A Simple Topographic Model of Gulf Stream Separation

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ABSTRACT

A simple, frictional, linear model is used to study the motion of the Gulf Stream over the continental shelf. It is found that the combination of frictional and topographic effects may provide a further mechanism by which the observed separation of the Gulf Stream may be achieved.

The model predicts separation in the form of a classical separated boundary layer and interrelates the slope of the bottom with the position of separation. Counter-circulations northwest of the stream and increased northward transport of the current are also predicted.

1. Introduction

Since the early 1950's many oceanographers have studied the intense western boundary current of the Atlantic Ocean. It is now well known that it is due primarily to the change in local rotation rate of the earth as one moves toward the poles. However, little is known of the role topography may play in the determination of the path and intensity of the Gulf Stream, and in particular, whether the observed separation of the Gulf Stream from the coast is in any way related to the bottom topography.

Early analytical theories of wind-driven ocean circulation (e.g., Sverdrup, 1947; Stommel, 1948; Munk, 1950) were able to predict the gross features of North Atlantic currents and to establish rough theoretical estimates of the transport but these theories were only applicable to oceans of constant depth. Pedlosky (1968, 1969) and Johnson (1968) have since modified the early linear, vertically integrated transport models to include three-dimensional detail but again the results were independent of bottom topography and no attention was paid to Gulf Stream separation. However, these models did provide important information about the role of surface and bottom Ekman layers and Johnson (1968) was able to show that there was transport of fluid from the western boundary current into the adjacent northern gyre.

Greenspan (1962, 1963), Carrier and Robinson (1962), Robinson (1965) and Spiegel and Robinson (1968) have investigated the feasibility of using inertial currents and jets to model the Gulf Stream circulation. Gulf Stream separation was predicted by joining the inertial boundary layer to the interior flow. Parsons (1969) used an inertial two-layer model developed by Morgan (1956) to predict separation of the Gulf Stream. His prediction rests upon the hypothesis that the lower of the two layers will surface within the model ocean under certain wind stress conditions. The separated boundary current will then flow out from the coast on a course adjacent to the surfacing line. As in the inertial models above Parson's criterion for separation is independent of frictional and topographic effects. This model has been further developed by Kamenkovich and Reznik (1972) who have allowed flow in the lower layer. They find separation similar to that of Parsons but a counter-current, circulating in the same sense as the main current, now appears northwest of the separated stream which is in the form of a narrow jet.

Even though inertial models successfully predict separation of the western boundary current from the coast, several analytical and numerical studies suggest that topographic effects may provide a further mechanism for separation. Nonlinear theories suggest that in the absence of frictional and wind stress forces, potential vorticity $[(f+\zeta)/H]$ is conserved (where f is the Coriolis parameter, H the depth and ζ the relative vorticity), and lines of constant f/H are important in determining the path of flow (Welander, 1966; Stommel, 1965; Holland, 1967). This concept was further developed by Warren (1963) to explain the apparent correlation between Gulf Stream meanders and bottom topography. He computed Gulf Stream paths for known bottom topographies and his results agreed very well with known observations of meander patterns.

Bryan (1963), Holland (1967) Schulman and Niiler (1970) and Holland (1973) employed numerical techniques to investigate large-scale ocean circulations. Holland (1967), in particular, noted the importance of f/H contours in the determination of the Gulf Stream path, and he suggested that the western boundary layer may separate due to topographic influences. In a later paper Holland (1973) showed that

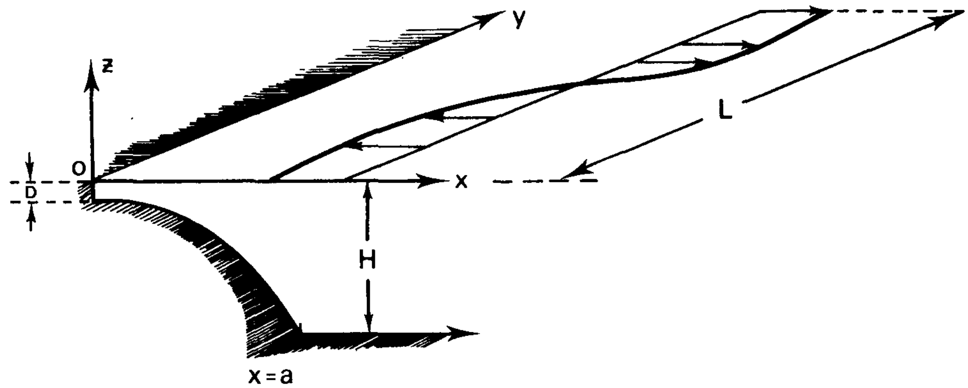

FIG. 1. The western boundary of the model ocean, showing the wind stress distribution.

the inclusion of both topographic and baroclinic effects leads to an increase in the transport of the Gulf Stream. By further reducing the grid size in the western boundary layer, Endoh (1973) was able to show that topography does lead to separation of the Gulf Stream. Endoh demonstrates that separation may be achieved solely through a vorticity balance between topographic, frictional and β-plane effects.

Few analytical solutions to this problem have been found due to the increased complexity of the topographic model. Killworth (1973) analyzed the importance of topography on Gulf Stream behavior, but since he assumed geostrophic balance over the continental slope, he was unable to find any tendency for the streamlines to deflect away from the coast. Korotayev and Shapiro (1972a,b) used a series of step functions for the depth of the ocean to study the effects of bottom topography in barotropic and baroclinic oceans. Their results showed deflection of streamlines across topographic bumps but their study sheds little light on the phenomenon of Gulf Stream separation.

In this paper a steady linear model will be used to study an ocean with a frictional western boundary layer. The results of Holland (1973) suggests that the retention of baroclinic effects is desirable, but in order to keep the analytical approach reasonably simple we are forced to assume a homogeneous ocean. In this model, topography will provide the mechanism by which a classical boundary layer type separation may be achieved.

2. The model

We will restrict the model to the western regime of the hypothetical ocean as shown in Fig. 1. The x, y, z coordinates are in the east, north and vertical directions respectively. The bottom topography is given by

$$z=\begin{cases} h(x), & x\leqslant a \\ -H, & x>a. \end{cases}$$

The appropriate steady linear momentum equations for an incompressible, homogeneous and hydrostatic ocean are

$$-fv=-\frac{1}{\rho}p_x, \qquad (1)$$

$$fu=-\frac{1}{\rho}p_y+\nu v_{xx}, \qquad (2)$$

$$0=-\frac{1}{\rho}p_z, \qquad (3)$$

$$(uh)_x+(vh)_y=0, \qquad (4)$$

where p is the hydrostatic pressure, ρ the density, $\mathbf{u}=(u,v,w)$ the velocity vector, ν the eddy viscosity, and f the Coriolis parameter which is a linear function of y (i.e., $f=f_0+\beta y$).

Two important assumptions have been made in the formulation of (1)–(4). First, as in the western boundary layers considered by Johnson (1968) and Killworth (1973), all second-order velocity derivatives have been neglected apart from the term $\nu(\partial^2 v/\partial x^2)$. This assumption may well be invalid if the shelf region becomes very shallow so that $\nu(\partial^2 v/\partial x^2)\sim\nu_v(\partial^2 v/\partial z^2)$ where ν_v is the vertical eddy viscosity. But for the purposes of this simple model, vertical diffusion terms will be ignored. A second assumption is that the slope of the bottom is small enough so that momentum contributions from the bottom Ekman layer can be neglected. [The importance of bottom Ekman layers has been studied by Killworth (1973)]. We thus assume that $w=-uh_x$ at $z=h(x)$.

Eq. (4) implies that a volume transport streamfunction ψ may be used, where

$$\left.\begin{aligned} \frac{\partial\psi}{\partial x}&=hv \\ \frac{\partial\psi}{\partial y}&=-hu \end{aligned}\right\}. \qquad (5)$$

Using (5), Eqs. (1) and (2) may be combined to give the vorticity equation

$$\left(\frac{\psi_x}{h}\right)_{xxx}-\frac{\beta\psi_x}{h}-\frac{f\psi_y h_x}{h^2}=0. \tag{6}$$

We will choose the boundary conditions so that the western boundary layer flow matches onto a flow similar to a Sverdrup balanced interior flow in which the applied wind stress is zonal and proportional to $\cos(\pi y/L)$, where L is the horizontal scale for the ocean. Thus, we have

$$\left.\begin{aligned}\psi&=A\sin\frac{\pi y}{L}\\ \psi_x&=0\end{aligned}\right\},\quad x=a, \tag{7}$$

where $a\ll L$. At the coast, no-slip conditions are used:

$$\psi=\psi_x=0\quad\text{at } x=0. \tag{8}$$

In order to solve (6) analytically we employ a very simple exponential function for the depth variation, i.e.,

$$h=-De^{\alpha x}, \tag{9}$$

where D is the depth at the coast and α a parameter which determines the steepness of the bottom ($\alpha=3\times10^{-7}$ cm^{-1} for an ocean that drops from a continental shelf depth of 200 m to an ocean depth of 4 km over a distance of 100 km from the coast). In order to ensure identical influx conditions for different topographies, D is adjusted so that $h(a)=-H=$constant for all α; that is $D=-H/e^{\alpha a}$. Substituting (9) into (6) leads to

$$\nu\psi_{xxxx}-3\alpha\nu\psi_{xxx}+3\nu\alpha^2\psi_{xx}-(\beta+\alpha^3\nu)\psi_x-f\alpha\psi_y=0. \tag{10}$$

Putting

$$\left.\begin{aligned}\psi&=P_1(x)\cos\frac{\pi y}{L}+P_2(x)\sin\frac{\pi y}{L}\\ P(x)&=P_1(x)+iP_2(x)\end{aligned}\right\} \tag{11}$$

leads to

$$\nu P''''-3\alpha\nu P'''+3\nu\alpha^2P''-(\alpha^3\nu+\beta)P'+\frac{f\alpha\pi i}{L}P=0. \tag{12}$$

A modified β-plane has been assumed so that f is constant except where it is differentiated. [The effect of this assumption has been discussed by Schulman and Niiler (1970).] The solution of (12) is straightforward once α and β are known and is of the form

$$P=\sum_{i=1}^{4}C_i\exp(\theta_i x).$$

The four values of θ_i are the roots of the characteristic equation:

$$\nu\theta^4-3\alpha\nu\theta^3+3\nu\alpha^2\theta^2-(\alpha^3\nu+\beta)\theta+\frac{f\alpha\pi i}{L}=0.$$

The constants C_i are determined from the four boundary conditions in (7) and (8), and are given by

$$C_i=A(-1)^{i+1}\sum_{\substack{j,k=1\\ j\neq k;\,j,k\neq i}}^{4}\theta_j\theta_k(-1)^{j+k+1}(e^{\theta_k a}-e^{\theta_j a}).$$

Typical values of the various parameters are

$$\left.\begin{aligned}\nu&=4\times10^6\text{ cm}^2\text{ s}^{-1},&\beta&=2.6\times10^{-13}\text{ s}^{-1}\text{ cm}^{-1}\\ L&=2\times10^8\text{ cm},&f&=1\times10^{-4}\text{ s}^{-1}\end{aligned}\right\}.$$

The value of A does not affect the dynamics so for convenience its value will be taken so that typical westward interior velocities are of the order of 5 cm s^{-1}.

3. Results

For the familiar problem of viscous flow over a flat plate, boundary layer separations occur when the shear stress on the plate becomes zero. In an analogous manner we assume that the western boundary layer separates from the coast where the stress at the coast

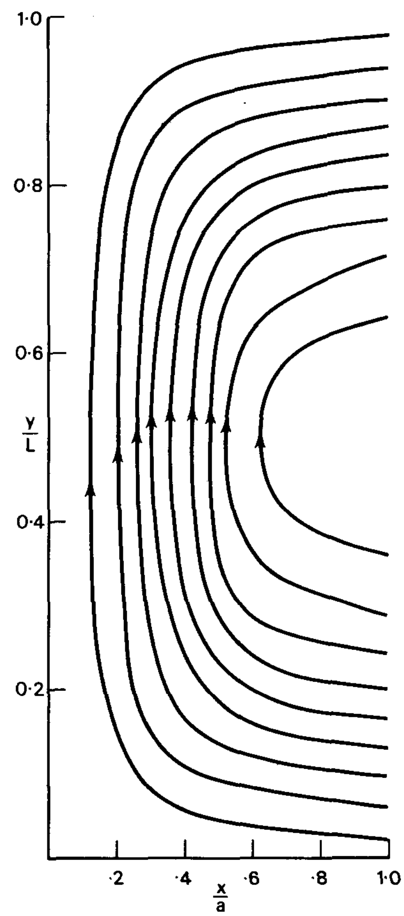

FIG. 2. Transport streamline pattern for $\alpha=0$ (Munk solution).

becomes zero. That is

$$\frac{\partial v}{\partial x}=0 \quad \text{at} \quad x=0.$$

From (5) and (11) this means

$$-\frac{h_x}{h^2}\psi_x+\frac{\psi_{xx}}{h}=0 \quad \text{at} \quad x=0;$$

or, using (8) and (11)

$$P_1''(0)\cos\frac{\pi y}{L}+P_2''(0)\sin\frac{\pi y}{L}=0.$$

The point of separation y_s is thus given by

$$y_s=\frac{L}{\pi}\tan^{-1}\left[-\frac{P_1''(0)}{P_2''(0)}\right]. \tag{13}$$

If separation occurs, the separated stream will leave the coast at $y=y_s$ and follow the streamline originating at this point. This type of boundary layer separation is well illustrated in the following examples.

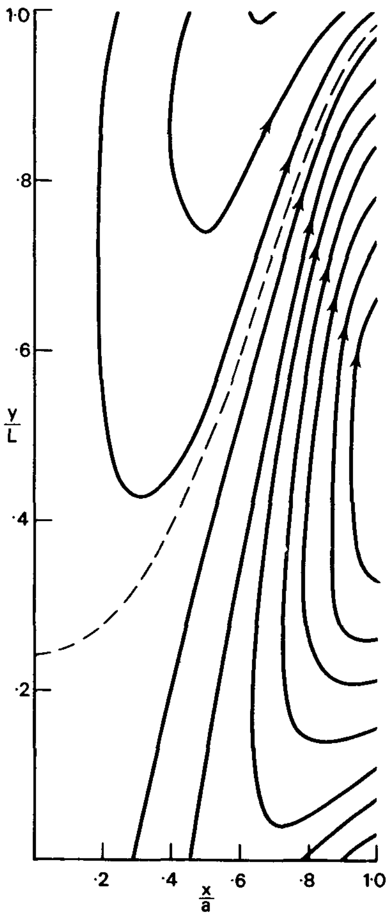

FIG. 3. Transport streamline pattern with $\alpha=2\times10^{-3}$ m^{-2} and $a=100$ km. The dotted line shows the line of separation. $y_s/L=0.24$. The contour interval is 0.10 A.

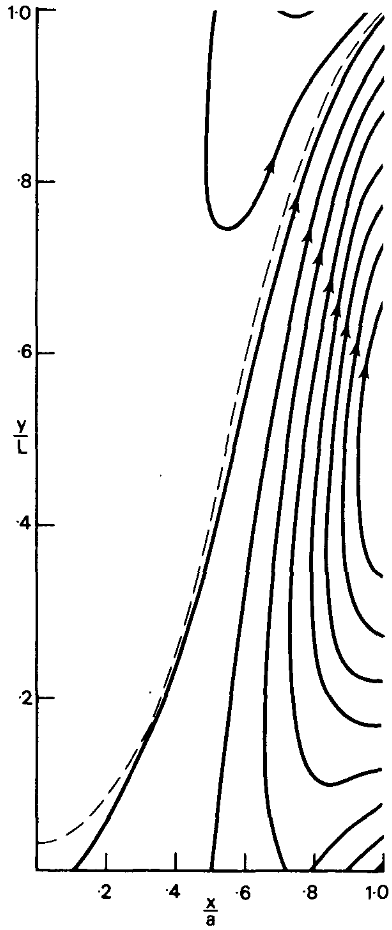

FIG. 4. As in Fig. 3 but with $\alpha=3\times10^{-3}$ m^{-2}. $y_s/L=0.03$. The contour interval is 0.131 A.

The familiar Munk solution ($\alpha=0$) is shown in Fig. 2, while Figs 3–5 ($\alpha=2, 3, 4\times10^{-7}$ cm^{-1}) give the corresponding topographic patterns. The streamlines for a flat-bottomed ocean are symmetrical about $y = L/2$, whereas the topographic patterns show no symmetry. Instead the western boundary current now separates from the coast and flows in a concentrated stream toward the northeast. The point of separation y_s is obviously dependent upon the steepness of the continental rise—the greater the slope, the further south the Gulf Stream separates. Fig. 6 shows the dependence of y_s on α. Notice that as α is increased past 3×10^{-7}, y_s moves into the adjacent gyre located between the latitudes $y=-L$ and $y=0$. A further zero stress point now appears in the northwest corner of the original gyre.

Associated with the separated Gulf Stream is a fairly intense counter-current located in the northwest. This counter-current does not carry a great volume of water but it does penetrate deep into the southernmost part of the gyre. As α is increased to 4×10^{-7} cm^{-1}, a further eddy appears in the northwest and the counter-current

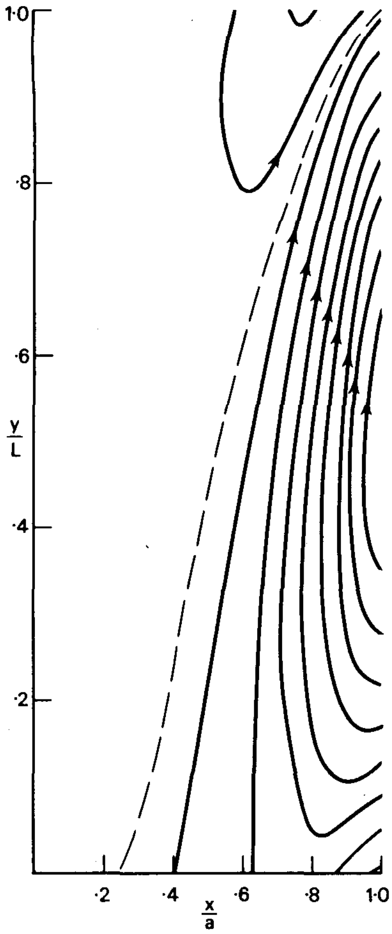

FIG. 5. As in Fig. 3 but with $\alpha=4\times10^{-3}$ m^{-2}. $y_s/L=0.11$. The contour interval is 0.123 A.

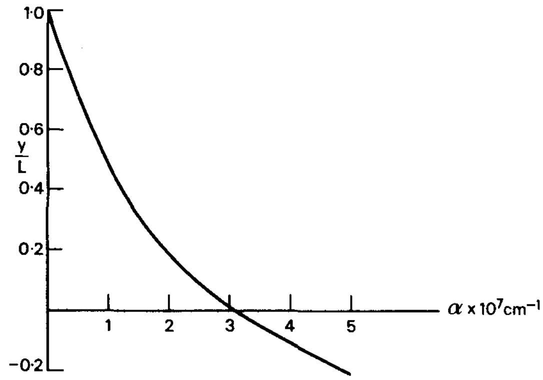

FIG. 6. The variation of separation point y_s with α.

itself separates from the coast. Observational data on coastal counter-current is sparse but they have been previously discussed by Stommel (1965), Korotayev and Shapiro (1971) and Sarkisyan and Ivanov (1971).

Northward flux profiles for several values of α are presented in Fig. 7. It is readily noticeable that the inclusion of topography tends to shift the stream toward the foot of the slope and the counter-circulation is located between the main stream and the west coast. The northward flowing stream is a great deal more concentrated in the topographic models when compared with the flat-bottomed model, and much of this increased intensity can be attributed to the induced eddies west of the stream.

Fig. 8 shows a profile of the balance of terms in the vorticity equation (6). Whereas Munk's solution ($\alpha=0$) predicts a balance between the planetary vorticity tendency and horizontal friction, our solution shows that

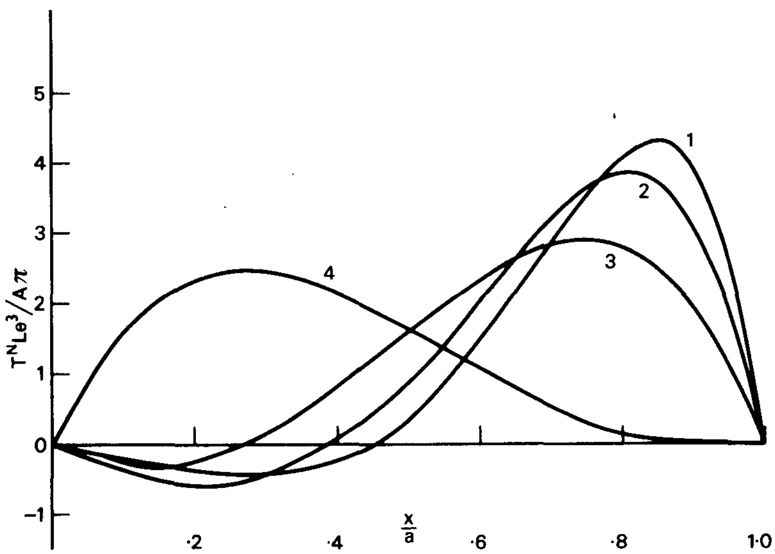

FIG. 7. Northward flux profiles for various topographies at $y/L=0.6$. The vertical scale is a non-dimensional form of the transport T^N.

1. $\alpha=3\times10^{-7}$ cm^{-2}
2. $\alpha=2\times10^{-7}$ cm^{-2}
3. $\alpha=1\times10^{-7}$ cm^{-2}
4. $\alpha=0.0$.

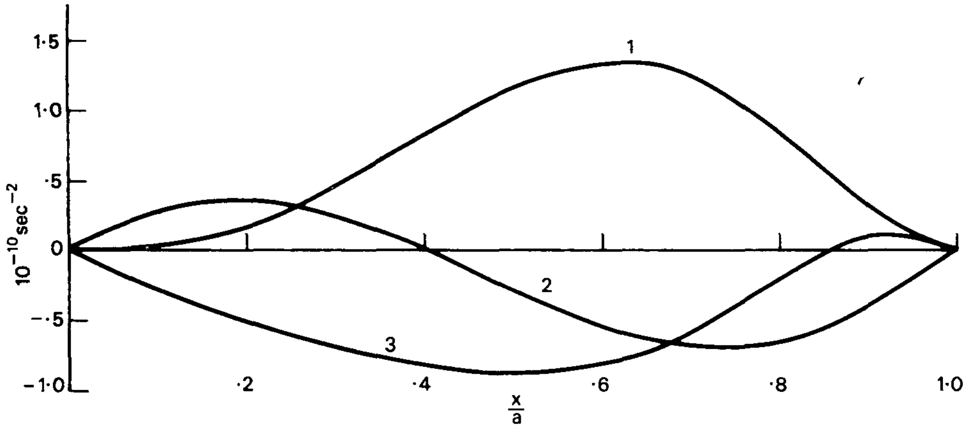

FIG. 8. Profile of vorticity terms in Eq. (6), at latitude $y=L/2$. Term 1 is the topographic term, 2 the planetary vorticity tendency, and 3 the frictional term.

induced topographic vorticity balances both the planetary vorticity tendency and horizontal friction. These results would suggest that the topographic vorticity contribution would be a major factor in the overall vorticity balance over the continental rise.

In a recent series of experiments reported by Knauss (1969), measurements were made of the volume transport of the Gulf Stream. The results showed a rate of increase in the transport of the Gulf Stream "equivalent to 7% per 100 km over a distance of 2000 km downstream from the Florida Straits." An analogous increase in transport has been found in the present model (see Fig. 9). As one moves northeast with the Gulf Stream, the overall northward flux of the stream increases as a result of the feeding effect of the induced countercurrent. This type of transport increase may well be a major factor explaining the observed increase reported by Knauss. The analogous diagram for the Munk solution shows no *such* increase since the flow is symmetric about $y=L/2$.

4. Summary

Through the use of a simple, barotropic, topographic boundary layer model we have been able to quantitatively assess the importance of topography in determining the path of the Gulf Stream. We found that Gulf Stream separation in a frictional western boundary layer is directly dependent upon the gradient of the continental rise—the greater the slope the further south the stream separates. Furthermore, we found that a continental rise induces a counter-current which tends to "feed" the main stream resulting in an overall increase in the Gulf Stream transport.

The interchange of fluid between adjacent gyres is shown in Fig. 10. While there is still no net transport between adjacent gyres, fluid entering the western boundary layer initially turns south and flows into the adjacent gyre before turning northward and returning to the boundary layer.

Although the simplicity of the theory restricts quantitative conclusions, the model has demonstrated that topography may provide a further mechanism by which separation of the Gulf Stream can be achieved. The recent results of Holland (1973) suggest, however, that the coupling of both baroclinic and topographic effects can be a major factor in the dynamics of the Gulf Stream. Baroclinicity could reasonably be ex-

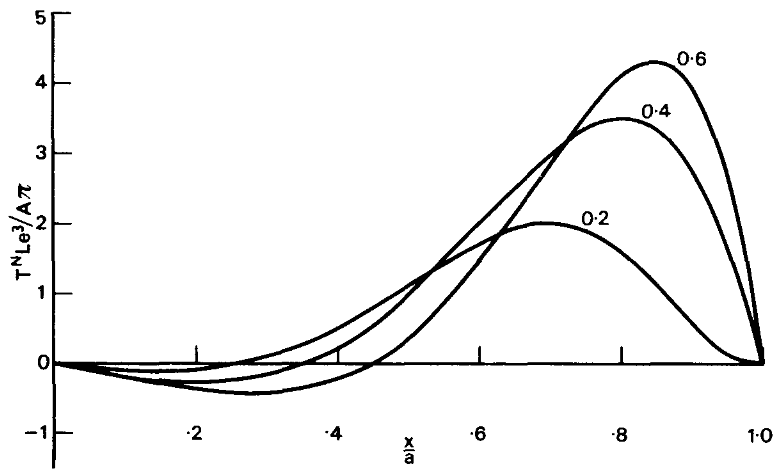

FIG. 9. Northward flux profiles for $\alpha=3\times10^{-3}$ m^{-2} at $y_L=0.2$, 0.4 and 0.6.

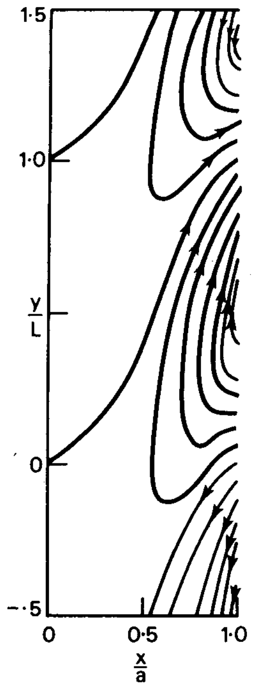

FIG. 10. Plan view of the western boundary streamline pattern for $\alpha = 3\times10^{-3}$ m^{-2}.

pected to alter the features of this model, but to what extent is uncertain. Inertial effects may also become important as the velocities in the Gulf Stream increase, and in this respect a more detailed approach is desirable.

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