorse measuring the bottom 100 m of the water column and a 75-kHz RDI Longranger for the adjacent 500 m above (Figs. 3a,b). Throughout the combined depth range of the ADCPs, each mooring was instrumented with around 40 Seabird (SBE)-56 thermistors and two SBE-37 MicroCAT CTDs. The vertical spacing of thermistors is highest over the bottom 150 m (10-m resolution), while they were deployed farther apart higher up on the mooring. The CTDs were deployed near the top and bottom of the mooring line, providing pressure records to estimate the depths of the thermistors. Moreover, moored and shipboard CTD data were used to calculate a third-order polynomial fit to estimate
salinity and potential density from the temperature data measured by the thermistors.

Mooring M5 was instrumented with an upward-looking 300-kHz RDI Workhorse ADCP, measuring the upper 100 m of the water column, as well as two McLane Moored Profilers (MMPs) profiling below (Fig. 3c). A few SBE-56 and RBR-Solo thermistors, an SBE-37 CTD, and an Aanderaa model 8 recording current meter (AA-RCM8) were deployed above and below the profiling depth limits of each MMP for assessing mooring knockdown and cross calibration. The nominal profiling depth ranges of the MMPs were 100–800 and 820–1130 m, and with a profiling speed of 0.25 m s$^{-1}$, the temporal resolutions at midprofile are about 45 and 20 min, respectively. Each MMP carried a standard Falmouth Scientific Inc. (FSI) CTD and velocity in the upper and lower MMPs was measured by an FSI acoustic current meter (ACM) and a Nortek Aquadopp profiler, respectively.

Large currents ($>0.4$ m s$^{-1}$) around yeardays 19 and 52 coincide with times when the MMPs at M5 could not profile along the mooring (Fig. 3c). At these times, a moored CTD near the surface indicates mooring knockdown of up to 300 m. Moreover, due to issues with the clock in a current meter, velocity measurements between 600 and 800 m in mooring M5 are only available when the MMP is profiling downward (Fig. 3c).

We also conducted a tidally resolving CTD-LADCP station (S2) at a water depth of 2505 m, about 15 km offshore from the moorings. The station was occupied for 23.5 h (on yearday 25) and a total of 32 nearly full water column profiles were taken. Along with the moorings, these data provide information on the cross-slope internal tide energetics.

### b. Linear scattering model

We will present internal tide energetics from the Coupling Equations for Linear Tides (CELT) model (Kelly et al. 2013) to address how the simple standing wave solution due to reflection from a vertical wall is modified by the slope of southeastern Tasmania. Briefly, CELT is a linear scattering model for one-dimensional topography, which is based on matching normal modes across topographic steps under the appropriate boundary conditions. The reader is referred to Kelly et al. (2013) for further details.

CELT was run with realistic bathymetry from southeastern Tasman slope (averaged in the along-slope direction over 10 km around the site of the observations) discretized with 500-m horizontal resolution and with adjacent flat bottoms in the deep ocean and the continental shelf. The buoyancy frequency is horizontally uniform and calculated from the World Ocean Atlas (Boyer et al. 2013). The imposed forcing is a mode-1, M2 internal tide propagating from the deep ocean and we have solved the scattering problem for 100 (surface and internal) modes (from a posteriori analysis, 100 modes is enough to calculate smooth velocity fields). The depth-integrated energy flux of the incoming wave was set to 1.7 kW m$^{-1}$, which is the same incident flux in the model by K16.

### c. Regional numerical model

In addition to CELT, we will also compare our observations with the model from K16 that has realistic two-dimensional bathymetry of the Tasman slope. The model is the MITgcm (Marshall et al. 1997), with lateral resolution of 1 km on the

![Table 1. Mooring (M5, T5, and T6) and CTD-LADCP station (S2) observations on the southeastern Tasman slope.](image-url)
slope and a stretched vertical coordinate with 200 grid cells. The forcing is an idealized internal-tide beam, tuned to have a width broadly consistent with that observed from satellite altimetry, $\sim 300$ km (Zhao et al. 2018), but with a relatively small incident energy flux to investigate the linear response of the continental slope. The incident energy flux at the moorings site is $1.7 \text{ kW m}^{-2}$ (Fig. 1b), which is a factor of 2 smaller than in situ (Waterhouse et al. 2018) and satellite altimetry (Zhao et al. 2018) observations. We refer the reader to K16 for further details.

To compare model with observations we regridded the model output onto a height above the bottom grid and averaged over $10$ km along the slope. We then calculated internal tide energy density and flux (as described below in section 2d). Along-slope averaging removes relatively high wavenumber waves, which are hard to compare with observations for several reasons. For example, with a 1-km horizontal resolution the model does not fully resolve the small-scale topographic corrugations. Therefore, along-slope averaged energetics provides a more robust comparison with the data than the model output extracted from a single location.

d. Data processing

Throughout this paper, buoyancy frequency $N$ refers to that calculated from the background temperature and salinity fields (using the Gibbs Seawater Oceanographic Toolbox; McDougall and Barker 2011). We define “background” as the temporal average over 3-day windows (denoted by the overbar in $\overline{N}$). Vertical isopycnal displacement $\eta$ is calculated from the difference between observed background and depths of potential density surfaces.

Semidiurnal velocity and isopycnal displacement were calculated from harmonic fits with a sliding 3-day window to depth-gridded data. For each segment, the harmonic fit solves for amplitudes of a linear trend as well as variability at semidiurnal, diurnal and inertial frequencies (with equivalent periods of 12, 24, and $17.5$ h, respectively). The bandwidth associated with this window length is narrow enough to separate semidiurnal and inertial frequencies (at the latitude of our observations), but wide enough to encompass all semidiurnal variability. Thus, though we do not filter in the frequency domain, we will refer to these semidiurnal time series as “bandpassed” (which will be denoted with the subscript $D_2$). Following Martini et al. (2011), this bandpass approach avoids time-gridding MMP data, which have uneven and depth-variable temporal resolution.

Under the flat-bottom approximation, the internal tide (baroclinic) horizontal velocity $u_{\text{BC}}$ can be calculated by subtracting the depth-averaged velocity (barotropic $u_{\text{BT}}$) from the total velocity. With observations throughout most of the water column, both the barotropic and baroclinic components can be directly calculated (e.g., Pickering et al. 2015). When data gaps in the vertical are significant, fits to standard vertical modes can be used to calculate low-mode internal tide velocities (e.g., Waterhouse et al. 2018). In both cases, observations throughout the water column, especially in the upper few hundred meters (Nash et al. 2005), are necessary for reliable estimates of $u_{\text{BT}}$ and $u_{\text{BC}}$.

Only one of our moorings (M5) meets this requirement. Even in that case, there are significant data gaps until yearday 21 and around 52, and no data from the ADCP near the surface is available in the last 3 weeks of deployment. Moreover, though $u_{\text{BC}}$ is a justifiable definition for internal-tide velocities over a flat bottom, either freely propagating or coastally trapped internal waves on a sloping bottom may have a non-zero depth-integrated velocity (Musgrave 2019).

Hereinafter we will refer to the total semidiurnal signal (i.e., $u$) as the internal-tide velocity. Though this is not ideal, we do not have a reliable estimate of $u_{\text{BT}}$ for most of our dataset and, even worse, $u_{\text{BC}}$ may not be a valid definition for internal-tide velocity on the slope. Moreover, our results do not change qualitatively by subtracting the surface-tide velocities from the TOPEX/Poseidon Global Inverse Solution (TPXO) 7.2 (Egbert and Erofeeva 2002) from the observed semidiurnal velocities.

The horizontal kinetic energy (HKE) and (linear) available potential energy (APE) are defined by

$$
\text{HKE}(z, t) = \frac{\rho_0}{2} \langle u \cdot u \rangle, \tag{1}
$$

and

$$
\text{APE}(z, t) = \frac{\rho_0}{2} \overline{\nabla^2 (\eta^2)}, \tag{2}
$$

respectively. The term $\rho_0$ is a constant reference density ($1025 \text{ kg m}^{-3}$), and the angle brackets denote temporal averaging over a semidiurnal period. This averaging procedure implies that energy only varies on time scales longer than a semidiurnal period.

The nearly full water column extent of observations at M5 also allows us to calculate the internal-tide horizontal energy flux, i.e.,

$$
\mathbf{F}(z, t) = \langle p \mathbf{u} \rangle, \tag{3}
$$

where $p$ is the pressure perturbation calculated from vertical integration of $\eta$ (Kunze et al. 2002).

Caveats analogous to those presented for the calculation of internal-tide velocity on a sloping bottom apply for the pressure perturbation estimates. Nevertheless, we will show our energy flux estimates are qualitatively consistent with the expected standing-wave pattern, which gives confidence on the validity of the calculations.

e. Stationarity metric

In addition to the bandpassed semidiurnal signals described earlier, we also solve for the amplitude and phase at $M_2$ and $S_2$ frequencies from harmonic fits to our entire time series, which are sufficiently long ($\sim 50$ days) to resolve these tidal frequencies. We define the stationary tidal signal as the results from these harmonic fits and we refer to their sum as the stationary spring–neap cycle (which will be denoted with the subscript $M_2S_2$).

Stationary HKE and APE are calculated from stationary velocity and displacement. For constant displacement variance, temporal variability of $N$ leads to changes in stationary
APE (and total energy). In our observations, this temporal variability is significantly smaller than the spring–neap cycle between semidiurnal constituents.

Similar to Nash et al. (2012), we quantify stationarity with the skill score (Murphy 1988), i.e.,

$$SS_{x} = \left[1 - \frac{\text{var}(x_{D_{2}})}{\text{var}(x_{M_{2}S_{2}})}\right]$$, \hspace{1cm} (4)

where $x$ is the variable for which the stationary metric is computed (e.g., HKE), the subscript $D_{2}$ denotes the semidiurnal band and $M_{2}S_{2}$ the stationary signal. If the stationary signal is equal to the semidiurnal energy, the former captures all of the variance of the latter and $SS_{x} = 1$. If $x_{D_{2}}$ and $x_{M_{2}S_{2}}$ are anticorrelated, the skill score is negative.

3. Results

a. Modeled interference patterns

Before presenting the observed internal tide, it is instructive to look at idealized expectations for energy density and flux from models with varying degrees of realism. Our goal in this section is to provide background for interpreting the observations by relating the modeled spatial structures of the internal tide to the setup and limitations of each model.

The well-known analytic solution of wave reflection from a vertical wall (dotted lines in Figs. 4a–d) shows that energy density and flux in a standing wave are oscillatory as a function of distance from the wall (LeBlond and Mysak 1981; Nash et al. 2004; Johnston et al. 2015). The cross-slope flux component does not have an oscillatory structure because it is identically zero. This result contrasts with the spatially constant energetics for a single wave, such that the ratio between HKE and APE is a useful diagnostic of wave interference (Nash et al. 2004; Martini et al. 2007; Alford and Zhao 2007; Klymak et al. 2011). For a complete derivation of this simple model’s solution, the reader is referred to appendix A in Johnston et al. (2015).\footnote{Note the derivation in Johnston et al. (2015) has two minor mistakes: in their notation, assuming $k$ is the magnitude of the zonal wavenumber component, the complex amplitude of $v_{w,c}$ should be $a_{o} \pm i\kappa$ in their Eq. (A1); in Eqs. (A3) and (A5), instead of a common amplitude factor $C$, velocity components $u_{t}$ and $v_{t}$ should have amplitude factors $C_{u} = \sqrt{(a_{o}k)^{2} + (fi)^{2}}$ and $C_{v} = \sqrt{(a_{o})^{2} + (fk)^{2}}$, respectively.}

Despite the sloping bottom, potentially different responses to one and two-dimensional topographies, and the partial mode-1 reflectivity of 83%, oscillatory patterns of energy density and along-slope energy flux in CELT and K16 are
broadly similar to the wall model (Figs. 4a–d). Around the sites of our observations (vertical bars in Figs. 4a–d), all solutions have primarily southward energy flux and the ratio between HKE and APE increases toward deeper water. In contrast to the wall model, reflection is distributed across the slope and little energy gets to the shelfbreak (as indicated by APE approaching 0 instead of a finite value at $x_0$, Fig. 4b).

From CELT (Figs. 4e–h), the internal tide response to one-dimensional bathymetry leads to bottom-enhanced energy density and flux around the moorings site (7 km from the shelfbreak). At this location, larger near-bottom tidal energy is a result of both (i) offshore-propagating scattered waves and (ii) the seafloor crossing the depth level of the deep-ocean mode-1 APE maximum (i.e., the profile of the incoming mode-1 wave at 3300 m has maximum APE at $\approx 1000$ m). The geometric effect (ii) can be thought of as reminiscent of the interference pattern from a vertical wall, where both incident and reflected waves are mode-1. As a result from two distinct mechanisms, the interference pattern is such that HKE is enhanced near the bottom at the mooring sites over a shorter vertical scale than APE.

Energy density and the along-slope energy flux in K16 is similar to CELT (Fig. 4) and can thus be explained by the internal tide response in the cross-slope direction only. However, the cross-slope energy flux in the two models have opposing directions (Figs. 4c,g,k). CELT has a (small) net onshore energy flux because the reflectivity is not 100%. K16 has a net offshore energy flux around the location of the observations, but the sign of this component varies in the along-slope direction.

FIG. 5. Spatial variability of the modeled (K16) depth-integrated energy flux in southeastern Tasmania. (a) Along-slope component of the energy flux, (b) cross-slope component, (c) cross-slope flux at $x_0$ and terms in the energy balance. Maps (a) and (b) are shaded for water depths less than 200 m and the dots show the sites of the observations. While a standing wave (Fig. 4d) explains the cross-slope structure of the along-slope flux, the cross-slope component has more complex spatial variability. Assuming small dissipation, the divergence of the depth-integrated energy flux in K16 is approximately 0. Due to large reflection, the cross-slope ($F^x$) at $x_0$ is proportional to the along-slope gradient of the mean along-slope flux between the shelfbreak and $x_0$ ($dF^x/dy$, where the overbar denotes the cross-slope mean).
direction (Fig. 5). The \(\sim100\)-km along-slope wavelength in the cross-slope energy flux is consistent with a slope wave on the Tasman slope, which can be excited by the incident internal tide (K16). Since the model is stationary, dissipation is relatively small, and reflectivity is large, an energy budget implies the cross-slope energy flux at some distance from the shelf-break is balanced by the along-slope gradient of the along-slope flux (Fig. 5c).

b. Overview of the observations

General features of the semidiurnal internal tide, such as bottom-enhanced energy, are highlighted by a 3-day subset of the 50-day mooring records (Fig. 6). Velocity and isopycnal displacements are enhanced over the bottom; 300 m and have amplitude maxima of \(>0.1\) m s\(^{-1}\) and \(\sim75\) m, respectively. These amplitudes are characteristic of the observed spring tides, four of which have been captured in the full 50-day mooring records (Figs. 8a–c). At neap tides, both semidiurnal velocity and displacements drop to about 0.05 m s\(^{-1}\) and 30 m, respectively.

From the full mooring records, near-bottom velocity spectra have statistically significant peaks at the diurnal, semidiurnal and two times the semidiurnal frequencies (Fig. 7). The beating between the diurnal and semidiurnal tides can be seen as diurnal inequalities of isopycnal displacements (e.g., around yearday 48 in Figs. 8a,b). Mooring M5, which was deployed on a crest of the corrugated bathymetry, shows larger near-bottom subtidal variability than currents inside the trough of the corrugation.

The diurnal internal tide, which is subinertial and thus coastally trapped at the latitude of our observations, has vertical isopycnal displacements of up to 50 m (Fig. 8d), which are comparable to the semidiurnal variability (Fig. 8c). The background cross-slope velocity within the trough of the corrugated bathymetry (below \(\sim1100\) m at the site of the T5 mooring) is consistently upslope (Fig. 8e).

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**Fig. 6.** Velocity (colors) and isopycnal surfaces (black curves) over a 3-day subset of the time series. (a) Cross-slope and (b) along-slope velocity component anomalies relative to (c) the 3-day temporal mean from mooring T5. (d)–(f) As in (a)–(c), but for mooring M5. The horizontal dashed lines indicate the bottom depth at mooring M5, which is on the crest of a bathymetric corrugation. Note velocity (both varying and temporally averaged) are topographically constrained within the corrugation trough (i.e., below the dashed line at mooring T5). For the time-mean velocity, that effect extends upward for another \(\sim150\) m.
c. Observed interference pattern

We now describe the observed semidiurnal energy density and flux and how it compares with the patterns in the modeled interference pattern. While our sparse measurements preclude a quantitative decomposition of each wave in the interference pattern, some of the features in the observations are qualitative consistent with the models and thus suggest the large reflectivity of the continental slope. Models and observations also have their limitations, and we will highlight differences between their results.

The cross-slope variability in depth-integrated energy density and flux is roughly consistent with the modeled predictions (Fig. 9). Both the model and mooring show that HKE/APE ≤ 1 at the same location while HKE is larger than APE near the CTD-LADCP station. Moreover, the along-slope energy flux is poleward and its magnitude increases from the shelfbreak to the deep ocean. Since these features in energy density and flux are consistent with a standing wave pattern, the energetics observed at the two sites across the slope are suggestive of the large reflectivity of the continental slope. While the offshore cross-slope flux at the station is not a feature of the standing wave, it has the same direction as in K16, which indicates the variability of the internal tide in the along-slope direction.

Furthermore, models and mooring observations show bottom-enhanced energetics (Fig. 10). Observed energy density (Figs. 10a,b) and along-slope flux (Fig. 10e) are larger in the bottom few hundred meters, which is the depth range that mostly contributes to the temporal variability in the correspondent depth-integrated quantities (Figs. 10c,f). While large values are also found near the surface, which is poorly covered by our observations, they have shorter vertical extent than the near-bottom energetics. These quantitative statements are only true for the three bottom-enhanced periods observed at M5, which are associated with the spring tides previously noted from mooring T5 (Fig. 8).

The predominantly along-slope direction in the moored energy flux (Figs. 10d–f) further corroborates the standing wave pattern predicted from the models. Since the pressure perturbation is estimated by vertically integrating isopycnal displacement, the along-slope energy flux can also be identified in the phase difference between the near-bottom velocity components and isopycnal displacements (Fig. 11). If the internal tide shoaled on the continental slope, near-bottom cross-slope velocity and isopycnal displacement would be anticorrelated, such that the energy flux would be on slope. However, model and observations show it is the along-slope velocity component that is anticorrelated with displacement (i.e., ellipses in Fig. 11b are polarized with negative slope) and the energy flux at the mooring site has a predominantly along-slope component. Therefore, the observed energy flux supports the high reflectivity of the continental slope because the net cross-slope energy flux is much smaller than the along-slope component, in agreement with the model results that we described earlier (Figs. 4 and 5).

Several features are also different between models and observations. The magnitudes of energy density and flux are not consistent with the model because the modeled energy density is similar to the observed minimum (Figs. 9a and 10c) while the average observed flux is close to the model prediction (Figs. 9b and 10f). Moreover, the partition between HKE and APE varies in time and only the third spring tide (around yearday 60, Fig. 10e) appears to be consistent with the model (see also Fig. 14d). Furthermore, our description of the cross-slope structure is also crude because the station data at S2 only provides a snapshot in time and we only have observations at two locations across the continental slope (the other moorings are close to M5 and differences among them are not due to the large-scale structure of the interference pattern).

d. Bathymetric corrugations: Spatial variability over small scales

Similar to the vertical structure from the M5 mooring, all (full-record) temporally averaged semidiurnal energy profiles are enhanced in the bottom few hundred meters (Fig. 12). From all moorings, the vertical scale of enhanced APE is longer than for HKE. These different length scales are qualitatively consistent with CELT (Figs. 4e,f), suggesting this feature is simply due to the broad standing wave structure at this location of the slope. Vertical profiles of modeled energy density (K16 in Fig. 12) are broadly consistent with observations. Consistent with the relatively weak forcing in the model, modeled energy density is about half of the observed.

Despite the proximity of the three moorings, energy density is larger along the trough of the corrugation (moorings T5 and T6) as opposed to the crest (mooring M5, Fig. 12). Enhanced total energy along the trough is primarily associated with the increase in APE (Fig. 12b). The different quantitative responses of HKE and APE to the corrugations may be associated with wave interference leading to different spatial structures for each quantity.
Although we have not investigated how corrugations modulate the energetics, one hypothesis is that high wavenumber waves are generated by topographic scattering (Legg 2004; Thorpe 2001) and contribute to the interference pattern at small spatial scales. Larger HKE on the trough (T5) than on the crest (M5) for heights above the rim of the corrugation might be evidence for internal tide scattering (Fig. 12a).

For this paper, the modulation of tidal energy by the corrugations is relevant because it creates substantial spatial variability at small horizontal scales. As a result, this leads to uncertainties on how to compare energy magnitudes between moorings as well as between observations and model. For consistency, in the remainder of this paper we restrict the vertical range of depth integration between 40 and 550 m above the bottom (for mooring M5, energy density is linearly interpolated between MMPs). Within this range, depth-integrated energy is higher from moorings deployed in the corrugation trough (T5 and T6) as opposed to the crest (M5).

e. Internal tide energy: Time mean and variability

The stationary tidal signal contributes most to the mean (≥80%) and variance (SS\(_{M2S2}\) ≈ 0.6) of the observed energy (Table 2). As expected, the amplitude of the M\(_2\) tide is much larger than that of the S\(_2\) (85% of the stationary mean, not shown). The large stationary fraction of the temporally averaged energy density is in qualitative agreement with concurrent offshore observations (Waterhouse et al. 2018) and model estimates (Savage et al. 2020).

The vertical profile of bandpassed and stationary velocity components (Fig. 13) illustrates the validity and caveats of our stationarity metrics (Table 2). Toward the bottom, where HKE is elevated, semidiurnally bandpassed \(u\) and \(v\) amplitude and phase are well predicted by the harmonic fits to the M\(_2\) and S\(_2\) frequencies, such that there is large skill score between the corresponding signals (SS\(_{M2S2}\) ≈ 0.9 and SS\(_{M2S2}\) > 0.7). Higher up in the water column, tidal velocities have smaller magnitude and the stationarity also decreases. In terms of the kinetic energy, the stationary metric is generally smaller than for each velocity component separately. However, because HKE is enhanced toward the bottom, the variance explained calculated from the depth-integrated kinetic energy is similar to the high values closer to the bottom.

The vertical profile shows additional features that are not addressed by the stationary depth-integrated energy. For example, the stationary component may be significant for only one velocity component at a single depth (Figs. 13b,c) or the spring–neap cycle may differ for each velocity component separately. However, because HKE is enhanced toward the bottom, the variance explained calculated from the depth-integrated kinetic energy is similar to the high values closer to the bottom.
Moreover, the relatively high amplitude and stationarity of the along-slope tidal velocity further away from the bottom is likely associated with the surface tide.

For depth-integrated energy density, the large stationary signal (M2 and S2) is apparent as the spring–neap cycle in the temporal variability (Fig. 14). HKE has a lower stationarity and spring–neap cycle amplitude than APE. However, due to the spatial variability of the partition between kinetic and potential energy in an interference pattern, variability in total energy is likely a more adequate measure of stationarity. Though the nonstationary variability does have a nonnegligible contribution, our observations are not sufficient to explain the

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underlying mechanisms at any specific time (e.g., a relatively more energetic neap tide around yearday 25, Fig. 14a).

Though we previously showed the consistency between the observed and modeled average HKE to APE ratio (Fig. 9), the temporal variability of this ratio is not consistent with a standing wave at all times (Fig. 14d). In particular, HKE has a smaller spring–neap cycle amplitude than APE, and their ratio tends to be lower at spring tides (Fig. 14). This pattern suggests

FIG. 12. Vertical structure of energy density in the bottom half of the water column: (a) horizontal kinetic energy (HKE), (b) available potential energy (APE), and (c) their sum. Each profile is a temporal average over the whole record of the semidiurnally bandpassed HKE and APE. Since the model has relatively weak forcing, the modeled profile (K16) is shown with twice its amplitude for quantitative comparison with the observations. For the locations of moorings T5 and T6, the horizontal dashed lines indicate the heights of the corrugation crests above the bottom depth at the adjacent trough. While the broad standing wave structure (K16) explains bottom-enhanced energy over the bottom 300–400 m, larger energy density is observed along the corrugation trough (T5 and T6) relative to the crest (M5).

FIG. 11. (a),(b) Hodographs from observed and modeled semidiurnal cross-slope velocity $u$, along-slope velocity $v$, and vertical isopycnal displacement $\eta$. Velocity and displacement are averaged in the bottom 100 m. Observations are taken from mooring M5 (blue ellipses) and modeled results from CELT (red ellipses). Mean hodographs for the observations are calculated from the phase-averaged semidiurnal velocity and displacement (black ellipses). Because the pressure perturbation is estimated from vertical integration of $\eta$, these two quantities are proportional to one another. Therefore the nonpolarized $u-\eta$ ellipses in (a) indicate small cross-slope energy flux, and the predominant along-slope energy flux is shown by largely polarized $v-\eta$ ellipses in (b).
that when the incident (and reflected) internal tides have larger amplitude, the standing wave predicted from the models (Fig. 4) is a better approximation to the total interference pattern. In contrast, the surface tide or locally generated internal tides may have a significant amplitude when the remote forcing is weak at neap tides.

However, this interpretation is not strictly valid because energy density and flux are nonlinear quantities. That is, because the observed energy depends on the relative phasing between all waves in the interference pattern, including locally generated waves, the time of maximum incoming flux does not necessarily coincide with maximum observed energy density on the slope. This nonlinear effect is highlighted by the mismatch between observed and predicted spring–neap cycles (Fig. 15). Based on the group velocity, calculated from climatological stratification, the remote internal tide reaches the southern Tasman slope in 9.23 ± 0.06 days (mean and standard deviation based on different ray path choices, appendix A). This is smaller than the average 12-day estimate from observed energy, which also differs from the fortnightly cycle of the surface tide by 2 days. Therefore, the phase of the observed spring–neap cycle cannot be explained by individually considering the remote internal tide or the surface tide.

In contrast to these data, offshore observations by Waterhouse et al. (2018) are in agreement with the predicted travel time from Macquarie Ridge (Fig. 15). This difference between datasets suggests that there should be an effect local to the Tasman slope that changes the phasing of the spring–neap cycle there, but not offshore.

f. Interference of remote and local waves

We attribute the unexpected phase of the observed fortnightly cycle to the complex interference of remote and local tides. Consider the four-wave interference

\[
y(x, t) = \cos(k_{M2}x - \omega_{M2}t) + \cos(k_{S2}x - \omega_{S2}t) + \cos(-\omega_{M2}t) + \cos(-\omega_{S2}t),
\]

(5)

---

**Table 2. Depth-integrated internal tide energy statistics.** Time mean values and standard deviations (inside parentheses) are in kJ m⁻². Depth integrals for the three moorings are taken between 40 and 550 m above the bottom. The skill score (SS) is defined by (4). The numerical model (K16) only has M₂ forcing and no temporal variability in tidal energy.

<table>
<thead>
<tr>
<th></th>
<th>Stationary (M₂ + S₂)</th>
<th>Bandpassed (D₂)</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HKE</td>
<td>APE</td>
<td>E</td>
</tr>
<tr>
<td>M5</td>
<td>0.4 (0.1)</td>
<td>0.8</td>
<td>(0.3)</td>
</tr>
<tr>
<td>T5</td>
<td>0.9 (0.2)</td>
<td>1.4</td>
<td>(0.6)</td>
</tr>
<tr>
<td>T6</td>
<td>0.6 (0.1)</td>
<td>1.3</td>
<td>(0.5)</td>
</tr>
<tr>
<td>K16</td>
<td>0.4</td>
<td>0.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>

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**Fig. 13.** Variance of semidiurnally bandpassed velocity and HKE explained by the correspondent stationary signals from mooring T5. (a) Vertical profiles of the skill score, defined by (4), for the cross-slope u velocity component, the along-slope v component, and HKE. The vertical black line is the skill score computed from the depth-integrated HKE. (b),(c) Time series of each velocity component farther away from the bottom. (d),(e) As in (b) and (c), but closer to the bottom. The depth of the time series and the skill score in (b)–(e) is shown on the top left of the corresponding panels. The larger variability toward the bottom is generally more stationary than higher above.
by the remote wave terms in (5). Moreover, additional constant phase factors could be included for a more realistic analogy with tides in the Tasman sea, but they do not qualitatively change the results and are set to zero for simplicity.

By adding lunar and solar tides together,\(^3\) we can rewrite (5) as
\[
y(x, t) = 2[\cos(kx - \omega t) \cos(\delta kx - \delta \omega t) + \cos(\omega t) \cos(\delta \omega t)].
\]

where \(\omega = (\omega_{S2} + \omega_{M2})/2\) and \(\delta \omega = (\omega_{S2} - \omega_{M2})/2\) (with analogous definitions for \(k\) and \(\delta k\)). While each term on the right-hand side of (6) is separable in terms of a carrier wave (with arguments \(\omega\) and \(k\)) and an envelope (with arguments \(\delta \omega\) and \(\delta k\)), this is not the case for the full four-wave interference.

The form of the envelope can be calculated from the energy-like quantity \((y^2)(x, t)\), where the angle brackets denote time averaging over a tidal period.\(^4\) Squaring and averaging (6) (see appendix B) gives
\[
\frac{1}{2} \langle y^2 \rangle (x, t) = 1 + \cos(\delta kx) \cos(kx) + \cos(\delta kx - 2\delta \omega t) \\
\times \left[ \cos(\delta kx) + \cos(kx) \right].
\]

It is relevant to see the behavior of (7) for \(x = \pi/(2\delta k) \pm (2\pi/k)\). This location corresponds to half the wavelength of the remote wave envelope. Since \(\delta k/k \ll 1\), we can make the approximation \(\cos(\delta kx) \approx \cos(\pi/2) = 0\). Under this approximation, (7) becomes
\[
\frac{1}{2} \langle y^2 \rangle (x, t) \approx 1 + \sin(2\delta \omega t) \cos(kx).
\]

From (8), the spring–neap cycle at distances where \(\cos(kx) > 0\) is out of phase relative to what is observed at locations where \(\cos(kx) < 0\). Qualitatively, this phase inversion happens because the total spring–neap cycle has an intermediate phase between the individual spring–neap cycles from local and remote tides, which are themselves out of phase around \(x = \pi/(2\delta k)\). Since the solutions are periodic, there are two intermediate phases for the total spring–neap cycle. For a given distance \(x\), the sign of \(\cos(kx)\) determines which intermediate phase is the correct one.

While (8) provides some insight on the complex interference, we calculate the phase of the envelope numerically to address the superposition of waves with different amplitudes. As opposed to (5), the amplitude of the \(M_2\) tide in the ocean is usually much larger than the \(S_2\). As for the relative contribution of remote and local tides on the Tasman slope, the agreement of standing-wave predictions with our mooring observations, as well as with previous glider

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\(^3\)With the trigonometric identity \(\cos(a) + \cos(b) = 2 \cos[(a + b)/2] \cos[(a - b)/2]\).

\(^4\)Much of the complexity in the spring–neap cycle with remote and local tides stems from averaging in time only. While an averaging operation in both time and space could be defined, it is not the relevant calculation to interpret pointwise observations.
surveys (Boettger et al. 2015; Johnston et al. 2015), indicates the remote wave is more energetic.

Numerical calculations of the fortnightly phase with fixed ratio between M2 and S2 amplitudes and varying local wave magnitude (see appendix C for details), show that, even with modest local waves, the spring–neap phase is sensitive to the travel time, especially for widths that are consistent the Tasman sea (Fig. 16). The larger sensitivity for these widths is reminiscent of the behavior of (7) at $x = \pi(2\delta k)$, which for the Tasman sea is approximately 1125 km. However, the different amplitudes between remote and local waves allow for phases of the envelope in between the two values predicted by (8).

The sensitivity of the spring–neap cycle implies that small errors in estimating the distance (or travel time) of propagation by the remote wave may result in large errors in determining the phase of the spring–neap cycle. For example, if the remote wave amplitude is 2 times the local one (corresponding to a factor of 4 in energy, orange line in Fig. 16a), an error of 5% leads to estimates of the spring–neap phase that range by an equivalent time of 4 days. In terms of the travel time, a 5% error of 9 days corresponds to 12 h.

While the error bound based on different choices of ray paths is only about $\pm 1.5$ h (appendix A), there are several other potential sources of error (both uncertainties and biases). For example, the group velocity can change in relatively small temporal and spatial scales due to the background currents and stratification (e.g., Duda et al. 2018) or basinwide over longer time scales as a result of climate variability (Zhao 2016).

The four-wave interference provides a mechanism that qualitatively explains the observations of the spring–neap cycle (Fig. 15c). On the continental slope, the interference of both local and remote tides is significant such that the timing of observed spring tides is in between the predictions. Moreover, changes in the travel time of a few hours between different spring tides can account for the variability of a few days in their timing. We also highlight this result does not change whether or not TPXO surface tide velocities are subtracted from the data, suggesting the observations are the result from the interference between locally generated internal tides and the remote internal tide. In contrast to the continental slope, the four-wave interference does not apply at the TBEAM site because the magnitude of the northwestward propagating internal tide is much larger than other waves (as shown from satellite altimetry; Zhao et al. 2018), and a simple travel time prediction agrees with the in situ observations (Waterhouse et al. 2018).

4. Incident energy flux on the Tasman slope

Because of the complex interference pattern, we cannot calculate the incident energy flux from the observed total energy (i.e., through the internal-wave polarization relations).

![Fig. 15. Predicted and observed semidiurnal spring–neap cycle. (a) Semidiurnal normalized barotropic tidal energy from TPXO. Local (dashed) refers to the Tasman slope and remote (solid) is the prediction at Macquarie Ridge lagged by the travel time based on the mode-1 group speed in the Tasman Sea [i.e., 9.24 days the value of black line in (c) at the distance corresponding to Tasmania]. (b) Total (dot–dashed) and stationary (solid) depth-integrated internal-tide energy from mooring T5 (other moorings are not shown for clarity) and (c) distance–time diagram with predicted and observed timing of semidiurnal spring tides taken from (a) and (b), as well as TBEAM data (Waterhouse et al. 2018). The stationary energy in (b) is not exactly periodic due to temporal variability in the buoyancy frequency. Horizontal dashed lines in (b) are depth integrated from the regional numerical model (K16). Gray shading is as in Fig. 14. In (c), time on the y axis is equivalent to fortnight phase, where day 0 is the time of a reference spring tide (from TPXO) at Macquarie Ridge (i.e., the generation site of the remote internal tide) and the period is ~14.77 days. Solid black curves (indistinguishable from one another) are theoretical predictions from mode-1 propagation along a set of ray paths (see appendix B). The mode-1 propagation correctly predicts the phase of the stationary spring–neap cycle in the deep ocean (TBEAM), but it does not explain the timing on the southeastern Tasman slope (T5) and neither does the surface tide (TPXO).]
Nevertheless, we can make a rough estimate of the incoming energy flux by scaling the energy density in the model relative to the observations. The observed average energy density is 2.1 kJ m\(^{-2}\), which is a factor of 2.3 larger than the modeled energy at the mooring sites (Table 2). Since the model has a prescribed forcing, with a corresponding energy flux of 1.7 kW m\(^{-1}\), we infer the average incident energy flux on southeastern Tasmania is 3.9 kW m\(^{-1}\). Based on a standard deviation of 40% in total energy density, we infer an incident energy flux range between 2.3 and 5.5 kW m\(^{-1}\).

The inferred mean incident energy flux is consistent with a concurrent 3.4 kW m\(^{-1}\) mode-1 energy flux based on offshore observations (Waterhouse et al. 2018) and larger than the 2.3 kW m\(^{-1}\) from a glider survey (Johnston et al. 2015) undertaken two years before our mooring deployments. Both of these estimates are based on spatially distributed data and have large uncertainties. Since the temporal variability of energy density on the slope is primarily due to the spring–neap cycle, our rough estimate of the incident energy flux range (2.3–5.5 kW m\(^{-1}\)) primarily corresponds to the stationary (i.e., \(M_2\) and \(S_2\)) component.

Nevertheless, the largest energy density observed on the slope seems to be associated with nonstationary variability observed offshore (Fig. 17). These energetic events at both locations suggest the remote internal tide experiences nonstationary variability, which leads to a larger incident energy flux on the Tasman slope. Though the anomalously high value on the slope is not as large as would be predicted from Waterhouse et al. (2018), the time lag between these observations is consistent with the mode-1 travel time. Presumably, the incident energy flux at this time is large enough to overwhelm the temporal lag induced by the interference between remote and local tides. For the rest of the record (i.e., before yearday 55 in Fig. 17), this interference cannot be neglected, and the remote wave travel time alone does not account for the delay between observations on the slope and offshore.

While this analysis shows some consistency between the internal tide observations, there are several caveats that preclude a quantitative comparison. Because of the spatial structure of the offshore internal tide beam, both the mean and variability of the energy flux based on a single mooring are different than their across-beam averaged counterparts (which may explain larger nonstationary variability at TBEAM relative to T5 in Fig. 17). Moreover, the inferred energy flux by scaling the model to match the observations does not take into account the enhancement of tidal energy along the troughs of the corrugations, which the model does not fully resolve with its 1-km resolution. Finally, the difference between the incident energy flux estimate from Johnston et al. (2015) and our rough calculation could be due to large uncertainties as opposed to temporal variability of the incident internal tide beam.

![FIG. 16. Sensitivity of the spring–neap envelope to the local wave amplitude and the propagation distance of the remote wave (L).](image)

(a) Spring–neap phase as a function of L for a few local wave amplitudes and (b) the solutions for all local wave amplitudes between 0 and 1. The curves in (a) are the solutions along the horizontal dashed lines in (b). The phase is defined as the time when the envelope peaks (i.e., spring tide). The parameters in the calculation are consistent with the internal tide across the Tasman Sea and the Tasman slope corresponds to L \(\sim\) 1400 km.

![FIG. 17. Timing of the internal tide energetics observed on the slope (T5) and ~500 km offshore (TBEAM; Waterhouse et al. 2018).](image)

Time series of depth-integrated energy on the slope (orange line) and magnitude of the mode-1 semidiurnal energy flux (black lines). The solid black line is temporally lagged by the travel time of energy between the TBEAM and T5 sites. As expected from the interference between locally and remotely generated internal tides, the group-velocity-based travel time does not explain the overall timing of energy maxima and minima on the slope. The last peak is an exception, potentially due to a substantially larger incoming energy flux, thus reducing the role of wave interference in setting the timing of spring tides on the slope.
5. Summary

Mode-1 internal tides can propagate for hundreds or thousands of kilometers with little dissipation (Ray and Mitchum 1996; Egbert and Ray 2000; Zhao et al. 2016; Alford et al. 2019). Previous work has studied the internal tide beam that is generated south of New Zealand and propagates northward for $\sim 1400$ km until impinging on the continental slope of Tasmania. In this paper, we have presented in situ observations from a region of the slope where previous studies have identified high reflectivity (Johnston et al. 2015; K16; Zhao et al. 2018).

As expected, in situ observations are roughly consistent with modeled standing wave patterns and thus suggest most of the remote internal tide reflects from the continental slope (Figs. 4 and 9–11). While previous work has focused on the standing wave extending hundreds of kilometers away from the boundary (Johnston et al. 2015), we highlight that the bottom-enhanced energetics, over a portion of the continental slope, is part of the standing wave on a sloping bottom. Model analysis indicates that this feature is observed where the rising seafloor on the continental is within the depth range of the deep-ocean mode-1 APE maximum. Even though most of the energy flux from the incident wave is reflected, this region of the continental slope with bottom-enhanced tidal energy may fuel near-bottom turbulence and account for the relatively small fraction of the incident wave that is dissipated on the slope.

In addition to the large-scale standing wave pattern, small-scale bathymetric corrugations lead to spatial variability of tidal energy over horizontal scales of $O(1)$ km. Our observations show larger tidal energy along a trough of the corrugation relative to the adjacent crest (Figs. 12 and 14). Although we have not investigated which processes are responsible for this pattern, we hypothesize that the more energetic tide along the troughs can lead to larger near-bottom turbulence there as opposed to the crests.

The temporal variability of tidal energy on the continental slope is primarily associated with the stationary spring–neap cycle over the $\sim 50$-day records (Table 2 and Fig. 14). Although the TBEAM moored observations do not show a large spring–neap cycle (presumably due to temporal variability of the internal tide over pointwise observations), a large fraction (93%) of the time averaged energy flux is explained by the M$_2$ internal tide (Waterhouse et al. 2018). Therefore, our observations indicate that a substantial fraction of the internal tide energy on the slope can be predicted.

The relatively large stationarity ($SS_{\text{avg},S} \geq 0.6$) on the southeastern continental slope of Tasmania is consistent with model estimates (Savage et al. 2017, 2020). Globally, altimetry-based estimates indicate that stationarity of more than 50% is a common feature of remote internals even far away from their generation sites, as long as they do not propagate near the equator (Zaron 2017). In situ estimates are scarce, but stationarity larger than 0.6 has been observed in the North Pacific, where energetic mode-1 internal tides propagate over $O(1000)$ km from both from Hawaii and Alaska (Zhao et al. 2010).

Despite the large stationarity on the Tasman Sea, anomalously large energetics both on the slope and 500 km offshore suggests that nonstationary variability of the remote internal tide can be nonnegligible (Fig. 17). The actual processes that create this variability are beyond the scope of this paper, but two potential candidates are wave–mean flow interactions and enhanced internal tide generation due to larger stratification at the generation site.

The observed spring–neap cycle on the slope has an intermediate phase relative to the spring–neap cycles of the remote internal tide and the local surface tide (Fig. 15). We show from simple trigonometric calculations that the timing of the spring tide that arises from wave interference is a function of both the relative magnitudes of local and remote tides as well as the travel time of the remote wave (Fig. 16). These effects can lead to changes in the timing of the spring tide by a few days, as observed in our mooring records. Although we only present the variability in the timing of the spring tides, our observations reinforce results from Kelly and Nash (2010) on the interference between local and remote waves. For example, modeling internal-wave generation on the continental slope of Tasmania, perhaps associated with the small-scale corrugations, would need to incorporate tidal forcing as the interference between the surface and the remote internal tide.

An important question that we are not able to address from our sparse measurements is the energy flux carried by each separate wave in the full interference pattern. Though scaling the model results based on the observations suggests the mean incident energy flux is about 3.9 kW m$^{-1}$, more observations would be required to obtain a direct estimate. Nevertheless, analysis of the in situ observations, along with models of varying degree of realism, allows us to explain certain features of the complex internal tide field on the southeastern Tasman slope: the standing wave captures broad features of the internal tide energetics, but interaction with small-scale topography and interference with locally generated waves also play a significant role.

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Data availability statement. Mooring (M5, T5, and T6) and shipboard (S2) data are available at [https://doi.org/10.6075/J0FT8JK1](https://doi.org/10.6075/J0FT8JK1). For access to TBEAM data, see Waterhouse et al. (2018). The source code to generate the figures is available at [https://github.com/olavobm/ITEnergySoutheastTasmania](https://github.com/olavobm/ITEnergySoutheastTasmania).

APPENDIX A

Range of Travel Time Estimates

We argued in section 3f that the temporal mismatch of $\sim 2$ days between the observed spring–neap cycle and the
predicted from the remote internal tide only is due to interference with the locally generated tide. An alternate hypothesis is that the theoretical estimate of the remote wave travel time that we used (9.25 days) as a temporal lag is physically inadequate. Here we present a range of travel time estimates and show that they can only account for a time mismatch of less than a day and are thus not able to explain the timing of the spring–neap cycle on the slope.

The theoretical group velocity we refer to is that derived from the dispersion relationship for linear internal mode waves. This formulation does not include nonlinear effects or wave–current interactions. Moreover, one of the parameters is stratification, which can vary in a wide range of spatial and temporal scales. Finally, the travel time is calculated based on a choice of ray path and is thus dependent on the spatial variability of the relevant parameters used to calculate the group velocity.

From altimetry observations and the same theoretical group velocity we use in this paper, Zhao et al. (2018) found the observed phase speed from satellite altimetry is at most 5% higher than the theoretical prediction (see their Fig. 8). This implies a shorter travel time by 5%, equivalent to 11 h. As argued by Zhao et al. (2018), this difference may be attributed to effects not included in linear theory or the different temporal coverages of the data in the altimetry record and climatology. Regardless of the dominant effect, the difference in travel time alone does not explain the ∼2-day lag in Fig. 15.

The altimetry-based energy flux of the westward propagating internal tide highlights the geographical locations where ray paths may be taken for appropriate travel time estimates (Fig. A1a). We chose several ray paths that end at southeastern Tasmania, but start at different locations along Macquarie Ridge (Fig. A1b). An additional ray path was taken following the middle of the internal tide beam and bending westward around the East Tasman Plateau, consistent with refraction by the topography. From the group velocity taken on the ray paths (Fig. A1c), we calculated a range of travel time estimates (Fig. A1d).

Due to relatively large-scale gradients of the climatological group velocity, the range of travel time estimates is less than 5 h and different choices of ray paths do not explain the observed timing of the spring–neap cycle on the slope. Even when scaling the climatological group velocity throughout the Tasman sea, such that it matches with the value obtained from moored observations at the TBEAM site, the difference in travel time is only 8 h.

Given all these travel time estimates, the value of 9.25 days used in Fig. 15 may be biased by about half a day at most. Therefore, this range of theoretical estimates is relatively short and cannot explain the 2-day lag between theoretical and observed timings of the spring–neap cycle.

APPENDIX B

Four-Wave Interference with Equal Amplitudes: Analytical Calculation

The form of the envelope of the four-wave interference (5) is given by the energy-like quantity \( \langle y^2 \rangle(x, t) \). Angle brackets denote averaging over a wave period, i.e.,

\[
\langle y^2 \rangle(x, t) = \frac{\omega}{2\pi} \int_{T/2}^{T/2} y^2(x, t') \, dt',
\]

(B1)

where \( \omega \) is the semidiurnal frequency and \( T = 2\pi/\omega \) is the equivalent period. The square of (6) is

\[
\frac{1}{4} y^2 = \cos^2(kx - \omega t) \cos^2(\delta kx - \delta \omega t) + \cos^2(\delta \omega t) \cos^2(\omega t) + \cos^2(kx - \omega t) \cos^2(\delta kx - \delta \omega t) \cos(\delta \omega t) \cos(\omega t).
\]

(B2)

We now phase average each term of (B2). Since \( \delta \omega \omega \ll 1 \), terms with frequency \( \delta \omega \) are approximately constant over a semidiurnal period. With this approximation, the average of term I is
The spring–neap phase in Fig. 16 is calculated from

\[ y(x) = \cos(kx - \omega t) \cos(\delta kx - \delta \omega t) \]

Similarly, the average of term II is

\[ \langle II \rangle = \frac{1}{2} \cos^2(\delta \omega t). \]  

(B4)

The average of the third term is

\[ \langle III \rangle = 2 \cos(\delta \omega t) \cos(\delta kx - \delta \omega t) \cos(kx - \omega t) \]

Therefore, \( y^3 \) is given by

\[ \frac{1}{4} (y^3)(x, t) = \frac{1}{2} \cos^2(\delta kx - \delta \omega t) + \frac{1}{2} \cos^2(\delta \omega t) \]

\[ + \cos(kx) \cos(\delta \omega t) \cos(\delta kx - \delta \omega t). \]  

(B6)

\[ \frac{1}{2} \cos^2(\delta kx - \delta \omega t) + \frac{1}{2} \cos^2(\delta \omega t) = \frac{1}{2} \left[ \frac{1}{2} \cos(2\delta kx - 2\delta \omega t) + \frac{1}{2} \cos(2\delta \omega t) + 1 \right] \]

\[ = \frac{1}{2} \left[ \cos \left( \frac{1}{2} (2\delta kx) \right) \cos \left( \frac{1}{2} (2\delta kx - 4\delta \omega t) \right) + 1 \right] \]

\[ = \frac{1}{2} \left[ \cos(\delta kx) \cos(\delta kx - 2\delta \omega t) + 1 \right]. \]  

(B7)

and the last term as

\[ \cos(kx) \cos(\delta \omega t) \cos(\delta kx - \delta \omega t) = \frac{1}{2} \cos(kx) \left[ \cos(\delta \omega t - \delta kx + \delta \omega t) + \cos(\delta \omega t + \delta kx - \delta \omega t) \right] \]

\[ = \frac{1}{2} \cos(kx) \left[ \cos(\delta kx - 2\delta \omega t) + \cos(\delta kx) \right]. \]  

(B8)

Substituting (B7) and (B8) into (B6) gives

\[ \frac{1}{2} (y^3)(x, t) = 1 + \cos(\delta kx) \cos(kx) + \cos(\delta kx - 2\delta \omega t) \]

\[ \times [\cos(\delta kx) + \cos(kx)]. \]  

(B9)

**APPENDIX C**

**Four-Wave Interference with Arbitrary Amplitudes: Numerical Calculations**

The spring–neap phase in Fig. 16 is calculated from

\[ y(t) = A \cos(k_{M2} L - \omega_{M2} t) + B \cos(-\omega_{M2} t) \]

\[ + C \cos(k_{S2} L - \omega_{S2} t) + D \cos(-\omega_{S2} t), \]  

(C1)

which is similar to (5), but with arbitrary amplitudes. In the idealized calculation, the remote wave has fixed amplitude and, since the \( M_2 \) tide is often much more energetic than the \( S_2 \), we set \( A = 0.9 \) and \( C = 0.1 \). For simplicity, we set \( B/D = A/C \), for \( B \) and \( D \) greater than 0. The spring–neap cycle is calculated for \( 0 \leq (B + D) < 1 \) (i.e., the \( y \) axis in Fig. 16b). Results are also presented for \( 0 \leq L \leq 2250 \, \text{km} \), where the upper bound is the periodicity of the results (see Fig. 16a) and \( L \approx 1400 \, \text{km} \) is analogous to the distance between Macquarie Ridge and Tasmania.

The mode-1 horizontal wavenumbers (\( k_{M2} \) and \( k_{S2} \)) for the respective semidiurnal tidal frequencies are calculated from the dispersion relationship.
\[ \omega^2 = \left( \frac{NH}{\pi} \right)^2 k^2 + f^2, \]  

(C2)

where \( H \) is the bottom depth, \( f \) the Coriolis parameter and \( N \) the buoyancy frequency. Equation (C2) assumes constant stratification and a rigid lid approximation (e.g., Wunsch 1975). We use parameters appropriate for the Tasman Sea: \( N = 2 \times 10^{-5} \text{s}^{-1} \) (i.e., the depth average from climatology; Boyer et al. 2013), \( H = 4000 \text{m} \) and \( f \approx 1.03 \times 10^{-5} \text{s}^{-1} \) (correspondent to a latitude of 45°S, where the sign is irrelevant). The implied group velocity (i.e., \( c_g = \omega / k \)) is 1.7 m s\(^{-1}\), which corresponds to the average value across the Tasman Sea along the path of the internal tide.

For a range of values of \((B + D)\) and \(L\), we then calculate \(\langle y^2 \rangle(t)\), where the angle brackets denote semidiurnal phase averaging, and find the time of maximum variance. We define this time as the spring–neap phase.

Note that when local and remote waves have the same magnitude \((A = B \text{ and } B + D = 1)\), there are values of \(L\) for which there is perfect cancellation at each frequency and the spring–neap phase is not defined. These are the same distances for which the derivative of spring–neap phase with respect to \(L\) is maximum in the case where \(B + D < 1\) (Fig. 16a).

REFERENCES


Boettger, D., R. Robertson, and L. Rainville, 2015: Characterizing the semidiurnal internal tide off Tasmania using glider data.


