The Arrested Topographic Wave

G. T. Csanady

Woods Hole Oceanographic Institution, Woods Hole, Mass. 02543

(Manuscript received 11 June 1976, in final form 8 August 1977)

ABSTRACT

The mean circulation in a coastal zone of variable depth may under certain circumstances be modeled by linear equations, including a bottom friction linear in the depth-averaged velocity. The resulting steady-state problem is similar to the problem of topographic wave generation by wind. The equation governing the pressure field has the form of a one-dimensional heat conduction equation, with longshore distance in the direction of topographic wave propagation playing the role of time. The effects of wind stress or of freshwater influx are felt only over the forward portion of a long shelf (i.e., that portion to which topographic waves propagate from the source region).

Various simple solutions can be written down for the arrested topographic wave problem in virtue of the heat conduction analogy. They generally show the presence of a pressure field trapped in a nearshore band. For periodic wind stress, for example, the scale width of the trapped pressure field is $L = (2\pi/\beta k)^{1/2}$, where $r$ is a bottom resistance coefficient (dimension velocity), $\beta$ the Coriolis parameter, and $k$ the longshore wavenumber of the wind stress and a bottom slope, assumed constant.

The model is applied to the eastern North American continental shelf in an attempt to determine the physical origin of the southwestward pressure gradient which has recently been identified as the main driving force of the observed longshore flow. It is shown that neither wind stress nor freshwater influx can reasonably be held accountable for this pressure gradient. On the other hand, a longshore surface elevation gradient imposed at the shelf break by the dynamics of a deepwater current produces exactly the type of response observed over the shelf.

The same model also accounts for the mean circulation in winter of Lake Ontario, as well as for some features of the summer circulation.

1. Introduction

Observations in Lake Ontario during the International Field Year on the Great Lakes (IFYGL) have suggested that a succession of topographic waves set up by a number of wind-stress impulses of mostly the same direction can establish a mean pressure field in the nearshore zone in such a way as to oppose the mean wind stress over a substantial portion of the shoreline. The mean longshore pressure gradient then sustains mean flow opposing the mean wind.

Wavelike motions do not produce a mean pressure gradient or mean flow in a linear theory unless they are subject to decay. In the case of the topographic waves observed in Lake Ontario, the decay period ($e$-folding time) is shorter than the wave period of about 14 days. The strong eastward coastal currents on the south shore generated by similarly directed storms propagate cyclonically to the north shore in a few days, where they become strong westward currents. They do not survive, however, for another half-cycle to allow significant eastward currents to develop again. The longshore currents in these waves are generated by a pressure field trapped within a coastal zone of 15 km width or so, which propagates cyclonically, and of course also decays. The net result is apparently a mean westward pressure gradient established on part of the north shore by a sequence of eastward storms.

To the extent that topographic waves conform to linear models [and they can be described by such models remarkably well (Csanady 1976a)], a succession of them should produce a result which can also be modeled by linear theory, by a steady-state solution representing mean wind-driven flow. The solution should have characteristics similar to topographic waves, i.e., it should be trapped in the shore zone. Friction, of course, must play an essential role in such a model.

A coastally trapped frictional flow model is not exclusively valid for enclosed shallow seas, but should be equally useful to model mean flow along open oceanic shelves. Topographic waves along the west coast of North America have been identified by Cutchin and Smith (1973) and in view of Lake Ontario experience, one immediately wonders about their average effect. Similarly,
Beardsley and Butman (1974) have shown that relatively large longshore pressure gradients appear during some storms along the east coast of North America. If these should be associated with decaying topographic waves, as one would expect, their average influence along the irregular east coast may be important. It is not inconceivable, for example, that the mean longshore pressure gradient which appears to drive east coast shelf waters southward (Stommel and Leetmaa, 1972; Scott and Csanady, 1976) is partly a shore-trapped field generated by decaying topographic waves. This would make the longshore pressure gradient part of the large-scale response of the shelf region to wind stress. Freshwater influx over a larger portion of the shelf may also produce a response having the character of a shore-trapped pressure field over a larger piece of the coast.

Any frictional steady-state circulation model must of course have some relationship to classical Ekman models. As pointed out by Welander (1957), problems of this kind may often be attacked in two parts: one, the "local" problem, by calculating the distribution of velocity with depth for prescribed horizontal pressure gradient, and two, the "global" problem, by calculating the pressure field over an entire basin or other identifiable region. The problems raised above call for the second type of calculation, which may be accomplished using depth-averaged equations of motion. By a specific set of idealizations a "global" type of Ekman model is arrived at below, which turns out to be useful in discussing the dynamics of the coastal zone and has a strong connection to simple topographic wave models.

2. Conceptual model

Consider a long and straight coastline, and choose coordinates so that the y axis coincides with the coast, positive x pointing offshore. As shown in Fig. 1, we suppose that the depth h is only a function of offshore distance, i.e., \( h = h(x) \) and that significant depth variations are confined to a nearshore band of width \( l \). This is a realistic idealization of the coastal zone of the Great Lakes or of the ocean in several locations, including, for example, the eastern and western North American continental shelves. The order of magnitude of the coastal zone width \( l \) may be 10 or 100 km, the depth at its outer edge of the order of 100 m.

It is desired to model mean flow (a monthly mean, say) along such a coast driven by the mean wind stress. The longshore component of the latter is expected to be the main driving force. The longshore mean wind stress will be regarded as varying over distances of order \( k^{-1} \), where \( k^{-1} \) is much larger than the width \( l \) of the coastal zone. The approximation

\[
kl \ll 1
\]

has been used by Gill and Schumann (1974) in a particularly illuminating study of topographic wave generation by wind. Although from the physical point of view the steady-state flow problem discussed below is rather different, mathematically it has some close similarities to the work of Gill and Schumann.

The water column will at first be assumed homogeneous and bottom friction will be taken to be proportional to the depth-average velocity. This is realistic if the water column is well mixed. A linear friction law is adequate for the mean flow problem, even though a quadratic friction law applies to the instantaneous velocity (Csanady, 1976b).

The linear friction hypothesis has the effect that the role of the acceleration term in the topographic wave problem is taken over in the steady-state problem by a linear bottom friction term. The dynamical equations for the depth-averaged velocities become [in close analogy with those written down by Gill and Schumann (1974)]

\[
\begin{align*}
-f\bar{v} &= -g \frac{\partial \bar{z}}{\partial x} \\
-f\bar{u} &= -g \frac{\partial \bar{z}}{\partial y} + \frac{F}{h} - \frac{ru}{h} \\
\frac{\partial (u\bar{h})}{\partial x} + \frac{\partial (v\bar{h})}{\partial y} &= 0
\end{align*}
\]

(1)

The notation is conventional, with \( F(y) \) the longshore component of the (kinematic) wind stress, a function of long-shore distance only, given the narrow width of the coastal zone, \( l \ll k^{-1} \). The kinematic bottom stress is \( ru \), so that \( r \) is a bottom resistance coefficient of the dimension of velocity.
The first of Eqs. (1) is valid only if the offshore component of the bottom stress \( F_{ox} \) is negligible compared to the Coriolis force of longshore flow, i.e., if

\[
|F_{ox}| \ll |fuh|.
\]  

(2)

Given a strong longshore flow, this is a reasonable approximation, as conventional estimates of turbulent friction quickly show.

Eliminating \( u \) and \( v \) from (1) a single equation for the surface elevation \( \zeta \) results:

\[
\frac{\partial^2 \zeta}{\partial x^2} + \frac{f}{r} \frac{dh}{dx} \frac{\partial \zeta}{\partial y} = 0.
\]

(3)

The boundary condition at the shore is that the normal transport vanishes, i.e., \( uh = 0 \). The second of \( \exists \) Eqs. (1) then shows that because the depth also vanishes at the coast, we have

\[
F = rv \quad (x = 0)
\]

(4)

which may be converted, using the first of Eqs. (1), into a condition on \( \zeta \), i.e.,

\[
\frac{\partial \zeta}{\partial x} = \frac{fF}{rg} \quad (x = 0).
\]

(4a)

At first, a steady-state analog of the topographic wave will be sought, i.e., a solution trapped at the shore. This is achieved by prescribing vanishing surface elevation at infinity as the second boundary condition:

\[
\zeta = 0 \quad (x \to \infty).
\]

(5)

In view of the close analogy with the equations governing topographic wave generation and propagation, it is appropriate to refer to this steady-state model as the “arrested” topographic wave.

3. Some general properties of the solutions

Qualitative features of the flow field in an arrested topographic wave may be elucidated from general arguments based on boundary conditions and other constraints given an arbitrary depth distribution \( h(x) \). The boundary condition at infinity and the first of Eqs. (1) imply that \( v = 0 \) as \( x \to \infty \), while the second then yields

\[
u = \frac{F}{fh} \quad (x \to \infty)
\]

(6)

which is the familiar Ekman flux.

An integration of the continuity equation leads to

\[
\frac{d}{dy} \int_0^\infty vhdx = - \frac{F}{f}
\]

(7)

For example, if \( F = F_x \), cosky,

\[
\int_0^\infty vhdx = - \frac{F_x}{k} \sin ky + \text{constant},
\]

(8)

independently of the depth distribution. Thus the total longshore transport across any \( y = \text{constant} \) section is independent of bottom topography, as is the longshore velocity \( v \) at small \( x \) or the offshore transport at large \( x \).

These constraints determine the general character of the transport streamline pattern. In view of the third of Eqs. (1) a transport streamfunction may be introduced by

\[
uh = \frac{\partial \psi}{\partial y}, \quad vh = - \frac{\partial \psi}{\partial x}.
\]

(9)

Eqs. (6)–(8) imply that, while the streamlines \( \psi = \text{constant} \) may be distorted in various ways over complex bottom topography, the total number of streamlines (range of \( \psi \) variation) and their asymptotic spacing both nearshore and at large distances from the shore remains the same for a given wind-stress field. Even a highly idealized coastal zone model should therefore give a realistic picture of the arrested topographic wave flow field. Physically, the constraints dictate that near shore the water moves alongshore, with the wind, while far from shore it moves perpendicular to the wind. The solution of Eqs. (1) determines the details of how the transition takes place. More important, these equations also determine how far the “coastal constraint” reaches: this is meant here to denote the condition of zero net onshore or offshore transport. In some simple “local” Ekman models of coastal circulation this constraint is imposed on the flow, although of course its validity is limited to some nearshore band. The solution of the arrested topographic wave problem should provide guidance in regard to the range of validity of such approximations.

The vorticity balance of the flow in the arrested topographic wave follows on taking the curl of Eqs. (1):

\[
\frac{\partial}{\partial x} \left( \frac{F}{h} - \frac{rv}{h} \right) + \frac{fu}{h} \frac{dh}{dx} = 0.
\]

(10)

The first term is the curl of wind stress minus bottom stress, which is balanced by the vortex stretching term consequent upon offshore flow to maintain a steady vorticity distribution.

Suppose that the coastal zone is approached from the open sea and assume for concreteness that \( dh/dx = 0 \) beyond some \( x = l \). Over the constant depth portion of the ocean Eq. (3) shows immediately that \( \zeta = \partial \zeta/\partial x = 0 \). The boundary condition at infinity in this case must be imposed at \( x = l \) where the depth begins to decrease. According to Eq. (6), a certain offshore velocity is imposed at
the outer edge of the coastal zone, which determines the vortex stretching term in Eq. (10). This equation may then be solved for \( \partial \theta / \partial x \), which can be used as a first step in a numerical integration procedure of Eqs. (1). In this sense the offshore Ekman flux at the outer edge of the coastal zone may be thought to force the flow distribution in the arrested topographic wave by the mechanism of vortex stretching, although of course this is a somewhat limited point of view.

4. The heat conduction analogy

Rather more insight into the properties of arrested topographic waves is gained by noting that Eq. (3) has the form of the heat conduction equation in one dimension, with negative \( y \) playing the role of time. The analog of thermometric conductivity is \( \kappa = r/(f \, dh/dx) \) and the analogy is close over a beach of constant slope, \( dh/dx = s = \) constant, because then \( \kappa \) is constant. Negative \( y \) is also the direction of propagation of topographic waves: according to the results of Gill and Schumann (1974), a given initial \( \zeta \) distribution propagates to negative \( y \) without change of shape, in a frictionless model. In the present steady-state model the same disturbance spreads out in the \( x \) direction, as one follows it along negative \( y \), in the manner of a hot spot spreading out in a conducting slab.

The analogy suggested by Eq. (3) is between surface elevation and temperature, negative \( y \) direction and time. According to the first of Eqs. (1) the longshore velocity \( v \) is proportional to the elevation gradient, which makes it the analog of heat flux. The boundary condition at the shore [Eq. (4)] is thus equivalent to a flux condition, the longshore wind stress being the analog of a prescribed heat flux at one surface of a conducting slab. The boundary condition at infinity [Eq. (5)] is equivalent to saying that the temperature far from the end surface of the semi-infinite slab remains constant.

From Eq. (7) and the first of Eq. (1) it is easily shown that

\[
\frac{d}{dy} \int_{0}^{\infty} \frac{\zeta}{x} \frac{dh}{dx} \, dx = - \frac{F}{g} .
\]

If \( dh/dx = s = \) constant and \( F = 0 \), the total excess volume of fluid present (the analog of the total heat content) remains conserved, i.e.,

\[
\int_{0}^{\infty} \zeta \, dx = \text{constant} \quad \left( \frac{dh}{dx} = \text{constant}, \; F = 0 \right).
\]

Any excess volume which may be present in the shore zone initially, due to any past cause, spreads out gradually as one follows the coast in the negative \( y \) direction, if along this portion of the coast there is no longshore stress (corresponding to zero heat flux at the \( x = 0 \) surface of a conducting slab or semi-infinite solid).

It is now easy to state precisely what is necessary to specify fully the mean elevation field and hence the mean flow field in the coastal zone considered. Boundary conditions at \( x = 0 \) and \( x \rightarrow \infty \) were seen to require the prescription respectively of wind stress and of sea level far from shore. The boundary condition at infinity may also be regarded as quantifying the interaction of the shelf with the open ocean. The specific condition stated in Eq. (5) prescribes the absence of any such interaction, which, of course is a particularly simple special case.

For a full solution of a heat conduction problem it is necessary to specify initial conditions as well, which translate into a \( \zeta(x) \) distribution at \( y = 0 \). By the first of Eqs. (1) this implies inflow or outflow of water through the \( x \) axis. Physically, this may be regarded as parameterizing the interaction of the part of the coastal zone considered, i.e., the semi-infinite strip \( y \leq 0 \), with the part \( y > 0 \). Note, however, that no boundary conditions at some second section \( y \leq 0 \) need (or can) be prescribed, on account of the parabolic nature of Eq. (3).

A prescribed initial temperature distribution, with no further heat flux into a conducting slab, spreads out in time. Eventually, the details of the initial distribution become unimportant, the only initial parameter which matters being the total heat content. In a semi-infinite solid the temperature becomes one-half of a Gaussian distribution, as if an instantaneous heat source at \( x = 0 \) had generated the field. Carrying this argument one step further, it is not difficult to see that a fairly complex initial temperature distribution may be the present effect of past heat flow through the exposed surface of a semi-infinite solid, appropriately distributed in time. The coastal zone analogy is a certain distribution of wind stress at \( y > 0 \), producing some specific elevation distribution \( \zeta(x) \) at \( y = 0 \), and a corresponding longshore velocity distribution.

In referring to the \( y \geq 0 \) region, one has the temptation to speak of "upstream" sources or sinks, but of course if they are predominantly sinks, the flow is in the positive \( y \) direction and the region \( y \geq 0 \) is downstream. It is upstream only from the point of view of a Kelvin or topographic wave traveling alongshore. To simplify the terminology, \( y \geq 0 \) will be referred to as the backward and \( y \leq 0 \) as the forward portion of the coastline, from the point of view of the cross section at \( y = 0 \).
5. Periodic wind stress

One set of circumstances under which the potentially troublesome initial conditions become unimportant is when history repeats itself, i.e., when the heat flux is periodic. The coastal zone analogy of periodic longshore wind stress (periodic in y) may be thought to be a more or less realistic idealization for two reasons: 1) the mean wind stress itself varies in space over distances of the order of 1000 km and 2) the orientation of most coastlines, even if smoothed to retain large-scale features only, changes with a wavelength not much longer than a few hundred kilometers. For the eastern or western North American continental shelves, assuming that the longshore wind stress is actually periodic, is of course a considerable idealization, amounting to a specific assumption of the distribution of \( \zeta \) sources over the backward portion of the coastline. The same idealization is perhaps most realistic for closed basins such as the Great Lakes or the Baltic Sea. In any case, periodic solutions are likely to offer some illuminating insight.

Periodic solutions of Eq. (3) are readily obtained by the method of the separation of the variables, the same method as used by Gill and Schumann (1974). Writing

\[
\zeta = Z_1(x) Z_2(y),
\]

one finds on substitution into Eq. (3) that in order for the separation to be possible

\[
\frac{Z_2}{Z_3} = \frac{r}{f h'} \frac{Z_1'}{Z_1} = \text{constant},
\]

where primes denote differentiation. The constant has the reciprocal dimension of length and it clearly scales the variation of \( \zeta \) alongshore. Given a periodic longshore wind stress of wavenumber \( k \), it is plausible to write

\[
Z_2 = e^{inky} \quad (n = 1, 2, 3, \ldots).
\]  

This describes the longshore variation of surface elevation, while the offshore distribution follows from

\[
\frac{d^2 Z_1}{dx^2} + \frac{inkf}{r} \frac{dh}{dx} Z_1 = 0.
\]

The boundary condition at infinity [Eq. (5)] requires

\[
Z_1 = 0 \text{ as } x \to \infty,
\]

while the boundary condition at the shore \( x = 0 \) [Eq. (4)] yields

\[
F = \frac{rg}{f} Z_1(0)e^{inky},
\]

a condition easily satisfied if

\[
F = F_0 e^{inky}.
\]

Since any reasonable periodic longshore distribution of wind stress may be represented by a series of terms of the above form, Eq. (18) will be used; thus the subsequent calculations are valid for a particular Fourier component of the wind stress. This then yields the boundary condition on the \( Z_1(x) \) distribution, i.e.,

\[
\frac{dZ_1}{dx} = \frac{fF_0}{rg} \quad (x = 0).
\]

![Fig. 2. Distribution of longshore wind stress and surface elevation at the shore.](image)
6. Solution for inclined plane beach

Eq. (15) may now be solved with the boundary conditions (16) and (19). With an arbitrary depth distribution \( h(x) \) a solution could be found by numerical means, but several simple shore zone models lead to easy analytical solutions. The beach of constant slope offers special simplicity, i.e.,

\[
h = sx,
\]

where \( s = \) constant. For this simple depth distribution the solution of Eq. (15) satisfying the boundary condition at infinity is

\[
Z_1 = A \exp[-(1 - i)(x/L)],
\]

where \( L = (2\pi/kfs)^{1/2} \) is a length-scale of the offshore distribution and \( A \) a complex constant, the value of which follows from Eq. (19):

\[
A = \frac{1 + i}{\sqrt{2}} \frac{F_n}{g} \left( \frac{f}{rks} \right)^{1/2}.
\]

With the aid of Eq. (1) all other variables may now be expressed in terms of \( F_n \). The fundamental wave, \( n = 1 \), of the stress distribution, for example, gives rise to the following pattern of surface elevation and flow, now in terms of real variables:

\[
\begin{align*}
\frac{\zeta gs}{F_1} &= \frac{2}{kL} \ e^{-x/L} \sin \left( ky + \frac{x}{L} - \frac{\pi}{4} \right) \\
\frac{uhf}{F_1} &= \cos ky - e^{-x/L} \cos \left( ky + \frac{x}{L} - \frac{\pi}{4} \right) \\
&- \sqrt{2} \frac{x}{L} e^{-x/L} \cos \left( ky + \frac{x}{L} - \frac{\pi}{4} \right)
\end{align*}
\]

\[
\begin{align*}
\frac{vr}{F_1} &= e^{-x/L} \cos \left( ky + \frac{x}{L} \right) \\
\frac{\psi fk}{F_1} &= \sin ky - e^{-x/L} \sin \left( ky + \frac{x}{L} \right) \\
&- \sqrt{2} \frac{x}{L} e^{-x/L} \sin \left( ky + \frac{x}{L} - \frac{\pi}{4} \right)
\end{align*}
\]

The transport streamfunction \( \psi \) has been defined in Eq. (9). From the solution it may be seen that, at the shore \((x = 0)\), the longshore velocity distribution is in phase with the wind stress. The physical reason is that, by the second of Eqs. (1), in very shallow water the wind stress must be balanced by the bottom stress, both longshore pressure gradient and Coriolis force of offshore flow being negligible in comparison. The bottom stress is, of course, in phase with the longshore velocity.

The longshore pressure gradient leads the longshore velocity by \( 3\pi/4 \), which means that at the shore the pressure gradient force opposes the wind stress over 75% of the shoreline (Fig. 2). Constant phase lines of surface elevation and longshore velocity have an inclination of \( \tan^{-1}kL \) against the \( x \) axis. Both \( \zeta \) and \( v \) decay to negligible values within a distance of order \( L \) from the coast.

The variation of offshore velocity and transport streamfunction is more complex. Far from the coast the transport becomes the offshore Ekman transport corresponding to the local longshore wind stress.

The \( x \) and \( y \) length scales in Eqs. (23) are different, and their ratio \( kL \) determines to what extent the streamlines are squeezed together near the coast. Once \( kL \) is fixed, however, Eqs. (23) may be illustrated by nondimensional contours, valid for any specific case characterized by the same \( kL \). Calculations have been made for \( kL = 0.183 \), which is four times the value of this parameter appropriate for Lake Ontario. Streamline and other diagrams for this value of \( kL \) may also be regarded as illustrations of the Lake Ontario coastal zone response to wind stress, but with the offshore length scale exaggerated four times. Figs. 3, 4 and 5 contain the results of these calculations, showing, respectively, the longshore velocity versus offshore distance, the pattern of transport streamlines and the distribution of surface elevation.

These figures illustrate how the flow satisfies the various integral constraints and boundary conditions discussed earlier. At the shore the flow is downwind, longshore velocity and wind stress being in phase. Over much of the coastline, a "return" flow region appears further offshore, where the flow is against the wind. This is driven by the longshore pressure gradient, which opposes the wind over much of the coastline. Far from shore the transport distribution becomes the Ekman drift, offshore velocity being here in phase with wind stress.

It is noteworthy that an intrinsic offshore length-scale \( L \) arises in the theory even for a beach sloping down indefinitely offshore. The definition [Eq. (21)] shows \( L \) to be the geometric mean of (twice) the analog of thermometric conductivity \([tr/fs]\), which has the physical dimension of length on account of the fact that the analog of time is here longshore distance \( L \) and the longshore length scale of the forcing, \( k^{-1} \). The distance \( L \) scales the offshore extent of the coastal influence in case of periodic forcing.

7. Wind stress along part of the coast

The periodic wind stress model is not very useful in dealing with pieces of an open coastline
between sudden changes in direction, which is the usual situation on continental shelves. The flow field in this case is best idealized as being due to a few sectors of piecewise constant longshore stress. To the accuracy of a linear model, each sector may be treated separately and the resultant flow field determined by linear superposition. In each sector a square half-wave wind stress is assumed, with constant $F$ along a portion of the coast, zero elsewhere. For each sector the initial condition may be taken to be zero influx across the $x$ axis, i.e., $\zeta(x) = 0$ at $y = 0$ for all $x$. After all the sectors are summed, the actual initial condition for any given sector arises as the sum of flows induced by all backward sectors.

An inclined plane beach is again assumed so that the heat conduction analogy may be used in its simplest form. The boundary conditions are equivalent to constant heat flux at $x = 0$ for a finite period, and zero heat flux afterward: constant

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**FIG. 3.** Variation of longshore velocity with distance from shore, at two longshore locations. Note "return" flow at larger distances from shore.

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**FIG. 4.** Pattern of streamlines in $x$-$y$ plane for scale ratio $kL = 0.183$. Values on contours are of $u/kF_1$. 
wind stress is prescribed for $-Y < y < 0$, zero stress for $y < -Y$.

The solution of the relevant one-dimensional heat conduction problem is known (Carslaw and Jaeger, 1959, p. 76) and is easily adapted to become the arrested topographic wave solution. Surface elevation and longshore velocity are

\[
\begin{align*}
\zeta &= -\frac{2F_0}{rg} \left[ (\kappa y)^{1/2} \operatorname{erf} \left( \frac{x}{2(\kappa y)^{1/2}} \right) \right] \\
&\quad - \left[ (\kappa (Y - y))^{1/2} \operatorname{erf} \left( \frac{x}{2(\kappa (Y - y))^{1/2}} \right) \right] \\
v &= \frac{F_0}{r} \left[ \operatorname{erfc} \left( \frac{x}{2(\kappa y)^{1/2}} \right) \\
&\quad - \operatorname{erfc} \left( \frac{x}{2(\kappa (Y - y))^{1/2}} \right) \right]
\end{align*}
\]

(24)

This is valid for $y \leq -Y$; for $Y < y \leq 0$ only the first term in the braces is present. In this expression, as before, $\kappa = r/f_s$ and $F_0$ is the constant amplitude of wind stress, positive along positive $y$, which produces a negative surface elevation disturbance. Eq (24) may be rewritten in a non-dimensional form, when the distributions depend only on the parameter $Y/\kappa$. Length scales of $x$ and $y$ are $\kappa$, the velocity scale is $F_0/r$, and $\zeta$ is scaled by $F_0/sg$. In Fig. 6 Eq. (24) is illustrated for $Y/\kappa = 180$, or, taking typical east coast parameters listed in Table 1, 1800 km.

As may be seen from this illustration, the longshore pressure gradient opposes the wind stress over its entire range of action, but its sign reverses at $y = -Y$ and in effect it replaces the wind stress as the driving force of the flow at greater negative $y$. The longshore velocity is maximum at the shore where the wind stress acts, but it drops to zero at the shore, and reduces in a growing nearshore band where the wind stress is no longer driving the flow. In terms of the heat conduction analogy, one may think of the temperature (surface elevation) equalizing and spreading out, while the heat flux (longshore velocity) drops to zero at the coast and diminishes gradually to zero in a growing nearshore band.

The accompanying pattern of streamlines has not been drawn but it is straightforward: for negative $F_0$ there is inflow over the square wave, and a longshore, boundary-layer-type flow along negative $y$ for $y < -Y$, the width of the boundary layer growing gradually as $(-y)^{1/2}$. There is also an internal boundary layer, within which the longshore velocity decays, of a width growing as $(-y + Y)^{1/2}$. There is no flow to the backward side of the square wave, to positive $y$; all the inflow turns into the forward direction. For positive $F_0$ the same flow pattern holds with the arrows reversed—outward flow over the square wave, supplied by a boundary-layer-type flow from the forward portion of the coastline.

A negative wind stress acts as a $\zeta$ source, a positive one as a $\zeta$ sink. Both affect flow only on the forward portion of the coastline. However, a source produces forward flow, a sink backward flow. The reader is reminded that in the Northern Hemisphere, forward flow leaves the coast to the right, backward flow to the left.

8. Inflow or outflow across backward boundary

As remarked before, the full solution of Eq. (3) for a given strip $y_i < y < 0$ of a coastline contains the influence of inflow or outflow through the "backward" boundary, $y = 0$. If the surface elevation distribution along this boundary is $\zeta_b(x)$, it induces a $\zeta$ field in the absence of wind stress (corresponding to an insulated surface boundary condition) given by the following well-known solution of the heat conduction equation (Carslaw and Jaeger, p. 56):

\[
\zeta = \frac{1}{2(\pi \kappa y)^{1/2}} \int_0^\infty \zeta_b(x') \left[ \frac{\exp \left( \frac{(x - x')^2}{4\kappa y} \right)}{4\kappa y} + \frac{\exp \left( \frac{(x + x')^2}{4\kappa y} \right)}{4\kappa y} \right] dx',
\]

(25)

valid for $y < 0$ of course. The inflow or outflow is related to $\zeta_b(x)$ through the first of Eqs. (1).

If significant $\zeta_b$ variations are confined to a strip $x \leq l$, than at distances large compared to $l^{1/\kappa}$ Eq. (25) approaches the simple source solution.
9. Freshwater influx

To broaden the conceptual model formulated in connection with Eqs. (1), it is possible to include the influence of another $\zeta$ source, freshwater influx at the shore. Such a step makes the present model the global counterpart of the local Ekman model discussed in Csanady (1976b).

As in the local model, the admixture of freshwater at the shore is supposed to result in small density variation $\rho'$, the total density being $\rho_0 + \rho'$ with $|\rho'| \ll \rho_0$. The density deficiency $\rho'$ due to freshening is supposed uniform over the depth of the water column, and to vary at a constant rate in the offshore direction, i.e.,

$$\frac{1}{\rho_0} \frac{d\rho'}{dx} = \frac{1}{\mathcal{L}} = \text{constant},$$

where $\mathcal{L}$ is the offshore length-scale of the density distribution.

As discussed in greater detail in Csanady (1976b) these idealizations are realistic for winter conditions on the eastern North American continental
shelf, when horizontal mixing is accomplished by tides and storms, more or less independently of the mean circulation.

In the dynamical equations (1) the offshore pressure gradient was represented by the term involving surface slope, \( \partial \zeta / \partial x \). With a horizontal density gradient present, there is also a contribution to the pressure gradient which is linear with depth, being \( gh/L \) at the bottom, zero at the surface. The depth-average value is \( gh/2L \), and this has to be added to the right of the first of Eqs. (1) to give

\[
-fv = -g \frac{\partial \zeta}{\partial x} - g \frac{h}{2L}.
\]

(27)

When the steps leading to Eq. (3) are again carried out, with the first of Eqs. (1) replaced by Eq. (27), the result is

\[
\frac{\partial^2 \zeta}{\partial x^2} + \frac{1}{\kappa} \frac{\partial \zeta}{\partial y} = - \frac{1}{2L} \frac{dh}{dx},
\]

(28)

where \( \kappa = \left( \frac{r}{f(dh/dx)} \right)^{-1} \) is the same analog of thermometric conductivity as used above.

The nonhomogeneous term on the right of Eq. (28) is the analog of an internal heat generation term in the heat conduction equation [Carslaw and Jaeger, (1959, p. 78); with a positive sign it would represent heat absorption]. Freshening at the shore therefore acts as a distributed positive \( \zeta \) source.

For a beach of constant slope, \( dh/dx = s \), the conductivity is a constant \( (\kappa = r/f s) \) and the distributed source strength is \( ks/2L = r/2fL \), also constant. If the offshore density gradient is confined to a coastal strip \( 0 \leq x \leq l \), at \( y = 0 \), this distributed \( \zeta \) source generates a surface elevation distribution (Carslaw and Jaeger, p. 80)

\[
\zeta = -\frac{ry}{2fL} \left[ \frac{l}{2(-\kappa y)^{1/2}} \right] \left( 1 - 2i^2 \text{erfc} \frac{l-x}{2(-\kappa y)^{1/2}} \right.
\]

\[
- 2i^2 \text{erfc} \frac{l+x}{2(-\kappa y)^{1/2}} \left( 0 \leq x \leq l \right)
\]

(29)

where \( i^2 \text{erfc} \) is the twice-integrated complementary error function.

This solution may be expected to have artificial features near \( y = 0 \) where the shore-side freshening abruptly starts. However, at large \( (-\kappa y)^{1/2} \) it should give a realistic picture of the effect of freshwater influx on a long shelf. At such large distances the nearshore \( \zeta \) distribution may be shown to be approximately

\[
\zeta = (\frac{sl}{L \pi^{1/2}})(-\kappa y)^{1/2} - (\frac{sx^2}{4L}).
\]

(30)

This gives \( \partial \zeta / \partial x = -sx/2L = -h/2L \), so that according to Eq. (27) the depth-integrated longshore flow vanishes. Physically, what happens is that a surface elevation distribution is set up such that the geostrophic flow points to negative \( y \) near the surface, positive \( y \) near the bottom. Vigorous vertical mixing then eliminates significant longshore flow in either direction.

Nearshore freshening thus produces no net longshore flow along a long shelf, even though it raises surface elevations near shore. This may still act as a \( \zeta \) source, generating longshore flow to negative \( y \), for a forward portion of the coastline where significant horizontal density gradients no longer exist, or where such gradients are less than over the backward portion. In other words an initial \( \zeta \) distribution at \( y = 0 \) could be partly due to such freshening influences over the backward part of the coastline, as well as to wind stress over the same region.

Other properties of the solution for distributed \( \zeta \) sources are in many respects similar to wind-stress-induced flow and are easily deduced from Eq. (29).

10. Oceanic influence

Up to now all the solutions of Eq. (1), in their original form or as modified in Eq. (27), were of the shore-trapped kind, the surface elevation becoming negligible at large distances from shore. Physically this corresponds to various forcing effects acting near shore, producing different flow patterns in the coastal zone, without interaction with the flow in the deeper ocean. In other words, in all the above models the deep ocean was regarded as inert, clearly an especially simple situation.

In reality, elevation gradients of the same order of magnitude as observed or inferred over continental shelves may well exist in the deep ocean. These involve, however, forces of much greater total magnitude owing to the much greater depth and are a consequence of larger scale phenomena, e.g., of wind stress acting over an entire oceanic basin. They may legitimately be regarded as extraneous factors from the point of view of a limited coastal zone model. To represent their effects on the coastal zone one may try to impose appropriate boundary conditions at the outer edge, in place of the simple hypothesis of vanishing elevation, \( \zeta \to 0 \) at \( x \to \infty \). Such boundary conditions may be regarded as modeling the influence of an "active" ocean on the coastal zone.
Because the idealizations of the simple coastal zone model adopted above become unrealistic at depths much in excess of 100 m, it is convenient to limit the model to some nearshore strip \(0 \leq x \leq l\). An arbitrary distribution of surface elevation \(\zeta(l, y)\) along the \(x = l\) line may then be used to represent the influence of the open ocean on the shelf. This constitutes a simple replacement for the previous \(\zeta \to 0\) boundary condition at infinity, and is clearly only one of several possible choices in this regard. It is not apparent, however, that any other choice would be superior.

For a coastal zone of constant slope the simple heat conduction analogy may again be invoked. The zone \(x = 0\) to \(l\) corresponds to a slab of constant conductivity. A specific \(\zeta(l, y)\) distribution at the outer edge corresponds to a prescribed variation of temperature at one surface of the slab. At the other surface, \(x = 0\), one may prescribe finite or zero heat flux, corresponding to whether or not a longshore mean wind is present. Internal heat sources and the initial temperature distribution have the same interpretation for the slab as for a semi-infinite conducting medium discussed before.

As an example of oceanic influence, we consider the case of a constant longshore pressure gradient at \(x = l\), imposed at \(y = 0\):

\[
\frac{\partial \zeta}{\partial y} \bigg|_{x=l} = \gamma = \text{constant} \quad (y \leq 0) \\
\zeta = 0 \quad (y > 0)
\]

(31)

In the absence of wind the boundary condition at the shore is \(\partial \zeta/\partial x = 0\), according to previous discussion. In the heat conduction analogy this problem is a slab of zero initial temperature, no heat flow across \(x = 0\), the temperature at \(x = l\) changing at a constant rate in time after \(t = 0\). The appropriate solution of the heat conduction equation is (Carslaw and Jaeger, p. 104):

\[
\zeta = \gamma y - \frac{\gamma(x^2 - l^2)}{2} - \frac{16\gamma l^2}{\kappa \pi^3} \sum_{n=0}^{\infty} \frac{\exp[(2n + 1)^2 \pi^2 y/4 l^2]}{(2n + 1)^3} \cos[(2n + 1)\pi x/2l].
\]

(32)

The exponential factors multiplying the cosine series are \(\exp(y/\alpha_n l)\), where \(\alpha_n = 4l/[\kappa(2n + 1)^2 \pi^2]\). The largest of these factors corresponds to \(n = 0\), but this also becomes negligible for

\[-y \gg \alpha_0 l.\]

At such large negative \(y\) the \(\zeta\) distribution in the coastal zone does not change in shape, the levels changing uniformly at all \(x\) in the \(y\) direction. The corresponding longshore velocity distribution is

\[
v = \frac{g}{f} \frac{\partial \zeta}{\partial x} = -\frac{g \gamma x}{f \kappa} = -\frac{g \gamma x}{r} = -\frac{gh}{r} \frac{\partial \zeta}{\partial y},
\]

(33)

having noted that

\[
\frac{\partial \zeta}{\partial y} \bigg|_{x=l} = \gamma
\]

at all \(x\). Eq. (33) expresses balance between bottom stress \(rv\) and longshore pressure gradient force \(-gh(\partial \zeta/\partial y)\). According to the second of Eqs. (1), and recalling that \(F = 0\), the offshore velocity \(u\) is zero everywhere. The simple physical situation is then that the “active” ocean impresses its longshore gradient on the entire coastal zone, the water flowing alongshore, down the gradient. The longshore velocity increases with depth, because the total pressure-gradient force does, and this has to be balanced by bottom stress.

11. Comparison with other theoretical work

Frictional steady-state models of flow over variable topography have been discussed in the literature many times. A few of these arrive at results similar to those above. The periodic wind stress model discussed above may be regarded as a simplification, and to some extent generalization, of work by Birchfield (1967) on wind-driven steady motions in a circular basin of parabolic depth profile. Birchfield used a slightly more complicated linear bottom friction law and solved the depth-averaged equations for uniform wind (as well as for some other stress distributions). His surface elevation and streamline plots for uniform wind are such that, when unwrapped and developed into a straight shoreline, they look very similar to those shown in Figs. 4 and 5.

In a more recent paper Birchfield (1973) discussed the same flow problem with the aid of the viscous boundary layer approximation, and the postulate that an \(E^{1/4}\) Ekman layer near the coast accomplishes the mass balance in a closed basin. In support of his postulate Birchfield gives some illuminating scaling arguments, which contain assumptions similar to what Eqs. (1) were based on. He also derives an equation for the pressure in the coastal boundary layer which is equivalent to Eq. (3) here. His boundary conditions are the same as Eqs. (4a) and (5), and his solution for circular geometry is the direct analog of Eqs. (23).

Another very similar study is that of Pedlosky
This work also uses boundary-layer approximations, postulating thin Ekman layers at top and bottom, and an $E^{1/4}$ layer near shore. The approach is aimed at sufficiently deep water: the region closest to the shore is excluded by a vertical wall so deep that the boundary layer approximation remains valid. Just seaward of this vertical wall there is, however, a sloping bottom, over which Pedlosky finds a "topographic boundary layer". For the pressure distribution over this boundary layer Pedlosky also derives an equation equivalent to Eq. (3) here. Thus, just as Birchfield's coastal boundary layer, Pedlosky's topographic boundary layer is equivalent to the arrested topographic wave model discussed here.

Pedlosky's results are embedded in other complexities, some of which seem to be due to the artificial presence of a vertical wall at some unspecified distance from the shore, but mostly they follow from other important physical effects considered in that analysis, i.e., internal friction and density gradients. The present investigation exhibited the properties of the arrested topographic wave in isolation. Pedlosky's work demonstrates that the same phenomenon survives under much more complex conditions, when it is superimposed on other flow components. Perhaps most importantly, these other studies effectively remove the depth limitation of the arrested topographic wave model and show that when the momentum is not uniformly mixed over a water column, very similar coastally trapped pressure fields arise as in the well mixed case.

### 12. Comparison with observation: Lake Ontario

The original purpose of this investigation was to determine whether a residue of successive topographic waves, excited by variable winds in Lake Ontario, could be held responsible for mean flow patterns observed in that lake during IFYGL. The arrested topographic wave is a linear model that should represent such residual flow due to frictionally decaying long waves. Other, more complex influences are also known to be at work in the Lake Ontario mean circulation problem (Csanyi and Scott, 1978) especially under stratified conditions. A flow pattern of the character of the arrested topographic wave is most likely to be in evidence in late fall–early winter, when the lake is unstratified and a number of intense storms pass over the Great Lakes region.

A good case can be made that the mean flow in Lake Ontario in the winter 1972–73 was in fact very much as represented by the arrested topographic wave model. Pickett (1977) has published a summary of all the current records avail-

### Table 1. Coastal zone parameters used in calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lake Ontario</th>
<th>East Coast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope $s$</td>
<td>$10^{-2}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Coriolis parameter $f$ (s$^{-1}$)</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Friction coefficient $r$ (cm s$^{-1}$)</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>Stress amplitude $F_s$ (cm$^2$ s$^{-2}$)</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>Stress wavenumber $k$ (cm$^{-1}$)</td>
<td>$10^{-7}$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>$\zeta$ diffusivity, $\kappa = r/s$ (km)</td>
<td>0.1</td>
<td>10</td>
</tr>
<tr>
<td>Length scale $L$ (km)</td>
<td>4.5</td>
<td>141</td>
</tr>
<tr>
<td>$kl$</td>
<td>0.045</td>
<td>0.14</td>
</tr>
<tr>
<td>Velocity amplitude $F_s/r$ (cm s$^{-1}$)</td>
<td>33</td>
<td>10</td>
</tr>
<tr>
<td>Elevation amplitude $2F_s/(kLgs)$ (cm)</td>
<td>1.5</td>
<td>14.1</td>
</tr>
</tbody>
</table>
able for this period. Fig. 7 here shows the January 1973 mean currents which are perhaps most easily interpreted. The mean wind was almost exactly eastward, i.e., directed along the long axis of the lake, its average speed having been 4.7 m s\(^{-1}\). Proceeding in a counterclockwise sense around the basin from its western end the arrested topographic wave model predicts the following mean flow pattern given uniform eastward wind stress. On the south shore there should be eastward flow, at first in a thin boundary layer along the coast, later broadening out. At the eastern end of the lake this broad flow should turn westward, proceeding against the mean wind. Along the shore, however, a thin and gradually thickening eastward flow should appear. By the western end of the lake the eastward flow should be broad, and it should there turn the corner, becoming a broad westward flow along the south shore, outside the thin eastward flowing boundary layer. Note that the thickening of the boundary layer along the north shore takes place against the sense of the flow, in the direction of a cyclonic circuit around the perimeter.

Quantitatively, the periodic longshore wind stress model may be invoked with appropriately chosen coastal zone parameters. Table 1 lists plausible idealizations. The friction parameter \( r \) is conjectural, arbitrarily taken to be one order of magnitude less than the empirical value found in tidal waters off Long Island. Fig. 4 may be taken to represent the nearshore flow field unwrapped into a straight line and the offshore length scale exaggerated by a factor of 4. The location of Olcott, N.Y., corresponds in this figure to about \( ky = -2.2 \). At this location there is downwind flow near the coast in a thin layer (less than one \( L \)) while a little further out strong return flow is in evidence, to about \( x/L = 2 \) or 3, i.e., somewhat beyond 10 km. The north shore of the lake is modeled by \( \pi/2 > ky > \pi/2 \), the south shore by the remainder. Oswego and Rochester are represented, respectively, by about \( ky = 2.20 \) and 3.14.

The data in Fig. 7 are more or less consistent with the theoretical interpretation, but note especially the pronounced westward flow 12 km north of Olcott. Pickett (1977) interprets this pattern in terms of the numerical model of Rao and Murthy (1970). However, that model shows negligible transport in the location corresponding to the Olcott current meters for any of three reasonable choices of the friction parameter. Westward (return) flow in the Rao-Murthy model coincides with the deepest part of the lake cross section instead of being a coastally trapped phenomenon. As pointed out elsewhere (Csanyi, 1973) the flow pattern calculated by Rao and Murthy is of the “topographic gyre” type, characteristic of the no-rotation limit of steady circulation or of the initial, transient response of a lake to suddenly applied wind stress.

The difference between the topographic gyre and the arrested topographic wave type of flow pattern lies in the relative importance of rotation and friction. If the length scale \( L \), which is a measure of the shore-trapped flow pattern in the arrested topographic wave model, were comparable to the lake width, this would be an indication that there is no central region uninfluenced by the presence of the coast, where transport is Ekman drift to the right of the wind. Given \( L = 4.5 \) km (Table 1), however, and a lake width of 60–70 km the basic hypotheses of the arrested topographic wave model are clearly satisfied, and Ekman drift at midlake should be present. There is considerable direct observational evidence in support of this view. In the Rao-Murthy numerical model no Ekman drift is discernible at mid-lake. It is difficult to say precisely why this is the case, because the same physical effects were taken into account in that model as in the discussion in this paper. Perhaps a grid spacing too coarse near shore suppressed the important rotational constraints on flow over variable depth, noting that bottom slope in Lake Ontario is only large within 10–15 km of the shore.

More detailed evidence on the mean circulation in the near-shore zone of Lake Ontario is available for the summer season when daily surveys were taken of the current field at five coastal transects (see, e.g., Csanady and Scott, 1974). The mean coastal circulation under stratified conditions is more complex than in homogeneous water and its details are discussed elsewhere (Csanady and Scott, 1978). However, the summer observations unmistakably show the presence of a shore-trapped mean pressure field and associated longshore flow, more or less as exemplified above in Fig. 6. One component of the summer mean circulation is produced by interfacial stress across the thermocline accelerating the cold bottom layer in the coastal zone, in the manner of wind stress acting over homogeneous water. Significant interfacial stress acts only over a finite portion of the coastal zone (about the eastern half of the south shore) but generates a shore-trapped current well beyond its range of action, which turns the corner at the eastern end of the lake and penetrates some 200 km along the north shore, against the mean wind. The properties of this shore-trapped bottom layer current are in good quantitative agreement with the model of Fig. 6 or Eq. (24).

13. The east coast shelf

In recent years considerable progress has been made in understanding the mean circulation of the
shallow sea covering the continental shelf east of North America. Extensive early studies of Bumpus (1973) carried out with the aid of drifters and drogues have been confirmed by many recent current meter observations (Beardsley et al., 1976). North of Cape Hatteras the mean flow was found to be toward the southwest, of an intensity generally increasing with distance from shore. Fig. 8 from Beardsley et al. is a typical example of the observed facts. Although there is some seasonal variability, mainly of intensity, essentially the same mean flow pattern prevails in summer or winter months.

Theoretical studies (Stommel and Leetmaa,
1972; Csanady, 1976b) have shown that while wind stress and freshwater influx are important influences on mean shelf circulation, they cannot by themselves account for the observed mean south- westward flow. From an analysis of nearshore current meter observations, Scott and Csanady (1976) inferred that a longshore surface elevation gradient of a magnitude about $1.4 \times 10^{-7}$ is the proximate cause of westward flow along the south coast of Long Island. There was no evidence of the origin of this pressure gradient, although Sturges (1977) suggested in discussion that it may be the coastal signature of a longshore current in deep water, much as some other coastal elevation gradients are. If a steady deepwater current of constant intensity flows along a coast crossing latitude circles, the variation of the Coriolis parameter with latitude gives rise to longshore elevation gradients. Why a deepwater current should behave this way is a question relating to larger scale dynamical problems in the ocean.

The frictional coastal flow model which was found to describe mean shelf circulation (Csanady, 1976b) is of the simple Ekman type, or more accurately, much as the model discussed by Jeffreys (1923). It is based on the postulate of zero cross-isobath flow. Sufficiently close to the coast this postulate is certainly valid and may therefore be labelled the "coastal constraint". By imposing this constraint on each streamtube a remarkably realistic description of the mean shelf circulation is obtained, if only an appropriate longshore elevation gradient is assumed present. A recent analysis of a somewhat larger body of observations by Flagg (1977) has shown that the same model with about the same longshore pressure gradient describes the flow not only nearshore, but as far as the 100 m isobath, i.e., more than 100 km from the shore.

Simple theories based on the postulate of coastal constraint cannot of course answer such important questions as how far from shore the coastal constraint remains valid, or what physical mechanism maintains the longshore pressure gradient. Imposing the coastal constraint and specifying a longshore gradient is theoretically equivalent to prescribing a local horizontal pressure gradient vector. As Welander (1957) pointed out, the circulation pattern of shallow seas may be separated into a local (depth dependent) solution for which a pressure gradient is specified, and a global pattern, derived as the solution of an equation for the horizontal distribution of pressure. A useful version of the latter may be obtained by a depth integration of the equations of motion. Eqs. (1) are of this type and represent a simplified version of the global problem appropriate for certain shallow seas. They should be useful for determining at least winter-mean circulation over the east coast shelf, when the water is nearly homogeneous in the vertical.

In this light consider the physical origin of the southwestward flow and associated longshore pressure gradient. On the "inert" ocean hypothesis, $\xi \to 0$ as $x \to \infty$, the theoretical discussion above has shown that a surface elevation distribution trapped nearshore may be evoked by either wind stress or by freshwater influx. These were identified in the theoretical discussion as $\xi$ sources, i.e., wind stress as a line source at the shore, acting much as heat flux at a boundary, and water freshening as an area source all over the shelf, acting as an internally distributed heat source. At a given section across the shelf (say, south of Long Island) a certain $\xi(x)$ distribution may be viewed as produced by all $\xi$ sources in the backward portion of the shelf, in this instance all of the shelf region northeastward of the chosen section. Moreover, over a given shelf width $l$, only sources within a longshore distance $l^2/2k$ backward of the section considered are effective. Realistic idealizations for the east coast shelf are also listed in Table 1. With $l = 100$ km and $k = 10$ km the backward influence zone comes to be of order 500 km, or not all that long. Even if one interprets such order of magnitude estimates with appropriate caution, this argument still rules out significant upstream effects from, say, beyond the Grand Banks of Newfoundland.

If the observed positive $\xi$ distribution at a given section (appropriate to southwestward flow) were due to wind stress, it would require positive $\xi$ sources within 500 or 1000 km of the section, northeastward. In other words, the wind would have to have a significant southwestward component over a substantial portion of the backward coastline. Actually, the mean wind has a strong eastward component along all of this coast, and acts as a negative $\xi$ source. In no way can the observed or inferred pressure distribution therefore be ascribed to wind stress, as some kind of larger scale response. The shore-trapped pressure field evoked by the mean wind stress is certainly such as to result, in combination with the direct wind effect, in northeastward mean flow at all distances from shore.

The pressure distribution associated with a given offshore density gradient (due to freshwater influx) was found to be such as to cause zero net longshore flow, the surface elevation gradient balancing exactly the dynamic height gradient, beyond some initial establishment region of order $l^2/2k$. A concentrated source of freshwater somewhere within a few hundred kilometers northeast of the section considered could still cause some southwestward net flow. However, no reasonable quantitative
assumptions lead to anything like the observed flow field.

If one now abandons the inert ocean hypothesis and prescribes a longshore gradient of given magnitude along the outer edge of a shelf region of width $l$, Eq. (32) shows that, beyond an initial region of $l^2/2\kappa$ in longshore extent, the same (constant) pressure gradient comes to drive the flow all over the shelf region. Moreover, the longshore velocity due to this effect alone is proportional to water depth [Eq. (33)]. Clearly, these predictions are qualitatively in good accord with the observed facts. Quantitatively, the agreement is good if the longshore elevation gradient is taken to be of order $10^{-7}$.

To examine the suggestion of Sturges (1977) to the effect that this gradient is due to the variation of the Coriolis parameter with latitude, arising from a deepwater boundary current, the observations of Luyten (1977) may be analyzed. There is indeed a boundary current present along the continental slope and rise, flowing to the southwest. Given a current of constant velocity $v$ along the isobaths the variation of the Coriolis parameter gives rise to a longshore gradient of

$$
\frac{\partial \xi}{\partial y} = \beta \frac{vl}{g} \cos \alpha,
$$

(34)

where $\beta \cos \alpha = df/dy$, $\alpha$ being the angle of the isobaths against north and $l$, the width of the slope/rise current. Luyten's data suggest $v = 5$ cm s$^{-1}$, $l = 200$ km, which, even with $\alpha = 0$, gives a longshore gradient of order $10^{-8}$, or an order of magnitude too small. The angle $\alpha$ is actually between $60^\circ$ and $90^\circ$ along the New England shelf, making $\partial \xi/\partial y$ due to this cause even smaller.

One concludes that, while the observed southward flow of the shelf appears to be driven by gradients imposed by a much more massive boundary current, the effect is more complex than the simple variation of the Coriolis parameter with latitude, acting on a uniform boundary current.

Acknowledgments. This work was supported by the Great Lakes Environmental Research Laboratory of NOAA, Ann Arbor, and by Brookhaven National Laboratory, under its ERDA sponsored program of coastal shelf transport and diffusion. Mr. Paul May kindly carried out the computations resulting in Figs. 3–5.

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