Horizontal Divergence and Vorticity Estimates from Velocity and Temperature Measurements in the MODE Region[1]

HARRY L. BRYDEN[2] AND NICK P. FOFONOFF

Woods Hole Oceanographic Institution, Woods Hole, Mass. 02543

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ABSTRACT

Estimates of horizontal derivatives of velocity made by differencing velocity measurements are used to show that the observed velocity field due to low-frequency mesoscale motions during the preliminary Mid-Ocean Dynamics Experiment (MODE-0) field program is horizontally nondivergent within estimated errors. The errors in horizontal derivatives of 0.15×10^{-6} s^{-1} are too large for direct estimates of horizontal divergence to be made accurately. The vorticity, however, can be estimated from these horizontal derivatives with an error small compared with its magnitude. Over the measurement period of 50 days, the advection of planetary vorticity balances only one-half of the local change of vorticity so these observations cannot be explained in terms of barotropic Rossby waves alone. There are indications that vortex stretching, estimated from a linear heat balance, may balance the remaining local change of vorticity as expected for baroclinic Rossby waves. Based on other measurements in this region, however, it is likely that the horizontal advection of relative vorticity is also important in the vorticity balance. A positive, but not significantly different from zero, correlation between estimates of relative vorticity and advection of planetary vorticity suggests that the enstrophy of the observed velocity field is decreasing with time. In conjunction with a similar result for the perturbation potential energy obtained in this region, this result supports the view that the MODE region is a region of decay, rather than growth, of the low-frequency mesoscale motions.

1. Introduction

The dominance of the geostrophic balance in the conservation of horizontal momentum for large-scale, low-frequency motions constrains the mass balance to be horizontally nondivergent to lowest order and makes it necessary to work with the vorticity balance in order to understand the dynamics of time-varying ocean currents. In this work tests for horizontal nondivergence and for two-term vorticity balances are made from measurements of current made in a moored array over a period of 50 days during the preliminary Mid-Ocean Dynamics Experiment (MODE-0) field program.

The principal obstacle to these tests is the size of the errors in estimates of horizontal derivatives of velocity. Because estimating derivatives involves differencing measured velocities, small errors in measuring velocity can cause large errors in horizontal derivatives especially for small separations. Past estimates of horizontal velocity gradients in high-current regions from drogue-tracking experiments have not been accompanied by detailed error estimates (Chew, 1974; Chew and Berberian, 1971; Molinari and Kirwan, 1975; Reed, 1971; Stevenson *et al.*, 1974). Chew (1974), however, has indicated that accurate estimates of horizontal derivatives of velocity could not be made from his drogue tracks. From an array of profiling current meter measurements in an upwelling region, Johnson (1977) found the error in his estimate of horizontal divergence to be as large as his estimate. Thus, the largeness of errors in estimates of horizontal derivatives of velocity may make tests for horizontal nondivergence and vorticity conservation meaningless.

In this work, errors in horizontal derivatives of velocity are estimated to be nearly an order of magnitude less than the typical magnitude of the derivatives. Whether these errors are realistic, however, is difficult to assess. Because horizontal nondivergence to lowest order is so well-determined on theoretical grounds that most oceanographers would attribute significant horizontal divergence to errors in estimates of horizontal derivatives of velocity, the test for horizontal nondivergence is used here also as a test of the error estimates. Confirmation of horizontal nondivergence within estimated errors is also confirmation of the accuracy of the error estimates.

It is important to recognize that, while the current field may be horizontally nondivergent to lowest order, the horizontal divergence of the current field may still be important in the vorticity balance. This apparent

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[2] Present affiliation: School of Oceanography, Oregon State University, Corvallis 97331.

inconsistency is resolved by using scale analysis to order the vorticity balance as shown in Eqs. (3)-(5). To lowest order, the horizontal divergence is much smaller than the individual derivatives of velocity [Eq. (4)]. Thus, to lowest order the current field is horizontally nondivergent. The horizontal divergence, however, need not be exactly zero. To next order, the vortex stretching, which is horizontal divergence multiplied by negative Coriolis parameter, may be as large as the other terms in the vorticity balance [Eq. (5)].

Once the observed current field is shown to be horizontally nondivergent to lowest order, the estimates of horizontal derivatives of velocity are used to test two-term vorticity balances. Because the relative importance of horizontal advection of relative vorticity ($u\partial\zeta/\partial x+v\partial\zeta/\partial y$), of advection of planetary vorticity (βv) and of vortex stretching ($f\partial w/\partial z$) in causing local time changes of vorticity ($\partial\zeta/\partial t$) is not known for ocean currents, models of low-frequency motions differ in the relative importance of these terms. Barotropic Rossby wave models have a balance between local change and advection of planetary vorticity (Longuet-Higgins, 1965); in baroclinic Rossby wave models, vortex stretching is also important (Rhines, 1970); and in models of waves in the presence of a mean current, horizontal advection of relative vorticity contributes to local changes of relative vorticity (Rossby *et al.*, 1939). Estimates of these terms from measurements then are valuable for understanding the dynamics of low-frequency motions and for assessing the applicability of the various models in explaining the observations.

In this work, a test is made for balance between local change of relative vorticity and advection of planetary vorticity within their estimated errors. Also, some reasonable but inaccurate estimates of vortex stretching are made using a linear heat balance. The measurements are not sufficient to estimate horizontal advection of relative vorticity. To test whether vortex stretching and horizontal advection of relative vorticity could be estimated accurately in future experiments, an extended error analysis is carried out. From a suitable array of measurements, all terms in the vorticity balance [Eq. (5)] can be estimated with errors smaller than the expected magnitude of the terms.

2. Theory

The conservation of mass can be written

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}=-\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right), \quad (1)$$

where (u,v,w) are the velocities in the (x,y,z) = (eastward, northward, upward) directions, ρ is the density of seawater and t is time. Batchelor (1967, pp. 167–171) has shown that the terms on the left side of (1) are small compared with those on the right provided 1) that the typical velocities considered, U, are smaller than the speed of sound, c; 2) that the time scale of change of the velocities, $1/\omega$, is longer than the time it takes a sound wave to travel through the depth of water, H; and 3) that the water depth is less than the scale height, c^2/g, where g is the vertical component of the gravitational acceleration. With these assumptions, Eq. (1) may be written in the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=O\left(\frac{U^2}{c^2},\frac{\omega^2H^2}{c^2},\frac{Hg}{c^2}\right)\left|\frac{\partial u}{\partial x}\text{ or }\frac{\partial v}{\partial y}\text{ or }\frac{\partial w}{\partial z}\right|. \quad (2)$$

When U^2/c^2, ω^2H^2/c^2 and Hg/c^2 are small, the fluid is said to be incompressible and the divergence of the velocity field, $\partial u/\partial x+\partial v/\partial y+\partial w/\partial z$, is zero to lowest order.

The equation for the conservation of the vertical component of relative vorticity, $\zeta=\partial v/\partial x-\partial u/\partial y$, is obtained by eliminating pressure in the horizontal momentum equations [Eqs. (2), Bryden, 1977]:

$$\underbrace{\frac{\partial\zeta}{\partial t}}_{\left(\frac{\omega}{f}\right)}+\underbrace{u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}}_{\left(\frac{U}{fL}\right)}+\underbrace{\beta v}_{\left(\frac{\beta L}{f}\right)}+\underbrace{(f+\zeta)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{(1)}=0, \quad (3)$$

where β is the northward derivative of the Coriolis parameter f and where nondimensional numbers characterizing the size of each term are listed below each term. To derive this equation the Reynolds stresses are neglected and the viscous terms and nonlinear terms involving vertical velocity are assumed small. Charney (1973) has discussed these assumptions and the derivation of this equation. For typical horizontal length scales (L = 60 km) and time scales ($1/\omega$ = 10 days) of the currents (U = 10 cm s^{-1}) observed in this region (Gould *et al.*, 1974), these nondimensional numbers are small: $\omega/f=0.02$, $U/fL=0.02$ and $\beta L/f=0.02$, so the current field must be horizontally nondivergent to lowest order, i.e.,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=O\left(\frac{\omega}{f},\frac{U}{fL},\frac{\beta L}{f}\right)\left|\frac{\partial u}{\partial x}\text{ or }\frac{\partial v}{\partial y}\right|. \quad (4)$$

Thus $\partial u/\partial x$ and $\partial v/\partial y$ should be of opposite signs and almost the same magnitude so that their sum is small compared to their individual magnitudes. According to (2) their sum is equal to $-\partial w/\partial z$ so the vorticity balance can be written

$$\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}+\beta v=f\frac{\partial w}{\partial z}. \quad (5)$$

Because the nondimensional numbers ω/f, U/fL and $\beta L/f$ are the same size for currents observed in the

MODE region, all terms may be of importance in this vorticity balance [Eq. (5)].

To estimate horizontal advection of relative vorticity, it is advantageous to use horizontal nondivergence to represent horizontal advection of momentum in terms of the speed R and the direction ϕ, measured positively counterclockwise, of the horizontal current [Eqs. (10), Bryden, 1977]:

$$\left.\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= -R^2\frac{\partial \phi}{\partial y}\pm O(\delta)u\frac{\partial u}{\partial x} \\ u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} &= R^2\frac{\partial \phi}{\partial x}\pm O(\delta)v\frac{\partial v}{\partial y} \end{aligned}\right\}, \qquad (6)$$

where δ is the largest of $(\omega/f,\ U/fL,\ \beta L/f)$ in Eq. (4). These equations relate the acceleration of a current to the convergence of streamlines. To estimate horizontal advection of relative vorticity, the curl of Eqs. (6) is taken to yield

$$u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}=\frac{\partial}{\partial x}\left(R^2\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(R^2\frac{\partial \phi}{\partial y}\right) \pm O(\delta)\sqrt{2}\left(u\frac{\partial u}{\partial x}\pm v\frac{\partial v}{\partial y}\right)L^{-1}. \qquad (7)$$

For typical scales of currents in this region (Gould *et al.*, 1974), the error

$$O(\delta)\sqrt{2}\left(u\frac{\partial u}{\partial x}\pm v\frac{\partial v}{\partial y}\right)L^{-1}$$

in this representation of horizontal advection of relative vorticity is 1.1×10^{-13} s^{-2}, which is small compared to the expected size of local changes of relative vorticity, $\omega U/L=2\times10^{-12}$ s^{-2}.

3. Observations and methods

The measurements of velocity and temperature were recorded by current meters on four subsurface moorings during a common time period of 52 days in autumn 1971 in a mid-ocean region southwest of Bermuda (Fig. 1). Each current meter was at a depth between 1502 and 1522 m and only the current meters on moorings 1 and 3 recorded temperature (Table 1). To eliminate high-frequency motions the time series of current and temperature were put through a Gaussian low-pass filter of half-width 24 h and were subsampled daily (Schmitz, 1974). These measurements are part of the data set collected during the MODE-0 field program and used by Gould *et al.* (1974) to estimate energy levels and spatial and temporal scales of mesoscale motions in this region. Other measurements recorded during the same period on nearby surface moorings are not considered in this study because of the contamination of velocity measurements by surface motions of the buoys (Gould and Sambuco, 1975).

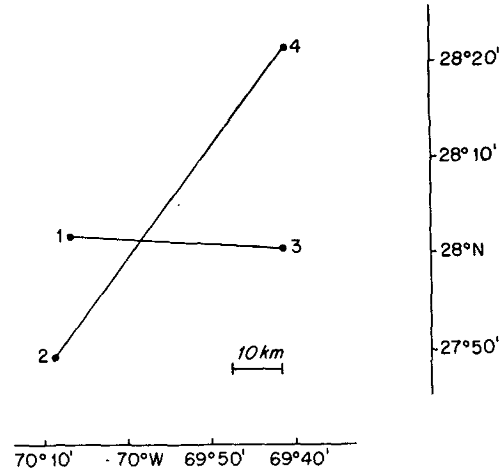

Fig. 1. Spatial distributions of MODE-0 array 1 moorings used in this analysis.

Estimates of horizontal derivatives are made by differencing velocities along diagonals of the array to obtain estimates of $\partial u/\partial\eta$, $\partial v/\partial\eta$, $\partial u/\partial\tau$, $\partial v/\partial\tau$ where τ and η are axes of the skewed coordinate system determined by the diagonals. The coordinate system then is changed to a rectangular system $(x,y)=$(east, north) to obtain estimates of $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$ and $\partial v/\partial y$.

Errors in these estimates are considered to be due to either measurement errors or sampling errors. From differences of velocities measured by current meters separated by small horizontal distances during the Internal Wave Experiment in this region, the measurement error in velocity component was estimated to be $\delta u=0.45$ cm s^{-1} (Bryden, 1976). Approximately one-half of the variance in current differences could be attributed to a bias between currents measured by each pair of current meters. The errors in horizontal derivatives of velocity due to measurement errors are estimated to be $\sqrt{2}\delta u/\Delta s$ where Δs is the horizontal separation. For this array (Fig. 1) the measurement errors in horizontal derivatives are estimated to be 0.15×10^{-6}

Table 1. MODE-0 array 1 data used in tests for horizontal nondivergence and vorticity balance.

Mooring	Depth of current meter (m)	WHOI data number	Position	Variables recorded
1	1522	4091	28°01.50′N 70°06.8′W	Current temperature
2	1503	4081	27°49.00′N 70°08.8′W	Current
3	1502	4121	28°00.2′N 69°41.5′W	Current temperature
4	1504	4101	28°21.5′N 69°41.5′W	Current

s^{-1}. This error is random with respect to the current meters selected for an array. Because of the bias between each pair of current meters, this error is not random over time for a particular array of current meters.

Sampling errors are estimated by assuming that the observations are of a phenomenon which locally has a form

$$F(z)\exp[i(\mathbf{k}\cdot\mathbf{x}-\omega t)].$$

To determine the sampling error in finite-difference estimates of horizontal derivatives of velocity, the wavenumber $\mathbf{k}$ and horizontal separation $\boldsymbol{\Delta}\mathbf{x}$ are used as follows:

$$\frac{\text{Finite difference estimate}}{\text{True value}}=\frac{F(z)\exp(i\mathbf{k}\cdot\mathbf{x}_0)[\exp(i\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\mathbf{x})-\exp(-i\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\mathbf{x})]/|\boldsymbol{\Delta}\mathbf{x}|}{F(z)\exp(i\mathbf{k}\cdot\mathbf{x}_0)(i\mathbf{k}\cdot\boldsymbol{\Delta}\mathbf{x})/|\boldsymbol{\Delta}\mathbf{x}|},$$

$$=\sin(\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\mathbf{x})/(\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\mathbf{x}),$$

where $\mathbf{x}_0$ is the mid point of the line segment joining the two observation points. This sampling error depends on the relative orientations of $\mathbf{k}$ and $\boldsymbol{\Delta}\mathbf{x}$. If $\boldsymbol{\Delta}\mathbf{x}$ and $\mathbf{k}$ are perpendicular, there is no sampling error; if they are parallel, the sampling error is maximum. Due to sampling errors for this array (Fig. 1), the estimates of $\partial u/\partial x$, $\partial v/\partial x$ may be small by 2% and the estimates of $\partial u/\partial y$, $\partial v/\partial y$ may be small by 4%. Because the variation in measurement depths is small (± 10 m, Table 1), the error is estimating horizontal derivatives from observations at different depths is considered small. Within the combined measurement and sampling errors, the estimates of $\partial u/\partial x$ and $\partial v/\partial y$ are tested to determine whether the observed current field is horizontally nondivergent to lowest order (Table 2).

TABLE 2. Tests for horizontal nondivergence from 4-day averaged estimates of $\partial u/\partial x$ and $\partial v/\partial y$. Estimates of $\partial u/\partial x$ and $\partial v/\partial y$ have standard deviation errors of 0.15×10^{-6} s^{-1} due to measurement errors in velocity. Estimates of $\partial u/\partial x$ may be small by 2% and of $\partial v/\partial y$ small by 4% due to sampling errors. The tests are done for the ranges of $\partial u/\partial x$ and $\partial v/\partial y$ determined by these sampling errors. One asterisk (*) denotes a discrepancy from horizontal nondivergence of 1–2 standard deviations. Two asterisks (**) denote a discrepancy of 2–3 standard deviations. No asterisk denotes a discrepancy of less than 1 standard deviation. All tabulated values should be multiplied by 10^{-6}.

Time (days)	$\frac{\partial u}{\partial x}$	$\frac{\partial v}{\partial y}$	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$	Range in $\frac{\partial u}{\partial x}$	Range in $\frac{\partial v}{\partial y}$
2	0.69	−0.69	0.00	0.69 −0.15 0.70 +0.15	−0.72 −0.15 −0.69 +0.15
6	0.78	−0.70	0.08	0.78 −0.15 0.80 +0.15	−0.73 −0.15 −0.70 +0.15
10	0.86	−0.63	0.23	0.86 −0.15 0.88 +0.15	−0.66 −0.15 −0.63 +0.15
14	0.67	−0.66	0.01	0.67 −0.15 0.68 +0.15	−0.69 −0.15 −0.66 +0.15
18	0.87	−0.61	0.26	0.87 −0.15 0.89 +0.15	−0.64 −0.15 −0.61 +0.15 *
22	0.46	−0.50	−0.04	0.46 −0.15 0.47 +0.15	−0.52 −0.15 −0.50 +0.15
26	0.41	−0.36	0.05	0.41 −0.15 0.42 +0.15	−0.38 −0.15 −0.36 +0.15
30	−0.11	−0.42	−0.53	−0.11 −0.15 −0.11 +0.15	−0.44 −0.15 −0.42 +0.15 **
34	−0.46	0.44	−0.02	−0.47 −0.15 −0.46 +0.15	0.44 −0.15 0.46 +0.15
38	−1.19	0.83	−0.36	−1.21 −0.15 −1.19 +0.15	0.83 −0.15 0.86 +0.15 *
42	−0.55	0.34	−0.21	−0.56 −0.15 −0.55 +0.15	0.34 −0.15 0.35 +0.15
46	−0.13	0.03	−0.10	−0.13 −0.15 −0.13 +0.15	0.03 −0.15 0.03 +0.15

Estimates of horizontal derivatives can be combined to estimate horizontal divergence ($\partial u/\partial x+\partial v/\partial y$), vorticity ($\partial v/\partial x-\partial u/\partial y$), shear distortion ($\partial u/\partial y+\partial v/\partial x$) and normal distortion ($\partial u/\partial x-\partial v/\partial y$). Kirwan (1975) has derived conservation equations for these quantities and reviewed observations of velocity gradients in the ocean. Of these conservation equations, the vorticity balance has proved most useful to modellers in exploring the dynamics of low-frequency oceanic currents because the time change is an important term in this balance. In order to compare the MODE-0 observations with theoretical models, emphasis in this work is placed on estimating horizontal divergence and vorticity to test for vorticity balance. Estimates of normal distortion and shear distortion are thus not included here. In Frankignoul (1974, Fig. 8), estimates of normal distortion from this array were correlated with variations in the internal wave field. Estimates of shear distortion were found to be generally smaller than their estimated errors.

By the divergence theorem, estimates of horizontal divergence obtained by integrating the velocity normal to the line segment joining each pair of moorings around the array and dividing by the area enclosed, i.e.,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=(1/\text{area})\oint\mathbf{u}\cdot d\mathbf{n},$$

are theoretically the same as estimates of horizontal divergence obtained from estimates of horizontal derivatives made above. Numerically, the values of horizontal divergence by the two methods are identical for a three- or four-mooring array provided the normal velocity is obtained by averaging the normal velocities at the two moorings determining each line segment. Estimates of vorticity obtained by application of Stokes theorem,

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=(1/\text{area})\oint\mathbf{u}\cdot d\mathbf{s},$$

are numerically identical to those obtained from estimates of horizontal derivatives above. Errors in estimates of horizontal divergence and vorticity are 0.22×10^{-6} s^{-1} due to measurement errors in velocity. Due to sampling errors there may be errors in horizontal divergence of $+2\%\partial u/\partial x+4\%\partial v/\partial y$ and in vorticity of $+2\%\partial v/\partial x-4\%\partial u/\partial y$.

To test for vorticity balance, the local change of relative vorticity $\partial\zeta/\partial t$ is estimated by differencing 4-day averaged values of vorticity. Due to measurement errors these estimates have errors of 0.90×10^{-12} s^{-2}. For typical velocities of 10 cm s^{-1} and horizontal scales of 60 km (Gould *et al.*, 1974) the sampling errors in these estimates may be as large as 0.10×10^{-12} s^{-2}. The advection of planetary vorticity βv is estimated by averaging 4-day averaged northward velocities for the four current meters. Due to measurement errors, these estimates of planetary advection have errors of 0.50×10^{-12} s^{-2}. Due to sampling errors in the spatial averaging, these estimates of planetary advection may be small by 9%.

Vortex stretching $f\partial w/\partial z$ is estimated from a linear, nondiffusive heat balance, $\partial T/\partial t+w\partial\bar{T}/\partial z=0$, and a linear decrease in vertical velocity from its value at 1500 m to the ocean bottom. Differences of 4-day averaged temperatures are averaged for the two current meters to obtain an estimate of local temperature change $\partial T/\partial t$. The vertical gradient of temperature $\partial\bar{T}/\partial z$ is estimated from a CTD station in this region (Millard and Bryden, 1973). The estimate of vertical velocity, $w=-(\partial T/\partial t)/(\partial\bar{T}/\partial z)$, is then divided by 3500 m, the distance from 1500 m depth to the bottom. The justification for the linear decrease in vertical velocity from 1500 m depth to the bottom is that the vertical profile of horizontal velocity in this region is similar to the theoretically determined first baroclinic mode (Gould *et al.*, 1974) which has a nearly linear decrease in vertical velocity from 1500 m to the bottom (Richman, 1972). The errors in these estimates of vortex stretching are large. For other observations in this region, local changes of temperature were balanced by horizontal advection of temperature (Bryden, 1976) so that vertical advection of temperature was much smaller than local temperature change, $w(\partial\bar{T}/\partial z)<(\partial T/\partial t)$. Thus these estimates of vortex stretching are order of magnitude estimates and have errors as large as the estimates. For vertical displacements of isotherms of 60 m in 10 days, these estimates of vortex stretching are of magnitude 0.7×10^{-12} s^{-2} and have errors of 0.7×10^{-12} s^{-2}.

From the above calculations, three estimates of horizontal divergence

$$-\frac{\partial w}{\partial z}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$$

$$-\frac{\partial w}{\partial z}=\frac{\partial T}{\partial t}\left(\frac{\partial\bar{T}}{\partial z}\right)^{-1}\Big/3500\text{ m}$$

$$-\frac{\partial w}{\partial z}=-\left(\frac{\partial\zeta}{\partial t}+\beta v\right)\Big/f$$

are compared (Table 3) and two vorticity equations

$$\frac{\partial\zeta}{\partial t}+\beta v=0$$

$$\frac{\partial\zeta}{\partial t}+\beta v=f\frac{\partial w}{\partial z}=-f\frac{\partial T}{\partial t}\left(\frac{\partial\bar{T}}{\partial z}\right)^{-1}\Big/3500\text{ m}$$

are tested for balance within estimated errors (Table 4). To reduce the errors in these vorticity balance tests,

TABLE 3. Comparison of direct and indirect estimates of horizontal divergence ($\times10^{-8}$ s^{-1}). Negative horizontal divergence $\partial w/\partial z$ is calculated by three methods: 1) directly from estimates of $\partial u/\partial x$ and $\partial v/\partial y$; 2) indirectly from temperature measurements assuming a conservation of heat equation of the form $\partial T/\partial t+w\partial\bar{T}/\partial z=0$ and a linear decrease of the vertical velocity from its value at 1500 m to zero at the bottom; and 3) indirectly from the linear vorticity equation.

Time (days)	$\frac{\partial w}{\partial z}=-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$	$\frac{\partial w}{\partial z}=-\frac{\partial T}{\partial t}\left(\frac{\partial\bar{T}}{\partial z}\right)^{-1}\Big/3500\text{ m}$	$\frac{\partial w}{\partial z}=\left(\frac{\partial\zeta}{\partial t}+\beta v\right)\Big/f$
4	−4.1	−0.74	−0.98
8	−15.5	1.87	0.04
12	−12.2	1.59	1.20
16	−13.7	2.22	0.54
20	−10.8	−2.89	−1.77
24	−0.1	−0.38	−0.01
28	+24.4	1.46	1.46
32	+27.6	0.56	−1.05
36	+18.8	−1.26	−5.73
40	+28.7	0.31	0.48
44	+16.0	0.32	−0.48
Average absolute magnitude	15.6	1.24	1.25

TABLE 4. Comparison of local time change of vorticity, advection of planetary vorticity and vortex stretching (all units, $\times 10^{-12}$ s^{-2}). The time change of vorticity ($\partial\zeta/\partial t$) is estimated by differencing 4-day averaged values of vorticity and dividing by 4 days. The advection of planetary vorticity (βv) is estimated by averaging 4-day averaged northward velocities over the four current meters and multiplying by $\beta = 2.0\times 10^{-13}$ cm^{-1} s^{-1}. Vortex stretching is estimated by assuming a linear heat balance where $w=-(\partial T/\partial t/\partial T/\partial z)$ and a linear decrease of vertical velocity from its value at 1500 m to zero at the ocean bottom.

Time (days)	$\frac{\partial\zeta}{\partial t}$	βv	$\frac{\partial\zeta}{\partial t}+\beta v$	$f\frac{\partial w}{\partial z}$
4	−0.57	−0.10	−0.67	−0.51
8	−0.22	0.25	0.03	1.28
12	0.21	0.61	0.82	1.09
16	−0.48	0.85	0.37	1.52
20	−2.11	0.90	−1.21	−1.98
24	−0.76	0.75	−0.01	−0.26
28	0.31	0.69	1.00	1.00
32	−1.28	0.56	0.72	0.38
36	−4.05	0.13	−3.92	−0.86
40	0.52	−0.19	0.33	0.21
44	0.13	−0.46	−0.33	0.22

time-integrated balances are also estimated:

$$\zeta_{\text{end}}-\zeta_{\text{start}}+\beta\int_{\text{start}}^{\text{end}} v dt=0$$

$$\zeta_{\text{end}}-\zeta_{\text{start}}+\beta\int_{\text{start}}^{\text{end}} v dt=-f(T_{\text{end}}-T_{\text{start}}) \div[(\partial\bar{T}/\partial z)\times 3500\text{ m}].$$

The error in local change of vorticity ($\zeta_{\text{end}}-\zeta_{\text{start}}$) is 0.31×10^{-6} s^{-1} due to measurement errors and -0.09×10^{-6} s^{-1} due to sampling errors. The error in time-integrated planetary advection

$$\beta\int_{\text{start}}^{\text{end}} v dt$$

is 0.17×10^{-6} s^{-1} due to measurement errors and -0.13×10^{-6} s^{-1} due to sampling errors. The error in the estimate of time-integrated vortex stretching is as large as the estimate.

4. Results and discussion

Daily estimates of $\partial u/\partial x$ and $\partial v/\partial y$ (Fig. 2) have a correlation coefficient of -0.93 which is significantly different from zero at a 95% confidence level (Pearson and Hartley, 1970). The regression line of these daily estimates is not significantly different from the line of horizontal nondivergence (slope $=-1$, intercept $=0$) at a 95% confidence level using methods described by Fofonoff and Bryden (1975). Thus, these estimates of $\partial u/\partial x$ and $\partial v/\partial y$ are not significantly divergent. Four-day averaged estimates of $\partial u/\partial x$ and $\partial v/\partial y$ are examined for horizontal nondivergence within estimated errors (Table 2). Nine of the twelve comparisons yield horizontal nondivergence within one standard deviation error of 0.22×10^{-6} s^{-1}, two within 1–2 standard deviations, and one comparison within 2–3 standard deviations. This last comparison occurs during a time period when estimates of $\partial u/\partial x$ and $\partial v/\partial y$ are changing rapidly. Thus, within expected errors the velocity field is horizontally nondivergent to lowest order.

To determine whether horizontal divergence can be estimated accurately from the horizontal derivatives of velocity, direct estimates of horizontal divergence are compared with indirect estimates of the order of magnitude of horizontal divergence from linear heat and vorticity balances (Table 3). These indirect estimates suggest that horizontal divergence in this region should be of order 10^{-8} s^{-1}, which is an order of magnitude less than the expected error of 0.22×10^{-6} s^{-1} in direct estimates. Thus, direct estimates of horizontal divergence are dominated by errors and are not accurate estimates of horizontal divergence.

To determine how small the errors in velocity measurements must be in order to make accurate estimates of horizontal divergence it is assumed that sampling and measurement errors should be equal and no larger than 1×10^{-8} s^{-1}. For horizontal separations of 27 km the sampling error in estimates of horizontal derivatives of magnitude 7 cm s^{-1} per 60 km ($=1.2\times 10^{-6}$ s^{-1}) is 1×10^{-8} s^{-1}. To obtain an error of 1×10^{-8} s^{-1} for horizontal separations of 27 km, the error in velocity measurements must be reduced to 0.02 cm s^{-1} which is substantially smaller than the error of 0.45 cm s^{-1} determined from other measurements in this region (Bryden, 1976). This error analysis suggests that direct estimates of horizontal divergence in the mid-ocean will be dominated by errors until an order of magnitude increase in the precision of velocity measurements is attained.

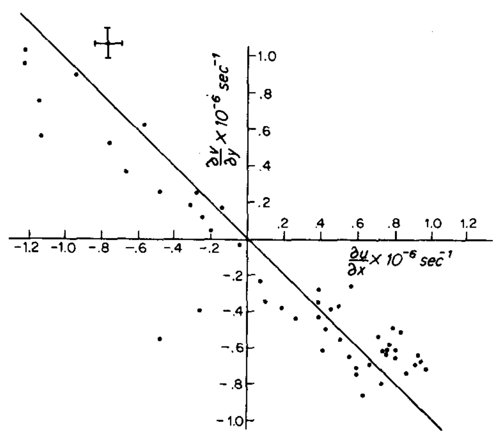

FIG. 2. Daily estimates of $\partial v/\partial y$ plotted against estimates of $\partial u/\partial x$. The correlation coefficient is calculated to be -0.93. The line is drawn to illustrate the condition of horizontal nondivergence. Typical errors are indicated.

Because indirect estimates of horizontal divergence are an order of magnitude smaller than the direct estimates, attempts at estimating horizontal divergence should be made from either a nonlinear heat balance or a nonlinear vorticity balance. Since it is likely that the horizontal divergence would be required for use in a vorticity balance, the error in horizontal divergence obtained from a nonlinear heat balance is estimated. Bryden (1976) showed that vertical velocity in the main thermocline in this region could be estimated by means of a nonlinear heat balance with an error of 0.5 m day^{-1} (0.6×10^{-3} cm s^{-1}). Estimates of horizontal divergence obtained by differencing vertical velocities over a depth interval of 600 m then would have an error of 1.4×10^{-8} s^{-1}. This error is the same size as the estimates of the order of magnitude of horizontal divergence and is more than an order of magnitude less than the error of 0.22×10^{-6} s^{-1} in direct estimates of horizontal divergence. Thus, indirect estimates of horizontal divergence from a nonlinear heat balance have errors small enough that the indirect estimates may be useful in vorticity balance tests.

Since the current field has been shown to be horizontally nondivergent, tests for two-term balances in the vorticity conservation equation (5) are made. The correlation between estimates of local time change of vorticity and of advection of planetary vorticity (Table 4) is -0.20 which is not significantly different from zero at a 95% confidence level. The correlation between estimates of local change plus planetary advection and of vortex stretching from a linear heat balance (Table 4) is 0.65 which is significantly different from zero at a 95% confidence level. The local change of vorticity over the measurement period, $-2.87(^{+0.31}_{-0.40})\times10^{-6}$ s^{-1}, is significantly different from the vorticity change due to the time-integrated planetary advection, $1.38(^{-0.17}_{+0.30})\times10^{-6}$ s^{-1}, at a 95% confidence level based on the estimated errors. The sum of local change plus planetary advection over the measurement period, $-1.48(^{+0.35}_{-0.48})\times10^{-6}$ s^{-1}, is of the opposite sign from the change in vorticity due to vortex stretching, $+0.72(^{+0.72}_{-0.72})\times10^{-6}$ s^{-1}.

Previous analyses of velocity measurements in this region have suggested that the low-frequency motions are manifestations of barotropic Rossby waves in which local change of vorticity is balanced by the advection of planetary vorticity. From a two-layer analysis of float trajectories, Phillips (1966) found 78% of the kinetic energy in the barotropic mode. From a modal analysis of velocity measurements on a surface mooring during MODE-0, Richman (1972) found more energy in the barotropic than the baroclinic modes although Gould *et al.* (1974) had reservations about this analysis because of possible contamination of the velocity measurements by surface mooring effects. Analyzing the same measurements used in this study, McWilliams and Flierl (1976) found that a wave fit consisting of two barotropic Rossby waves could account for 78% of the amplitude of the observed velocities. The direct test for a vorticity balance between local change of vorticity and advection of planetary vorticity, however, reveals that less than half of the local change can be attributed to planetary advection. Moreover, based on the error analysis there is a significant imbalance between local change and planetary advection so that these velocities cannot be explained in terms of barotropic Rossby waves alone. These results demonstrate the power of direct tests for vorticity balance and the conflict with the results of the wave fit points out the danger in extending a wave analysis from a description of the spatial and temporal scales of the velocity measurements to a description of their dynamics.

After consideration of planetary advection, half of the local change of vorticity remains to be balanced. From an analysis of the heat balance in this region, Bryden (1976) found that the nonlinear horizontal advection balanced local changes of temperature. Based on this result, it is likely that horizontal advection of relative vorticity, $u(\partial\zeta/\partial x)+v(\partial\zeta/\partial y)$, is important in balancing the remaining local change of vorticity. The sharpness with which the vorticity changes in time from positive to negative vorticity (Fig. 3) also suggests the importance of nonlinear effects. Unfortunately, from four moorings it is not possible to estimate the horizontal advection of relative vorticity.

To determine whether vortex stretching could balance the remaining local change of vorticity as expected for baroclinic Rossby waves, estimates of vortex stretching are made from a linear heat balance. Because temperature was measured on only two moorings, estimates of horizontal advection of temperature cannot be made. Because of the importance of horizontal advection in the heat balance in this region (Bryden, 1976), these estimates of vortex stretching are expected to have large errors. Nevertheless, the correlation between these estimates of vortex stretching and of the sum of local change of vorticity plus planetary advection is significantly different from zero suggesting a vorticity balance like that of baroclinic Rossby waves. The estimate of time-integrated vortex stretching, however, has sign opposite from that required to balance the remaining local change of vorticity. In addition, the significant correlation may be due to the expected correlation between warm water and negative vorticity above a level of no motion. Therefore from this analysis it is uncertain whether vortex stretching balances the sum of local change plus advection of planetary vorticity.

In exploring the dynamics of these low-frequency mesoscale motions, it would be valuable to estimate each of the terms in the vorticity conservation equation (5). As shown by the error analysis, the local change of vorticity and advection of planetary vorticity can be estimated at 4-day intervals with errors less than

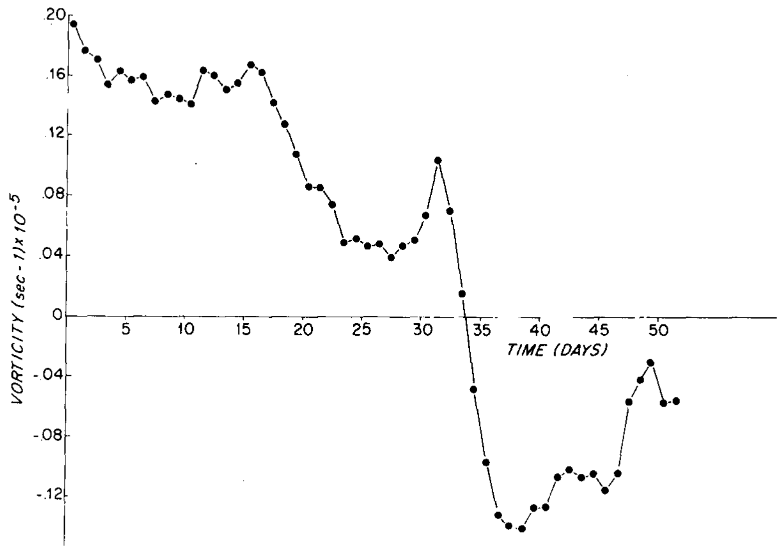

FIG. 3. Daily estimates of relative vorticity $\zeta=\partial v/\partial x-\partial u/\partial y$.

1×10^{-12} s^{-2}. It is suggested above that the horizontal divergence could be estimated with an error of 1.4×10^{-8} s^{-1} by means of a nonlinear heat balance. This implies that vortex stretching in this region ($f=0.7\times10^{-4}$ s^{-1}) can also be estimated with an error less than 1×10^{-12} s^{-2}. By using the representation in Eq. (7), the horizontal advection of relative vorticity can be estimated from a five mooring array in the shape of a plus

$$\begin{matrix} & \times & \\ \times & \times & \times. \\ & \times & \end{matrix}$$

The error due to the assumption of horizontal nondivergence to lowest order is only 0.11×10^{-12} s^{-2}. For a basic measurement separation of 50 km, an error analysis indicates that the horizontal advection of relative vorticity can be estimated with an error of 0.77×10^{-12} s^{-2} due to measurement errors in current speed and direction and an error of 20% due to sampling errors from the finite-difference procedures. Thus, all terms in the nonlinear vorticity conservation equation (5) can be estimated with errors less than 1×10^{-12} s^{-2} from a five-mooring array of current and temperature measurements.

Such estimates of terms in the vorticity balance are also useful in exploring changes in enstrophy ζ^2. Multiplying the vorticity equation (5) by vorticity yields the equation for the conservation of enstrophy

$$\frac{\partial}{\partial t}(\zeta^2/2)+(\mathbf{u}\cdot\nabla\zeta)\zeta+\beta v\zeta=fw_z\zeta. \qquad (8)$$

In conjunction with energy analysis, enstrophy analysis is useful in understanding nonlinear transfers between motions of different temporal and spatial scales (Rhines, 1975). In a baroclinic instability model, the enstrophy as well as the energy of the perturbation wave must increase. As the enstrophy increases, the enstrophy production must be positive, i.e., $fw_z\zeta-\beta v\zeta-(\mathbf{u}\cdot\nabla\zeta)\zeta>0$. For this analysis no accurate estimates of w_z or $\mathbf{u}\cdot\nabla\zeta$ are available but a correlation between planetary advection and relative vorticity is calculated to be +0.50 which is not significantly different from zero at a 95% confidence level. The positive sign of the correlation indicates negative enstrophy production which is in agreement with a decrease in the amplitude of vorticity over the observation period suggested in Fig. 3. Bryden (1976) found an indication of decay rather than growth of perturbation potential energy in this region from an analysis of the heat balance. Thus, there are indications from two sets of measurements that the MODE region is a region of decay of the low-frequency mesoscale motions, although longer-term observations are necessary to establish the statistical significance of this decay.

5. Conclusions

Estimates of horizontal derivatives of velocity are made by differencing velocity measurements made on four moorings during the MODE-0 field program. The correlation between estimates of $\partial u/\partial x$ and $\partial v/\partial y$ is -0.93 which is significantly different from zero. A linear regression indicates that these estimates of $\partial u/\partial x$ and $\partial v/\partial y$ are not significantly divergent. The errors in these derivatives are estimated to be 0.15×10^{-6} s^{-1} and within these errors the velocity field is found to be horizontally nondivergent to lowest order. Comparison of estimates of horizontal divergence calculated from these derivatives with indirect estimates of horizontal divergence from linear heat and vorticity balances indicates that horizontal divergence cannot be estimated accurately within these errors.

The errors in horizontal derivatives, however, are small enough that estimates of vorticity can be made and used to test for vorticity balance. Over the measurement period of 50 days only one-half of the local change of vorticity is balanced by the advection of planetary vorticity. Within estimated errors, this imbalance is significant so that these observations cannot be explained in terms of barotropic Rossby waves alone. Estimates of vortex stretching obtained from a linear heat balance are significantly correlated with the sum of local change of vorticity plus planetary advection. This correlation suggests a vorticity balance like that of baroclinic Rossby waves. The average vortex stretching over the measurement period, however, has sign opposite from that required to balance the remaining local change of vorticity.

An extended error analysis indicates that local time change of vorticity, advection of planetary vorticity, horizontal advection of relative vorticity and vortex stretching each can be estimated over 4-day periods with an error less than 1×10^{-12} s^{-2} from a five-mooring array of current and temperature measurements. Such estimates would be useful in understanding the vorticity and enstrophy balances for the low-frequency mesoscale motions.

Finally, a positive correlation between planetary advection and relative vorticity indicates that the enstrophy of the velocity field is decreasing during the measurement period. Although not significantly different from zero, this correlation supports the view that the MODE region is a region of decay of the low-frequency motions.

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