ABSTRACT: Intraseasonal waves in the tropical Atlantic Ocean have been found to carry prominent energy that affects interannual variability of zonal currents. This study investigates energy transfer and interaction of wind-driven intraseasonal waves using single-layer model experiments. Three sets of wind stress forcing at intraseasonal periods of around 30, 50, and 80 days with a realistic horizontal distribution are employed separately to excite the second baroclinic mode in the tropical Atlantic. A unified scheme for calculating the energy flux, previously approximated and used for the diagnosis of annual Kelvin and Rossby waves, is utilized in the present study in its original form for intraseasonal waves. Zonal velocity anomalies by Kelvin waves dominate the 80-day scenario. Meridional velocity anomalies by Yanai waves dominate the 30-day scenario. In the 50-day scenario, the two waves have comparable magnitudes. The horizontal distribution of wave energy flux is revealed. In the 30- and 50-day scenarios, a zonally alternating distribution of cross-equatorial wave energy flux is found. By checking an analytical solution excluding Kelvin waves, we confirm that the cross-equatorial flux is caused by the meridional transport of geopotential at the equator. This is attributed to the combination of Kelvin and Yanai waves and leads to the asymmetric distribution of wave energy in the central basin. Coastally trapped Kelvin waves along the African coast are identified by alongshore energy flux. In the north, the bend of the Guinea coast leads the flux back to the equatorial basin. In the south, the Kelvin waves strengthened by local wind transfer the energy from the equatorial to Angolan regions.

KEYWORDS: Inertia-gravity waves; Kelvin waves; Planetary waves; Waves, oceanic; Intraseasonal variability

1. Introduction

Significant intraseasonal (10–90 day period band) variance in currents, sea levels and sea surface temperatures (SST) has been observed in the tropical Atlantic in the last century (Düing et al. 1975; Houghton and Colin 1987; Weisberg 1984; Weisberg and Weingartner 1988; Katz 1997). In the equatorial Atlantic, evidence has been found that the intraseasonal variability and the associated equatorial waves are crucial for the maintenance of zonal currents. They are closely linked to the interannual variability of equatorial deep jets (Greatbatch et al. 2018; Perez et al. 2012; Wu and Bowman 2007), which is able to transfer the energy upward, affect the SST pattern and eventually feed back to the atmosphere (Brandt et al. 2011). In the southeastern Atlantic, extratropical regions, intraseasonal variability also has its remote influence on upwellings and currents through coastally trapped waves, which is believed to be an important source of the El Niño–like climate mode in the Benguela upwelling region (Illig and Bachéley 2019; Richter et al. 2010). Hence, in recent years, increasing concerns are raised in the tropical Atlantic, on the energy transfer of those intraseasonal waves.

The intraseasonal variability of currents, sea levels, and SSTs in the equatorial Atlantic Ocean is excited by either wind forcing (Athie and Marin 2008; Polo et al. 2008) or instability of zonal currents and SST fronts (Yu et al. 1995; Grodsky et al. 2005). At periods longer than 40 days, wind-driven equatorial Kelvin waves are found to be the most prominent intraseasonal signal of zonal velocity in the equatorial Atlantic Ocean (Han et al. 2008; Polo et al. 2008; Brandt et al. 2006). For shorter periods (10–40 days), tropical instability waves (TIWs) attain their maximum power density, prone to transfer their energy into Yanai waves (Han et al. 2008; Tuchen et al. 2018; Athie and Marin 2008). At quasi-biweekly periods, wind-driven Yanai waves are also prominent in the eastern Atlantic basin (Garzoli 1987; Bunge et al. 2007).

Previous research has used equatorial basin mode (EBM) by Matsuno (1966) and McCreary (1985) to explain those intraseasonal waves including the wind-driven equatorial waves and TIWs in the Atlantic and other oceans. Based on observations, Bunge et al. (2007) detected a quasi-biweekly signal of meridional velocity in the equatorial Atlantic and regarded it as wind-driven Yanai waves for the first baroclinic mode based on the phase speed. Polo et al. (2008) highlighted the propagation of intraseasonal waves within the 25–90 days period band in the tropical Atlantic by filtering altimeter data and concluded that equatorial and coastally trapped Kelvin waves of the first two baroclinic modes dominate. Tuchen et al. (2018) utilized the full-depth moored velocity data at 23°W of the equator and found the peak energy of Yanai waves in the 30–40 days period band from the second to fifth baroclinic mode. Miyama et al. (2006) used linear, continuously stratified (LCS) models for the first to tenth baroclinic modes in the Indian Ocean with idealized wind forcing at biweekly period, finding that the largest oceanic response is restricted to biweekly Yanai waves in the presence of vertical mixing. Han et al. (2008) used both an ocean general circulation model (OGCM) and an LCS model...
to reveal the role of wind-driven equatorial waves in the tropical Atlantic. They found that Kelvin waves dominate sea surface height anomaly (SSHA) and thermocline variability in 40–60 days period band, and in the east basin, wind-driven waves dominate even in shorter 10–40 days period band. Greatbatch et al. (2018) added the meridional flux of intra-seasonal zonal momentum to an LCS model associated with a high baroclinic mode (around the fifteenth) designed to support deep jets, showing the evidence that the intraseasonal variability can maintain the zonal jets. As featured zonally propagating signals of currents, sea levels and temperatures by wind-driven Kelvin and Rossby waves are present in equatorial basins, the influence of meridional variances on them is still unclear. In particular, energy transfer issues have been little addressed in the combined event of Kelvin and Yanai waves, since traditional ray tracing through Wentzel–Kramers–Brillouin (WKB) approximation (Philander 1978; Schopf et al. 1981) can be applied only in the presence of a single type of waves. Aiki et al. (2017, hereafter AGC17) have proposed a general expression for the horizontal component of the group-velocity-based energy flux for model diagnostic. As the scheme will not be bothered by the selection of dispersion relations and propagating latitudes of waves, it has been applied in the tropical Atlantic, Indian, and Pacific Oceans (Song and Aiki 2020; Li and Aiki 2020; Toyoda et al. 2021) for annual and semiannual variabilities. Song and Aiki (2020) presented a complete annual cycle of wave energy due to both wind-driven Kelvin and reflection-generated Rossby waves in the equatorial Atlantic by exciting the gravest three baroclinic modes with monthly climatological wind forcing in a linear shallow water model in the tropical Atlantic. Nevertheless, the studied wave is limited in seasonal periods, and they have utilized what is called level-2 (i.e., approximated) version of AGC scheme, abandoning a time-variance term with assuming low-frequency waves.

This study hence adopts another version, which is called level-0 (i.e., exact) from AGC17 with full time-variance terms, previously omitted, to address wave energy transfer problems of wind-driven intraseasonal waves in the tropical Atlantic, especially in the presence of notable meridional velocity variability. The structure of this manuscript is as follows: section 2 explains numerical experiments including the employed wind forcing, numerical model, and implementation of the AGC level-0 scheme; section 3 analyses the simulated waves in the view of wave energy, discusses the influence of meridional velocity anomaly on the zonally propagating signals, and identifies coastally trapped waves along the African coast; and section 4 summarizes the addressed problems and conclusions of this study.

2. Methodology

a. Intraseasonal wind forcing

Kelvin and Yanai waves are thought as the dominant wind-driven intraseasonal equatorial waves. The propagation of Yanai waves acts close to Rossby waves at low frequencies and to Kelvin waves at high frequencies, subject to the dispersion relation for certain baroclinic mode following the theoretical solution of equatorial waves (Matsumoto 1966). Hence, our numerical experiments in the tropical Atlantic were separately forced by winds at different intraseasonal frequencies to investigate corresponding wave features.

Daily wind stress data utilized in this study are from ERA-40 dataset provided by European Center of Medium-Range Weather Forecasts (ECMWF). The results of Fourier analysis for variances of both zonal and meridional wind stress (with the original data from January 1993 to June 1994 covering 512 days after high-pass filter of 90 days) in intraseasonal period band (from 10 to 90 days) are shown as Fig. 1. Three peaks of the variance appear at periods of 20, 36 and 51 days for the zonal component and two peaks appear at 85 and 36 days for the meridional component. Hereafter, for convenience, the peak periods are referred to as 80D for 85 days, 50D for 51 days, 30D for 36 days and 20D for 20 days. Although the quasi-biweekly meridional wind stress can directly cause the first-mode Yanai wave in the eastern tropical Atlantic (Garzoli 1987), it is noted from Fig. 1 that the variance of both meridional and zonal stress components at 20D period is about one order of magnitude smaller than variance at periods longer than 30 days.

The variance of wind stress components is capable of generating multiple types of intraseasonal waves in the tropical region. Zonal wind stress $\tau^x$ can excite Kelvin waves, while Yanai waves are thought to be induced by meridional wind stress $\tau^y$ and meridional gradient of zonal wind stress $\partial\sigma^x/\beta$ (Miyama et al. 2006). The horizontal distribution for the variance of zonal and meridional wind stress at the four intraseasonal periods, mentioned above, is hence checked along the equator (Fig. 2). Meridional wind stress is revealed to be stronger than zonal wind stress in the 80D scenario (Fig. 2a), whereas in 20D, 30D, and 50D scenarios, zonal wind stress dominates the western basin (west of 30°W) (Figs. 2b–d). It is
also seen that, the zonal wind stress presents a decline trend from west to east, while the meridional wind stress reveals a two-peak structure and even stronger in the eastern basin for 50D and 80D scenarios. From Fig. 2, the meridional gradient of zonal wind in the tropical Atlantic may not be a considerable energy source for Yanai waves as in the Indian Ocean (Miyama et al. 2006).

b. Numerical experiments

To investigate the wave energy transfer concerning different types of intraseasonal waves, it is straightforward to excite one certain baroclinic mode by the wind stress of the above peak periods independently. As the evidence of prominent intraseasonal signals by Kelvin and Yanai waves in the first and second baroclinic modes has been found in the tropical Atlantic (Polo et al. 2008; Han et al. 1999), we checked the theoretical dispersion relation of equatorial waves by Matsuno (1966), shown as Fig. 3, where the intraseasonal wave periods have been labeled in the dimensionless dispersion curve for the first and second baroclinic modes. On the one hand, the second mode has been found to contain more wave energy at intraseasonal periods compared with the first mode through the whole basin (Illig and Bachêlery 2019). On the other hand, for the first mode, unless we apply the 20D wind forcing, which is rather weak as shown by Fig. 1, the long “Kelvin-wave-like” Yanai wave will not be present (Fig. 3a). Hence, the second baroclinic mode is more suitable for studying “different” Yanai waves, and we performed numerical experiments in a single-layer model for this mode following Song and Aiki (2020) with the governing equations as

\[
\begin{align*}
\frac{\partial u^{(n)}}{\partial t} - f v^{(n)} + \frac{\partial p^{(n)}}{\partial x} &= \frac{\tau^x}{H^{(n)} \rho_0} + D_x, \\
\frac{\partial u^{(n)}}{\partial t} + f u^{(n)} + \frac{\partial p^{(n)}}{\partial y} &= \frac{\tau^y}{H^{(n)} \rho_0} + D_y, \\
\frac{\partial p^{(n)}}{\partial t} + (c^{(n)})^2 \left( \frac{\partial u^{(n)}}{\partial x} + \frac{\partial u^{(n)}}{\partial y} \right) &= 0,
\end{align*}
\]

which has been written in Cartesian coordinates for simplicity, although the model is set up in spherical coordinates. The symbols \((u^{(n)}, v^{(n)})\) are the horizontal components of velocity and \(p^{(n)}\) is the geopotential for the \(n\)th baroclinic mode, specifically in this study \(n = 2\). The symbol \(D\) in Eq. (1) represents the horizontal eddy viscosity term estimated by Smagorinsky (1963) with a coefficient of 0.1 after squaring. The symbol \(c^{(n)}\)
in Eq. (1) is the phase speed of nonrotating gravity waves. The symbol $H^{(n)}$ in Eq. (1) has often been called the wind-coupling thickness, which represents the portion of wind forcing that projects into the $n$th baroclinic mode. We used the climatological annual mean vertical profiles of temperature and salinity data in the World Ocean Atlas (WOA), averaged in the area of 75°W–25°E and 40°S–40°N, to calculate $c^{(n)}$ and $H^{(n)}$ in the classical eigenvalue problem. Readers can refer to supporting information by Song and Aiki (2020) for details. As the experiments in this study are all carried out for the second baroclinic mode, $H^{(n)}$ is no longer necessary to indicate the contribution of wind forcing in different basin modes, but we still applied the same $H^{(n)}$ and $c^{(n)}$ for the second mode by Song and Aiki (2020) for consistency (see Table 1).

In our experiments, the latitude of the Atlantic basin ranges from 44°S to 72°N with coastlines given by 100-m isobath in the bathymetry data from the GEBCO (General Bathymetric Chart of the Oceans) dataset. The horizontal resolution in either the zonal or meridional direction is 0.25°. We reconstructed daily time series of monochromatic wind stress with the Fourier components at corresponding frequencies introduced in section 2a. Three numerical experiments for the second gravest baroclinic mode driven by the reconstructed wind forcing at 80D, 50D, and 30D periods are hence performed independently. Each of the experiments has been integrated for 5 years, including the spinup from the state of no motion. It has been confirmed that an equilibrium state is reached (not shown).

c. Level-0 energy flux scheme

An energy budget equation for shallow water waves in the presence of wind forcing and eddy viscosity reads

$$\frac{\partial E^{(n)}}{\partial t} + \nabla \cdot \left( \mathbf{V}^{(n)} p^{(n)} \right) = \frac{\mathbf{V}^{(n)} \cdot \mathbf{\tau}}{H^{(n)} \rho_0} + \mathbf{D} \cdot \mathbf{V}^{(n)},$$

(2)

where $E^{(n)} = (1/2)[(u^{(n)})^2 + (v^{(n)})^2 + (p^{(n)} c^{(n)})^2]$ is the sum of kinetic and gravitational energies, $\mathbf{V}^{(n)} = (u^{(n)}, v^{(n)})$ and $\mathbf{D} = (D_x, D_y)$ are the velocity and viscosity force vectors, respectively, $\mathbf{V}^{(n)} \cdot \mathbf{\tau} = (u^{(n)} \tau_x + v^{(n)} \tau_y)$ is the wind forcing power, and $\mathbf{D} \cdot \mathbf{V}^{(n)}$ is the eddy viscosity stress power. The overline symbol in Eq. (2) is a phase average operator provided in AGC17. For inertia gravity waves and Kelvin waves, the pressure flux $\mathbf{V}^{(n)} p^{(n)}$ is equal to group velocity times wave energy. However, for Rossby waves or Yanai waves, $\mathbf{V}^{(n)} p^{(n)}$ is no longer parallel to the group velocity. AGC17 hence obtained a general expression for the group-velocity-based energy flux for waves at all latitudes to read,

$$c_E E = \mathbf{V}^{(n)} p^{(n)} + \nabla \times \left( p^{(n)} \mathbf{\varphi}^{(n)} / 2 + u^{(n)} v^{(n)} / \mathbf{\varphi}^{(n)} \right) \mathbf{\varphi},$$

(3)

which includes a scalar quantity $\varphi$. This quantity is defined as

$$\Delta \varphi^{(n)} - (\mathbf{\varphi}^{(n)} \cdot \mathbf{\varphi}^{(n)}) \varphi^{(n)} - \left( \sqrt{3} \mathbf{\varphi}^{(n)} \cdot \mathbf{\varphi}^{(n)} \right)^2 \varphi^{(n)} = \varphi^{(n)},$$

(4)

where $\varphi^{(n)} = (\mathbf{\varphi}^{(n)} / \mathbf{\varphi}^{(n)}) - (\mathbf{\varphi}^{(n)} / \mathbf{\varphi}^{(n)}) - [\mathbf{\varphi}^{(n)} / \mathbf{\varphi}^{(n)}]^2$ is the potential vorticity. The rotational flux in Eq. (3) can redirect the vector $\mathbf{V}^{(n)} p^{(n)}$ to head in the direction of group velocity for different types of waves in tropical basins. Equation (3) was named as the level-0 expression in AGC17 (hereafter referred to as AGC-L0 energy flux). In previous studies, for wave oscillation far slower than the equatorial inertial frequency, a reasonable simplification is to remove the second-order time derivative in both Eqs. (3) and (4) (AGC17). Then Eq. (4)

**TABLE 1.** List of wave constants associated with the gravest second baroclinic mode and the numerical model for the tropical Atlantic Ocean. The equatorial deformation radius follows $R_{edd} = \sqrt{c^{(n)}} H$ and the equatorial basin mode period follows $T_{eob} = 4L_{eob} / c^{(n)}$.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth</td>
<td>$H_b$</td>
<td>4500 km</td>
</tr>
<tr>
<td>Basin length</td>
<td>$L_{eob}$</td>
<td>6111 km</td>
</tr>
<tr>
<td>Baroclinic mode number</td>
<td>$n$</td>
<td>2</td>
</tr>
<tr>
<td>Wind-coupling thickness</td>
<td>$H$</td>
<td>911 km</td>
</tr>
<tr>
<td>Nonrotating gravity wave speed</td>
<td>$c$</td>
<td>1.21 m s$^{-1}$</td>
</tr>
<tr>
<td>Equatorial deformation radius</td>
<td>$R_{edd}$</td>
<td>230 km</td>
</tr>
<tr>
<td>Equatorial basin mode period</td>
<td>$T_{eob}$</td>
<td>234 days</td>
</tr>
</tbody>
</table>
FIG. 4. Zonal wavenumber–frequency spectrum of zonal velocity anomaly at the equator in the scenarios driven by the wind forcing of (a) 30D, (b) 50D, and (c) 80D periods associated with the second baroclinic mode. The same nondimensionalization as in Fig. 3 has been performed in axes; dashed black line indicates the theoretical dispersion relation of equatorial waves. The spectrum density (color shading) has been normalized with the peak density for both axes in the $\omega-k$ domain.

FIG. 5. As in Fig. 4, but for meridional velocity anomaly; $\overline{S_p}$ is the averaged spectral density of meridional velocity anomaly over the $\omega-k$ domain and $\overline{S_u}$ is that of zonal velocity anomaly.
becomes a diagnostic equation by which \((q^{(n)})^{\text{app}}\) can be obtained directly from \(q^{(n)}\). The energy flux in Eq. (3) is hence approximated by

\[
c_{g}\mathcal{E} \approx \nabla \cdot \mathbf{p}^{(n)} + \nabla \times \left[ \mu^{(n)} \left( \nabla \times q^{(n)} \right)^{\text{app}}/2 \right],
\]

which has been referred to as level-2 expression in AGC17 (hereafter referred to as AGC-L2 energy flux). Readers can refer to AGC17 for more details. The above simplification is inappropriate in this study, as the revelation of energy flux for intraseasonal waves expects the solution of Eq. (3) in the presence of the time derivative term. However, directly employing an elliptic partial differential equation solver, such as BiCGSTAB (biconjugate gradient stabilized method), to solve Eq. (4) leads to nonconvergence. We therefore applied Fourier transform to \(q^{(n)}\) and \(u^{(n)}\) as

\[
q^{(n)}_{\omega} = \frac{1}{\sqrt{2\pi}} \int q^{(n)}(t) e^{-i\omega t} dt,
\]

\[
\varphi^{(n)}_{\omega} = \frac{1}{\sqrt{2\pi}} \int \varphi^{(n)}(t) e^{-i\omega t} dt.
\]

Substituting Eq. (6) to Eq. (4), at certain frequency \(\omega\), after rearrangement, it reads

\[
\Delta \varphi^{(n)}_{\omega} + (3\omega^2 - f^2)(\varphi^{(n)}_{\omega}) = \varphi^{(n)}_{\omega}.
\]

It is obvious that, as long as \(3\omega > f\) holds, Eq. (7) becomes the elliptic partial differential equation which can be solved by BiCGSTAB certainly. We can apply inverse Fourier transform to solution \(\varphi^{(n)}_{\omega}\) of Eq. (7) to yield \(\varphi^{(n)}\) as

\[
\varphi^{(n)} = \frac{1}{\sqrt{2\pi}} \int \varphi^{(n)}_{\omega} e^{i\omega t} d\omega.
\]

Then the AGC-L0 flux can be determined by using the obtained \(\varphi^{(n)}\) for each experiment. Practically, fast Fourier transform (FFT) algorithm with a data length of 128 covering the final year for both \(q^{(n)}\) and \(\varphi^{(n)}\) is applied in the study.

3. Results

a. Intraseasonal waves

We first investigated the simulated velocity anomaly in the tropical Atlantic to identify intraseasonal waves. At the
equator, a two dimensional FFT algorithm is employed with 128 horizontal grids from 25°W to 6.75°E to yield frequency–wavenumber spectra of the excited waves under the three groups (30D, 50D, and 80D) of wind stresses (see Figs. 4 and 5). As the realistic distribution of wind forcing is applied in the experiments, the band of wavenumber is broad; hence different types of waves including Kelvin, Yanai, and Rossby waves are possible to coexist. By seeing Figs. 4 and 5, the simulated results agree well with the theoretical solutions. For the zonal velocity $u^{(n)}$, Kelvin waves are dominant in all the three scenarios (Fig. 4).

For the meridional velocity $v^{(n)}$, it would be the Yanai wave with westward phase speed that dominates in the 30D and 50D scenarios, which is short in the 50D scenario and long in the 30D scenario, subject to the curve of dispersion relation. In the 80D scenario, Rossby waves in the second and fourth meridional mode dominate the variability of $v^{(n)}$, and one harmonic wave at the period of around 40 days acting as weak and high-frequency Yanai waves is detected as well (Fig. 5).

The results reveal that, by resonance mechanism, higher-frequency variability of meridional wind component is easier to excite meridional velocity anomaly. As in the 30D scenario, the variance of $v^{(n)}$ is one order of magnitude larger than $u^{(n)}$, while the meridional wind forcing is not (Fig. 1 and Figs. 6a,b). However, in the 80D scenario, the variance of $v^{(n)}$ is one order of magnitude smaller even though the meridional wind forcing is greater than the zonal (Fig. 1 and Figs. 6e,f). It is noted that the coexistence of the Yanai wave and the Kelvin wave in the 30D and 50D scenario provides an opportunity to study their interaction.

b. Horizontal energy flux

The FFT algorithm is applied to 128 snapshots covering one year from each numerical experiment. The yielded $q_w$ is used to calculate AGC-L0 energy flux.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Phase speed (m s$^{-1}$)</th>
<th>Group velocity (m s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelvin wave ($m = -1$)</td>
<td>1.21</td>
<td>1.21</td>
</tr>
<tr>
<td>Yanai wave ($m = 0$, 30D)</td>
<td>0.32</td>
<td>0.71</td>
</tr>
<tr>
<td>Yanai wave ($m = 0$, 50D)</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>Long Rossby wave ($m = 1$, 80D)</td>
<td>0.36</td>
<td>0.29</td>
</tr>
<tr>
<td>Long Rossby wave ($m = 2$, 80D)</td>
<td>0.21</td>
<td>0.14</td>
</tr>
<tr>
<td>Long Rossby wave ($m = 3$, 80D)</td>
<td>0.13</td>
<td>0.07</td>
</tr>
<tr>
<td>Long Rossby wave ($m = 4$, 80D)</td>
<td>0.09</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**TABLE 2.** Table of theoretical phase speed and group velocity for equatorial waves in the second baroclinic mode at the selected periods.

![Fig. 7](image_url)
The obtained horizontal distribution of cycle-mean AGC-L0 flux is presented in Fig. 6. In the 30D and 50D wind forcing scenarios, strong eastward energy fluxes dominate the whole basin, while southward and northward fluxes are alternately distributed (Figs. 6a–d). In the 80D wind forcing scenario, the eastward energy flux becomes strong from the central basin, while the meridional AGC-L0 flux is rather weak and irregularly distributed in the presence of both Rossby and Yanai waves (Figs. 6e,f). Since strong Kelvin waves are found in the experiments (Fig. 4), the eastward energy propagation is as expected. Also, the strength of Kelvin waves is affected by the horizontal distribution of zonal wind stress. Namely, as the increase of zonal wind in the 80D scenario starts from 35°W rather than 50°W in the other two scenarios (see Figs. 2a–c), the corresponding zonal flux by Kelvin waves is relatively weak in the western basin (Fig. 6e). However, the equatorial Kelvin wave, whose group velocity is eastward, is not able to transfer the energy meridionally, implying that the alternate structure of meridional flux should be associated with Yanai waves.

It is noted that the phase speed of Yanai waves (0.12 m s\(^{-1}\) for 50D and 0.71 m s\(^{-1}\) for 30D in the second mode, see Table 2) is less than that of Kelvin waves (1.21 m s\(^{-1}\) in the second mode), which gives rise to the concern on the time evolution of energy transfer in this basin. Thus we investigated the time evolution of the zonal and meridional components of the AGC-L0 flux in the 30D and 50D scenarios at the equator (see Fig. 7). The zonal energy transfer route (propagation of wave packet by wave group velocity) subjects to the Kelvin waves, taking around 50 days to pass the basin (Figs. 7a,c). Unexpectedly, from Figs. 7b,d, the zonal propagation of the meridional flux structure also subjects to the group velocity of Kelvin waves rather than Yanai waves (see Table 2). Furthermore, the phase evolution of meridional flux is detected as well (see blue solid line in Figs. 7b,d). The alternate structure of meridional flux appears to evolve slowly with the phase of Yanai waves, but the sign of meridional flux at certain latitudes is nearly fixed during wave events (see Figs. 7b,d).

The asymmetric distribution of zonal AGC-L0 flux with respect to the equator should be noted as well. Indeed, inspecting the AGC-L2 flux (see Fig. 8) which has a relatively weaker meridional component, this asymmetric structure becomes less observable. This fact implies that the nearly stationary alternate structure of meridional flux may spatially modulate the zonal energy transfer contributing to the asymmetric distribution of wave energy.

c. Yanai waves and cross-equatorial flux

It is uncertain from the above analysis that the meridional component of AGC-L0 flux is caused by Yanai waves. To confirm and evaluate the influence of the Yanai wave on the...
meridional flux, we investigated the 3D wind forcing scenario in which the magnitude of meridional velocity anomaly is one order larger than the zonal velocity anomaly implying more observable Yanai waves (see the magnitude of velocity anomaly in Fig. 5).

The general analytic solution of \( \phi^{(o)} \) derived by AGC17 in dimensionless form (indicated by the tilde symbol) at a certain frequency is as follows:

\[
\tilde{\phi} = \tilde{v}_y / (\tilde{k} + 2\tilde{\omega}) \tag{9}
\]

where \( \theta = \tilde{k}x - \tilde{\omega}t \) is the wave phase and \( \tilde{v}_y = \tilde{\omega} \tilde{\phi} / \theta \). The theoretical solution for the Yanai wave by Matsuno (1966) is as follows:

\[
\begin{align*}
\tilde{v} &= \exp \left[ -\frac{1}{2} (\tilde{y})^2 \right] \sin \theta, \\
\tilde{u} &= \frac{\tilde{v}}{\tilde{\omega} - \tilde{k}} \exp \left( -\frac{1}{2} \tilde{y}^2 \right) \cos \theta, \\
\tilde{p} &= \frac{\tilde{v}}{\tilde{\omega} - \tilde{k}} \exp \left( -\frac{1}{2} \tilde{y}^2 \right) \cos \theta.
\end{align*}
\tag{10}
\]

We have obtained the analytic AGC-L0 flux by substituting Eq. (10) to Eq. (9) (as shown in Fig. 9).

In Fig. 9, the alternate structure of cross-equatorial flux does not appear in the analytic AGC-L0 flux; however, the meridional velocity itself presents the structure. This is because the geopotential anomaly by Yanai waves is actually zero at the equator. We hence suggest that the cross-equatorial meridional flux may originate from the Kelvin wave rather than from the Yanai wave. When the Kelvin wave with strong geopotential at the equator propagates to the east, it will be distorted by the meridional velocity associated with the Yanai wave producing considerable meridional energy flux. It explains why the propagation of the structure subjects to the Kelvin wave and meanwhile the phase evolution of the Yanai wave is presented in a Kelvin-wave ray (Figs. 7b,d). To confirm the speculation, we performed an additional experiment, in which zonal wind stress in the 30D scenario has been eliminated to prevent prominent Kelvin waves (see Figs. 10a,b). With no zonal wind forcing, even the zonal velocity anomaly is mostly caused by the Yanai wave (the Yanai wave still shows a small amplitude in zonal velocity anomaly because the region used for FFT is not exactly at the equator but at 0.25°N to fit the computational grids and to avoid the island). The new distribution of meridional flux is shown as Fig. 10c, where the cross-equatorial flux disappears in the central basin and the pattern approximates the analytical meridional flux of pure Yanai waves (see Fig. 9). As it is also impossible to generate this cross-equatorial flux without the meridional velocity anomaly of Yanai waves certainly, the flux should be caused by the coexistence of Yanai and Kelvin waves rather than by the Yanai wave or the Kelvin wave individually.

We then focused on the influence of the approximately stationary structure of the cross-equatorial distribution of wave energy and zonal energy flux. The 50D scenario is selected because the energy of the Yanai wave and the Kelvin wave in this scenario is of the same order of magnitude. A central equatorial region from 29° to 21°W covering two pairs of the alternate meridional flux is shown in Fig. 11. The asymmetric distribution of \( E \) with respect to the equator corresponds to an alternate flux pair (a northward and a southward ray), confirming the influence of the cross-equatorial flux and the Yanai wave on the distribution. Once the cross-equatorial meridional flux is triggered, the zonal energy flux will be adjusted as well (see the contour in Fig. 11b). The zonal velocity anomaly by the Yanai wave serves as an extra zonal energy flux causing a convergence (divergence) of wave energy in the north (south) of the equator. Therefore, as long as the mean meridional flux over a background wave cycle is nonzero, the cross-equatorial asymmetric distribution of wave energy would appear. In the 50D scenario, the sign of the meridional flux at a certain longitude is nearly fixed owing to the fast Kelvin wave and the
slow Yanai wave (see Table 2 and Figs. 7c,d). The asymmetry is hence notable.

d. Coastally trapped Kelvin waves

In the eastern basin (to the east of 10°W), the tilting coastline of Africa can lead the coastally trapped waves returning to the equatorial region (Song and Aiki 2020; Clarke 1983). In addition, land coverage will cause an asymmetric wind field as well. Thus, apart from the cross-equatorial meridional flux, the horizontal boundary becomes rather important to distort the wave energy transfer in the eastern basin. To investigate the coastal influence, we hence focused on the 80D scenario, for which the AGC-L0 flux in the coastal region is relative prominent compared to the meridional flux in central basin (as shown in Fig. 6f).

The distribution of $E$ manifests a peak region at the equator from 2°W to 2°E (which is slightly leaning toward the northern coastline) where the local wind forcing power is negative (see Fig. 12a). North of this region nearby the Gulf of Guinea, convergent fluxes are found: one is a southwestward flux from the coastally trapped wave led by the slant coastline; the other is the eastward energy flux from the central basin (see black arrows in Fig. 12b). The convergence causes the entrance of wave energy from the north to the peak region leading to the high wave energy there. It suggests that the invasion of coastally trapped waves is crucial for the distribution of wave energy in this region. However, the invasion is also affected by the cross-equatorial meridional flux (alternating structure along the equator). In the proximity of the peak region, the meridional flux encourages the energy returning to the equatorial area, but surrounding the cape in the west around 8°W, it suppresses the returning oppositely (see the gray arrows in Fig. 12b). Islands in the Gulf of Guinea deviate the energy flux from the zonal direction affecting the development of coastally trapped waves (as shown in Fig. 12). Hence we now compare the impacts of local power from wind forcing and energy...
transfer from equatorial waves on the coastally trapped waves by checking the tangential AGC-L0 flux along the African coastline.

We separated the coastline into two sections (C1 and C2 in Fig. 12) and projected the AGC-L0 flux to the tangential direction of each section (see Fig. 13). From Fig. 13, along the sections C1 and C2, the group velocity of the wave is around 1.2 m s\(^{-1}\) confirming the dominance of coastally trapped Kelvin wave. As Fig. 13a suggests, indeed only a small fraction of wave energy is transferred from the equator to both southward and northward coastally trapped waves. Instead, the coastal waves are notably strengthened from around 28\(^\circ\)S at section C1 and 7\(^\circ\)E at section C2, which confirms the deviation of energy flux by the islands (see Figs. 12a, b). At section C1, the power input of local wind forcing stays positive along the coast (see contours in Fig. 12a). While at section C2, the positive wind power is mainly found in the upwave slopes of the coastline corresponding to the southward group velocity, which suggests the meridional work by the local wind forcing (see contours in Fig. 12a). We also noted the difference of the alongshore flux in two phases at both sections. It may be attributed to the time evolution of meridional flux in the upwave region (see Fig. 12b), as in the eastern basin of the 80D scenario, the Yanai wave is generated locally and the above mentioned fixed sign of meridional flux does not remain.

4. Summary

This study utilized monochromatic wind variance at three typical intraseasonal frequencies to drive the second vertical baroclinic mode in the tropical Atlantic Ocean. Those numerical experiments were performed in a single-layer shallow water model. The intraseasonal variability induced by Rossby, Kelvin, and Yanai waves has been reproduced in the study basin. The AGC-L0 scheme for energy flux by AGC17 was applied to investigate wave energy transfers in the intraseasonal period band. The results reveal that Yanai waves featured by prominent meridional velocity anomaly at the equator are easier to be excited by higher-frequency wind forcing (the 50D and 30D scenario in this study) in the equatorial Atlantic. The transfer route of Kelvin waves distorts in the presence of cross-equatorial meridional flux caused by Yanai waves in such a way as to form the asymmetric distribution of wave energy with respect to the equator. The energy
flux also reveals the development of both southward and northward coastally trapped Kelvin waves along the coast of Africa and its feedback to the equatorial region in the eastern basin.

We have proved that the founded cross-equatorial flux requires the participation of both prominent Kelvin and Yanai waves. As the cross-equatorial flux is essentially the momentum transport due to inertia by meridional velocity of Yanai waves at the equator, the absence of strong meridional velocity anomaly (as in the 80D scenario) or zonally propagating geopotential anomaly (as in the no-Kelvin wave scenario) will vanish this flux. We also see that, as the consequence of the meridional flux, the zonal flux mainly caused by the Kelvin wave is distorted (see Figs. 6a,c). In that sense, these fluxes reveal the possible nonlinear interaction between Yanai and Kelvin waves, although the applied numerical model in this study is full linear.

The Yanai wave, by virtue of its prominent variability of meridional velocity, is important to the formation of meridional and vertical structures of zonal currents in the tropical Atlantic (d’Orgeville et al. 2007; Hua et al. 2008). The equatorward momentum transport caused by the Yanai wave is proved to be necessary in those formation mechanism (Ascani et al. 2010; Greatbatch et al. 2018). However, previous studies usually isolate the influence of Yanai waves, or focus on the interaction between intraseasonal Yanai waves and longer-period Kelvin waves (Hua et al. 2008; Ascani et al. 2015). For example, in the research by Ascani et al. (2010), they applied fictitious meridional wind forcing to yield Yanai waves merely. Although they found the dissipated Yanai wave is able to form the meridional structure of Equatorial Intermediate Current system, this mechanism is relative weak. As in real oceans, intraseasonal wind or SST instability normally excites both Kelvin and Yanai waves, the cross-equatorial flux presented here is appealing to concerns on the nonlinear interaction between the two types of intraseasonal waves. Nevertheless, to further study on the formation of equatorial deep jets, the AGC-L0 scheme is required to be promoted to the vertical direction.

The asymmetric structure of SST fluctuation with respect to the equator was reported in the tropical Atlantic (Athie and Marin 2008; Yu et al. 1995). Athie and Marin (2008) obtained a spatial pattern for SST in the intraseasonal band from 20 to 50 days with similar zonal distortions as the flux in the 30D
scenario (see Fig. 6a). Indeed their SST mode is dominated by TIWs. The TIW in tropical oceans has been proved having similar property as wind-driven Yanai waves nevertheless (Lyman et al. 2005; Tuchen et al. 2018). Hence, the cross-equatorial flux should be accounted for as another possible cause for the SST pattern, apart from the asymmetric source of instability in the two sides of the equator as pointed by Yu et al. (1995).

Owing to the applied AGC-L0 scheme, we have distinguished the alongshore energy flux of coastally trapped Kelvin waves for the first time. The equatorial triggering of the Benguela Ninö through coastally trapped waves has been noticed since Shannon et al. (1986), however, the influence from the equator versus the local wind is still unclear (Richter et al. 2010). The observation of the Angola Current near 11°S has proved that the remote equatorial effect can be reproduced by exciting EBM simply (Kopte et al. 2018). Illig and Bächler (2019) analyzed the contributions of waves in the first and second mode, respectively, and found that only the first-mode wave can affect the region south of the 15°S. As none of previous studies illustrate the connection of the equatorial and coastal waveguides, here we have presented a new diagnostic tool to reveal both the remote equatorial and local influence on the regional climate mode, Benguela Ninö, in the view of wave energy transfer when the interannual modulation of the intraseasonal wind forcing is applied for the first and second baroclinic mode in the future.

It should be also noted that, compared with the AGC-L2 scheme, the retainment of the time variance in the AGC-L0 scheme gives an extra dissipation term by 3/2(1/c0) in Eq. (7). With low frequencies, it becomes neglectable thereby making the level-0 scheme back to the level-2 scheme. However, for intraseasonal waves, the level-2 scheme becomes inappropriate. It underestimates the meridional flux and the coastal flux so as to make the distorted transfer route placid (see Fig. 8). The results suggest that, for the study concerning waves without rapidly directional change in energy transport such as long Rossby and equatorial Kelvin waves or their combined events in low-latitude regions, AGC-L2 scheme is applicable. Meanwhile the level-0 scheme is highly expected to be applied in studies on high-frequency waves or interactions between quadrature wave signals, for example, the remote effect of equatorial waves through coastally trapped waves and the energy transfer of Yanö waves in tropical oceans.

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