Simulation of an Observed Squall Line with a Meso-Beta-Scale Hydrostatic Model

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ABSTRACT

This paper reports on simulations of an observed squall line with the meso-beta-scale hydrostatic PERIDOT model. An attempt is made to answer the question of what model changes are necessary to roughly simulate a squall line with a hydrostatic model lacking explicit microphysics. A qualitatively satisfactory simulation is obtained by modifying the physical package of the model and by increasing the resolution compared to that of the operational model. Sensitivity experiments are conducted in order to determine the necessary components for the simulation of the squall line. The authors find that both increased resolution and a downdraft parameterization are required.

The energetics of the inner circulation of the squall line are investigated using the best simulation. The main supply for the rear to front flow is identified as conversion of potential energy into kinetic energy, which is linked to the line-normal horizontal gradient of perturbation pressure. This result agrees with the conclusions of the sensitivity experiments and with previous studies on the important role played by the density current in squall-line propagation.

1. Introduction

In the afternoon of 7 June 1987, a squall line passed over southwestern France. The region was devastated by severe winds and there were casualties. This kind of weather event is exceptional in France. The French Weather Service operational limited-area model PERIDOT failed to forecast this event. Subsequent studies of this case have been undertaken at Météo-France. An objective of this paper is to answer the following two questions. Is it possible to predict, even roughly, a squall-line phenomenon with a hydrostatic model, such as PERIDOT, which does not have explicit microphysics? What model changes are necessary? For example, Zhang et al. (1989) have reported a successful simulation of a squall line with a hydrostatic model, but with predictive equations for cloud water, ice, rainwater, and snow. Recently, the same squall line has been successfully simulated by Bélair et al. (1994) with a research version of the hydrostatic Canadian regional finite-element model. They found that their explicit moisture scheme significantly influences the internal circulation of the squall line and only weakly influences the surface features.

Midlatitude squall lines are quite similar to tropical squall lines; it is not surprising that different values of the Coriolis parameter cause no significant differences in the structure of squall lines since the space scales and timescales of the squall line are small (Ogura and Liou 1980). The typical surface signatures of a squall line include the following: a sudden change in wind direction, increases in wind intensity and surface pressure, a decrease of dry and equivalent potential temperatures, and an increase of latent and surface heat fluxes (Houze 1977; Chaline et al. 1988). Observational studies have found the following common characteristics in the vertical structure of squall lines: a low-level high-θe inflow near the leading edge of the line, a rear inflow of low-θe at midlevels below the stratiform cloud, and low θe air above the low-level high θe inflow ahead of the line. As the midlevel inflow progresses toward the leading edge, it slopes downward, assuming the characteristics of a density current near the surface. The downward motion is sustained by the evaporation and melting of stratiform precipitation and by convective downdrafts acting on the flow as it enters the convective region. Convective-scale downdrafts are initiated by the loading of convective precipitation and sustained by the evaporation of the convective precipitation (Zipser 1977; Houze and Betts 1981). The downdrafts are colder than their environment at the ground since they come from a higher level and have been cooled by evaporation. As the air descends toward the surface, it is more dense than the environmental air, creating a density current. The front of this density current produces the well-known cold pool, resulting in the gust front, which plays an important role in forcing the low-level high-θe air upward. Nonhydrostatic simulations have outlined the role of the cold pool in squall-line systems. Rotunno et al. (1988) determined

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that a balance between the negative alongline vorticity generated by the cold pool and the positive alongline vorticity of the low-level shear is optimal for long-lived convection. On the other hand, Lafore and Moncrieff (1989) showed that not only is the local triggering of convection by the gust front important, but the whole inner circulation is also important in accounting for the longevity of the system, especially when pronounced mesoscale circulations exist (see their Fig. 22).

As evidenced by the important role played by the cold pool, simulation of the density current seems to be required for a successful simulation of a squall line. Since the cold pool is directly linked to the convective-scale downdrafts, it seems that an accurate downdraft parameterization is necessary. We have designed a downdraft parameterization that proved to be an essential component for the simulation of the squall line. Similarly, Zhang et al. (1989) used a cumulus parameterization (based on Fritsch and Chappell’s 1980 scheme) that included a parameterization of the convective downdrafts. Taking the dimensions of the squall line into account, the horizontal grid size has been refined from the 38-km operational value to 9.5 km. Additional sensitivity tests have been carried out in order to determine the relevance of some parameters of the model and of the initial data.

After we succeeded in qualitatively simulating the squall line, the structure and energetics of the simulated squall line were studied. In this paper, only some brief results about energetics are presented. A complete discussion of these results can be found in Ducrocq (1994) and will be the subject of a subsequent paper.

The case under study is presented in section 2. Section 3 describes the numerical model, the downdraft parameterization, and the main characteristics of the simulations. The results of the best simulation, together with the sensitivity tests, are presented in section 4. Energetics of the simulated squall line are briefly discussed in section 5.

2. Case description

The 7 June 1987 meteorological situation has been described in many forecasting reports at Météo-France (Anne and Vassal 1987; Beauvais et al. 1987; Ruchon 1988). The following description is largely based on these previous studies. The basic available data are those of the Spanish and French national networks of surface stations and radiosoundings. These data are augmented by satellite pictures, reflectivities from the Bordeaux radar, and reports from a regional network of automatic ground stations over Southwestern France. (Geographical names are shown in Fig. 1.) Nonetheless, we lack data at upper levels and over the ocean where the phenomenon first appeared.

a. Squall-line development and life cycle

At 0000 UTC 6 June 1987, an upper-level pressure trough, associated with a northerly flow of cold air from Greenland, is located over the North Atlantic. During the following night, the trough deepens and develops a north–south orientation. At 1200 UTC 7 June, it extends from the United Kingdom to the Iberian Peninsula. The southwesterly flow ahead of the trough (Fig. 2a) brings a deep layer of warm air over northern Spain and the Bay of Biscay. The Bordeaux sounding at 1200 UTC, just prior to the squall-line development, shows a deep layer of conditional instability. At 500 hPa, the quasi-stationary low-pressure center extending from Scotland to southern Norway generates a cold-air flow behind the trough \( T = -24^\circ\text{C} \). At the surface, a low-pressure center forms off southern Brittany by 0300 UTC. As it moves northeastward, it deepens quickly. A frontal system is associated with this surface low-pressure center (Fig. 2b). The warm front undulates over northern France. The cold front, which was over the Atlantic Ocean in the morning, moves eastward at a speed of approximately 15 m s\(^{-1}\). Based on satellite and radar pictures, it does not seem very strong.

The phenomenon under investigation develops on the leading edge of the cold front, about 80 km off the French coast. Intense radar echoes are first observed at 1415 UTC (Fig. 3a) at the forward edge of the advancing cold front. The clouds associated with these intense echoes are not detectable on the infrared satellite pictures at this time; the cloud tops are not higher than those of the cloudy frontal system. Shortly afterward, the front accelerates as it approaches the mainland, reaching a propagation speed of around 20–25 m s\(^{-1}\). The intense radar echoes rapidly organize into a narrow line of convection, oriented parallel to the coast. The line progressively
FIG. 2. For 7 June 1987 (a) 500-hPa analysis at 1200 UTC; (b) Surface analysis at 1200 UTC.
Fig. 3. Reflectivities from the radar of Bordeaux: (a) at 1415 UTC; (b) at 1515 UTC; (c) at 1645 UTC; and (d) at 1800 UTC. The map is deformed [see scale at the right side of figure (a)]. Terms CF and SL indicate, respectively, the cold-front radar echoes and the squall-line radar echoes.

moves away from the front. As the line of convection reaches the coast (at 1510 UTC), it grows significantly in size (about 200 km long and 10 km wide) and in intensity (Fig. 3b). Very strong surface winds are produced, with gusts reaching 30–40 m s⁻¹ on the coastline. Clearly, the line of convection has become a squall line.

Inland, the squall line continues to progress eastward at a speed of about 20–25 m s⁻¹. As can be seen on the Bordeaux radar display of 1645 UTC, the squall
line is well separated from the front at this time (Fig. 3c). The squall line is strengthened and reaches its mature phase. The most intense radar echoes associated with the squall line are approximately 450 km long and 10 km wide. All squall-line radar echoes are approximately 50–70 km wide. After the separation of the squall line from the front, the latter moves eastward at a slower speed of about 15 m s\(^{-1}\). The squall line has a north–south orientation, whereas the front has a north-northeast–south-southwest orientation.

After 1700 UTC, the squall line begins to dissipate and consists of isolated and fairly intense thunderstorms with surface wind gusts of around 30 m s\(^{-1}\) near 1800 UTC (Fig. 3d). The length of the squall line as shown by radar is reduced (about 250 km long) and its width is increased (about 70–110 km wide) compared to the size of the squall line during its mature stage. The convective elements continue to decrease in intensity as they progress eastward through the remainder of southern France in the evening. The first cold front rainfall is recorded on land near 1630 UTC. Then, the surface cold front reaches the coast near 1700 UTC.

b. Presquall and postsquall characteristics

As usually observed in squall lines, the parameters that undergo the most spectacular evolution are the temperature and the wind. Before the squall-line passage, the prevailing surface winds are weak (5–10 m s\(^{-1}\)) from east to southeast. The winds above 1000 m are southwesterly with speeds around 15–20 m s\(^{-1}\). At the squall-line passage, the surface winds veer to west-northwest and increase to 25–35 m s\(^{-1}\), with gusts reaching 30–40 m s\(^{-1}\). The surface temperatures ahead of the line are high, reaching nearly 30°C in Basque Country at 1500 UTC (Fig. 4a). This results from diurnal warming and the Foehn effect in the foothills of the Cantabrian and Pyrenean ranges, which is caused by southwesterly flow over these mountains. The convective instability that initiates the squall line over the ocean is due to air coming from Spain, warmed by the Foehn effect on the lowest 1500 m and surmounted by a northwesterly flow of cold air. The temperatures continue to rise inland before the arrival of the squall line. As the gust front passes the coast, the temperature falls by more than 10°C in less than 10 minutes. Inland, the temperature falls by similar amounts but less rapidly (Fig. 4b).

Rainfall is far from exceptional, at most 10–15 mm with the passage of the squall line. The low rainfall may be explained by significant evaporation of precipitation before it reaches the ground.

Occurring two to three hours after the squall-line passage, the cold frontal passage is generally associated with a slight wind shift, a weak temperature drop (1°–2°C), and around 5 mm of recorded precipitation.

**Fig. 4.** Screen-level temperatures (in °C) over Southern France from the regional PATA network of automatic ground stations: (a) at 1500 UTC; (b) at 1700 UTC. Terms CF and SL lines indicate, respectively, the cold-front surface and the squall-line gust front.
3. Model description

a. The operational package

The model used for this study is a research version of the French Weather Service limited-area model PERIDOT described by Imbard et al. (1986), Bougeault (1986), and Bougeault et al. (1991). The two-dimensional version has also been used by Bougeault and Lacarrère (1989) and Ducrocq (1993). The model is a gridpoint, σ-coordinate, primitive equation model, with horizontal winds \((U, V)\), temperature \(T\), specific humidity \(q\), and the logarithmic surface pressure \(P_s\) as prognostic variables. The top and bottom boundary conditions are \(\sigma = do/\sigma_t = 0\). A relaxation scheme (Davies 1976) is applied to the six outermost grid points of the lateral boundaries. The numerical formulation is carried out on a horizontal Arakawa C-grid. A semi-implicit leapfrog scheme is used for the time integration, with implicit treatment of the vertical advection and the linear part of the gravity wave modes. A time filter (Asselin 1972) is applied to all prognostic variables in order to prevent decoupling between consecutive time steps. Second-order horizontal diffusion is applied on the momentum fluxes \(U_P\) and \(V_P\), the potential temperature, and the specific humidity. The diffusion coefficient is proportional to the horizontal grid size.

The operational version of the model has a complete physical package.

- A radiation parameterization is called at each time step. Clouds are diagnosed from the relative humidity.
- The subgrid-scale mixing scheme of Louis et al. (1981) is used. The vertical diffusion coefficients are functions of the Richardson number. Surface temperature and humidity are obtained by Deardorff’s (1978) method.
- The explicit precipitation scheme is described in appendix A. For this scheme, as soon as a grid element is saturated, the liquid water formed falls as precipitation, and latent heat is released. A fraction of the rain is evaporated in subsaturated lower layers.
- The deep convection scheme uses a Kuo-type formulation with a self-regulated partition between precipitation and moistening (Geleyn 1985). This scheme will be referred to later as G85.

In some of our numerical integrations, Bougeault’s (1985) convective mass flux scheme (referred to later as B85) has been selected instead of the G85 scheme. The B85 scheme was used in the French EMERAUDE global model and is now also used in the successor of EMERAUDE called ARPEGE.

b. Downdraft parameterization

As part of this squall-line study, a simple parameterization of convective downdrafts has been designed. It is analogous to the updraft convective mass flux of the B85 scheme. Although this parameterization has been used so far only in conjunction with B85, it may be used in conjunction with any updraft parameterization scheme that supplies the amount of convective precipitation as output.

The effects of convective downdrafts on the environment are linked to the moisture and static energy transport by the downdraft mass flux \(\omega_d\) and therefore also to the evaporation \(E\) of the precipitation needed to sustain the negative buoyancy of the downdrafts. The profiles of specific humidity and static energy in the downdraft are referred to, respectively, as \(q_d\) and \(S_d = \phi_d T_d + gZ\). It is assumed that the heat source \(Q_1\) and the moisture sink \(Q_2\) are given by

\[
Q_1 = -\frac{\partial}{\partial P} [\omega_d (S_d - S)] - LE
\]

(1)

\[
Q_2 = -\frac{\partial}{\partial P} [\omega_d (q_d - q)] + E.
\]

(2)

It is also postulated that the downdraft moisture and energy budgets are closed without mixing with the environment; that is, \(E = \omega_d \frac{\partial q_d}{\partial P} = -1/L \omega_d \frac{\partial S_d}{\partial P}\). Then we can rewrite (1) and (2) as follows:

\[
Q_1 = \omega_d \frac{\partial S_d}{\partial P} - \frac{\partial \omega_d}{\partial P} (S_d - S)
\]

(a)

(3)

\[
Q_2 = \omega_d \frac{\partial q_d}{\partial P} - \frac{\partial \omega_d}{\partial P} (q_d - q).
\]

(b)

(4)

The term (a) represents the cooling by the large-scale updraft that compensates the sink of mass due to the convective downdraft. Term (b) is negative at lower levels and represents the cold pool generated when the downdraft spreads out near the surface. Term (c) is positive and is linked, as is (a), to the compensation by the environment. Term (d) corresponds to the drying when the dry air is expelled. Therefore, this simple downdraft formulation simulates the transport of cold and dry air from midlevels toward the lower levels that is made possible by the evaporation of the convective precipitation.

The thermodynamic structure of the downdrafts is determined in a manner similar to the convective updraft in the B85 scheme. The specific humidity \(q_d\) and the static energy \(S_d\) are obtained by going downward along the saturated adiabat that originates at the coldest point—with respect to moist static energy—of the environmental sounding. Finally, an assumption on the mass flux \(\omega_d\) is required to close the scheme. Obviously, \(\omega_d\) must depend on the static stability and the amount of precipitation available for the initiation and maintenance of the downdraft. Therefore, we postulate that \(\omega_d\) is proportional to the square of the difference between the moist static energy \(h\) of the environment and the moist static energy \(h_d\) of the downdraft. Then we
assume that a given proportion \( \epsilon \) of precipitation is used to produce the downdraft. Thus, the closure is written as

\[
\int \omega_d \frac{\partial \theta}{\partial \theta} dP = \epsilon g P,
\]

where the integral is taken over the whole depth of the downdraft, \( P \) is the amount of convective precipitation (i.e., as produced by the updraft scheme), and \( g \) is the gravitational constant. The resulting amount of precipitation is therefore \((1 - \epsilon) P\), where the evaporation coefficient \( \epsilon \) is a degree of freedom of this downdraft scheme.

In this convective parameterization, the effect of deep convection on momentum is taken into account in a same manner as on the temperature or humidity. The wind structure of the downdrafts is determined in a way similar to the convective updraft in the B85 scheme; that is, as the vertical average of the wind profile.

c. Set of simulations

Both two- and three-dimensional simulations have been performed. A summary of the eight 3D experiments is shown in Table 1. Most simulations use a horizontal resolution of 9.5 km, which is about four times higher than the resolution of the operational model at this period. Only experiments \( S L .2 \) and \( S L .5 \) use the operational mesh length of 38 km. Figure 5 shows the model domains. Domain \( \mathcal{O} \) corresponds to the operational domain used until 1991. All fine mesh experiments have been performed on domain \( \mathcal{A} \). Most results are displayed on the smaller region \( \mathcal{R} \). The vertical grid for the 3D simulations is the operational grid of 15 levels (A in Table 2).

Since we wanted to investigate the model’s sensitivity to certain parameters at limited cost, we have also made 2D runs. The characteristics of the 2D simulations are displayed in Table 3. These simulations are performed on a vertical cross section normal to the French Atlantic coast since the observed phenomenon has a 2D structure parallel to the coast. The vertical cross section of the 2D simulations is taken along the line (A–A') given in Fig. 5. This cross section is approximately 870 km in length. These simulations have a horizontal grid size of approximately 9.5 km, comparable to the 3D simulations. The vertical grids are described in Table 2. The physical package of the 2D simulations is the same as for the 3D simulation \( S L .1 \) except that the evaporation coefficient is changed in some 2D simulations.

The initial conditions are supplied by a 1200 UTC analysis from the operational model on the domain \( \mathcal{O} \). This analysis is significantly improved compared to the real-time operational analysis. At this time, the PERIDOT system had a sequential data assimilation based on optimal interpolation every 12 h, with a first guess based on the latest 12-h forecast of the model. About 15 days after the event, the computational domain for analysis and forecast was increased in size. This change allowed reanalyses of the situation taking advantage of important additional data, like the La Coruna (Spain) sounding, which had not been used in

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**Table 1.** Description of the three-dimensional numerical simulations. The horizontal mesh length \( \Delta x \) is given at 60°N for the polar stereographic projection. Term \( \epsilon \) is the evaporation coefficient used in the downdraft parameterization.

<table>
<thead>
<tr>
<th>Simulations</th>
<th>Resolution</th>
<th>Parameterization</th>
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<tbody>
<tr>
<td>( S L .1 )</td>
<td>( \Delta x = 9.525 ) km &lt;br/&gt;vertical grid A &lt;br/&gt;domain A</td>
<td>B85 convection scheme &lt;br/&gt;downdraft parameterization ( (\epsilon = 0.3) )</td>
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<tr>
<td>( S L .2 )</td>
<td>( \Delta x = 38.1 ) km &lt;br/&gt;vertical grid A &lt;br/&gt;domain O</td>
<td>G85 convection scheme</td>
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Table 2. The vertical grids. Grid A: 15 $\sigma$-levels; grid B: 21 $\sigma$-levels; grid C: 27 $\sigma$-levels; grid D: 40 $\sigma$-levels.

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The real-time analysis because it was well out of the domain. Several data-assimilation experiments on this case were performed by Juvan du Vachat et al. (1988), including experiments that also used satellite data over the Atlantic Ocean. They demonstrated the positive impact of the additional data and of the increased domain size. For the present study, we use the best of their subsequent analyses.

All simulations last 6 h. The lateral boundary conditions are linear interpolations in time between the improved 1200 UTC analysis and the 1800 UTC forecast of the operational model starting from this analysis. When a mesh length smaller than the operational mesh length is used, the initial data and lateral boundary conditions are simply interpolated in space without any addition of meso-$\beta$-scale information.

4. Simulation results

a. The control simulation SL.1

Simulation SL.1 is the experiment that best approximates the observations. Its physical package includes the BR 5 convection scheme and the downdraft parameterization. The gust front is successfully simulated: the surface winds change from southeasterly to westerly behind the line and increase considerably (Fig. 6). The gust front separates from the front and progresses more rapidly than the cold front. The cross section along line (A–A') of the vertical velocity at 1500 UTC shows the existence of two systems (Fig. 7a): the cold front (CF in Fig. 7), which is characterized by a core of upward motions of 10–20 cm s$^{-1}$, and ahead of it, the squall line (SL in Fig. 7) with upward motions greater than 30 cm s$^{-1}$ with a maximum around 60 cm s$^{-1}$. The cooling behind the line is also simulated as shown by the strong gradient in the temperature field (Fig. 8 compared with Fig. 4). The convection is nearly aligned with the coast, as can be seen by the well-defined banded structure of the convective precipitation and the upward vertical velocities (Fig. 9). The simulated rainfall is similar to the observed rainfall recorded by the regional network of automatic ground stations, that is, about 10 mm during the passage of the squall line. The location and extent of the simulated rainfall agree with the distribution deduced from radar (Fig. 10 compared with Fig. 3).

Table 3. Description of the two-dimensional numerical simulations. Term $f$ is the coriolis parameter and $\epsilon$ is the evaporation coefficient.

<table>
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<tr>
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<th>Resolution</th>
<th>Characteristics</th>
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<tr>
<td>SL2.1</td>
<td>$\Delta_x = 9.525$-km vertical grid A</td>
<td>$f \neq 0$ $\epsilon = 0.3$</td>
</tr>
<tr>
<td>SL2.2</td>
<td>same as SL2.1</td>
<td>$f \neq 0$ $\epsilon = 0.5$</td>
</tr>
<tr>
<td>SL2.3</td>
<td>same as SL2.1</td>
<td>$f = 0$ $\epsilon = 0.5$</td>
</tr>
<tr>
<td>SL2.4</td>
<td>$\Delta_x = 9.525$-km vertical grid B</td>
<td>same as SL2.3</td>
</tr>
<tr>
<td>SL2.5</td>
<td>same as SL2.1</td>
<td>$f = 0$ $\epsilon = 0.3$</td>
</tr>
<tr>
<td>SL2.6</td>
<td>$\Delta_x = 9.525$-km vertical grid C</td>
<td>same as SL2.2</td>
</tr>
<tr>
<td>SL2.7</td>
<td>$\Delta_x = 9.525$-km vertical grid D</td>
<td>same as SL2.5</td>
</tr>
<tr>
<td>SL2.8</td>
<td>same as SL2.7</td>
<td>$f = 0$ $\epsilon = 0.3$ increase of initial easterly winds</td>
</tr>
<tr>
<td>SL2.9</td>
<td>same as SL2.7</td>
<td>$f = 0$ $\epsilon = 0.3$ increase of initial easterly winds</td>
</tr>
</tbody>
</table>
The observed presquall-line characteristics are also qualitatively simulated: before the initiation of the convective line, at 1300 UTC, the low-level air is warm and moist near the Atlantic coast. [We note an area (not shown) 1500 m high and 200 km wide, with $\theta_e = 322$ K.] Above this area, the air has a lower $\theta_e$ of around 318 K. The region of development of the line therefore exhibits a convectively unstable layer over a depth of 2500 m. In agreement with the observations, a southeasterly flow is simulated inland up to approximately 1000 m, while the flow is southwesterly over the ocean and above 1000 m inland.

The surface features and vertical structure of the simulated system are similar to other midlatitude squall lines. The sensible heat flux is increased just behind the convective line (Fig. 11); this is due to the cooling caused by the downdraft parameterization and the increasing wind speeds. A north–south line of convergence is also produced near the surface, and the global radiation drops behind the line by more than 300 W m$^{-2}$ (Fig. 11). The vertical cross section of potential temperature and relative humidity along line (B–B') shows the cold pool near the ground (Fig. 12). The downdraft parameterization produces the core of downward motion behind and beneath the upward motions (Fig. 7). A vertical cross section of the line-normal velocity (relative to a constant propagation speed $C$ of the squall line taken equal to 10 m s$^{-1}$) is displayed at 1500 and 1700 UTC in Fig. 13, with equivalent potential temperature superimposed. A rear
edge of the squall line, the atmosphere at this time is convectively unstable up to 4000 m. Just behind the leading edge of the squall line, the pool of cold air with low $\theta_e$ is also visible. With time, the low-level rear to front flow intensifies and expands (Fig. 13b). Meanwhile, the front to rear flow decreases in intensity. After 1600 UTC, the potential instability at the leading edge of the squall line extends only up to 1500 m. During the decaying stage, the upward motions separate into two cores: one at the leading edge of the squall line, and the other above the downdraft around 3000 m. The rear core seems to result from the potential convective instability generated by the upward and rearward transport of high $\theta_e$. At 1600 UTC, there are downward motions up to 3500 m between the two up-

![Diagram](image)

**FIG. 7.** Vertical cross sections along line (A–A) of vertical velocity from simulation SL1: (a) at 1500 UTC (contour interval: 0.10 m s$^{-1}$); (b) at 1600 UTC (contour interval: 0.10 m s$^{-1}$). Terms CF and SL indicate, respectively, the position of the cold front and the location of the squall line. The left axis indicates height in meters above sea level.

to front flow begins to develop at 1500 UTC (Fig. 13a). It is located just above ground level at the front side of the downward motion core. It represents the forward-flowing part of the density current generated by the downdraft parameterization. The rear outflow of the density current is also simulated. The warm and moist air at low levels has been carried away by the upward front to rear inflow, as can be seen in Fig. 13a by a tongue of $\theta_e$ greater than 322 K. At the leading edge of the squall line, the atmosphere at this time is convectively unstable up to 4000 m. Just behind the leading edge of the squall line, the pool of cold air with low $\theta_e$ is also visible. With time, the low-level rear to front flow intensifies and expands (Fig. 13b). Meanwhile, the front to rear flow decreases in intensity. After 1600 UTC, the potential instability at the leading edge of the squall line extends only up to 1500 m. During the decaying stage, the upward motions separate into two cores: one at the leading edge of the squall line, and the other above the downdraft around 3000 m. The rear core seems to result from the potential convective instability generated by the upward and rearward transport of high $\theta_e$. At 1600 UTC, there are downward motions up to 3500 m between the two up-

![Diagram](image)

**FIG. 8.** Screen-level temperatures from simulation SL1 on the area $R$ (contour interval is 1°C): (a) at 1500 UTC; (b) at 1700 UTC.
draft cores (Fig. 7b). This area can be compared with the transition zone often found in radar reflectivity fields in observed squall lines. Zhang and Gao (1989) also mentioned such an area in their study of the squall line that had been simulated by Zhang et al. (1989). However, the size of the whole system in their simulation is larger than in our simulation: near 200 km wide and up to 200 hPa in height, with a downward area extending up to 5000 m. Our simulated system in growing to mature stages is only 70–110 km wide and up to 8 km and in dissipating stage about 140 km wide and 6000 m high.

Although the squall line of 7 June 1987 has been qualitatively simulated, we should also mention the less realistic aspects of the simulation. The squall line initiates too early in our experiment, since the phenomenon is already inland at 1400 UTC. This deficiency may be due to the convection scheme being too sensitive to the moisture convergence or to deficiencies in the initial analysis. After 1500 UTC, the intensity of the upward and downward motions decreases. At 1800 UTC, the northern part of the simulated squall line lags the observed one. The mean propagation speed of the simulated squall line is about 10–13 m s⁻¹, less than the observed propagation speed of about 20–25 m s⁻¹. The maximum wind strength is smaller than the observed by 5–10 m s⁻¹. This underestimation seems to be linked to the overestimation of the temperature behind the line; the simulated downdrafts are not sufficiently cold compared to the observed. Indeed, the temperature drop is only 3°–6°C compared with the 10°C drop observed. The temperature ahead of the line is also underestimated by 3°–4°C in the Basque Country and in the Landes region and by 1°–2°C in the remaining part of southern France. The lack of ice microphysics in the simulation may explain some of the underestimation of wind intensities and temperature drop. Indeed, Zhang and Gao (1989) reported that surface pressure perturbations and vertical motions are weaker in their hydrostatic simulation without ice microphysics.

b. Sensitivity tests

Experiment SL.2 has been performed with the horizontal grid size and physical package of the operational model using the improved analysis. Thus, this experiment differs from SL.1 in the horizontal resolution, the convection scheme, and the downdraft parameterization. No squall-line system is simulated in this experiment (not shown). Therefore, although the improvement of the initial data is a necessary element to simulate the large-scale fields well, it is not a sufficient condition to simulate the squall line. For experiment SL.3 (not shown), the horizontal grid size is decreased from the operational 38.1 km (at 60°N) to 9.525 km, and the physical package is the same as that of the operational model. As in experiment SL.2, no squall line is simulated. Some convective events are simulated ahead of the front, but they are not organized in a line of convection. Though their location is correct at 1800 UTC, this is not due to a good forecast of the propagation speed but rather to a triggering of these convective events that happen too early in the afternoon; the propagation speed of the convective area is about 10–15 m s⁻¹. The surface temperature behind the convective events remains too high. The convective precipitation is too large and not organized in a band. Moreover, at 1600 UTC, an unrealistic microscale low-pressure center is simulated south of Bordeaux. It looks like a gridpoint storm. These unrealistic aspects are frequently obtained in fine-mesh simulations with the G85 scheme. This scheme has been designed for coarser grids and appears to be inadequate when a finer mesh is used. This may be due to an overly large sensitivity of the scheme to local moisture convergence, which can be more intense in fine-mesh simulations. Therefore, simulation SL.3 shows that an increase of resolution alone does not lead to simulation of the squall line.

In experiment SL.4, the B85 scheme is used instead of the G85 convective scheme. Experiment SL.4 gives a better organization of the frontal precipitation (not shown) but also fails to forecast the squall line. The front moves eastward too rapidly; it is 50 km ahead of its observed position at 1800 UTC. This may be explained by the lack of a simulated squall line ahead of it. However, features are more realistic in SL.4 than in SL.3, particularly a well-structured front. As in ex-
experiment SL.3, the precipitation is too large. Most of the precipitation is due to the convective parameterization (about 90% of the total amount). Evaporation of the convective precipitation that sustains the convective-scale downdrafts is not represented in simulations SL.3 and SL.4; this may explain the large amount of precipitation in these simulations. Although the large-scale precipitation scheme allows for some evaporation of the rainfall in the unsaturated lower levels, this is not sufficient to generate a density current in these experiments. The convective-scale downdrafts must also be taken into account as shown by experiment SL.1, which differs from SL.4 only in the inclusion of the downdraft parameterization.

To show that successful simulation of the squall line is not only due to the parameterization changes but also to a smaller horizontal grid size, experiment SL.5 has been performed. In this experiment, the B85 scheme and the downdraft parameterization are used with the operational horizontal grid size. Although the simulated winds are stronger than in experiment SL.2, they do not reach the realistic values simulated in experiment SL.1. The line of convergence simulated at 1300 UTC weakens over time.

These four sensitivity experiments establish that both an increased resolution and more advanced physics are required to simulate the squall line. Clearly the downdraft parameterization is a crucial component in the simulation of the cold pool and the gust front. Since the downdraft parameterization has one degree of freedom, which is the evaporation coefficient $\epsilon$, we have investigated the sensitivity of simulations to this coef-
Fig. 11. Sensible heat flux in thick lines (contour interval is 100 W m$^{-2}$) and global radiation (the gray scale given at the right of the picture is incremented by steps of 300 W m$^{-2}$) at 1500 UTC from simulation SL.1.

In experiment SL.1, it was taken equal to 0.3. In experiment SL.6 the value of $\epsilon$ is reduced to 0.15, and in experiment SL.7 it is increased to 0.5. When the evaporation coefficient is decreased (experiment SL.6), the propagation speed is also decreased to about 11 m s$^{-1}$, compared to 13 m s$^{-1}$ in the reference experiment SL.1. When the evaporation coefficient is increased (experiment SL.7), the system in SL.7 first moves faster than SL.1. With larger evaporation, the convective rainfall is reduced compared with that of experiment SL.1. However, as time progresses, the system in SL.7 moves slower than SL.1. At the end of simulation SL.7, winds behind the line are, in some areas, unrealistic in both direction and intensity. Therefore, the value of 0.3 for $\epsilon$ seems to be a good choice in order to obtain a reasonable propagation speed without unrealistic winds.

Observations show that the precipitation efficiency (ratio of rainfall to water vapor inflow) depends on the vertical shear in the cloud (see, for example, Fig. 2 of Fritsch and Chappel 1980). However, the kind of relation between precipitation efficiency and vertical shear is an open question as found, for example, by Weisman and Klump (1982) with two kinds of regimes. In Fritsch and Chappel's (1980) convective downdraft–updraft parameterization, the evaporation is taken as a function of the vertical shear. By analogy, the evaporation coefficient $\epsilon$ has been expressed as a

Fig. 12. Vertical cross sections along line (B–B') from simulation SL.1 at 1500 UTC of potential temperature in dashed lines (contour interval is 2 K) and relative humidity in thick lines (contour interval is 10%). Term SL indicates the location of the squall line. The left axis indicates height in meters above sea level.

Fig. 13. Vertical cross sections along line (B–B') of normal-line velocity relative to the squall-line system ($C = 10$ m s$^{-1}$) (contour interval is 1 m s$^{-1}$) and equivalent potential temperature in half-tone areas (the intensity scale is given at the right of the picture; it is incremented by 2 K) from simulation SL.1: (a) at 1500 UTC; (b) at 1700 UTC. Terms CF and SL indicate, respectively, the position of the cold front and the location of the squall line. The left axis indicates height in meters above sea level.
function of the vertical shear in experiment SL.8. From Fig. 2 of Fritsch and Chappel (1980), the precipitation efficiency $E$ is approximated by a quadrilateral hyperbola,

$$E = \frac{-0.217 \frac{\Delta V}{\Delta z} + 2}{\frac{\Delta V}{\Delta z} + 0.7143},$$

where $\Delta V/\Delta z$ is the vertical shear over the cloud depth. The total evaporation rate is $(1 - E)$. Some observations have shown that convective precipitation can constitute 60%–80% of the total precipitation (Houze 1977; Chalon et al. 1988). Therefore, in experiment SL.8, we have used the following expression $0.6(1 - E)$ for the evaporation coefficient $\epsilon$ in the convective-scale downdrafts. The results of simulation SL.8 are similar to those of experiment SL.1. Our formulation of $\epsilon$ based on a dependence upon the vertical shear does not bring any improvement. Indeed, comparison of SL.1 and SL.8 show that our heuristic value of $\epsilon$ is in good agreement with values deduced from observation (Fankhauser 1971; Marwitz 1972). However, (6) may be useful for other cases.

To test the sensitivity to vertical resolution and initial low-level winds and temperatures, some 2D simulations have been performed (see Table 3). The results of these simulations are not described in detail here. However, they established that the solution is not significantly sensitive to minor changes in the detail of the initial state or to an increase of the vertical resolution. Therefore, the results do not appear to be fortuitous since for a large number of these sensitivity tests the results are similar to those of simulation SL.1.

5. Energetics

In order to study the energetics of quasi-two-dimensional, meso-$\beta$-scale phenomena (e.g., rainbands or squall lines), some diagnostics have been designed (Ducrocq 1994). We have applied these diagnostics at three stages of the lifecycle of the simulated squall line. The approach of this study is as follows. We first isolate the circulation at the scale of the squall line, which exhibits the characteristic rear to front (hereafter RTF) and front to rear (hereafter FTR) flows. The main sources and sinks of kinetic energy for these flows are then determined. The temperature and pressure fields at the scale of the squall line exhibit the characteristic features of the squall line. The kinetic energy equation and method used to separate the squall-line circulations from the base state are described in appendix B. A schematic representation of the energy exchanges is given in Fig. 14 for the development stage, in Fig. 15 for the mature stage, and in Fig. 16 for the decaying stage.

The conversion term $PP$ of potential energy into kinetic energy is the primary source of energy for the RTF flow during the development and mature stages (Figs. 14a and 15a). This term contributes to the forward progression of the RTF flow. The perturbation pressure field near the ground shows a presquall mesoslow and a mesohigh behind the leading edge of the squall line, which results from the cooling produced by the downdraft parameterization. The induced line-normal pressure gradient is responsible for the largest part of term $PP$. Gallus and Johnson (1992) and Gao
et al. (1990) have examined the mesoscale momentum budget of a midlatitude squall line in the United States. The first study uses observed data and the second the previously quoted simulation of Zhang et al. (1989). In agreement with our results, they found for the mature stage that the term linked to the line-normal pressure gradient is important to strengthen the FTR flow near the surface.

The other sources of kinetic energy during the mature stage for the RTF flow are the terms $ADV$ (parallel advection), $HS_1$ (horizontal shear), and $ADW_1$ (vertical advection). They tend to increase the flow above and ahead of its maximum and consequently contribute to the upward and forward propagation of the RTF flow.

The sources of kinetic energy for the FTR flow (Figs. 14b and 15b) are terms $ADW$, $VS$ (vertical shear), and $HS_1$ during the mature stage. Terms $ADW$ and $VS$ tend to increase the FTR flow above and ahead of its maximum; this leads to the ascent of the FTR flow. The term $PP$ tends to decrease the FTR flow near the ground and also where the FTR flow is stronger. This term results from the presquall mesolow near the ground and from a second mesolow located just above the leading edge of the RTF flow. This conversion term $PP$ of kinetic to potential energy seems to be linked to the transport of high-$\theta_e$ air from the lower levels by the upward FTR flow, which leads to potential instability in the upper right part of the maximum of FTR flow.

During the decaying stage (Fig. 16), the main sources of kinetic energy identified during the development and mature stages for the RTF and FTR flows disappear.
6. Discussion

The forecast of the 7 June 1987 squall line has been much improved compared to that of the operational model that does not simulate any convective system. We found in this study that a squall line can be qualitatively simulated and predicted with a hydrostatic, meso-β-scale model without an elaborate microphysical scheme. The inclusion of a crude representation of the convective-scale downdrafts has permitted a fair simulation of the event. We found that both the more advanced physics and an increased resolution were required. The computational cost of the reference simulation is 64 times that of the operational model on the same domain. The computational cost of the downdraft parameterization is only 10% of the total time, that is, the same as that of the updraft convective scheme. With increases in computer power, the use of an operational fine-resolution model, such as the one used in this study, is not far from being possible. It is planned to be in operation at Météo-France in 1995. Our study shows the potential for operational models to improve their forecasts if increased resolution and more elaborate physics are used.

In the reference simulation, a convective parameterization is used. As quoted by Molinari and Dudek (1992), it remains uncertain whether a general solution exists for the parameterization of convective precipitation for grid spacing between 3 and 20–25 km. We have done a simulation without a convective scheme; this simulation was of poor quality compared to the simulations with a convective scheme. Zhang et al. (1988) have also argued that a convective scheme is needed even with a grid spacing of 10 km. Since most of the precipitation associated with the convective line in simulations with a convective scheme is due to the convective parameterization, a double counting of the convective feedback seems unlikely.

The mesoscale and convective circulations of the simulated squall line have been isolated: a downward cold RTF flow in the low levels, surmounted by a high-θ_e upward FTR flow, both intensifying during the development of the squall line. The boundary between the upward and downward motions is tilted at the mature stage, as found by Zhang and Gao (1989) and also in Gallus and Johnson (1992). The downward RTF flow forces the low-level warm inflow to rise. As high-θ_e air is carried upward and rearward by the FTR flow, potential instability is generated at the rear of the system. This midlevel potential instability could explain the development after 1600 UTC of a core of upward vertical velocities at the rear of the system and above the downward RTF flow. The main source of kinetic energy for the RTF flow during the growth and mature stages is the conversion of the perturbation potential energy, due primarily to the line-normal pressure gradient. The latter results from the juxtaposition of the cold subsiding air generated by the downdraft param-eterization and the warm air lifting just ahead of it. The important role played by this low-level generation term has been found in previous studies (Lafont and Moncrieff 1989; Gao et al. 1990). The FTR flow is supported at low levels during the growth and mature stages by an exchange of kinetic energy between the large-scale flow and the squall line, which is linked to the horizontal shear and the local tendency of the large-scale wind. The vertical advection of perturbation kinetic energy contributes to the upward propagation of the FTR flow. At the mature stage, the term linked to the vertical shear of line-normal large-scale wind also tends to lift up the FTR flow. These two sources of FTR flow are linked to the upward velocity. During the dissipation of the squall line, these sources vanish and the FTR flow progressively disappears. It could be explained by the following sequence of events: as the squall line encounters a less favorable environment to convective instability, the intensity of the upward velocity at the leading edge decreases below the low-level unstable area. Therefore, the sources of FTR flow that are directly linked to the upward velocity decrease. We mention that at 1600 UTC (i.e., at the beginning of the dissipation stage), the maximum of FTR flow is located where vertical velocities are downward or near zero (i.e., in the transition zone). Hence, the maxima in absolute value of negative u' and positive w' are completely separated, and this is detrimental for the production of FTR flow by term V.S. Since the FTR flow disappears, the upper core of upward velocity at the rear of the system is cut off from its low-level source of high-θ_e air, that is, its potential convective instability source. As a consequence, the air tends to be stabilized in this area and the intensity of the upward velocities decreases at the rear of the system.

Although we have qualitatively simulated the squall line, the propagation speed and the temperature drop are underestimated. Zhang and Gao (1989) have shown that the surface pressure perturbations and vertical motions are considerably weakened in their squall-line simulation when ice microphysics is removed from their model. The absence of detailed microphysics in our simulation may be one reason for the underestimation of the propagation speed. However, the similarity between our study and others concerning the terms responsible for the generation and maintenance of the squall line demonstrates that our model captures some fundamental aspects of this event.

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APPENDIX A

The Explicit Precipitation Scheme

The precipitation scheme used in our model parameterizes both condensation and evaporation. The following description is based on Bougeault (1986).
The saturated state \((T_w, q_w)\) (at isobaric equal energy) is defined for each point of the model at the temperature \(T\) and specific humidity \(q\) by

\[
\begin{align*}
C_{ph}T_w + Lq_w &= C_{ph}T + Lq \\
q_w &= q_{sat}(T_w),
\end{align*}
\]

where \(C_{ph}\) is the specific heat at constant pressure of the moist air.

Following Kessler (1969), all vapor exceeding the saturated specific humidity \(q_w\) is assumed to condense, and the specific humidity \(q\) is restored to the saturated value \(q_w\) in one model time step \(\Delta t\). Therefore, the tendency of \(q\) due to condensation for a saturated level is given by

\[
\frac{\partial q}{\partial t}_{\text{cond.}} = \min\left(0, \frac{q_w - q}{\Delta t}\right). \tag{A.1}
\]

The algorithm considers all layers of depth \(\Delta P\) and of mean pressure \(\bar{P}\) below the highest level where saturation occurs. For each saturated layer, the precipitation flux \(R\) is increased by

\[
\Delta R = \frac{\Delta P}{g} \frac{dq}{\partial t}_{\text{cond.}} \tag{for } q > q_w. \tag{A.2}
\]

For each unsaturated layer, it is assumed that \(R\) should decrease in response to evaporation. To determine this decrease, some assumptions are made for the rain drops.

- Terminal velocity:

\[
W = A(P, T)D^\alpha. \tag{A.3}
\]

- Evaporation:

\[
\frac{dM}{dt} = B(P, T)D^\beta \rho \max(0, q_w - q). \tag{A.4}
\]

- Marshall–Palmer (1948) distribution:

\[
dN = N(D)dD = N_0 \exp(-\lambda D)dD. \tag{A.5}
\]

Here \(D\) is the drop diameter, \(M\) is the raindrop mass, \(N\) is the number of drops per unit of volume, \(\rho\) is the density, \(\lambda\) is the rainfall intensity, \(A\) and \(B\) are parametric functions of \(P\) and \(T\), and \(\alpha, \beta, N_0\) are constant coefficients. Calibrations made in the operational model have established that

\[
\begin{align*}
\alpha &= 0.7706 \quad \beta = 1.3853 \\
A &= 654.5(P_0/P)^\alpha(T_0/T)^\alpha \text{ m s}^{-1} \\
B &= 0.005655(P_0/P)(T_0/T)^{7.095} \text{ m}^3 \text{s}^{-1}.
\end{align*}
\]

Since the rainfall is given by

\[
R = \int_0^\infty WMdN \tag{A.6}
\]

and the rainfall variation due to evaporation by

\[
\frac{dR}{dz} = \int_0^\infty \frac{dM}{dt} dN, \tag{A.7}
\]

one obtains by elimination of \(\lambda\) between Eq. (A.7) and Eq. (A.6) and using the previous assumptions on the rain drops

\[
\Delta [R^{-(\beta+1)/(\alpha+4)}] = \mathcal{C} \max(0, q_w - q) \Delta P, \tag{A.8}
\]

where \(\mathcal{C}\) is a function of \(A, B, \alpha, \beta\).

The calibrated values of \(A, B, \alpha, \beta\) give, for a standard atmosphere, \(\mathcal{C} \approx -K_{\text{evap}} \bar{P}^{-2}\) with \(K_{\text{evap}} = 4.8 \times 10^6\) sl. Since \(1 - (\beta + 1)/(\alpha + 4) \approx 0.5\), the precipitation flux decreases due to evaporation for a subsaturated lower layer are given, insofar as \(R\) is positive, by the formula

\[
\Delta (R^{1/2}) = -K_{\text{evap}} \max(0, q_w - q) \frac{\Delta P}{\bar{P}^2}. \tag{A.9}
\]

The specific humidity tendency due to the rainfall evaporation is then given by

\[
\frac{\partial q}{\partial t}_{\text{evap.}} = -\Delta R \frac{g}{\Delta P}. \tag{A.10}
\]

Finally, for a given layer and with the assumption of static energy balance between the precipitation flux and the environment, the temperature tendency due to phase changes is computed from

\[
\frac{\partial T}{\partial t} = -\frac{L}{C_{ph}} \left[ \frac{\partial q}{\partial t}_{\text{cond.}} + \frac{\partial q}{\partial t}_{\text{evap.}} \right] + (C_{pv} - C_{pa}) \frac{R}{\rho C_{ph} \bar{P}} \frac{\partial T}{\partial z}, \tag{A.11}
\]

where \(C_{pa}\) and \(C_{pv}\) are the specific heat at constant pressure of the dry air and of the vapor, respectively.

### APPENDIX B

#### The Energetic Diagnostics

The energetic diagnostics are similar to those in Ducroq (1993), where the energetics of two-dimensional circulations due to symmetric instability were studied. However, in the present case, the diagnostics are extended to take into account the alongline gradients and the ageostrophy of the basic state. Each variable \(\alpha = (U, V, W, T_x, P_x)\) is decomposed into \(\tilde{\alpha}\) (the large-scale contribution) and \(\alpha'\) (the contribution of the smaller scales). This partition between small- and large-scale variables is based on a Fourier filter.

**a. Energetics equation**

Following Gao et al. (1990) and Zhang and Wang (1993), the kinetic energy budget is evaluated in a reference frame following the leading edge of the squall.
line, which is assumed to propagate at a constant speed C. The equation of the perturbation kinetic energy \( K = (u'^2 + v'^2) / 2 \) is given by

\[
\frac{DK}{Dt} = HS_r + TS_r + VS + CA + PP \\
+ DI + ADU_r + ADV + ADW, \quad (B.1)
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + C \frac{\partial}{\partial x}
\]

\[
HS_r = -u'(U - C) \frac{\partial \tilde{U}}{\partial x} - v'(U - C) \frac{\partial \tilde{V}}{\partial x} \\
\quad - u'v \frac{\partial \tilde{U}}{\partial y} - v'v \frac{\partial \tilde{V}}{\partial y}
\]

\[
TS_r = -u' \frac{D \tilde{U}}{Dt} - v' \frac{D \tilde{V}}{Dt}
\]

\[
VS = -u'W \frac{\partial \tilde{U}}{\partial z} - v'W \frac{\partial \tilde{V}}{\partial z}
\]

\[
CA = fu \tilde{V}_a - f v \tilde{U}_a \quad PP = - \frac{1}{\rho} u' \frac{\partial P'}{\partial x} - \frac{1}{\rho} v' \frac{\partial P'}{\partial y}
\]

\[
DI = u' \Delta u + v' \Delta v \quad ADU_r = -(U - C) \frac{\partial K}{\partial x}
\]

\[
ADV = -V \frac{\partial K}{\partial y} \quad ADW = -W \frac{\partial K}{\partial z}.
\]

The terms \( HS_r, TS_r, \) and \( VS \) represent exchanges of kinetic energy between the perturbation and the large-scale state. The term \( HS_r \) corresponds to the exchange of kinetic energy due to the horizontal shear of the large-scale flow without the advective contribution associated with the translation of the squall-line system. The term \( TS_r \) represents the kinetic energy exchange due to the time change of the large-scale flow evaluated in coordinates following the leading edge of the squall line. The sum of terms \( HS_r \) and \( TS_r \), can be rewritten as the following:

\[
HS_r + TS_r \\
= -u'U \frac{\partial \tilde{U}}{\partial x} - v'U \frac{\partial \tilde{V}}{\partial x} - u'V \frac{\partial \tilde{U}}{\partial y} - v'V \frac{\partial \tilde{V}}{\partial y} \\
\quad - u' \frac{\partial \tilde{U}}{\partial t} - v' \frac{\partial \tilde{V}}{\partial t}, \quad (B.2)
\]

where \( HS \) is the kinetic energy exchange due to the horizontal shear of the large-scale flow, and \( TS \) is linked to the local tendency of the large-scale flow. The term \( VS \) is related to the vertical shear of the large-scale state. The term \( CA \) is linked to the ageostrophic of the large-scale flow. It takes into account both the kinetic energy exchange between the perturbation and the large-scale flow (which is linked to the Coriolis force), and the conversion of potential energy (which is linked to the horizontal gradient of the large-scale pressure). The term \( PP \) is a conversion term between the potential energy and the kinetic energy of the perturbation. The term \( DI \) is the diabatic term due to diffusion. Term \( ADU_r \) corresponds to the horizontal advection of perturbation kinetic energy, except for the advective contribution associated with the translation of the squall-line system that is retained in \( DK / Dt \).

b. Separation between larger and smaller scales

The basic state is assumed to result from large-scale motions. The determination of the basic state is based on a Fourier analysis that is performed in vertical cross sections normal to the squall line. We define \( \tilde{\alpha} \) as the filtered variable (i.e., the contribution of the wave-numbers smaller than or equal to a cutoff wavenumber \( m \) in the Fourier analysis) and \( \tilde{\alpha} \) as the basic-state variable.

The virtual temperature \( T_v \) and the wind \((U, V, W)\) on the \( \sigma \)-grid are interpolated to a \( z \)-grid. The surface pressure is reduced to sea level \((\bar{P}_s)\). The basic state is determined as follows.

- Terms \( \bar{U}_g \) and \( \bar{V}_g \) are obtained by the Fourier filter. The virtual temperature \( T_v \) of the basic state is taken equal to \( \bar{T}_v \). By means of the hydrostatic relation, the pressure \( \bar{P} \) at each level of the vertical \( z \)-grid is obtained from \( \bar{T}_v \) and \( \bar{P}_s \).

- Then the geostrophic part of the basic-state wind is defined by

\[
\bar{U}_g = - \frac{Ra \bar{T}_v}{f \bar{P}} \frac{\partial \bar{P}}{\partial y} \quad (B.3)
\]

\[
\bar{V}_g = \frac{Ra \bar{T}_v}{f \bar{P}} \frac{\partial \bar{P}}{\partial x}. \quad (B.4)
\]

- Contrary to Ducrocq (1993), we allow the basic state to be ageostrophic. The ageostrophic part of the basic-state wind is determined by a Fourier filter on \( U_a = U - \bar{U}_g, V_a = V - \bar{V}_g, \) and \( W \):

\[
\bar{U}_a = \bar{U}_a = \bar{U} - \bar{\bar{U}}_g \\
\bar{V}_a = \bar{V}_a = \bar{V} - \bar{\bar{V}}_g \\
\bar{W} = \bar{\bar{W}}.
\]

Therefore, the basic wind is \( \bar{U} = \bar{U}_a + \bar{U}_g, \bar{V} = \bar{V}_a + \bar{V}_g, \) and \( \bar{W} \).

- The perturbation is then defined by \( \alpha' = \alpha - \tilde{\alpha} \).

c. Use on control simulation

The Fourier analysis is carried out along the \((A-A')\) direction—our \( x \) axis—over a distance of 760 km and
for seven parallel vertical cross sections separated by one grid length. The energetic terms are averaged on the three middle vertical cross sections. A schematic representation of the energetic exchanges on two different domains, including the RTF and FTR flows, is given in section 5. The zero-isotach of the perturbation line-normal velocity \( u' \) is used to define the boundaries of these domains. The domain including the FTR flow encompasses the negative values of \( u' \), and the domain including the RTF flow encloses the positive values of \( u' \). Obviously, since the size and the location of the RTF and FTR flows vary with time, the domains move with the squall line and their shape changes with time. The energetic terms are spatially averaged on these domains to give a comprehensive representation of the energetic exchanges for the FTR and RTF flows. A representation of the energetic exchanges for the whole inner circulation (RTF and FTR flows together) is also computed. The diffusive term \( Df \) is not displayed since it is not available from the standard model. This term can be important near the ground. The propagation speed \( C \) has been taken equal to \( 10 \text{ m s}^{-1} \). The cutoff wavenumber \( m \) of the Fourier filter that is used to separate the large- and small-scale variables has been chosen in accordance with the features of the spectra of the variables when the squall line is well developed. Indeed, the spectra of \( U, V, W, T_r, \) and \( P \) at 1500 UTC at the lower levels exhibit large amplitude for wave-numbers 0 and 1 and a secondary peak at wavenumbers 6 and 7, which correspond to the width of the simulated couplet of updraft and downdraft. Consequently, the cutoff wavenumber is chosen equal to 1, that is, the large-scale state is assumed to be the result of motions that have a wavelength larger than 760 km. If \( m \) is taken equal to 2, the results are qualitatively the same.

REFERENCES


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