NOTES AND CORRESPONDENCE

The Errors Involved in Inferring Salinity from Sound Velocity

William J. Emery

Department of Oceanography, Texas A&M University, College Station 77843

12 August 1976 and 6 December 1976

ABSTRACT

Four empirical equations relating sound velocity, salinity, temperature and pressure are examined to determine the errors in computing salinity from measurements of temperature, pressure and sound velocity. The measurement errors in these variables lead to rms salinity errors of 0.2‰ for an expendable device and 0.04‰ for a moored instrument. The magnitude of the salinity error appears independent of the equation used. Salinities computed using Lovett's and Del Grosso's equations agree well and differ from those computed using either of Wilson's equations.

1. Introduction

Measurements of salinity are essential for the determination of density structure in the ocean. Traditionally salinity has been computed from chlorinity measured by titration of a water sample taken as part of a hydrographic cast. The development of electronic sensors has made possible the *in situ* measurement of salinity as a function of conductivity. An empirical equation (Ribe and Howe, 1975) is used to compute salinity from measurements of conductivity, pressure and temperature.

Empirical equations also exist which relate salinity to sound velocity, temperature and pressure. The recent development of an expendable sound velocity probe (J. Feeney, personal communication) suggests that it might be useful to determine salinity structure from measured profiles of temperature and sound velocity. Sound velocity measurements might also be used to monitor salinity from moored arrays thus eliminating the problems of drift and fouling inherent in moored conductivity sensors.

The purpose of this study is to briefly examine the effect of measurement errors in temperature, pressure and sound velocity on four equations relating these variables to salinity. In this way the accuracies can be specified for inferring salinity using these equations. Also the correspondence between equations will be determined by comparison between salinities inferred from the different equations for triples of temperature, pressure and sound velocity.

2. Propagation of error

Two equations relating sound velocity to temperature, pressure and salinity were developed by Wilson (1960a, b). Both equations, fourth-order polynomials in temperature, pressure and salinity, were fit to repeated measurements of sound velocity for various seawater samples over a range of temperature and salinity. The first equation, hereafter referred to as Wilson's June equation, was developed for salinities between 33 and 37‰. The later equation, referred to as Wilson's October equation, was extended to cover salinities down to 0‰.

Assuming the errors in temperature, pressure and salinity are small, we can in general write the resultant error in computed sound velocity as

$$\epsilon_v=\frac{\partial v}{\partial s}\epsilon_s+\frac{\partial v}{\partial t}\epsilon_t+\frac{\partial v}{\partial p}\epsilon_p, \tag{1}$$

where ϵ_s, ϵ_t and ϵ_p are errors in salinity, temperature and pressure. If we further assume that the measurement errors in temperature, pressure and sound velocity are independent we can rewrite (1) to express the mean square error in salinity as

$$\overline{\epsilon_s^2}=\frac{\overline{\epsilon_v^2}+\left(\dfrac{\partial v}{\partial t}\right)^2\overline{\epsilon_t^2}+\left(\dfrac{\partial v}{\partial p}\right)^2\overline{\epsilon_p^2}}{\left(\dfrac{\partial v}{\partial s}\right)^2}. \tag{2}$$

Hereafter ϵ_s, ϵ_t, ϵ_p, ϵ_v will be used to denote rms values of these errors.

In order to simplify the presentation of the rms salinity error (ϵ_s) it was decided to evaluate the errors in (2) for a typical vertical profile in the ocean. In this way the dependence of ϵ_s on the combination of the

TABLE 1. Values of pressure, temperature, salinity and sound velocity at a spot in the North Pacific (from Reid, 1965).

Triple no.	Pressure (db)	Temperature (°C)	Salinity (‰)	Sound velocity (m s^{-1})
1	0	25.0	35.4	1534.9
2	50	25.0	35.3	1535.6
3	100	21.0	35.2	1526.2
4	150	19.0	35.1	1521.4
5	200	17.0	34.7	1515.9
6	250	15.0	34.5	1510.3
7	300	13.0	34.3	1504.4
8	350	11.0	34.2	1498.2
9	400	10.0	34.1	1495.4
10	450	9.0	34.1	1492.5
11	500	7.0	34.1	1485.7
12	550	7.0	34.1	1486.5
13	600	6.0	34.0	1483.3
14	650	5.5	34.1	1482.2
15	700	5.0	34.2	1481.1
16	750	4.5	34.2	1479.9
17	800	4.0	34.2	1478.6
18	850	4.0	34.2	1479.4
19	900	3.0	34.2	1476.0
20	950	3.0	34.2	1476.8
21	1000	3.0	34.2	1477.6

three independent variables (s,t,p) in (2) could be evaluated and displayed. A vertical profile to 1000 m in the North Pacific located at 25°N, 160°W was selected and values of salinity, temperature and pressure were taken from Fig. 2 in Reid's monograph (1965). These values are given in Table 1.

Different sets of ϵ_s, ϵ_t, ϵ_p were used in (2) to examine the applications suggested above. The accuracies of an expendable temperature probe were estimated as $\epsilon_t=0.03$°C and $\epsilon_p=2\%$ of *in situ* pressure. These errors were computed from tests of existing XBT (expendable bathythermograph) probes (J. Feeney, personal communication). The accuracy of the expendable sound velocity probe was given as 0.15 m s^{-1} (R. Bixby, personal communication).

The resultant salinity errors (ϵ_s) are given in Tables 2 and 3 as functions of the triples in Table 1. Surprisingly the salinity errors are smaller for Wilson's June equation than for his October equation. Both equations show salinity errors $<0.2‰$ down to 500 m. It was suggested that ϵ_p in an expendable device could be reduced to 1% of *in situ* pressure (J. Feeney, personal communication). Both of Wilson's equations yielded salinity errors $<0.2‰$ over the entire range of triplets for this reduced pressure error.

The errors appropriate for more precise moored instruments were taken as $\epsilon_t=0.005$°C, $\epsilon_p=0.04\%$ of full scale and $\epsilon_v=0.05$ m s^{-1}. Values of ϵ_s computed from (2) in Tables 2 and 3 range between 0.03 and 0.05‰. This is far below the accuracy available with profiling conductivity sensors but may be of sufficient accuracy to resolve salinity features from moored arrays.

TABLE 2. Root-mean-square salinity errors for Wilson's June equation.[1]

Triple no.	Pressure (db)	Expendable probes		Moored conductivity (MC) probe
		Ex 2%	Ex 1%	
1	0	0.1262	0.1262	0.0394
2	50	0.1253	0.1248	0.0389
3	100	0.1284	0.1267	0.0387
4	150	0.1312	0.1275	0.0383
5	200	0.1306	0.1245	0.0368
6	250	0.1336	0.1246	0.0361
7	300	0.1372	0.1251	0.0355
8	350	0.1428	0.1271	0.0353
9	400	0.1480	0.1284	0.0350
10	450	0.1551	0.1312	0.0351
11	500	0.1636	0.1352	0.0353
12	550	0.1706	0.1373	0.0353
13	600	0.1771	0.1392	0.0350
14	650	0.1870	0.1435	0.0354
15	700	0.1974	0.1481	0.0358
16	750	0.2061	0.1514	0.0358
17	800	0.2150	0.1548	0.0359
18	850	0.2237	0.1577	0.0358
19	900	0.2334	0.1619	0.0359
20	950	0.2424	0.1651	0.0359
21	1000	0.2515	0.1684	0.0359

[1] Ex 2%: ($\epsilon_p=0.03$°C, $\epsilon_p=0.02$P, $\epsilon_v=0.15$ m s^{-1}). Ex 1%: ($\epsilon_t=0.03$°C, $\epsilon_p=0.01$P, $\epsilon_v=0.15$ m s^{-1}). MC: ($\epsilon_t=0.005$°C, $\epsilon_p=0.4$ db, $\epsilon_v=0.05$ m s^{-1}). These definitions apply to Tables 2–5.

TABLE 3. Root-mean-square salinity errors for Wilson's October equation.

Triple no.	Pressure (db)	Expendable probes		Moored conductivity (MC) probe
		Ex 2%	Ex 1%	
1	0	0.1539	0.1539	0.0480
2	50	0.1547	0.1541	0.0480
3	100	0.1558	0.1536	0.0469
4	150	0.1587	0.1541	0.0463
5	200	0.1627	0.1549	0.0458
6	250	0.1675	0.1560	0.0452
7	300	0.1729	0.1573	0.0447
8	350	0.1787	0.1588	0.0442
9	400	0.1855	0.1608	0.0439
10	450	0.1927	0.1628	0.0436
11	500	0.1991	0.1647	0.0431
12	550	0.2078	0.1674	0.0431
13	600	0.2156	0.1699	0.0429
14	650	0.2241	0.1726	0.0427
15	700	0.2329	0.1755	0.0426
16	750	0.2418	0.1785	0.0425
17	800	0.2508	0.1816	0.0423
18	850	0.2609	0.1851	0.0423
19	900	0.2690	0.1880	0.0421
20	950	0.2794	0.1918	0.0421
21	1000	0.2898	0.1956	0.0421

A similar analysis was followed using a more recent equation developed by Del Grosso (1974). For both the expendable and moored cases the salinity errors

(Table 4) are very similar to those for Wilson's October equation.

Working with a device which measured sound velocity, pressure and temperature, Lovett (1963, 1968) developed an equation expressing salinity as a function of sound velocity, temperature and pressure. This equation was formulated using the experimental data of Wilson and is a fourth-order polynomial in sound velocity, temperature and pressure. Lovett gives the mean deviation between the computed salinity and the experimental data as 0.005‰. For this equation the mean square salinity error due to measurement errors can be written as

$$\overline{\epsilon_s^2}=\left(\frac{\partial s}{\partial t}\right)^2\overline{\epsilon_t^2}+\left(\frac{\partial s}{\partial p}\right)^2\overline{\epsilon_p^2}+\left(\frac{\partial s}{\partial v}\right)^2\overline{\epsilon_v^2}. \tag{3}$$

To evaluate the terms in (3) it was necessary to input values of sound velocity. Del Grosso's equation was used to compute sound velocity from the triples of temperature, pressure and salinity in Table 1. The values of sound velocity are included in Table 1. Sound velocities were also computed using both of Wilson's equations and the results of the error analysis were found to be independent of the set of sound velocities used. The difference between these sets will again be discussed when the four equations are compared.

The rms salinity errors from (3) given in Table 5 are smaller than those for Del Grosso's and Wilson's October equations and larger than those for Wilson's June equation. The differences between the errors for all four equations are not large enough however to single out any one equation. The similarity between errors suggests that the salinity error is primarily dependent on the input measurement errors rather than on the equation used.

TABLE 4. Root-mean-square salinity errors for Del Grosso's equation.

Triple no.	Pressure (db)	Expendable probes Ex 2%	Expendable probes Ex 1%	Moored conductivity (MC) probe
1	0	0.1539	0.1539	0.0480
2	50	0.1547	0.1541	0.0480
3	100	0.1558	0.1536	0.0469
4	150	0.1587	0.1540	0.0463
5	200	0.1629	0.1551	0.0458
6	250	0.1679	0.1563	0.0453
7	300	0.1734	0.1578	0.0448
8	350	0.1794	0.1594	0.0443
9	400	0.1864	0.1614	0.0440
10	450	0.1937	0.1636	0.0438
11	500	0.2004	0.1655	0.0432
12	550	0.2093	0.1683	0.0433
13	600	0.2174	0.1709	0.0430
14	650	0.2262	0.1737	0.0429
15	700	0.2352	0.1767	0.0427
16	750	0.2445	0.1798	0.0426
17	800	0.2539	0.1831	0.0425
18	850	0.2644	0.1868	0.0425
19	900	0.2731	0.1899	0.0422
20	950	0.2839	0.1938	0.0422
21	1000	0.2949	0.1979	0.0422

TABLE 5. Root-mean-square salinity errors for Lovett's equation.

Triple no.	Pressure (db)	Expendable probes Ex 2%	Expendable probes Ex 1%	Moored conductivity (MC) probe
1	0	0.1497	0.1497	0.0467
2	50	0.1504	0.1499	0.0467
3	100	0.1560	0.1538	0.0475
4	150	0.1526	0.1481	0.0446
5	200	0.1561	0.1487	0.0440
6	250	0.1605	0.1495	0.0434
7	300	0.1655	0.1506	0.0428
8	350	0.1711	0.1520	0.0422
9	400	0.1776	0.1538	0.0420
10	450	0.1846	0.1558	0.0417
11	500	0.1912	0.1578	0.0412
12	550	0.1995	0.1603	0.0412
13	600	0.2074	0.1628	0.0409
14	650	0.2158	0.1655	0.0408
15	700	0.2245	0.1684	0.0407
16	750	0.2333	0.1713	0.0405
17	800	0.2424	0.1745	0.0404
18	850	0.2522	0.1779	0.0404
19	900	0.2608	0.1810	0.0402
20	950	0.2709	0.1846	0.0401
21	1000	0.2815	0.1886	0.0402

In order to understand how the individual errors combine to produce the rms salinity errors the various terms in (3) are plotted for the expendable case in Fig. 1 as functions of triple number. As expected, the increasing pressure error (2% of *in situ* pressure) becomes the predominant term and is responsible for salinity errors greater than 0.2‰ at depths below 500 m.

3. Comparison of equations

The four equations being studied were used to compute salinities for the values of t, v, p given in Table 1. Both of Wilson's equations and Del Grosso's equation required solution by a numerical technique. The Newton-Raphson (Scarborough, 1966) method employed for this purpose contributed only a very small error (<0.001‰) to the salinity values.

Plotted together in Fig. 2 the curves for the four equations agree in form but differ in magnitude. Both of Wilson's equations yield systematically lower salinities than either Lovett's or Del Grosso's equations. The latter equations correspond best, having a mean difference less than 0.02‰. The differences between equations increase with increasing triple number.

To test the effect of using the sound velocities in Table 1 derived from Del Grosso's equation, the above

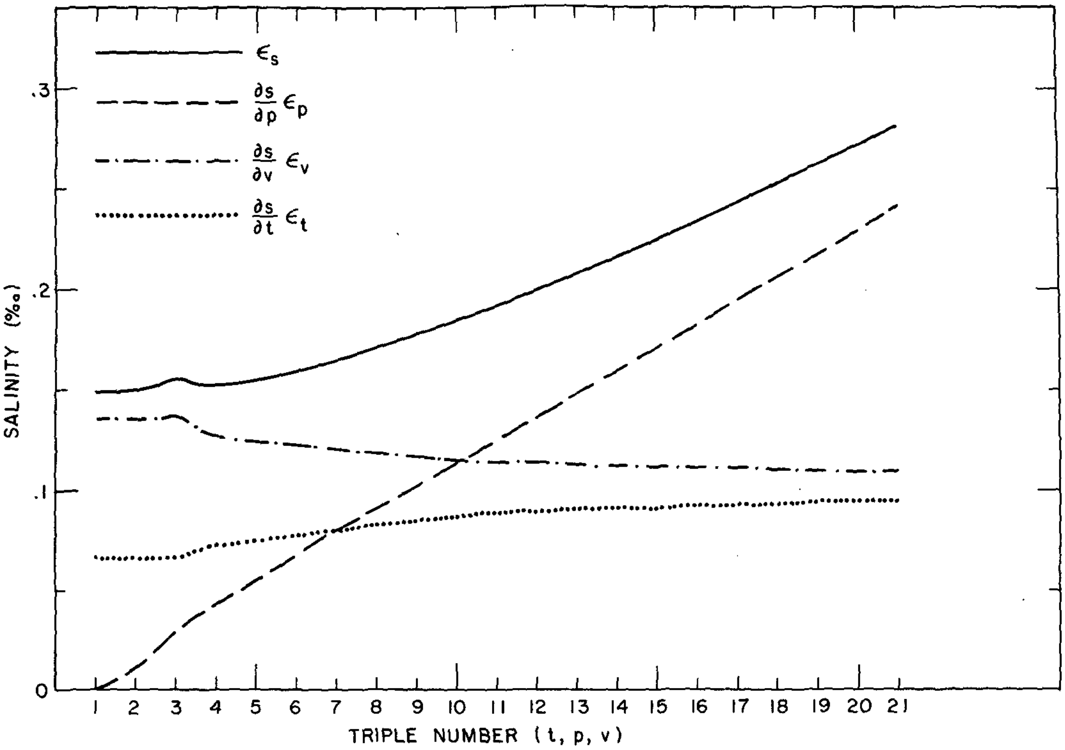

FIG. 1. Error components of Lovett's equation for the expendable case (Ex 2%).

calculations were repeated using sound velocities from both of Wilson's equations. Little change was observed in the relative agreement between the curves shown in Fig. 2.

4. Discussion

The errors in Tables 2–5 suggest that for many applications it may be useful to infer salinity from mea-

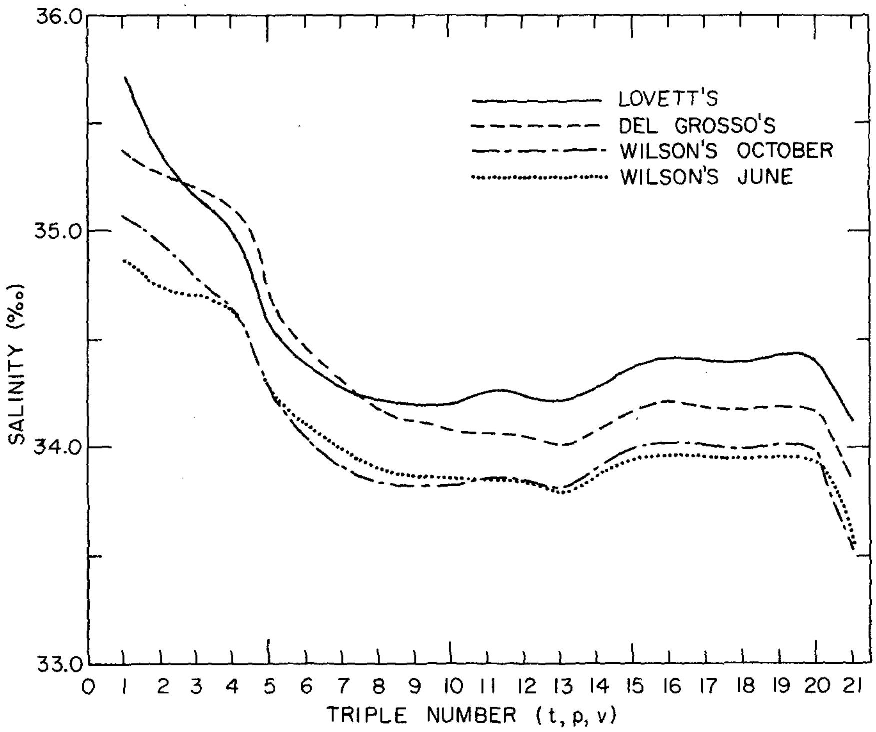

FIG. 2. Comparison of salinities computed from the four equations.

surements of sound velocity. An accuracy on the order of 0.2‰ for an expendable device would be adequate to resolve many of the important salinity features in the ocean. Depending on the cost per probe such an expendable instrument may prove to be a cost effective means of collecting salinity data. As for moored sensors further experiments with conductivity sensors are needed to determine if the operational advantages of sound velocity sensors outweigh the lower salinity accuracies indicated in Tables 2–5.

An important consideration for both applications, however, is the lack of agreement between the various equations. It is difficult to accept any of the above error tables when the salinities computed from the different equations agree so poorly. The relative agreement between Lovett's and Del Grosso's equations does suggest some improvement in these more recent equations.

To resolve this lack of agreement and to verify the error studies it is suggested that an experiment be carried out where temperature, salinity, pressure and sound velocity are all accurately measured with continuous profiling instruments. At the same time expendable instruments would measure temperature, pressure and sound velocity. The disparity between equations would be examined by computing salinity from each of the four equations and comparing it with the measured salinity. The accuracy of the expendable system would be found by computing salinity for the "best" equation and comparing it to that measured. Repeated profiles would then be made to allow computation of an rms estimate of the salinity errors for both the expendable and moored cases. In these computations care must be taken to allow for the different time constants of conductivity, temperature and sound velocity sensors. The accuracy to which these time constants are known and can be compensated for will contribute measurement errors not considered in this study.

Acknowledgments. The author would like to thank R. O. Reid, W. D. Nowlin and A. D. Kirwan for their help in preparing this paper. The important changes suggested by J. Lovett are also gratefully acknowledged. Support for the author was provided by the National Science Foundation as part of the International Southern Ocean Studies.

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