On Estimating Insolation over the Ocean

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ABSTRACT

The results of two studies that reported procedures for estimating insolation at sea are summarized. The insolation under clear skies may be computed reliably with a formula derived from the Smithsonian Meteoro-
logical Tables, using a transmission coefficient of 0.7, or with a formula derived by Lumb. For estimating the reduction of insolation by clouds, the factor $1 - 0.62C + 0.0019\alpha$ is suggested, where $C$ is cloud amount in tenths and $\alpha$ is noon solar altitude. Random errors of estimate within 95% confidence limits are less than 10% for mean monthly data and are about $\pm 20\%$ for weekly periods.

Heat budget studies of the oceans have long been hindered by the uncertainty in specifying the radiative flux. Atmospheric scientists have developed methods of some sophistication for computing this flux, but over the ocean the basic data (such as from upper air soundings) required as input to these models are generally lacking. Hence oceanographers use various formulas, needing only inputs such as solar altitude and cloud cover, to estimate the insolation that reaches the sea surface. Unfortunately, the results from the formulas vary so greatly that heat budget studies tend to be rather speculative exercises. This note summarizes the results of two studies (Reed, 1975, 1976) which attempted to evaluate the methods and formulas for computing insolation at sea. These reports were not widely distributed, and it was felt that the main conclusions would be of interest to oceanographers investigating the heat content of the upper ocean.

In the absence of clouds, the turbidity of the marine atmosphere generally varies over rather narrow limits as compared to air over land surfaces (Ainsworth and Monteith, 1972), which implies that a single formula might be suitable for computing insolation under clear skies over much of the world ocean. In order to test the estimates of clear-sky insolation, Reed (1975) used the data from 322 clear days at five nonurban, coastal sites in the National Weather Service radiation network (Apalachicola, Fla.; Santa Maria, Calif.; Cape Hatteras, N. C.; Astoria, Ore.; Annette Island, Alaska), and he also used some recent measurements off the Oregon coast and on a beach in northwest Africa. These data were compared to a formula given by Seckel and Beaudry (1973) that was derived from the data in the Smithsonian Meteoro-
logical Tables (List, 1958), using an atmospheric transmission coefficient of 0.7; the results are given in Table 1. The weighted mean difference between the formula and the National Weather Service data is $-2\%$, and five of the eight groups have differences of 4% or less. The relatively large difference (and large $\sigma$) at Annette Island results mainly from a few values in winter that are appreciably larger than estimates from the formula; this is believed to be the result of outbreaks of dry continental air over this site that create an atmosphere not typical of that over the open ocean. The reason remains obscure for the large difference for the second group of data at Cape Hatteras; perhaps there were undetected errors in the calibration factor of the pyranometer used for this period. The relatively large standard deviation at Apalachicola appears to result from a consistent seasonal trend in the differences that Reed (1975) concluded was the result of a seasonal reversal in the atmospheric circulation at the site.

One may obtain an estimate of the random error in an individual computed daily value from the standard deviations in Table 1. Neglecting the two largest values, which appear to result from systematic processes not typical of a marine atmosphere, the standard deviations

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**Table 1. Comparison of clear-sky insolation at various sites obtained by the National Weather Service with that computed from the Smithsonian formula.**

<table>
<thead>
<tr>
<th>Station</th>
<th>Dates</th>
<th>Number of values</th>
<th>Mean difference (formula - observed) (%)</th>
<th>$\sigma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apalachicola</td>
<td>22 Sept 61-</td>
<td>87</td>
<td>-1</td>
<td>$\pm 6$</td>
</tr>
<tr>
<td></td>
<td>11 May 62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Santa Maria</td>
<td>23 July 73-</td>
<td>55</td>
<td>+1</td>
<td>$\pm 3$</td>
</tr>
<tr>
<td></td>
<td>24 June 74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cape Hatteras</td>
<td>17 Mar 62-</td>
<td>43</td>
<td>0</td>
<td>$\pm 4$</td>
</tr>
<tr>
<td></td>
<td>24 Jan 63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>24 Sept 67-</td>
<td>42</td>
<td>$-10$</td>
<td>$\pm 4$</td>
</tr>
<tr>
<td></td>
<td>15 Sept 68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Astoria</td>
<td>18 Apr 62-</td>
<td>13</td>
<td>$-4$</td>
<td>$\pm 4$</td>
</tr>
<tr>
<td></td>
<td>5 Mar 63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 Jan 66-</td>
<td>16</td>
<td>$+6$</td>
<td>$\pm 3$</td>
</tr>
<tr>
<td></td>
<td>11 Apr 67</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30 June 67-</td>
<td>24</td>
<td>$-1$</td>
<td>$\pm 2$</td>
</tr>
<tr>
<td></td>
<td>8 May 68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annette Island</td>
<td>9 Jan 68-</td>
<td>42</td>
<td>$-7$</td>
<td>$\pm 9$</td>
</tr>
<tr>
<td></td>
<td>11 June 69</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $\sigma$ = standard deviation from the mean difference. Weighted mean difference $= -2\%$. 
are 2–4%; hence the random error of the estimate is 4–8% (two standard deviations at 95% confidence limits). For periods of several days or longer the random error of estimate should be less than 5%.

The formula given by Seckel and Beaudry (1973), hereafter called the Smithsonian formula, is shown in (1) with the values of the coefficients for two latitude belts:

\[ Q_0 = A_0 + A_1 \cos \phi + B_1 \sin \phi + A_2 \cos 2\phi + B_2 \sin 2\phi \]  

(1)

**Latitude 20°S to 40°N**

- \[ A_0 = -15.82 + 326.87 \cos L \]
- \[ A_1 = 9.63 + 192.44 \cos (L + 90) \]
- \[ B_1 = -3.27 + 108.70 \sin L \]
- \[ A_2 = -0.64 + 7.80 \sin 2(L - 45) \]
- \[ B_2 = -0.50 + 14.42 \cos 2(L - 5) \]

**Latitude 40°N to 60°N**

- \[ A_0 = 342.61 - 1.97L - 0.018L^2 \]
- \[ A_1 = 52.08 - 5.86L + 0.043L^2 \]
- \[ B_1 = -4.80 - 2.46L - 0.017L^2 \]
- \[ A_2 = 1.08 - 0.47L + 0.011L^2 \]
- \[ B_2 = -38.79 + 2.43L - 0.034L^2 \]

Here \( Q_0 \) is the clear-sky mean daily insolation (W m\(^{-2}\)), and \( \phi = (t - 21) (360/365) \), where \( t \) is time of year (days) and \( L \) the latitude.

Reed (1975) then compared the Smithsonian formula with other formulas or values that have been used for oceanic applications [Kimball, 1928; Mosby, 1936; T. G. Berland (given in Budyko, 1974); Laevastu, 1960; Lumb, 1964]. Kimball’s and Mosby’s results gave values appreciably greater (generally 20–50%) than the Smithsonian formula. A comparison of the other formulas computed for 25°N is shown in Fig. 1. Laevastu’s values are satisfactory at the lower solar altitudes but are about 20% too great at the higher altitudes; on the other hand, Berland’s values are too large at all solar altitudes, and the differences range from about 5–10% at the highest angles to over 20% at very low angles. Lumb’s formula, which was derived from an extensive set of data at the British-manned Atlantic weather stations, was the only one examined that gave consistently good agreement with the Smithsonian formula, and the agreement shown in Fig. 1 would be even better if Lumb’s formula were adjusted for seasonally varying earth-sun distance. Lumb’s (1964) formula for clear-sky conditions is

\[ Q_0 = 1353s(0.61 + 0.20\,s), \]  

(2)

where \( Q_0 \) is the clear-sky insolation (W m\(^{-2}\)) and \( s \) the sine of the solar altitude. Since this formula was derived for hourly computations, it is necessary to make the computations for each hour and average them to derive a daily mean.
Table 2. Comparison of the observed reduction in insolation $Q_s/Q_o$ versus that computed from Eq. (3) based on data at the National Weather Service stations.

<table>
<thead>
<tr>
<th>Station</th>
<th>Dates</th>
<th>Difference (computed - observed, $\pm$)</th>
<th>$\sigma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swan Island</td>
<td>Jan–Nov 64</td>
<td>-1 $\pm$3</td>
<td></td>
</tr>
<tr>
<td>Cape Hatteras</td>
<td>Apr–Dec 62</td>
<td>+1 $\pm$2</td>
<td></td>
</tr>
<tr>
<td>Astoria</td>
<td>May 62–Feb 63</td>
<td>-3 $\pm$7</td>
<td></td>
</tr>
<tr>
<td>Astoria</td>
<td>July 67–Apr 68</td>
<td>+2 $\pm$4</td>
<td></td>
</tr>
</tbody>
</table>

* $\sigma$ = standard deviation from the mean difference.

The major variations in insolation are caused by clouds, and a cloud factor must be used to adjust the clear-sky insolation. After noting considerable disagreement among the various factors that have been proposed, Reed (1976) examined 40 months of data at three coastal sites in the National Weather Service network (Swan Island, Caribbean; Cape Hatteras, N. C.; and Astoria, Ore.) and computed the following empirical relation:

$$Q_s/Q_o = 1 - 0.62C + 0.0019\alpha,$$ (3)

where $Q_s$ is the insolation under cloudy conditions, $Q_o$ the insolation under clear skies from the Smithsonian formula, C cloud cover (tenths) and $\alpha$ is noon solar altitude. The fit of this relation to the data for two groups of months with similar noon solar altitude at Swan Island is shown in Fig. 2, and statistics for the Weather Service stations are given in Table 2. It is apparent that the mean differences are quite small; the largest difference (and standard deviation) is at Astoria in 1962–63, and it results mainly from the data for two winter months with unusually high insolation, probably as a result of continental air over the site. The linear relation (3) appears to be valid for cloud cover from 0.3 to 1.0, and the random error of estimate (ignoring the largest value, $2\sigma = 4–8\%$) in monthly mean insolation should be less than $\pm 10\%$ within 95% confidence limits.

In addition, Eq. (3) was checked against 125 days of data recently obtained at sea aboard the NOAA ship Oceanographer in the eastern Pacific from the tropics to the Gulf of Alaska. In the mean, computed insolation was 2% less than observed insolation for 14 periods (ranging in length from 5 to 17 days, mean cloud cover $> 0.67$ except for one period, $2\sigma = 18\%$); five of

![Fig. 3. Comparison of the reduction in insolation $Q_s/Q_o$ and cloud cover (tenths) computed by the formulas of Berliand, Kimball, Laevastu, Tabata and Eq. (3). Results from Tabata’s formula and Eq. (3) are shown for solar altitudes of 40° and 60°.](image-url)
the observed values were less than those computed, and eight of those observed were greater than those from Eq. (3). Thus it appears that mean insolation for periods as short as one week could be computed with a random error of estimate of only about ±20%.

A number of cloud factors have been previously proposed with perhaps the most widely used being the following:

Kimball (1928) \[ Q_v/\theta_0 = 1 - 0.71C \]
T. G. Berland, 1960 \[ Q_v/\theta_0 = 1 - aC + 0.38C^2, \text{a varies with latitude} \]
Laevastu (1960) \[ Q_v/\theta_0 = 1 - 0.60C^3 \]
Tabata (1964) \[ Q_v/\theta_0 = 1 - 0.716C + 0.00252\alpha \]
Lumb (1964) eight formulas for various cloud conditions.

These formulas, except for those of Lumb, are compared in Fig. 3. Kimball's linear factor and Berland's nonlinear formula both appreciably underestimate the insolation received at see; this appears to at least partially result from the fact that they were derived primarily from data over inland terrestrial locations where height and density of clouds are generally different than at sea (see, e.g., Holle and MacKay, 1975). Laevastu's nonlinear formula gives results that are greater than from Eq. (3) except at large cloud amounts. Tabata's formula has the same form (i.e., the reduction in insolation is a linear function of cloud amount and solar altitude) as (3), but it gives somewhat smaller values at large cloud amounts; this seems to be caused by his use of clear-sky values that were systematically larger than those used in deriving Eq. (3). Finally, Lumb's formulas, which are also functions of cloud amount and solar altitude, were compared with the data from the Oceanographer with agreement generally within 10%; their use, however, requires very reliable information on cloud amount and type, which perhaps makes them unsuitable for use with routine ship's observations.

Eq. (3) appears to be appropriate for use from the tropics to high latitudes and seems valid for cloud covers of 0.3 and greater. For mean cloud amounts 0.2 and less, the reduction in insolation can be neglected for practical purposes. In the event of large amounts of cirrus clouds in the absence of other types, a reduction of about 5% is suggested rather than use of Eq. (3). It should be stressed that (3) was derived with the use of visual cloud estimates; comparison of visual estimates with estimates from satellite (NOAA 4 and SMS 2) photographs for the 125 days of observations aboard the Oceanographer showed that the satellite-derived values were consistently about 0.20 less than visual amounts. Consequently, data from satellites should be adjusted for use with Eq. (3).

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REFERENCES