Observation and Simulation of Storm-Induced Mixed-Layer Deepening

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Two observed cases of mixed-layer (ML) deepening due to storms are analyzed and simulated. The primary goal is to learn whether the relevant scale velocity in the parameterization of wind-driven deepening is $U_w$, the wind stress friction velocity, or $\delta V$, the magnitude of the horizontal mean velocity difference across the base of the ML. ML deepening is isolated from air-sea exchange and horizontal advection by diagnosing the entrainment tendency of ML temperature.

ML deepening is found to be highly intermittent on the storm time scale. Deepening in response to a wintertime atmospheric cold front occurred as $\delta V$ was accelerated during the initial rise in wind stress. Deepening abruptly ceased as wind stress began to decelerate $\delta V$, though the stress magnitude continued to increase. A similar relationship between wind stress $\delta V$ and ML deepening was also observed in a summertime case and is evidence that the relevant scale velocity is $\delta V$ not $U_w$.

In both cases the observed phase and extent of ML deepening are simulated realistically by the parameterization of Pollard et al. (1973) in which an overall Richardson number, $R_v = g' h / \delta V^2$, where $g'$ and $h$ are the ML buoyancy and thickness, sets a lower bound on the ML thickness. The value of $R_v$ is $\approx 0.65$.

1. Introduction

Upper ocean density profiles often show a neutrally stable surface layer, or mixed layer (ML), which is capped below by a stably stratified layer, the diurnal or seasonal thermocline. The neutrally stable ML is easily stirred by a surface stress or destabilizing surface buoyancy flux and, excepting very deep ML’s, air-sea exchanges are rapidly distributed through the ML. The stable layer at the base of the ML is turbulent only intermittently, hence, air-sea exchanges are effectively trapped in the ML. The ML thickness is thus proportional to the thermal and inertial mass of the surface layer in contact with the atmosphere and must be known to predict sea surface temperature (SST) and the speed of the directly wind-driven current.

The local time rate of change of ML thickness $h$ is (with the vertical coordinate positive up)

$$\frac{\partial h}{\partial t} = -\nabla \cdot (\nabla h) - W_e,$$  \hspace{1cm} (1)

where $\nabla$ is the depth-averaged horizontal velocity in the ML and $W_e$ is the entrainment velocity, the rate at which the ML depth cuts through material surfaces. (ML depth and thickness are used interchangeably. A glossary of symbols follows in Appendix A.) The change of $h$ due to advection is termed stretching; the change due to entrainment is termed deepening. Stretching may cause $h$ to increase or decrease. Deepening can only cause ML thickness to increase, i.e., $W_e \leq 0$.3

ML deepening is fundamentally a turbulent process (Phillips, 1977) and must be parameterized in any routinely useful model. The relevant variables of the parameterization include the ML buoyancy and thickness $g'$ and $h$ which form a velocity scale $(g' h)^{1/2}$. A second velocity scale $U$ is required to complete a nondimensionalization and write

$$\frac{W_e}{U} = E (g' h / U^2),$$ \hspace{1cm} (2)

where $g' h / U^2$ is the overall Richardson number of the ML and $E$ is a function to be determined. For the wind-driven ocean surface ML the scale velocity is usually assumed to be either $U_w$, the wind stress friction velocity, or $\delta V$, the magnitude of the mean (hourly average) velocity difference across the base of the ML. The primary goal of this study is to determine whether $U_w$ or $\delta V$ is the relevant scale velocity by simulating observed oceanic cases of ML deepening. Secondary goals are to determine the approximate form of $E$ and the importance of deepening in the ML response to storms.

Laboratory studies of ML deepening have been conducted by simultaneously observing $W_e$ and the

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3 A stabilizing air-sea buoyancy exchange may cause ML thickness to sharply decrease as shown in Section 4b. This process is the genesis of a new, distinct ML and is not represented in (1).

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overall Richardson number. Cases in which $\delta V$ was significant have been scaled with some success using both $U_*$ (Kato and Phillips, 1969) and $\delta V$ (Ellison and Turner, 1959) as the scale velocity. No laboratory measurements have been made to explicitly test the choice of the scale velocity.

Specific forms of the We parameterization have previously been tested by simulating observed cases; cf. Denman and Miyake (1973), Pollard et al. (1973) and Thompson (1976, 1977). Collectively, these studies have also been ambiguous; seemingly either $U_*$ or $\delta V$ may be used with success. It is suspected that previous tests via simulation have not been sufficiently sensitive to the parameterization of deepening because (all studies have not suffered from both of these problems) 1) free constants in the parameterization have been arbitrarily adjusted to fit model simulations to available observations and the information content of the simulations has not been critically examined; and most importantly, 2) the method of model testing has been a simple comparison of observed and simulated ML depth and temperature; the effects of ML deepening have not been isolated from the often very important effects of air-sea buoyancy exchange and advection.

In Section 2 the conservation equations for ML temperature, salinity and momentum and a parameterization of ML deepening are introduced and the simplifications of a one-dimensional simulation model are discussed. In Section 3 the information content of the model is considered and the full ML temperature equation is used to develop a diagnostic technique for indicating ML deepening present in field observations. In Section 4 two independent cases of storm-induced ML deepening are simulated and analyzed using the model and diagnostic technique. The results are summarized and discussed in Section 5. Readers familiar with ML modeling as reviewed by Niler and Kraus (1977) may wish to proceed directly to Section 3.

2. Mixed-layer model

a. Model geometry

The ML thickness $h$ (Fig. 1) is defined as the depth above which density is homogeneous to 0.02 units of sigma-$t$. This degree of density homogeneity is arbitrary but is consistent with the precision of the available observations. The transition layer or interface between the ML and the interior has thickness $d$. The bottom of the interface, $z = -h - d$, is the depth at which mixing ceases and is extremely difficult to observe when the interior is stratified. The conservation equations below are written for the case $d = 0$. If variables have linear depth dependence within the interface, then the conservation equations for finite $d$ are identical provided $h$ is replaced by $h + d/2$. The interface will therefore be ignored until model results are compared with observations. Model-computed $h (d = 0)$ should then be compared to observed $h + d/2$.

b. Thermodynamic and momentum equations

Density is taken to be a linear function of temperature and salinity,

$$\rho(T,S) = \rho_r[1 + \alpha(T - 24) + \beta(S - 36)], \quad (3)$$

where $\rho_r$ is a reference density, $\alpha = -3.0 \times 10^{-4}$ $^\circ$C$^{-1}$ and $\beta = 7.6 \times 10^{-4}$ $^\circ$C$^{-1}$.

The ML temperature equation (ignoring horizontal turbulent fluxes) is

$$\frac{\partial T_i}{\partial t} + V_i \cdot \nabla T_i = \frac{Q}{\rho_r C_p h} - \frac{\delta T}{h} \text{We}, \quad (4)$$

where $Q = SF + RS(0) - RS(-h)$ is the sum of the heat fluxes which originate directly from the sea surface (longwave radiation, latent and sensible heat fluxes) SF, and the fraction of solar insolation which is absorbed within the ML, $RS(0) - RS(-h)$. (See Appendix B for calculation of all air-sea exchanges.) The last term $-\delta T \text{We}/h$ is called the entrainment tendency of temperature and will be central to the subsequent analysis. When $\delta T < 0$ (ML warmer than the interior, the usual case) the entrainment tendency must always be non-positive, i.e., entrainment of cooler water can only cause the ML to become cooler.

The full ML conservation equations together with (1) have been used in process models to study the interaction between stretching, surface buoyancy flux and ML deepening (Thompson, 1974). For the simulation model integrated here, advection is necessarily ignored. It is impossible (and probably uninteresting) to simulate horizontal advection in
this study because 1) the model initial condition would have to include the initial (x,y) dependence of fluid advected to the study site at any time during the experiment, and 2) the advecting velocity includes contributions from internal and external tides, nonlocally forced longwaves, etc., which are beyond the scope of this study. The important kinematic effect of horizontal advection is treated as a data analysis problem in the next section.

The simplified (one-dimensional) temperature, salinity and momentum equations used for the ML simulation are

\[
\frac{\partial T}{\partial t} = \frac{1}{h} \left( \frac{Q}{\rho_r C_p} + \delta T \frac{\partial h}{\partial t} \right),
\]

\[
\frac{\partial S}{\partial t} = \frac{1}{h} \left[ S_r(E - P) + \delta S \frac{\partial h}{\partial t} \right],
\]

\[
\frac{\partial V_i}{\partial t} = -f \times V_i + \frac{1}{h} \left( \tau_{ir} + \delta V \frac{\partial h}{\partial t} \right),
\]

where \( E - P \) is evaporation minus precipitation, \( f \) is the Coriolis parameter times the vertical unit vector, and \( \partial h/\partial t = -We \) because stretching is neglected. The ML density equation is formed by substitution of (5) and (6) into the time derivative of (3) yielding

\[
\frac{\partial \rho_t}{\partial t} = \frac{B}{gh \rho_r} + \frac{\delta \rho}{h} \frac{\partial h}{\partial t},
\]

where the air-sea buoyancy flux is \( B = g \rho_r [\alpha Q + \beta S_r(E - P)] \).

All turbulent fluxes are assumed to vanish below the ML. If a given level is within a relic ML it will have acquired a velocity which is assumed to rotate inertially. Temperature in the interior can increase due to the penetration of solar insolation. The balances in the interior are

\[
\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} RS,
\]

\[
\frac{\partial S}{\partial t} = 0,
\]

\[
\frac{\partial V}{\partial t} = -f \times V.
\]

Since there is no spatial dependence, the partial time derivatives in (5) to (11) are equivalent to ordinary time derivatives.

Because stretching is ignored, wave radiation into the interior is not stimulated and kinetic energy is trapped within the ML. Pollard and Millard (1970) used an \( \textit{a priori} \) damping term \( C \delta V \) to simulate the temporal decay of \( |V_i| \) by dispersion. With \( C^{-1} = 5 \) days (their value), damping has a negligible effect on ML deepening in the cases discussed here because the time scale of ML deepening after current generation by a wind event is much less than 5 days, and because there is no significant resonant buildup of ML current due to successive wind events. Other consequences of ignoring stretching are discussed in Section 3. The model ML momentum equation does retain one important feature—resonance with clockwise (Northern Hemisphere) rotating wind stress at the inertial frequency (Pollard and Millard, 1970).

c. Parameterization of mixed-layer deepening

Because \( h \) is time dependent a parameterization of \( -We = \partial h/\partial t \) is required to close the set (7) and (8). The form used and tested here was suggested by Niler and Kraus (1977) who modeled the turbulent energy budget of the ML and derived what will be termed the ECM (energy conservation model),

\[
- g'(h/2) We = m_1 \frac{h(h/2)H(B)}{m_2 W e \delta V^2/2 + m_3 U^3}. \tag{12}
\]

The left-hand side is the rate of increase of potential energy during deepening (to a factor \( \rho_r \)) which is supplied by, from left to right, energy released during free-convection, energy released from the mean flow by the reduction of vertical shear during deepening and the work of wind stress on the surface drift current. \( H(B) \) is the Heaviside function; \( H = 1 \) if \( B > 0, H = 0 \) if \( B < 0 \). The parameters \( m_1, m_2, m_3, \) are presumably constants which represent the energy conversion efficiency of the three sources. They are regarded as unknowns. The following two special cases of (12) will be considered in detail:

1) TEM (turbulent erosion model). With \( m_1 = m_2 = 0 \), (12) reduces essentially to the Kraus and Turner (1967) and Denman (1973) model

\[
We = \frac{2m_3 U^3}{g'} \tag{13},
\]

which is formally equivalent to the commonly accepted result of the Kato and Phillips (1969) experiment \( We/U^3 \propto (R_T)^{-1} \), where \( R_T = g'/U^2 \).

2) DIM (dynamic instability model). With \( m_1 = m_3 = 0 \), (12) reduces essentially to the Pollard \textit{et al.} (1973) model

\[
\frac{g'}{\delta V^2} = R_V = m_2, \tag{14}
\]

which they interpreted as a stability limit on the ML depth (they assumed that the critical value \( m_2 = 1 \)). There is no sound theoretical or experimental evidence that a true critical bulk Richardson number exists for a turbulent ML (Phillips, 1977). However, the Ellison and Turner (1959) laboratory experiment showed that entrainment by a surface half-jet de-
creases by a full order of magnitude as $R_v$ increases from 0.4 to 0.8, suggestive of an approximate critical $R_v$ in that range.

Profiles of $T$, $S$, $V$ were stored on a grid having 20 cm vertical spacing, and time steps of 1 h were used in the numerical integrations. This very high grid density was required to resolve the rate of ML deepening (discussed below). For most purposes ML depth and temperature may be computed satisfactorily on a much coarser grid. The time step was commensurate with the sampling interval of the available air-sea data and gave sufficient accuracy. The method of integration follows Thompson (1976). Details are given by Price (1977). Solar insolation was absorbed with depth according to (B5) and all other buoyancy fluxes were absorbed in the shallowest grid level. The density profile was adjusted to relieve static instability by mixing downward from the surface. Hence, even with $m_1 = m_2 = m_3 = 0$, the model ML depth could increase by non-penetrative deepening. [For a discussion of penetrative versus non-penetrative ML deepening see Turner (1973, pp. 304–305).]

d. Notable differences in the response of the models

Before examining the complex natural cases of Section 4 it is instructive to briefly compare the response of TEM and DIM (or any two closures based on $U^*$ and $\delta V^2$) to a stepfunction wind stress which rises to a constant value at $t = 0$. In general, We is large when $U^*_{10}$ or $\delta V^2$ is large; hence, only the response of the scale velocities need be compared. The friction velocity is, trivially,

$$U^*_{10} = \text{constant}. \quad (15)$$

Assuming the mean flow to vanish at $t = 0$, then from (7),

$$\delta V^2 = \frac{2 U^*_{10}^4}{k^2 f^2} (1 - \cos(ft)) \quad (16)$$

There are two notable differences between (15) and (16). First, the $h^{-2}$ dependence of (16) should clearly distinguish DIM from TEM if time-dependent cases having a broad range of ML depth are simulated. Thompson (1976, 1977) simulated the annual ML cycle at Ocean Weather Ship November and found that a model very similar to TEM deepened insufficiently in the heating season when $h$ was small and deepened excessively in the cooling season when $h$ was large. A model very similar to DIM performed somewhat better, suggesting that the $h^{-2}$ dependence of $\delta V^2$ is more nearly correct. Our analysis will show a similar result. Second, $\delta V^2$ is oscillatory in time. The mean flow response increases during the first half inertial period when the ML current has a component in the direction of the wind stress and is accelerated by the stress, and decreases to zero during the next half inertial period when the ML current has a component anti-parallel to the wind stress. This dependence of $\delta V^2$ on the phase of the ML current with respect to the wind stress causes a qualitative difference in the deepening response of parameterizations based on $\delta V^2$ and $U^*_{10}^2$ provided the wind stress magnitude is sustained for more than half an inertial period. For a stepfunction wind stress and the idealized case $h = 0$ at $t = 0$ and $g'h = \text{constant}$ (two-layer fluid), the response of TEM is $We = \text{constant}$ for $t > 0$; the response of DIM is $We \propto \sin(ft) [1 - \cos(ft)]^{-1/2}$ for $0 < t < \pi/f$ and $We = 0$ for $t > \pi/f$.

3. Model testing and method of data analysis

The method of model testing must take into account that the model is 1) less than fully predictive because the constants $m_1$, $m_2$, $m_3$ are considered adjustable, and 2) highly idealized because all forms of advection are ignored.

a. Model information content

Even a single free constant in the closure ensures that simulated $h$ or $T_I$ can be adjusted to match $h_0$ or $T_{O_0}$ (observed variables are subscripted with 0) at any single time. Any reasonable parameterization can thus be made to simulate a given event with some success. Here, the free constant(s) is adjusted so that simulated ML depth matches observed ML depth at, arbitrarily, the end of the observational period. In both cases this is several inertial periods beyond storm onset and simulated and observed ML deepening have virtually ceased. The case-to-case stability of the free constant is then an important measure of closure performance; if the free constant is required to assume significantly different values from one case to another in order to simulate the observed net deepening, then the closure must be considered physically wrong or incomplete. A second test is made by comparing the observed and simulated rate of ML deepening. As noted in Section 2d, this will clearly reveal the difference between closures based on $\delta V^2$ and $U^*_{10}^2$. It would be a strong test of a parameterization with one free constant even if only a single observed case were available (provided wind stress is sustained for more than half the inertial period).

b. Diagnosing the rate of deepening and the effects of advection

The entrainment velocity computed as $-\frac{\partial h_0}{\partial t}$ is a useful measure of the rate of deepening

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* By rate is meant rate and phase or time evolution.
if no significant advection occurs. It is not useful here because internal wave motions cause large amplitude fluctuations in \( h_0 \); the ratio \( \text{We}/(h \nu V) \approx \frac{1}{2} \). For potential energy, the ratio is \( \approx \frac{1}{4} \) (two-layer fluid); hence, the increase in potential energy due to ML deepening is swamped by fluctuations due to stretching caused by internal waves. [A similar case is shown by Halpern (1974, Fig. 11).] Stretching enters the conservation equation for an intensive ML variable only by changing the ML thickness (thermal mass). The entrainment tendency depends only on the entrainment velocity and is expected to be a useful measure of the rate of deepening if it can be computed with sufficient accuracy from the available field observations. The entrainment tendency of temperature is used here because salinity was not measured in one case and precipitation was not measured in either case. From the full ML temperature equation (4), the entrainment tendency of temperature \( \Omega \) is

\[
\Omega = \frac{-\delta T}{h} \text{We} \\
= \frac{(-1/h \rho_C C_p)[(RS(0) - RS(-h) + SF]}{\nu V} \\
+ \frac{\partial T}{\partial t} + \frac{\partial T}{\partial z} .
\]

Each term on the right-hand side of (17) may be easily evaluated from the available air–sea data except the horizontal advection term for which accurate estimates of \( \nabla T(t) \) are unavailable.

A depth-averaged estimate of horizontal advection is therefore computed from a heat budget. A Reynolds averaged temperature equation is introduced,

\[
\frac{\partial T}{\partial t} + W \frac{\partial T}{\partial z} + V \cdot \nabla T \\
= -\frac{\partial}{\partial z} \frac{\partial T}{\partial t} + \frac{\partial}{\partial z} \frac{\partial T}{\partial z} \rho_C C_p ,
\]

in which molecular and horizontal turbulent fluxes have been ignored. The averaging is assumed to be over a period of roughly 30 min and, therefore, the entrainment heat flux is represented as a turbulent flux. Almost all of the vertical advective flux in the interior is due to inertial-internal wave motion which is well defined by the available data. [When (18) is integrated from \( z = -h \) to the sea surface, the result is equivalent to (4), where \( Q = \frac{-\rho_C C_p W^2 T(0)}{\nu V} + RS(0) - RS(-h) \) and \( \delta T \text{We} = -W^2 T(-h) \).] To solve for horizontal advection, (18) is integrated from the surface to a depth \( z = -c \) in the interior where the turbulent flux \( W^2 T(-c) \) is assumed to be negligible.

\[
\int_{-c}^{0} V \cdot \nabla T dz = \int_{-c}^{0} \frac{\partial T}{\partial t} dz + \frac{1}{\rho_C C_p} [RS(0) - RS(-c)] \\
- \frac{W^2 T(0)}{\nu V} - \int_{-c}^{0} W \frac{\partial T}{\partial z} dz. \tag{19}
\]

No term involving \( W \) appears in (19) because ML deepening only redistributes heat within the column. Horizontal advection from (19) is used to close the ML temperature equation (17) for the entrainment tendency. Terms \( \text{1}, \text{2} \) and \( \text{3} \) are easily evaluated from the air–sea data. The vertical advective flux term \( \text{3} \) is evaluated approximately by assuming that \( W \) has linear depth dependence above \( z = -c \) and vanishes at the sea surface,

\[
W(z) = -W(-c) z/c , \tag{20}
\]

and thus,

\[
\int_{-c}^{0} W \frac{\partial T}{\partial z} dz = W(-c) [\bar{T}_z^z - T(-c)] . \tag{21}
\]

\( \bar{T}_z^z \) denotes the depth average of \( T \) from \( z = -c \) to \( z = 0 \). The assumption (20) is consistent with assuming that horizontal velocity is well mixed within the ML and with the shape of low-order internal wave modes. \( W(-c) \) is evaluated by further assuming that horizontal advection is negligible compared to vertical advection in the presumably adiabatic temperature balance at \( z = -c \) as

\[
W = \frac{\partial T/\partial t}{\partial T/\partial z} = \frac{\partial z}{\partial t} . \tag{22}
\]

Vertical velocity is computed by following the motion of the isotherm which has the time-averaged value of temperature at \( z = -c \). The isotherm may be as much as \( \pm 5 \text{ m} \) from \( z = -c \) at its maximum excursion; hence, vertical velocity is interpolated to \( z = -c \) by (20). Using (21) and the definition of the surface heat flux \( \text{SF} \) in (19) gives the vertical average of horizontal advection

\[
V \cdot \nabla \bar{T}_z^z = \frac{1}{c} \left[ \int_{-c}^{0} \frac{\partial T}{\partial t} dz \\
+ \frac{1}{\rho_C C_p} [RS(0) - RS(-c) + \text{SF}] \\
- W(-c) [\bar{T}_z^z - T(-c)] \right] . \tag{23}
\]

The left-hand side of (23) is the residual of an approximate heat budget. A residual could arise from horizontal diffusion or measurement errors in temperature. In either event, use of (23) to compute a virtual horizontal advection would be appropriate. It is shown in Section 4a that the residual probably
is horizontal advection. The residual of (23) is an approximation of horizontal advection within the ML because of the assumptions required to evaluate the vertical advective flux term of (19), and because the residual is averaged to $z = -c$, where $c > h$. In the case studied here in which horizontal advection of temperature is most important (February 1973 case, Section 4a), the diagnosed value of $V \cdot \nabla T$ in the ML is typically $5 \times 10^{-5}$ °C s$^{-1}$. The local rate of change of temperature below the ML at $z = -c$ is typically $3.0 \times 10^{-5}$ °C s$^{-1}$, suggesting that (22) may be in error by 15%. The vertical advective flux term is comparable to the estimated residual; hence, (22) may lead to an error of 15% in the value of $V \cdot \nabla T^z$ diagnosed from (23). To estimate the error from averaging, we assume that the true value of $V \cdot \nabla T^z$ in the layer $-h > z > -c$ vanishes. Averaging over depth $c$ then leads to a relative error $(c-h)/c$. Hence, the depth $c$ should not be excessive, but must be below the interface layer (Fig. 1). In Price (1977) it is shown that the interface thickness $d$ observed during a short period of the case discussed in Section 4b follows

$$g'd/\delta V^2 = \frac{\Omega}{c}.$$  \hspace{1cm} (24)

Using (24) as a guide, and taking $\delta V = 20$ cm s$^{-1}$, $\delta T/\delta z = 1 \times 10^{-1}$ °C m$^{-1}$ and computing $\delta \rho$ as $\alpha \delta T/\delta z$, then $\delta \rho \approx 5$ m. The depth $c$ has therefore been estimated to be 5–10 m below the ML depth and averaging leads to an error of about 20% in the estimate of $V \cdot \nabla T^z$. Horizontal advection is roughly the same size as the diagnosed entrainment tendency. The two errors in horizontal advection estimated above thus lead to an error of $(15^2 \times 20^2)^{1/2} \approx 25\%$ in $\Omega$.

Combining (17) and (23), $\Omega$ may be diagnosed as

$$\Omega = \frac{\partial T}{\partial t} - \frac{1}{c} \left[ \int_c^0 \frac{\partial T}{\partial t} dz + W(-c)[T^z - T(-c)] \right]$$

$$- \frac{1}{\rho_p C_p} [RS(0) - RS(-c) + SF]$$

$$+ \frac{1}{\rho_p C_p} [RS(0) - RS(-h) + SF].$$  \hspace{1cm} (25)

An important consistency check of (25) is that while physically $\Omega$ must always have the same sign as $\delta T$, the diagnostic calculation is not constrained to have this property. A lower limit on the noise level of diagnosed $\Omega$ may thus be estimated independently of the calculation above by noting the occurrence of $\Omega \geq 0$ when $\delta T \geq 0$.

4. Observations and simulations

Two cases of storm-driven ML deepening are studied. The cases were observed at approximately the same location on the west Florida continental shelf (Fig. 2) but during different seasons. They differ greatly with respect to storm type and range of ML depth and represent independent tests of the model closure. Both data sets include time series of current and temperature profiles through almost the entire water column produced by the Cyclesonde (Van Leer et al., 1974), a moored autonomous profiling system (February 1973 case, Section 4a); or by the profiling current meter (PCM) (Curtin, 1974), a shipborne profiling system operated from an anchored vessel (June 1972 case, Section 4b). Profiles were made at time intervals $\leq 1$ h and had a vertical sampling interval of $\sim 5$ m.

a. Wintertime, February 1973 case

1) Atmospheric conditions and air-sea exchange

Winds at the Cyclesonde mooring were northwestward at approximately 5 m s$^{-1}$ (Fig. 3a) on 8 February. A strong atmospheric cold front passed the mooring late on 9 February, and wind speed increased to 15 m s$^{-1}$ while wind direction rotated clockwise through 150° at roughly the turning rate of an inertial oscillation [1 cycle(27.4 h)$^{-1}$]. Winds of 15 m s$^{-1}$ or greater persisted for 30 h. Dry-bulb and dew-point temperatures decreased roughly 10°C after the front passed (Fig. 3b); the air-sea temperature differences were approximately $-10^\circ$C and $-17^\circ$C on 11 February. Cloud cover, estimated from satellite photographs, was 0.5–1.0 during 9–12 February.

The sum of the turbulent surface heat flux and solar insolation, called the net surface heat flux, and the entrainment heat flux computed by DIM are
showed in Fig. 3c. The time average (over the record length) surface heat flux was $-1070$ cal cm$^{-2}$ day$^{-1}$, while the time average entrainment heat flux was $-220$ cal cm$^{-2}$ day$^{-1}$. Air-sea heat exchange was thus responsible for $\sim 80\%$ of the net local (non-advective) cooling of the ML. The temporal evolution of the entrainment heat flux is a sensitive model prediction discussed in detail below. Any model which is adjusted to correctly simulate the final ML depth predicts a very similar time average of the entrainment flux.

2) HYDROGRAPHY

STD sections were sampled along 26°N on 8 and 12 February. The instantaneous ML depth and temperature, and the change in ML depth and temperature (Fig. 4) showed large cross-shelf gradients on both occasions. On average, ML depth increased and ML temperature decreased between 8 and 12 February as expected. But west of 84°35'W, the ML depth decreased while the ML temperature increased. The atmospheric forcing was roughly uniform, though not in phase, over distances of hundreds of kilometers and is probably not the cause of the observed horizontal inhomogeneity. The two likely causes are spatial-temporal aliasing of ML depth fluctuations caused by inertial-internal waves and horizontal advection by subinertial frequency currents. The internal tide had an amplitude of roughly 5 m over the shelf and could account for a portion of the variability of $h_0$. These sections caution that advection may dominate the local rate of change of a ML property even when atmospheric forcing is intense. It is therefore essential that advection and ML deepening be separated before attempting to compare an idealized model with these field observations.

3) TIME-DEPTH SECTIONS

The Cyclesonde mooring was located in a region where the sense of the ML depth and temperature change from 8 to 12 February (deepening and cooling) suggests that local atmospheric forcing was dominant. The Cyclesonde profiled from 20 m depth to within a few meters of the bottom. Only the upper 100 m from the second half of the record is discussed here; the full record is discussed by Price (1976). ML depth fluctuated in phase with isotherms until about 2200 LT 9 February, the time of the frontal passage, and then began to cut through isotherms suggesting ML deepening. Isotherms continued to surface at a slower rate due to surface cooling and horizontal advection (demonstrated below). The initial (at 0200 LT 9 February) ML depth is estimated to be 30 ± 2 m. The final (at 0400 LT February) ML depth $h_{f0}$ is estimated to be 46 ± 2 m.

Vertical shear of horizontal velocity was large $[N^2/(\partial V/\partial z)^2 < 1]$ at and just below the base of the ML for a 6 h period centered on the frontal passage.

![Fig. 3a](image1)

![Fig. 3b](image2)

![Fig. 3c](image3)

![Fig. 4a](image4)

![Fig. 4b](image5)
due to a downwind acceleration of current within the ML (Figs. 5b and 5c). Shear reached high values again from 0300 to 1200 LT 10 February at a depth well within the thermocline. The shear at that time was not due to relatively high current speeds within the ML, but apparently to vertically propagating inertial-wave motion excited by the frontal passage, and perhaps superimposed internal tidal motion.

4) SIMULATIONS

The initial temperature profile, based on Cyclesonde data, was \( T_i = 24.05^\circ \text{C}, h = 30 \text{ m}, \delta T = 0, \) and \( \partial T/\partial z = 1.0 \times 10^{-1} \text{ }^\circ \text{C m}^{-1} \) below the ML. The initial salinity profile, based on STD data, was \( S_i = 36.30\%, \delta S = 0, \) and \( \partial S/\partial z = 7.7 \times 10^{-3} \text{ }\% \text{ m}^{-1} \). The behavior of simulated entrainment tendency was not sensitive to \( \delta \rho \) specified in the initial condition. Internal tidal currents of \( \approx 10 \text{ cm s}^{-1} \) were present before the frontal passage but the vertical shear of these motions was small compared to the vertical shear of the motions generated by the storm. The initial velocity profile was therefore taken as zero throughout the column.

The final ML depth \( h_T \) computed by ECM over a range of \( m_2 \) and \( m_3 \) between 0.0 and 1.0 is shown in Fig. 6a. The values written just above the abscissas are from DIM which deepens slightly less than ECM at the same \( m_s \). For \( m_2 = 0 \) (TEM), the increase in ML depth \( (h_T - 30) \) is proportional to \( m_3^{1/3} \). For \( m_3 = 0 \) (DIM), the increase in ML depth is proportional to \( m_2^{1/4} \). The requirement that \( h_{T0} \) be simulated correctly is satisfied by any \( m_2, m_3 \) pair along the line \( h_T = 46 \text{ m} \). The best \( m_2, m_3 \) pair is taken to be the one which minimizes the mean squared deviation (MSD) between observed and simulated entrainment tendency, \( \text{MSD} = (\Omega_{T0} - \Omega)^2 \) (Fig. 6b). Observed entrainment tendency was computed at hourly intervals from the Cyclesonde temperature data and \( \text{MSD} \) was computed over the duration of the model simulation. The minimum of \( \text{MSD} \) lies on the locus \( h_T = 46 \text{ m} \) at very small \( m_3 \). That \( h_T = 46 \text{ m} \) is consistent with minimum \( \text{MSD} \) suggests that \( h_{T0} \) was not seriously contaminated by horizontal advection. The analysis of \( \text{MSD} \) selects a narrow range of \( m_2, m_3 \); the best simulation has \( \text{MSD} = 2.0 \times 10^{-12} \text{ }^\circ \text{C}^2 \text{ s}^{-2} \) and occurs at \( m_2 = 0.65 \pm 0.05 \) and \( m_3 < 0.05 \). The DIM simulation with \( m_3 = 0.65 \) is equivalent to the best ECM simulation. The variance of \( \Omega_{T0} \) is \( \text{MSD} \) at \( m_2 = m_3 = 0 \) (non-penetrative deepening only, \( \Omega = 0 \)) and is \( 5.3 \times 10^{-12} \text{ }^\circ \text{C}^2 \text{ s}^{-2} \). Thus the best simulation accounts for about 60% of the variance in \( \Omega_{T0} \). (The noise level of \( \Omega_{T0} \) is discussed below.)

Denman and Miyake (1973) found that \( m_3 = 1.0 \) gave the best fit of their ML model (very similar to TEM but with \( m_2 = 1 \)) to their data. In the present case, TEM with \( m_3 = 1.0 \) gives \( h_T = 54 \text{ m} \) which is significantly deeper than \( h_{T0} \) and \( \text{MSD} \) is relatively large. The values of \( m_3 \) deduced from oceanic observations by Turner (1969) \( (m_3 = 10.4 \) and 5.2) and by Halpern (1974) \( (m_3 = 2.8) \) give still larger \( h_T \) and \( \text{MSD} \).

The DIM and TEM simulations of \( T_i \) (Fig. 7a)
the penetrative deepening. The interface could not be resolved in the Cyclesonde temperature profiles. The 5–10 m thick layer of relatively low $N^2$ found beneath the ML from approximately 0000 LT 10 February to 1200 LT 10 February (Fig. 5b) may be a manifestation of the interface and may account for a portion of the phase difference between simulated $h$ and $h_0$.

The simulated entrainment tendencies are qualitatively different and clearly distinguish DIM from TEM even though simulated $T_i$ and $h$ were similar (compared to the discrepancy between either simulation and the observations). The most prominent feature of $\Omega_0$ (Fig. 7c) is a negative peak of magnitude $9 \times 10^{-6}$ °C s$^{-1}$ which occurred from about 1800 LT 9 February to 0400 LT 10 February. This corresponds with the cold front passage and is the time that ML depth began to cut through isotherms (cf. Fig. 5a). For this case $\beta T < 0$ at all times, and the noise level of $\Omega_0$, $\Omega_n$, may be estimated by noting the magnitude of $\Omega_0$ at the times when $\Omega_0 > 0$; $\Omega_n = 1.5 \times 10^{-6}$ °C s$^{-1}$, somewhat less than the error $[0.25 \times (9 \times 10^{-9}) = 2.2 \times 10^{-6}]$ estimated in Section 3c. The contribution of $\Omega_n$ to the variance of $\Omega_0$, is estimated roughly as $\frac{1}{2} \Omega_n^2 = 1.2 \times 10^{-12}$ °C$^2$ s$^{-2}$ (assuming the noise is uncorrelated with the signal) which is about 25%.

Fig. 6a. ML depth (m) at 0400 LT 12 February ($h_0$) computed by ECM as function of $m_0$, $m_i$. Values just above abscissa are from DIM. 6b. Mean-square deviation of $\Omega$ from $\Omega_0$ (°C$^2$ s$^{-2}$) $\times 10^{-2}$ as function of $m_0$, $m_i$.

(assuming $h_f = 46$ m) are quite similar$^5$ because the net surface heat flux is the same for all simulations and accounts for approximately 80% of the net local heat flux into the ML. Simulated $T_f$ matches $T_{10}$ well until 0600 LT 10 February. Subsequently, $T_{10}$ decreases more rapidly than either simulation and by 12 February the simulations differ from $T_{10}$ by roughly 0.6°C. In Section 4b it is shown that this discrepancy is due largely to horizontal advection.

DIM deepens rapidly to $h_f$ and then remains steady, while TEM deepens gradually (Fig. 7b). Neither simulation matches $h_0$ well because inertial-internal wave advection causes large-amplitude (5 m) fluctuations in $h_0$ which effectively obscure

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$^5$ $T_i$ and $h$ simulated by ECM are qualitatively very similar to TEM if $m_0 < 0.05$, $m_i > 0.1$, and very similar to DIM when $m_0 > 0.3$ and $m_i < 0.1$. For intermediate values of $m_0$ and $m_i$ ($m_0 = m_i = 0.3$), the simulation of ECM appears, qualitatively, to be an average of DIM and TEM.

---

Fig. 7a. Observed and simulated ML temperature. Simulations are from DIM ($m_0 = 0.70$) and TEM ($m_0 = 0.35$). 7b. ML depth. 7c. Entrainment tendency. All variables have 1 h time intervals. The control volume depth (depth c) was 35 m until 1800 LT 9 February and 50 m thereafter. Observed entrainment tendency was smoothed with a five-point polynomial weight taper.
of the variance of $\Omega_0$. An ideal simulation can thus be expected to account for only about 75% of the variance of $\Omega_0$. DIM gives the best MSD$\Omega$ because it simulates the strong deepening which occurred during the frontal passage. The magnitude of DIM simulated $\Omega$ is somewhat low and the peak is 2 or 3 h after the peak in $\Omega_0$. However, TEM fails to simulate the major peak in $\Omega_0$ and predicts significant deepening to continue after 0400 LT 10 February when, apparently, no deepening occurred. From Figs. 6 and 7 it is apparent that the model cannot deepen as strongly as required during the cold front passage and then continue to deepen after the frontal passage as does TEM ($m_3 = 0.3$) without significantly exceeding both $h_{10}$, and the net, observed entrainment tendency. The maximum value $m_3$ can assume without noticeably degrading this simulation is estimated to be $m_3 = 0.15$.

Simulations were also run using ECM with $m_1$ nonzero (penetrative convection). For $m_1 = 0.03$ (Farmer, 1975; Deardorff et al., 1969), penetrative convection makes no significant contribution to ML deepening. The intense heat loss on 10 and 11 February does not coincide with the observed entrainment tendency, and if $m_1$ is made large enough to have any noticeable effect, $m_1 > 0.2$, the simulations are degraded compared to the case $m_1 = 0$.

5) Wind stress, mixed-layer current phase relation to mixed-layer deepening

The very rapid decrease in the magnitude of $\Omega_0$ at about 0400 LT 10 February occurred before wind speed reached its maximum value. This may be interpreted by examining the rate of work, $X_0$, of wind stress on the velocity difference across the base of the ML, $X_0 = \tau_0 \cdot \delta V_0$. Wind stress was computed directly from wind observations; $\delta V_0$ was estimated from directly observed current, $V_0$ (cf. Fig. 10b) as

$$\delta V_0 = \frac{1}{h_0 - 20} \int_{-h_0}^{-20} V_0 dz - \frac{1}{h_0} \int_{-h_0}^{-2h_0} V_0 dz.$$ (26)

(The sign convention of the $\delta$ operator is reversed for this discussion. No data are available above 20 m depth.) This is expected to yield a satisfactory estimate of the velocity difference (on the scale $h_0$) across the base of the ML. The rate of work $X_0$ was strongly positive during the increase in wind speed and the clockwise rotation of wind direction from 0800 LT 9 February to 0200 LT 10 February ($r_0$ and $\delta V_0$ in near resonance) (Fig. 8). The ML current generated on 9 February continued to rotate as an inertial motion on 10 February when wind direction was nearly constant. By 0300 LT 10 February, $\delta V_0$ had rotated 90° to the right of the wind stress and $X_0$ vanished. This corresponds closely with the cessation of observed entrainment tendency (cf. Fig. 7c). Wind stress magnitude was maximum at 1300 LT 10 February, but wind stress was then acting to decrease $\delta V_0$ ($X_0 < 0$). There was little or no entrainment tendency at that time. The strong correlation between $\Omega_0$ and $X_0$ is taken as strong evidence that mean flow shear, and not wind stress per se, dominated the ML deepening observed here.

Simulated and observed ML current, $V_t$ and $\delta V_0$, agree well only during the acceleration which occurred late on 9 February (Fig. 9). Thereafter, $\delta V_0$ appears to rotate clockwise roughly 25% faster than $V_t$. The amplitude of $V_t$ consistently exceeds the amplitude of $\delta V_0$. This, together with the large vertical shear in the main thermocline noted above, suggests that a substantial momentum flux accompli-
Fig. 10a. Horizontal temperature advection computed from heat budget at 1 h intervals and smoothed with a five-point polynomial weight taper. 10b. Directly observed velocity components within the ML. 10c. Observed and simulated (by DIM) ML temperature and simulated ML temperature plus the horizontal advection shown in Fig. 10a.

panying inertial-internal wave vertical propagation occurred after the frontal passage. The failure of the model to simulate accurately the phase and magnitude of ML current beyond 9 February would probably have resulted in a serious error in the simulation of ML deepening if a second wind event had occurred on 10 or 11 February.

6) HORIZONTAL TEMPERATURE ADEPTION

Horizontal temperature advection computed by the heat budget method [Eq. (23)] was comparable in magnitude to surface cooling (Fig. 10a) and was important in the calculation of entrainment tendency. It is not possible to strongly verify the estimated temperature advection by direct calculation of $V_0 \cdot \nabla T_0$ because $\nabla T_0$ changed significantly on a daily time scale and over horizontal scales of 20 km or less (Fig. 4a). It is possible to roughly verify the magnitude by estimating the east-west ML temperature gradient from Fig. 4 as $|\partial T/\partial x| = 3^\circ C(100 \text{ km})^{-1} = 3 \times 10^{-7} \text{ C cm}^{-1}$ (typical) and $2^\circ C(30 \text{ km})^{-1} = 7 \times 10^{-7} \text{ C cm}^{-1}$ (extreme). Current speeds, estimated from directly observed east-west velocity component (Fig. 10b), were $|u| = 10 \text{ cm s}^{-1}$ (typical) and 25 cm s$^{-1}$ (extreme). These gave $|u| |\partial T/\partial x| = 3.0 \times 10^{-6} \text{ C s}^{-1}$ (typical) and $16 \times 10^{-6} \text{ C s}^{-1}$ (very extreme), which are con-
sistent with Fig. 10a. The corresponding north-south (alongshore) temperature advection was roughly half the magnitude of cross-shelf advection. Also, the time scale of $(\nabla \cdot \nabla T)^0$ fluctuations is similar to that of the directly observed velocity components. These are evidence that the quantity computed from (23) is a plausible representation of horizontal advection.

A second consistency check is made by adding temperature advection to simulated $T_i$ (Fig. 10c) as

$$T_i(t) = T_i(t) + \int_0^t (\nabla \cdot \nabla T)^0 dt.$$ (27)

The difference between $T_i$ and $T_{10}$ arises from error in the simulation of entrainment tendency or the estimation of horizontal temperature advection within the ML. The difference is very small during the first two days of the simulation (errors could fortuitously cancel). The maximum difference is 0.1°C and occurs at 1500 LT 11 February when deepening had ceased in all of the simulations. The

Fig. 11a. Wind speed and direction measured by the research vessel at one-hour intervals. The slope of the dashed line "f" equals the rate of change of direction of an inertial oscillation. Numbered tick marks on abscissas are 0000 LT of the day marked. 11b. Dry-bulb and dew-point temperatures and cloud cover. SST taken from STD data. 11c. Net surface heat flux computed from air-sea data above using methods of Appendix B and the simulated (by DIM) entrainment heat flux.
magnitude and rate of development of the difference indicates that estimated horizontal advection was too large by $2 \times 10^{-6} \, ^\circ \text{C} \, \text{s}^{-1}$ during the preceding several hours. This same error is evident as the relatively large positive value of $\Omega_0$ at 1200 LT 11 February (Fig. 7c).

b. Summertime, June 1972 case

1) Atmospheric conditions and air-sea exchange

During the first two days of the experiment, 8–9 June, winds were light and variable (Fig. 11a). Wind speed increased to 5 m s$^{-1}$ by 11 June and the direction shifted to east-southeast. Wind speed increased rapidly to 13 m s$^{-1}$ early on 12 June, then eased to about 10 m s$^{-1}$ by midday. The dominant spatial scale of the wind field, estimated from sea level pressure charts, was in excess of 200 km. Dry-bulb temperature exhibited large ($\approx 8^\circ \text{C}$) diurnal variations during 8 and 9 June when cloud cover was light (Fig. 11b). Cloud cover increased to nearly 1.0 by 11 June, and dry-bulb temperature stabilized at $27^\circ \text{C}$, very nearly equal to SST. Dewpoint temperature remained steady at about $24^\circ \text{C}$ after wind speed had increased.

The average net surface heat flux was $-75$ cal cm$^{-2}$ day$^{-1}$ while the average entrainment heat flux computed by DIM was $-155$ cal cm$^{-2}$ day$^{-1}$ (Fig. 11c). Hence, most of the local ML cooling in this case is attributable to the entrainment heat flux.

2) Hydrography

Hydrographic sections along 26$^\circ$N were sampled before and after the anchor station (Fig. 12). On 5 to 6 June, ML temperature and depth appeared to have significant cross-shelf gradients as in the previous case though the subsurface fields (not shown) were relatively level. Winds were light on 5 to 6 June, and the shallow ML depths and high ML temperatures from 83$^\circ 25'$W to 84$^\circ 00'$W were observed during midday. Hence, the apparent spatial variations were probably caused by a diurnal oscillation. On 14 and 15 June the ML was, on average, 10 m deeper; however, west of 84$^\circ$W longitude the ML temperature increased by roughly 0.8$^\circ \text{C}$. An increase of that magnitude must have been caused by horizontal advection. The anchor station was located in a region where the change in
ML depth and temperature, deepening and cooling, were in the sense expected from the local atmospheric forcing.

3) Time-depth section of STD temperature

Fluctuations of the ML depth appear to be in phase with isotherm motion except at roughly 0000 on 10, 11, and 12 June when the ML depth cut through isotherms which then surfaced (Fig. 13). As shown below, these were times when the ML was deepening. Isotherms just beneath the ML moved downward in advance of the ML, indicating that gradual warming occurred at a given level before the ML arrived. This is probably a manifestation of an interface layer at the base of the ML. Aside from internal wave-induced stretching, ML depth shows a quantitative correlation with wind speed (cf. Fig. 11a).

Significant warming of the ML is apparent during midday of 10 and 11 June. The ML thickness decreased at those times but never vanished. On two occasions, at 1300 LT 12 June and at 0700 LT 13 June, the ML depth changed discontinuously on the 1 h sampling time scale when precipitation (2 and 6 cm) caused the ML to reform at the surface. The new ML's rapidly deepened and merged with the relic ML formed late on 11 June.

From 8 to 14 June the range of the diurnal SST cycle decreased from ~1.3°C to <0.1°C. The change was due in part to the increase in cloud cover which reduced insolation by roughly a factor of 2. Of greater importance was the increase in the effective thermal mass \( \lambda \) of the surface layer. On 8 and 9 June when \( h < 1 \text{ m} \), \( \lambda \) is somewhat arbitrarily estimated as the \( e \)-folding scale of \( R/S(z) \), \( \lambda \approx 3 \text{ m} \). On 12 and 13 June, \( \lambda \) was the ML depth, \( \lambda = 25 \text{ m} \). This factor of 8 increase in \( \lambda \) accounts for most of the decrease in the range of the diurnal SST cycle.

The ML was well mixed with respect to density at all times after 1400 LT 11 June. There was no clearly observable temperature gradient within the ML; the maximum gradient (except very near the base of the ML) was <10^{-3} °C m^{-1}.

Near-surface velocity measurements made with the PCM are subject to surface wave contamination. Because the ML was often very thin in this case, a time-depth section of \( (\partial V/\partial z)^2 \) is not shown. Unlike the previous case, there was never a prominent shear maximum within the thermocline.

4) Simulations

The initial temperature and salinity profiles for all but the upper 3 m were taken to be the time average over the first 6 h of the STD data. Temperature in the upper 3 m varied rapidly during 8 June and was taken to be the instantaneous observed temperature at 1200 LT 8 June, the beginning of the simulation. There was no initial ML. Initial velocity was taken as zero. The initial density profile included a layer of relatively low static stability extending from 15 to 25 m depth. Because \( g' h \) increased only slightly as the ML deepened from 15 to 25 m, this "thermocline duct" (Tully and Giovando, 1963) was an important feature of the initial condition.

Observed ML temperature (Fig. 14a) has an estimated uncertainty of ±0.3°C during the diurnal maxima of 8 and 9 June when \( h_\theta < 1 \text{ m} \) and there were strong near-surface gradients. The uncertainty is ±0.05°C at other times. Observed ML depth, Fig. 14b, has an estimated uncertainty of ±1 m except during the period of weak winds and strong heating on 9 June when the uncertainty was ±2 m. Equivalent ML depth \( h_\theta + d / 2 \) is shown when the ML reformed at the surface.

Only DIM and TEM were used to simulate this case. The free constants in DIM and TEM were adjusted to give \( h_\theta = 26 \text{ m} \) at 1000 LT 14 June which required \( m_2 = 0.6 \) in DIM, and \( m_\theta = 0.9 \) in TEM. With TEM and \( m_\theta = 0.3 \) as in the February 1973 case, \( h_\theta = 18 \text{ m} \), significantly less than \( h_\theta \). The
greatest deviation of $T_i$ from $T_{io}$ for both simulations occurred during the diurnal heating of 8 and 9 June, when winds were very light and $h_o$ was $< 1$ m thick. Under such conditions, SST simulation is very sensitive to the form of the insolation absorption function, $\gamma(z)$ (see Appendix B) which was chosen to be similar to that of average oceanic water (Sverdrup et al., 1942, p. 107).

Aside from the first diurnal cycle, both models satisfactorily simulate the main features of $h_o$ and $T_{io}$ in this case (Figs. 14a and 14b). However, there are significant, detailed differences between the simulations. For example, both models correctly predict the reformation of the ML during the precipitation events of 12 and 13 June. DIM accurately simulates the subsequent rate of ML deepening; TEM underestimates the rate of deepening of those shallow MLs by more than a factor of 2. (The temperature difference $\delta T$ of the rain-formed ML's was small ($\approx 0.05^\circ C$) and the entrainment tendency of temperature was not clearly resolved.)

Observed entrainment tendency (Fig. 14c) was highly intermittent as in the previous case. Deepening occurred strongly during periods centered on 0400 LT 10 June, 2200 LT 10 June and 2100 LT 11 June. Deepening either did not occur or was below the noise level of observed entrainment tendency at other times. This qualitative feature is matched by the DIM simulation which deepens at approximately the observed rate, but with a phase error of 2 to 3 h on each of the three occasions. Such a phase error may not be significant given the 1-h sampling interval of the air-sea data and 1 h time step of the numerical simulations. The greatest difference between TEM and DIM occurs between 1200 LT 10 June and 0000 LT 11 June. TEM produced a pulse of entrainment tendency centered at 1300 LT 10 June in response to increased wind speed. DIM predicted no deepening until approxi-

![Image](image.png)

**Fig. 15.** Observed and simulated (by DIM) east-west (a) and north-south (b) component of $\delta V$. No observed velocity is shown until $h_o$ exceeded 5 m. Gaps in observed velocity due to instrument malfunction are noted by ND.

![Image](image.png)

**Fig. 16.** Horizontal temperature advection computed from heat budget at one-hour intervals and smoothed with a five-point polynomial weight taper.

mately 5 h later when wind speed increased slightly and wind direction rotated clockwise. DIM more closely matches the observed entrainment tendency.

The deepening at 2100 LT 11 June was simulated by DIM and TEM with roughly equal success. Deepening was in response to a sudden rise in wind speed which persisted slightly less than half an inertial period, while wind direction remained approximately constant. The ML current generated by the sudden increase in wind stress did not rotate through $180^\circ$ before wind stress substantially decreased, causing deepening to virtually cease in both simulations. Hence, the simulations of this event are not qualitatively different.

Directly observed and simulated ML velocity are in substantial agreement (Fig. 15). $V_i$ has nearly the correct magnitude after the wind event early on 12 June, but lags $V_{io}$ by approximately 3 h, consistent with the phase error in the DIM simulation of entrainment tendency. The simulated entrainment momentum flux $\delta V e$ (not shown) reached 2 dyn cm$^{-2}$ during the deepening at 2100 LT 11 June and significantly influenced the magnitude of $V_i$.

The magnitude of horizontal temperature advection (Fig. 16) was almost as large as that of the previous case, but was relatively small compared to the entrainment tendency. Horizontal advection oscillated about an average value of nearly zero and never seriously affected ML temperature.

c. Comparison of cases

The value of the free parameter $m_3$ of DIM required to simulate the final observed ML depth was nearly the same in both cases (Table 1). The value of $m_3$ of TEM decreased by a factor of 3 from the wintertime to the summertime case. The sense of the change in $m_3$ is consistent with assuming that the $h^{-2}$ dependence of the time-dependent response of $\delta V e$ is roughly correct (Section 2d). That is, if $m_3$ is adjusted so that the deepening response

<table>
<thead>
<tr>
<th>ML depth (m)</th>
<th>$m_3$ of DIM</th>
<th>$m_3$ of TEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wintertime case</td>
<td>30/46</td>
<td>0.7</td>
</tr>
<tr>
<td>Summertime case</td>
<td>0/26</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**Table 1.** Stability of free parameters.
of TEM is roughly correct for a deep ML, then TEM would underpredict the deepening of a shallower ML, e.g., the summertime case as a whole, and the very shallow rain-formed ML's of that case in particular.

There were two occasions when the simulated rate and phase of deepening by DIM and TEM were distinctly different (the wintertime case and the second deepening event of the summertime case). On both occasions, the DIM simulation was cooler to the observed rate and phase of deepening. In the wintertime case, deepening was shown to occur strongly only when wind stress accelerated the observed velocity difference $\delta V_0$ independent of the wind stress magnitude.

The time averages of the heat fluxes due to entrainment, air-sea exchange, and horizontal advection (Table 2) are, of course, not stationary. There is one significant feature, however; the wintertime case was dominated by air-sea exchange. The entrainment heat flux was as large as the air-sea exchange for a short period (~8 h) but was not sustained. In the summertime case, the air-sea temperature difference was relatively small, and the ML was initially very thin. Under those circumstances the entrainment heat flux was dominant.

### Table 2. Time-averaged (percentage) heat fluxes (cal cm$^{-2}$ day$^{-1}$).

<table>
<thead>
<tr>
<th></th>
<th>Entrainment</th>
<th>Air-sea</th>
<th>Horizontal advection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wintertime case</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0200LT 9 February to 0400LT 12 February</td>
<td>$-220$ (13)</td>
<td>$-1070$ (61)</td>
<td>$-450$ (26)</td>
</tr>
<tr>
<td>Summertime case</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200LT 8 June to 1000LT 14 June</td>
<td>$-155$ (62)</td>
<td>$-75$ (30)</td>
<td>$-20$ (8)</td>
</tr>
</tbody>
</table>

5. Summary, conclusions and remarks

Two independent cases of ML deepening have been analyzed and simulated to find a parameterization of ML deepening. Because surface heat exchange and advection are generally important processes, a straightforward comparison of simulated and observed ML depth and temperature does not constitute a sensitive test of a parameterization. A method was therefore developed to compute the entrainment tendency of temperature, a sensitive measure of ML deepening, from field data. DIM gave a realistic simulation of net ML deepening and the rate and phase of ML deepening of both cases. Because the cases differed widely with respect to range of ML depth and storm type, the basic feature of DIM, dependence on $\delta V$ (through $R_v$) seems to be verified. By contrast, the simulations of TEM were much less realistic, and indeed, there was no clear evidence of any significant ML deepening driven by wind stress alone ($U_{\tau}^3$ parameterization).

The failure of TEM does not exclude the possibility that a weak $U_{\tau}^3$ mechanism exists. Even if $m_3 = 0.15$, the upper bound estimated in Section 4a5, the contribution of the $U_{\tau}^3$ mechanism to the annual ML cycle would be important. Neither should the success of DIM be interpreted as showing the existence of a critical $R_v$ in a literal sense. It does imply that entrainment decreases rapidly as $R_v$ exceeds about 0.6, as observed in the Ellison and Turner (1959) experiment. It is shown in Price (1977) that the Kato and Phillips (1969) and Kantha et al. (1977) laboratory experiments also support the choice $R_v \approx 0.65$.

No dynamical explanation of why $R_v \approx 0.65$ is offered. The interpretation of $R_v$ as the mean kinetic energy to potential energy conversion efficiency appears unlikely since our result would then imply 65% efficiency which is considerably greater than direct measurements indicate (cf. Thorpe, 1973). It is speculated that much of the work required to lift dense fluid through the ML during deepening probably is performed by turbulence generated near the sea surface but that such motions alone cannot cause detachment of fluid from the interface.

DIM may not be appropriate for ML simulation of a case in which the wind stress has been filtered or averaged in a way that removes energy at the local inertial frequency. If only low-pass filtered winds are available, the $U_{\tau}^3$ parameterization used alone may be adequate provided the coefficient $m_3$ is given an ad hoc depth dependence. For example, TEM gives a satisfactory prediction of final ML depth in the cases studied here if $m_3 = m_4 \exp(h/m_5)$, and $m_4 = 2.0$ and $m_5 = 20$ m. A number of investigators, including Garwood and Camp (1977), have recognized the need for some form of depth dependence for $m_3$. It has been argued that the depth dependence is required to model the spatial decay of turbulence generated near the sea surface. The present results suggest that the depth dependence of $m_3$ is necessary primarily to parameterize the $h^{-2}$ dependence of $\delta V^2$. The modified TEM does not, of course, accurately simulate the observed rate and phase of deepening since it neglects the phase between wind stress and ML current. For that reason $m_4$ may have to be given seasonal and zonal dependence to reflect characteristic storm patterns.

A wider sample of observed cases is required to confirm and refine these results. A primary goal of future field experiments must be to acquire measurements which allow an unambiguous separation of ML deepening from stretching and air-sea buoyancy exchange. Because the ML is very nearly vertically homogeneous, measurements made at any depth in the ML suffice to define ML properties. Pumping
systems may thus be employed to acquire continuous
time series of ML temperature, salinity, nutrient
concentration, etc. The ideal tracer for ML model
verification using the budget method developed
here is one which 1) can be continuously monitored
(as with a pumping system), 2) can be vertically
profiled, and 3) has no significant unknown surface
flux or local source within the ML. With con-
temporary pumping, profiling, and sensing systems,
several degrees of freedom are available to aid in
quantifying the entrainment flux.

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APPENDIX A

Glossary of Symbols and Acronyms

\[ x, y, z \] positive east, north, up co-
ordinates

\[ T, S, \rho, V, W \] temperature, salinity, density,
horizontal velocity vector, vertical velocity

\[ \text{entrainment velocity} \]

\[ T_i, S_i, \rho_i, V_i \] ML value of temperature, salinity, etc.

\[ h, h_f \] depth (or thickness) of ML
final (end of study period) ML

\[ d \] thickness of interface layer

\[ \delta T, \delta S, \text{etc.} \]

\[ T_o, S_o, \text{etc.} \]

\[ \nabla \] horizontal gradient operator

\[ \rho_r \] reference density (1.025 g cm\(^{-3}\))

\[ \alpha \] coefficient of thermal expansion,

\[ \beta \] coefficient of haline expansion,

\[ Q = RS(0) - RS(-h) + SF \]
net surface heat flux absorbed
in ML, equal to flux of insola-
tion at surface, RS(0), minus
flux at \( z = -h \),

\[ C_p \]
heat capacity of sea water

\[ g \]
acceleration of gravity

\[ f \]
Coriolis parameter at 26\(^\circ\)N

\[ \tau \]
wind stress vector

\[ R_v \]
overall Richardson number of
the ML, scaled with

\[ \text{nondimensionalized entrain-
ment, } [= \text{We/} \delta V, \text{We/} U_s^2] \]

\[ c \]
depth of control column used
for heat budget calculation
of horizontal advection

\[ \Omega \]
entrainment tendency of tem-
perature \([= -\delta T \text{We}/k] \)

\[ m_1, m_2, m_3 \]
ML

\[ \text{SST} \]

\[ \text{ECM} \]

\[ \text{TEM} \]

\[ \text{DIM} \]

\[ \text{MSD} \]

APPENDIX B

Computation of Turbulent and Radiative Fluxes

The formulas used in estimating turbulent and
radiative surface fluxes are

\[ \tau = \rho_o C_a U_o |U_o|, \quad \text{(B1)} \]

\[ QL = \rho_o C_q |U_o| (q_{10} - q_{18}) L, \quad \text{(B2)} \]
\[ QS = \rho_a C_q |U_a| (T_{ss} - (T_{10} + \gamma))C_p, \]  
\[ RL = Sr(T_{10} + 273)^4 (0.39 - c \sqrt{e_{10}} / (1 - 0.9n)) - [(T_{ss} + 273)^4 - (T_{10} + 273)^4], \]
\[ RS(z) = RS_{\text{air}}(z), \]  
\[ RSo = (RS_{\text{air}} + RS_{\text{dir}})(1 - 0.71n)(1 - RL(\alpha)), \]
\[ \gamma(z) = b_1 \exp(z/\gamma_1) + b_2 \exp(z/\gamma_2), \quad z < 0. \]

In formulas (B1) to (B5), the symbols have the following meaning, units and constant values (where appropriate):

-  \( \tau \): wind stress vector (dyn cm\(^{-2}\))
-  \( \text{QL} \): latent heat flux (cal cm\(^{-2}\) s\(^{-1}\))
-  \( \text{QS} \): sensible heat flux (cal cm\(^{-2}\) s\(^{-1}\))
-  \( \text{RL} \): net longwave radiation flux (cal cm\(^{-2}\) s\(^{-1}\))
-  \( \text{RSo} \): solar insolation absorbed by ocean (cal cm\(^{-2}\) s\(^{-1}\))
-  \( \text{RS}_z \): solar insolation flux at depth \( z \) (cal cm\(^{-2}\) s\(^{-1}\))
-  \( \rho_a \): air density \( = 1.23 \times 10^{-3} \) (g cm\(^{-3}\))
-  \( U_a \): wind vector at 10 m level (cm s\(^{-1}\))
-  \( |U_a| \): wind magnitude at 10 m level (cm s\(^{-1}\))
-  \( C_d \): drag coefficient \( = 1.5 \times 10^{-3} \)
-  \( C_q \): exchange coefficient for moisture and heat \( = 1.3 \times 10^{-3} \)
-  \( L \): latent heat of evaporation \( = 590 \text{ cal g}^{-1}\)
-  \( q_{ss} \): specific humidity of saturated air at sea surface temperature and pressure
-  \( q_{10} \): specific humidity of air at 10 m level
-  \( C_p \): specific heat of air at constant pressure \( = 0.24 \text{ cal gm}^{-1} \text{ C}^{-1}\)
-  \( T_{ss} \): sea surface temperature (°C) (assumed to be ML temperature)
-  \( T_{10} \): dry-bulb temperature at 10 m level (°C)
-  \( \gamma \): adiabatic lapse rate correction \( = 0.1 \text{ °C} \)
-  \( \sigma \): Boltzman constant \( = 1.35 \times 10^{-12} \text{ cal cm}^{-2} \text{ K}^{-4} \text{ s}^{-1}\)
-  \( S \): relative emissivity \( = 0.98 \)
-  \( c \): a constant \( = 0.05 \text{ mb}^{-1}\)
-  \( e_{10} \): vapor pressure at 10 m level (mb)
-  \( n \): cloud cover \((0 \leq n \leq 1)\)
-  \( \text{RS}_{\text{dir}} \): direct solar insolation \( = \text{Ro sin} (\alpha) \delta_c (\text{cal cm}^{-2} \text{ s}^{-1})\)
-  \( \text{RS}_{\text{dif}} \): diffuse solar insolation \( = \text{Ro sin} (\alpha) (0.91 - \delta_c (\text{cal cm}^{-2} \text{ s}^{-1}))\)
-  \( \text{Ro} \): solar constant \( = 0.032 \text{ cal cm}^{-2} \text{ s}^{-1}\)
-  \( \sin(\alpha) \): sine of sun’s altitude \( = \cos(\theta) \cos(\text{lat}) \cos(\phi) + \sin(\theta) \sin(\text{lat})\)
-  \( \theta \): sun’s declination \( = 23(2\pi/360) \cos(2\pi(357 - t)/365)\)
-  \( \phi \): sun’s hour angle \( = 2\pi(t + 0.5)/365\)
-  \( t \): Julian date from 0000 LT 1 January
-  \( \text{lat} \): latitude
-  \( \delta \): atmospheric transmission coefficient, taken as 0.85

A thorough discussion of the parameterization of turbulent fluxes (B1), (B2) and (B3) may be found in Pond et al. (1971). Computation of the net longwave radiation flux (B4) follows the method described by Kondratyev (1969, pp. 571–573). Solar insolation is computed according to List (1958, p. 420). Absorption of solar insolation follows Sverdru et al. (1942, pp. 104–110) and Kraus (1972, p. 92).

REFERENCES


