Comments on "The Arrested Topographic Wave"

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In an illuminating paper (hereafter referred to as ATW) Csanady (1978) considers the mean circulation in a coastal zone of variable depth. For steady flow, and using a linear parameterization of bottom friction (in the longshore direction only) an equation for the sea surface elevation field is obtained

$$\frac{\partial^2 \zeta}{\partial x^2} + \frac{f}{r}\frac{dh}{dx}\frac{\partial \zeta}{\partial y} = 0$$

which simply reduces to the heat equation for $h = sx$ or a linear bottom slope (the notation here is identical to that in ATW). The parameter $\kappa = r/fs$ is the analog of thermal conductivity, but has the dimension of a length.

The influence on the mean coastal circulation of wind stress, freshwater influx and deep ocean current systems is considered, and with regards to wind forcing, two different longshore distributions of the wind stress are envisioned, one periodic with longshore wavenumber k and amplitude F_1 and the other a boxcar of length Y and amplitude F_0.

In the case of periodic wind stress, the scales of the coastal motion are found to be as follows:

Longshore length scale	k^{-1}
Cross-shore length scale	$L = (2\kappa/k)^{1/2}$
Longshore velocity	F_1/r
Cross-shore velocity	F_1/hf
Sea surface elevation	$F_1/kLgs$.

The cross-shore length scale L arises naturally as the geometric mean of the conductivity and the longshore length scale, in exact analogy to the depth d to which heat due to a periodic heat source (frequency ω) will penetrate a rod (conductivity α), which is $d \sim (\alpha/\omega)^{-1/2}$.

In the case of the boxcar forcing, the scales of motion are given as follows:

Longshore and cross-shore length scales	κ
Velocity scale	F_0/r
Sea surface elevation	F_0/sg.

These scales are vastly different from those found for periodic forcing, particularly insofar as the implication of similar length and velocity scales in both longshore and cross-shore direction. This is surprising, since intuition suggests that results for both distributions should not be greatly different if the boxcar length is on the same order as one half the period of the periodic distributions. It is the purpose of the present note to reformulate the boxcar solution in ATW in a more general way and find more useful length scales which are equivalent to those found in ATW for the periodic solutions.

For the sea surface elevations, the boxcar solution as given in ATW [Eq. (24)] should be written as

$$\zeta = -\frac{2fF_0}{rg}(\kappa Y)^{1/2}\left\{(-y/Y)^{1/2}i\ \mathrm{erfc}\left[\frac{x/(\kappa Y)^{1/2}}{2(-y/Y)^{1/2}}\right] - (1 - y/Y)^{1/2}i\ \mathrm{erfc}\left[\frac{x/(\kappa Y)^{1/2}}{2(-1 - y/Y)^{1/2}}\right]\right\}.$$

Similar expressions can be written for u and v.

In this form, the natural length scales which appear are as follows:

Longshore length scale	Y
Cross-shore length scale	$L_0 = (\kappa Y)^{1/2}$
Longshore velocity scale	F_0/r
Cross-shore velocity scale	F_0/fh
Sea surface elevation	F_0Y/gsL_0.

It is clear that these scales are exactly analogous to those found for the periodic forcing. If variables are nondimensionalized by these scales ($\tilde{x} = x/L$, etc. . . .) the solutions for $\tilde{\zeta}$ and $\tilde{v}$ are

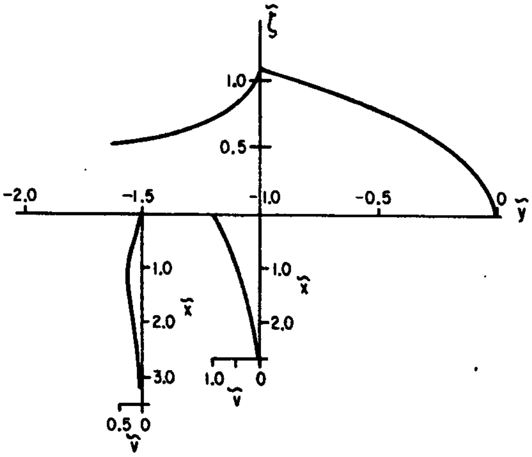

FIG. 1. Effects of isolated square half wave of longshore wind stress, showing nondimensional surface elevations $\tilde{\zeta}$ as a function of longshore distance $\tilde{y}$. Bottom half of illustration shows distribution of longshore velocity $\tilde{v}$ as a function of offshore distance $\tilde{x}$.

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$$\bar{\zeta} = -2\left\{(-\bar{y})^{1/2}i\ \mathrm{erfc}\left[\frac{\bar{x}}{2(-\bar{y})^{1/2}}\right]\right.$$

$$\left. - (1-\bar{y})^{1/2}i\ \mathrm{erfc}\left[\frac{\bar{x}}{2(-1-\bar{y})^{1/2}}\right]\right\}$$

$$\bar{v} = \left\{\mathrm{erfc}\left[\frac{\bar{x}}{2(-\bar{y})^{1/2}}\right] - \mathrm{erfc}\left[\frac{\bar{x}}{2(-1-\bar{y})^{1/2}}\right]\right\}$$

for $\bar{y} \leqslant -1$ and, as in ATW, for $-1 \leqslant y \leqslant 0$ only the first term in the braces is present. The advantage of this formulation is clear: these solutions do not explicitly depend on the parameter Y/κ. The sea surface elevation and longshore velocity presented in Fig. 6 of ATW are replotted in Fig. 1 in terms of the nondimensionalization suggested here. In this form the results are not dependent upon the specific value of Y chosen. An important result of this reformulation, is that it emphasizes the result already suggested by the periodic forcing study in ATW: that the important cross-shore length scale is the geometric mean between κ and the longshore length scale of the forcing. Typical values of κ, Y and L are shown in Table 1. Values of L on the east and west coast of the United States are close to typical shelf widths, suggesting that bottom friction effects are important over most of the shelf.

TABLE 1. Typical cross-shelf length scales for the east and west coast of the United States.

Location	$\kappa = r/fs$ (km)	Y (km)	$L = (\kappa Y)^{1/2}$ (km)
East coast United States	10	500	70
West coast United States	1	500	25

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REFERENCE

Csanady, G. T., 1978: The arrested topographic wave. *J. Phys. Oceanogr.*, **8**, 47–62.

Reply

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Winant's comments are appreciated. For further clarification of the results of ATW the three-dimensional plots of Figs. 1 and 2 may be useful. These were prepared by Mr. James Churchill. Note that the scales of the two axes are different, the nondimensional variables ky, y/Y, x/L having been used in the programming.

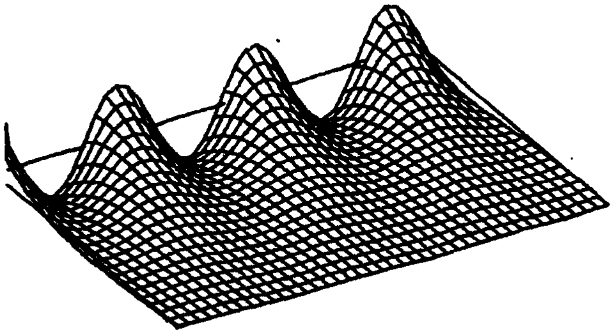

FIG. 1. Three-dimensional illustration of surface elevations induced by sinusoidally varying longshore wind. Variables plotted along the third axes are nondimensional, as defined in ATW.

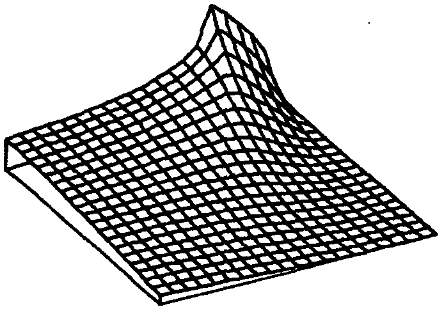

FIG. 2. As Fig. 1, for wind stress acting along finite pieces of coastline.

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