Freezing Fraction in Freezing Rain

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ABSTRACT: Freezing rain can cause significant tree damage with fallen trees and branches blocking roads and taking power distribution lines out of service. Power transmission lines are designed for ice loads from freezing rain, using models to estimate equivalent radial ice thicknesses from historical weather data. The conservative simple flux model assumes that all the freezing rain that impinges on a horizontal cylinder, representing vegetation or components of the built infrastructure, freezes. Here I present a simplified heat-balance formulation to calculate the fraction of the impinging precipitation that freezes, using parameters measured at ASOS weather stations and an estimate of solar heating. Radial ice thickness estimates from this approach are compared with the simple model and those generated from the ASOS icing sensor. These estimates can all be tested by comparing to measurements on cylinders at weather stations. A link to an Excel spreadsheet that calculates freezing fraction using user-input weather data is provided. In forecast freezing rain events, this tool could be used by utility crews and emergency response teams to estimate the likely range of equivalent radial ice thicknesses over the affected region and plan their response accordingly.

SIGNIFICANCE STATEMENT: Freezing rain can cause significant tree damage with fallen trees and branches blocking roads and taking power distribution lines out of service. Power transmission lines are designed for ice loads from freezing rain calculated from historical weather data. This paper provides an algorithm for computing ice loads on trees and power lines, using weather data to determine the fraction of the precipitation that freezes on them rather than dripping off. This freezing fraction result is compared to estimates reported by weather stations and to a simple model that assumes all the wind-blown freezing rain freezes on the wires, twigs, and branches. A link is provided to an Excel tool that calculates freezing fraction. This could be used with freezing rain forecasts to estimate the likely severity of the event.

KEYWORDS: Model comparison; Field experiments; Icing; Freezing precipitation

1. Introduction

Freezing rain can cause severe tree and infrastructure damage and lead to disaster declarations. When ice-covered trees and branches fail they can block roads and cause outages to overhead power distribution lines. In rural areas where miles of distribution lines may have to be repaired to restore power to a widely dispersed population, outages may be prolonged. While high-voltage power transmission lines and towers for television, radio, cell phone, and emergency communication are designed for ice loads and concurrent winds (TIA 2019; ASCE 2020), they can still fail in severe freezing rain storms. Models of ice accretion on trees and infrastructure, for a forecast or observed freezing rain event, require, at the least, information on the precipitation type, precipitation rate, and wind speed. More realistic ice accretions can be obtained if there are also air and dewpoint temperature data and an estimate of shortwave radiation to determine the

fraction of the freezing rain impinging on trees and infrastructure that freezes. The focus of this paper is the spatial variation of accreted ice loads in freezing rain events associated with the varying meteorological conditions across the affected area. The load of ice is represented by the equivalent radial ice thickness on the cylinders, representing wires and conductors of overhead lines, structural sections of lattice towers, and branches and twigs of trees. This paper presents a physical model for determining the equivalent radial ice thickness in freezing rain using parameters that are measured at weather stations.

Numerous articles address the upper air conditions associated with freezing rain and frozen precipitation (snow or ice pellets) as well as mixed-phase particles (e.g., Stewart and King 1987; Zerr 1997; Rauber et al. 2000; Hux et al. 2001; Thériault et al. 2006). Stewart and King (1987) use a model of the melting of snowflakes to investigate their evolution as they fall through the upper warm air layer, using a variety of snowflake properties, maximum air temperatures, and lapse rates. Their results suggest that mixed-phase particles may occur often in freezing rain/ice pellet conditions. Zerr (1997) relates the vertical structure of the atmosphere to the phase of the particle at the surface. He uses a melting model and a freezing model to investigate the fate of snowflakes with a range of masses falling in various ambient conditions and finds that the precipitation type at the surface is more closely related to the temperature and depth of the melting layer than to the characteristics of the refreezing layer. Rauber et al. (2000) investigate soundings in the United States between

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1970 and 1994 associated with freezing rain or drizzle significant enough to be included in *Storm Data* (NOAA 1970–1994) and determine that between 47% and 75% of them indicate that the warm rain process (Huffman and Norman 1988) was responsible for the freezing precipitation. The other 25% of the soundings had the classic warm air layer above a layer of subfreezing air. Hux et al. (2001) present an operational forecast approach for winter precipitation that incorporates climatological data. They find that soundings having two 0°C isotherms are associated with nearly all observations of freezing rain and freezing rain mixed with any other precipitation type. Pure ice pellets occur rarely and are not associated with a particular sounding type. Thériault et al. (2006) run a 1D cloud model with a double-moment microphysics scheme to examine the evolution of snowflakes as they fall through a melting layer and then a refreezing layer. They find that ice pellets and freezing rain are associated with a wide range of melting layer and subfreezing layer depths and temperatures while precipitation that includes mixed-phase particles results from a narrow range of melting layer depths with maximum temperatures less than 1°C.

Cortinas et al. (2004) take a retrospective approach using weather station data in the United States and Canada from 1976 to 1990 to characterize the occurrence of freezing rain, freezing drizzle, and ice pellets. The beginning date eliminates data that was archived only every third hour and the end date ensures that human observers specified the precipitation type, but with no option to specify mixed-phase particles. They find that freezing rain and drizzle occur most frequently before sunrise with the rate decreasing sharply in the morning to a minimum in midafternoon, while the occurrence of ice pellets is unrelated to time of day. The occurrence of multiple precipitation types at the same time depends on the type: 1) freezing rain occurs without other precipitation types 69% of the time, but occurs with ice pellets 18% of the time and snow 14%, and 2) ice pellets tend to occur concurrently with other precipitation types, with snow 37% of the time, and with freezing rain and rain each 18% of the time.

Beginning in the mid-1990s Automated Surface Observing System (ASOS) stations began widespread deployment. Reeves et al. (2014) and Elmore et al. (2015) deal with the ability of forecast models (NAM, GFS, and RAP) to distinguish snow, rain, freezing rain, and ice pellets by comparing with augmented ASOS stations observations and citizen science meteorological Phenomena Identification Near the Ground (mPING) reports. The ASOS stations augmented with human observers (service levels A and B) and the mPING participants can report ice pellets and multiple precipitation types, while the unaugmented stations (service levels C and D) cannot. They find that in contrast to snow and rain, freezing rain and ice pellets are often misdiagnosed by all three forecast models because they occur with similar wet-bulb temperature profiles. This difficulty is compounded by the likely significant spatial variability in precipitation type because of local temperature, precipitation rate, and drop size variations. Reeves (2016) finds that mPING observations of freezing rain are underreported compared to ASOS and suggests that this is likely due to the warmer ground

temperatures at the mPING observer locations compared to the ASOS location.

In the next section I provide background information on models for ice accretion on cylinders in freezing rain, field observations of the spatial variability of ice accretion in freezing rain, and the ASOS icing sensor, used both to specify precipitation type and to report ice thickness on horizontal surfaces in freezing rain. In section 3 I present a basic freezing fraction formulation to calculate the fraction of freezing rain impinging on a cylinder that freezes. Section 4 provides comparisons of simple model equivalent radial ice thicknesses with those from the freezing fraction calculation and ASOS icing sensor remarks in 20 freezing rain events. Section 5 includes discussions of the icing sensor in marginally freezing conditions, the effect of solar radiation, mixed precipitation, and mixed-phase drops on ice accumulation, and testing modeled equivalent radial ice thicknesses with measurements at weather stations. This is followed by the conclusions.

2. Background

a. Models for ice accretion in freezing rain

Freezing rain that freezes on trees, overhead lines, and communication towers can cause significant damage, resulting in road closures, communication and power outages, and sometimes disaster declarations. The basic elements of all these structures are cylinders: branches, twigs, cables, wires, and structural sections (e.g., angle, channel, H section). A simple model for the accretion of ice in freezing rain on a round horizontal cylinder with axis perpendicular to the wind direction assumes that 1) the precipitation is intercepted by the diameter of the cylinder and 2) the impinging water freezes in a uniformly thick layer around the circumference to form the equivalent radial ice thickness (Jones 1998):

$$R_{eq} = \frac{1}{\pi \rho_i} \sum_{j=1, n} \left[(0.1 P_j \rho_0)^2 + (0.36 W_j U_j)^2 \right]^{1/2} \Delta t, \quad (1)$$

where P (mm h⁻¹) is precipitation rate, U (m s⁻¹) is wind speed, W (g m⁻³) is liquid water content of rain-filled air, $\rho_0 = 1$ g cm⁻³ is density of water, $\rho_i = 0.9$ g cm⁻³ is density of ice, Δt (h) is time increment, and $\pi = 3.14$ is the ratio between the cylinder circumference and diameter. The summation in (1) is over the n hours in the freezing rain event. The 0.1 and 0.36 factors convert to a consistent set of units with R_{eq} (cm). The cylinder is assumed to be infinitely long to avoid the consideration of end effects.

I use $W = 0.067 P^{0.846}$ (Best 1950) for the liquid water content of the rain-filled air. Best uses drop distribution data in rain from three sites in England and Wales as well as data from Lenard (1904), Laws and Parsons (1943), and Marshall and Palmer (1948) to compute an overall best-fit distribution, in terms of the fraction of liquid water in the air that exceeds the drop diameter d . He points out that the fitted distributions fit the data fairly accurately if the 10% of the water comprising the smallest drops and the 5% comprising the largest drops is excluded. Crawford and Stewart (1995) present

FIG. 1. Comparison of the Crawford and Stewart (1995) ($C + S$) drop distributions in freezing rain for $P = 0.5$ and 1.8 mm h^{-1} , with the distributions fitted to the data provided by Best (1950) for similar precipitation rates measured at Shoeburyness and East Hill in England and Ynyslas, Wales, and from measurements compiled by Laws and Parsons (1943). The values in the legend are precipitation rates (mm h^{-1}).

measurements of drop distributions in freezing rain. They show representative drop concentration distributions for $P = 0.5$ and 1.8 mm h^{-1} and suggest that differences from the Marshall and Palmer (1948) distribution, either excess small drops or excess large drops, are due to the height of the melting layer. Figure 1 shows the two Crawford and Stewart distributions, along with Best's fits to the data he used for 12 precipitation rates somewhat less than, between, and somewhat greater than the Crawford and Stewart values. This comparison suggests that those freezing rain distributions are similar to the distributions underlying Best's parameterization of $W(P)$. The tendency of Best's fits to underestimate the amount of water in the smallest drops means that the calculated flux of water from the wind-blown rain may be slightly smaller than actual.

Collision efficiency of cloud drops, which do not fall and only move when they are carried by the wind, must be considered in determining the amount of ice that accretes during icing in supercooled clouds and fog. Finstad (1995) shows that collision efficiency should also be considered for drizzle-size drops ($d \sim 200\text{--}500 \mu\text{m}$) for low winds. For example, for a 0.0254-m diameter cylinder the collision efficiency of $500\text{-}\mu\text{m}$ drops decreases to 0.98 between 2 and 7 m s^{-1} . The collision efficiency of drops with $d = 200 \mu\text{m}$ is about 0.9 for wind speeds between 0.5 and 4 m s^{-1} . This could be taken into account for freezing rain cases in which the drop size distribution is measured along with all the other weather parameters.

This basic geometry in which precipitation is intercepted by the diameter and spread over the circumference is used by Fowle (1910), but without considering wind-blown precipitation, and by Goodwin et al. (1983), but with a user-specified fall speed for the precipitation in the wind term. The assumption of a uniformly thick layer of ice on the cylinder is an idealization of reality. The water may all freeze on the upper windward side, or may freeze slowly as it flows around the cylinder, and some may freeze as icicles as it starts to drip off. The R_{eq} is the thickness the ice would have if the actual accretion was molded into a uniformly thick layer around the cylinder. The weight of ice on a cylinder per unit length is calculated from R_{eq} and the bare cylinder diameter.

This simple model can be adapted for various other geometries including a vertical cylinder [drop the first term in (1)], horizontal cylinder with axis parallel to the wind (drop the second term), horizontal cylinder at a fixed angle (include angle relative to the wind direction in the second term), non-round cylinder (Jones and Peabody 2006), flat surface perpendicular to the trajectory of the wind-blown raindrops (drop the division by π), and flat horizontal surface (drop the second term and the division by π). These adaptations of the simple model, shown in Table 1, all assume that all the impinging freezing rain freezes: freezing fraction $f = 1$.

There are also models for the accretion of ice from freezing rain on cylinders that take into account the possibility that not all the impinging precipitation freezes, including Lozowski et al. (1983), Makkonen (1984), Finstad et al. (1988), Szilder (1994), and the Cold Regions Research and Engineering Laboratory (CRREL) model (Jones 1996). The latter model uses the same geometry as (1) but includes detailed heat balance calculations to determine how much of the freezing rain freezes. If not all the impinging water freezes on the cylinder, a second heat balance calculation determines how much of the remaining water freezes as icicles. The water that is not incorporated in icicles drips off. With one exception (shortwave radiation) the terms in the heat balance calculations rely on weather parameters reported by ASOS stations. At the few sites where shortwave radiation is measured I use that data in running the CRREL model, otherwise I have ignored that term. Ignoring solar radiation makes no difference at night and during daytime hours when the cooling rate is much greater than the rate needed to freeze the flux of precipitation. However, heating from solar radiation can have a significant effect on the heat balance calculation in daylight hours in conditions with barely enough cooling to freeze the impinging precipitation.

b. Field measurements

Significant spatial variations in the amount of ice that accretes in freezing rain events are related to variations in weather conditions. For many years the CRREL freezing rain team deployed when freezing rain events severe enough to result in ice accretions causing significant tree damage were forecast. These deployments included New Hampshire (February 1995), Tennessee and the Carolinas (February 1996), Louisiana and Texas (January 1997), Vermont and New York

TABLE 1. Adaptation of simple model to other geometries; T_i is the ice thickness on a flat surface.

Geometry	Equation
Simple model	$R_{eq} = \frac{1}{\pi \rho_i} \left[(0.1 P \rho_0)^2 + (0.36 W U)^2 \right]^{1/2} \Delta t$
Vertical	$R_{eq} = \frac{0.36 W U}{\pi \rho_i} \Delta t$
Parallel to wind	$R_{eq} = \frac{0.1 P \rho_0}{\pi \rho_i} \Delta t$
Angle to wind	$R_{eq} = \frac{1}{\pi \rho_i} \left[(0.1 P \rho_0)^2 + (0.36 W U \sin \theta)^2 \right]^{1/2} \Delta t$
Non-round cylinder	Jones and Peabody (2006)
Flat surface perpendicular to rain	$T_i = \frac{1}{\rho_i} \left[(0.1 P \rho_0)^2 + (0.36 W U)^2 \right]^{1/2} \Delta t$
Flat horizontal surface	$T_i = \frac{0.1 P \rho_0}{\rho_i} \Delta t$

(January 1998), Texas (December 2000), Arkansas (December 2000), Illinois and Missouri (January 2002), and New Hampshire (December 2008). We learned to expect significant tree damage with $R_{eq} > 0.6$ cm. During each deployment we drove around the region affected by the ice storm

FIG. 2. Ice samples collected by CRREL (a) from tree on Lookout Mountain, Chattanooga, TN, 2 Feb 1996; and (b) on electric fence in Hamilton, NY, 9 Jan 1998 (photos Nathan Mulherin).

collecting ice samples on natural and man-made cylinders including trees and other vegetation, bars, wires, and cables. Figure 2 shows two of the samples, one from a tree and the other from an electric fence. We weighed and measured each sample and calculated R_{eq} from the ice mass, sample length, and average substrate diameter. In most of these events we noticed significant local spatial variability in the amount of accreted ice. The sample in Fig. 2a is from 6 February 1996 on Lookout Mountain, Tennessee, at an elevation of about 600 m, 14 km from the Chattanooga airport at 200 m. While there was enough ice to cause significant tree damage on Lookout Mountain, our contemporaneous notes indicate that we saw little ice and no tree damage in the vicinity of the airport an hour later. This observation is supported by the Chattanooga METAR data that reports ten hours with ice pellets and three hours with ice pellets and freezing rain up to that time. The sample in the photo in Fig. 2b, on an electric fence in Hammond, New York, on 9 January 1998, has $R_{eq} = 1.7$ cm. The scale behind the sample indicates a maximum ice accretion dimension of about 8 cm. In articles about damaging freezing rain storms, newspapers often report these dramatic maximum dimensions, which are not representative of the weight of ice bringing down trees and power lines. The R_{eq} data for the 1998 ice storm in Vermont and New York from samples collected during 8–10 January at 33 sites are shown in Fig. 3. During that time the type of precipitation along the route varied among none, rain, freezing drizzle, freezing rain, ice pellets, and snow. There was significant tree, distribution line, and transmission line damage in the area. Simple model R_{eq} values for 5–10 January 1998 for the Watertown, New York; Massena, New York; and Burlington, Vermont, weather stations are 4.4, 4.1, and 2.6 cm, respectively. Note that the R_{eq} values in Fig. 3 are from samples at human distances above ground at various exposures and orientations to horizontal and the wind, different from the simple model assumption of horizontal and perpendicular to the wind at the 10-m anemometer height at weather stations. There are many articles about the meteorology of this storm and the associated damage (e.g., Irland 1998; Milton and Bourque 1999; DeGaetano 2000; Roebber and Gyakum 2003).

c. ASOS icing sensor

The components of ASOS stations are described in NOAA (1998). The ASOS icing sensor is used with the Light Emitting Diode Weather Indicator (LEDWI) and the temperature sensor to determine the type of precipitation. The LEDWI uses

FIG. 3. Maximum R_{eq} (cm) measured at each CRREL site in the 5–10 Jan 1998 freezing rain storm.

the fall speed of the precipitation to distinguish between liquid drops and snowflakes. Drizzle is not detected by the LEDWI because the drops are too small, so drizzle and freezing drizzle can be reported only at augmented stations. Because ice pellets and rain have similar fall speeds, ice pellets are usually reported as rain but may be reported as UP (unknown precipitation). In mixed snow and ice pellets, snow will be reported. When ice accretes on the icing sensor probe its vibration frequency decreases proportional to the mass of ice. When the LEDWI detects precipitation, a frequency drop of 33 Hz or more and a temperature 36°F (2.2°C) or colder creates a tentative report of freezing rain. A report of UP or rain from the LEDWI with freezing rain reported by the icing sensor generates an ASOS report of freezing rain. If, however, the LEDWI reports snow with an icing sensor report of freezing rain, the reporting hierarchy (snow, freezing rain, and rain in descending order) results in an ASOS report of snow.

Ryerson and Ramsay (2007) describe the calibration of the icing sensor to provide ice thickness information in addition to precipitation type. Because the icing sensors were originally intended only to provide precipitation type information they are not precision instruments. A test of a sample of the ASOS sensors indicated that their response is uniformly distributed within $\pm 20\%$ of the standard. In the calibration experiments, personnel measured ice accretion masses and thicknesses on an hourly or 2-hourly basis and compared to icing sensor output in 21 freezing rain episodes over four winters (1996–2000) at five sites using 16 icing sensors altogether. Based on this calibration, ASOS added a remark generated from the icing sensor that reports ice thickness on a horizontal surface in 0.01-in. (0.25 mm) increments (NOAA 2013). Ice thickness reports can be the result of ice deposited by fog, freezing fog, mist, freezing drizzle, or freezing rain. The probe is heated to deice when the frequency drops by 527 Hz corresponding to 0.08 in. (2 mm) of ice on a horizontal surface. NOAA (2013) notes some limitations of the ice thickness report: 1) in conditions in which the impinging water freezes slowly, it may run off the small icing sensor probe before freezing and will be underreported and 2) because of possible local micrometeorology variations there may be ice accretion at the ASOS but not at the observer. Note that a significant cooling element for the icing sensor probe is the 2-kg anodized aluminum heat sink (Fig. 4). It rapidly removes heat from the probe after a deicing cycle (All Weather Inc. 2008). The heat sink will also cool the probe during the icing cycle whenever it is colder than the probe.

In addition to comparing the icing sensor output to ice accreted on an elevated horizontal plate, Ryerson and Ramsay (2007) calculate R_{eq} on horizontal cylinders with axes perpendicular to the wind from measurements of the mass of ice on the cylinders. They find that R_{eq} is 0.394 times the ice thickness on the plate ($R^2 = 0.97$). Jones et al. (2004) compare icing sensor R_{eq} values with the CRREL and simple model R_{eq} values in a December 2002 freezing rain storm that was severe enough to cause outages from Oklahoma to the Carolinas. A total of 79 ASOS stations reported freezing rain. Data anomalies at these stations include gaps likely caused by power outages and hours with zero wind speeds, possibly

FIG. 4. ASOS icing sensor. The finned mass between the probe and the electronics is the heat sink.

caused by frozen anemometers. At the 19 stations where the icing sensor was available, Ramsay calculated sensor R_{eq} values from the 1-min data and the calibration done for Ryerson and Ramsay (2007). Half of these R_{eq} values are greater than the CRREL model R_{eq} values and most are smaller than the simple model R_{eq} values.

The Freezing Rain Accumulation Model (FRAM) (Sanders and Barjenbruch 2016) relates the ASOS reported ice thickness on a horizontal surface to precipitation, wind speed, and wet bulb temperature in hours with at least 0.02 in. (0.51 mm) of freezing rain and 0.01 in. (0.25 mm) of ice using data from ASOS stations across the United States in the winters of 2013/14 and 2014/15. Their ice-to-liquid ratio (ILR) function, based on fits to the ice thickness reports, is the sum of a quadratic wind polynomial, a cubic wet-bulb temperature polynomial, and the inverse of precipitation raised to a power, rather than being based on physically relevant heat transfer relationships that can be found in textbooks. The FRAM ice thickness on a horizontal surface in each hour is the ILR multiplied by the precipitation.

3. Method

a. Freezing fraction

The freezing fraction for ice accretion on a horizontal cylinder with axis perpendicular to the wind is developed in

Jones (1996). It is the ratio of the sum of the cooling and heating fluxes to the cooling needed to freeze the falling and wind-blown precipitation:

$$f = \frac{Q_w + Q_c + Q_e + Q_l + Q_s}{Q_f}, \quad (2)$$

where Q_f is the cooling needed to freeze the impinging rain, Q_w is cooling from warming supercooled drops, Q_c is convective cooling, Q_e is evaporative cooling, Q_l is longwave radiative cooling, and Q_s is shortwave radiative heating (all in W m^{-2}). The terms in the numerator are taken to be positive if they are cooling fluxes. This calculation is done in every hour with freezing rain and $P > 0$. The freezing fraction R_{eq} increment for the hour is f from (2) multiplied by the simple model R_{eq} increment.

In the following, the equations for each of the terms in (2) are provided following Jones (1996) which relies primarily on Incropera and DeWitt (1985). A number of the parameters that describe the properties of air depend on temperature T and/or atmospheric pressure P_a . The parameter values provided here are for $P_a = 975$ hPa, consistent with the average elevations in the states, excluding Florida, east of the Rocky Mountains, and $T = -2.5^\circ\text{C}$, the middle of the 0° to -5°C range of air temperatures in which freezing rain often occurs, based on data from 540 stations in the United States and in Canada along the U.S. border. Full expressions for these parameters are provided in appendix A.

b. Cooling needed to freeze the precipitation

The cooling flux needed to freeze the precipitation is the product of the latent heat of fusion and the flux of falling and windblown precipitation:

$$Q_f = L_f \left[\left(\frac{P\rho_0}{3.6} \right)^2 + (0.067P^{0.846}U)^2 \right]^{1/2}, \quad (3)$$

where $L_f = 334 \text{ J g}^{-1}$ is the latent heat of fusion for water. This is the denominator of the freezing fraction ratio.

c. Cooling from supercooled rain

I assume that the impinging rain drops are at the air temperature T . The cooling flux as this water warms to 0°C before freezing is

$$Q_w = c_w(0 - T) \left[\left(\frac{P\rho_0}{3.6} \right)^2 + (0.067P^{0.846}U)^2 \right]^{1/2}, \quad (4)$$

where $c_w = 4.22 \text{ J g}^{-1} \text{ }^\circ\text{C}^{-1}$ is the specific heat of water and T ($^\circ\text{C}$) is air temperature. The magnitude of $c_w T$ is much smaller than L_f , so supercooling of the rain drops makes only a small contribution to their freezing on the cylinder. Drops that have cooled to the wet-bulb temperature make a larger contribution, but in either case only a small fraction of the impinging precipitation freezes instantly. Convective and evaporative cooling are each more significant.

d. Convective cooling

Cold air blowing by the water on the cylinder acts to remove heat. Convective cooling depends on air temperature and on wind speed through the heat transfer coefficient in air h ($\text{W m}^{-2} \text{ }^\circ\text{C}^{-1}$):

$$Q_c = \pi h(0 - T), \quad (5)$$

with h given by

$$h = 0.583 \frac{k}{D} \text{Re}^{0.471}, \quad 35 < \text{Re} < 1600 \\ = 0.18 \frac{k}{D} \text{Re}^{0.63}, \quad 1600 < \text{Re} < 1 \times 10^6, \quad (6a)$$

where $D(m)$ is cylinder diameter, $k = 0.024 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ is thermal conductivity of air, $\text{Re} = UD/\nu$ is the Reynolds number, and $\nu = 1.4 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ is kinematic viscosity of air.

When there is no wind, cooling can still occur by free convection with h given by

$$h = 1.02 \frac{k}{D} \text{Ra}^{0.148}, \quad \text{for } 10^{-2} < \text{Ra} < 10^2 \\ = 0.85 \frac{k}{D} \text{Ra}^{0.188}, \quad \text{for } 10^2 < \text{Ra} < 10^4 \\ = 0.48 \frac{k}{D} \text{Ra}^{0.250}, \quad \text{for } 10^4 < \text{Ra} < 10^7, \quad (6b)$$

and Raleigh number

$$\text{Ra} = \frac{g|0 - T|D^3\rho_a c_p}{(T_0 + T)\nu k}, \quad (7)$$

where $g = 9.8 \text{ m s}^{-2}$ is the acceleration of gravity, $c_p = 1.006 \text{ J g}^{-1} \text{ }^\circ\text{C}^{-1}$ is the specific heat of air, $\rho_a = 1.3 \times 10^{-3} \text{ g cm}^{-3}$ is the density of air, and $T_0 = 273.15^\circ\text{C}$. Note that for $T > 0$, $Q_c < 0$ making it a warming rather than cooling term.

e. Evaporative cooling

The water on the cylinder is also cooled by evaporation:

$$Q_e = \pi L_e h_m (\rho_{w0} - \text{RH } \rho_{wT}), \quad (8)$$

where h_m (m s^{-1}) is vapor transfer coefficient in air; $L_e = 2501 \text{ J g}^{-1}$ is latent heat of vaporization for water; RH (decimal) is relative humidity; and ρ_{w0} and ρ_{wT} (g m^{-3}) are saturated vapor densities at 0°C and T , respectively. Evaporative cooling depends on air temperature, dewpoint temperature, and wind speed. The vapor transfer coefficient h_m depends on the Sherwood number Sh , which in turn depends on the Prandtl number $\text{Pr} = \mu_a c_p / k = 0.72$, Schmidt number $\text{Sc} = \nu / \kappa_m = 0.65$, and Reynolds number:

$$h_m = \frac{\text{Sh } \kappa_m}{D} \\ = 0.583 \frac{\kappa_m}{D} \left(\frac{\text{Sc}}{\text{Pr}} \right)^{0.37} \text{Re}^{0.471}, \quad 35 < \text{Re} < 1600 \\ = 0.18 \frac{\kappa_m}{D} \left(\frac{\text{Sc}}{\text{Pr}} \right)^{0.37} \text{Re}^{0.63}, \quad 1600 < \text{Re} < 1 \times 10^6, \quad (9)$$

where $\kappa_m = 2.1 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ is molecular diffusivity of water vapor in air. The saturated vapor densities in (8) are

$$\begin{aligned}\rho_{wT} &= \frac{m_w e_T}{R} = \frac{m_w}{R} \frac{e_0 e^{17.502T/(240.97+T)}}{T_0 + T} \\ \rho_{w0} &= \frac{m_w}{R} \frac{e_0}{T_0},\end{aligned}\quad (10a)$$

where $m_w = 18.016 \text{ g mol}^{-1}$ is molecular weight of water vapor, $R = 8.3144 \text{ J mol}^{-1} \text{ K}^{-1}$ is gas constant, and $e_0 = 614 \text{ N m}^{-2}$ is saturation vapor pressure of air over water at 0°C . For T between 0° and -5°C , (10a) is well fit ($R^2 > 0.999$) by the quadratic

$$\rho_{wT} \approx 4.87 + 0.334T + 8.91 \times 10^{-3} T^2, \quad (10b)$$

giving $\rho_{w0} = 4.87 \text{ g m}^{-3}$. Finally, relative humidity RH is related to dewpoint depression using Sargent (1980) provided in Lawrence (2005):

$$\text{RH} = 17.9/18 - (T - T_d)/18, \quad (11)$$

where $T_d(^{\circ}\text{C})$ is dewpoint temperature. For $T > 0^\circ\text{C}$ this is a cooling term only if the dewpoint depression $T - T_d$ is big enough.

When there is no wind, evaporative cooling can still occur by free convection with h_m given by

$$\begin{aligned}h_m &= 1.02 \frac{\kappa_m}{D} \left(\text{Ra} \frac{\text{Sc}}{\text{Pr}} \right)^{0.148}, \quad 10^{-2} < \text{Ra} < 10^2 \\ &= 0.85 \frac{\kappa_m}{D} \left(\text{Ra} \frac{\text{Sc}}{\text{Pr}} \right)^{0.188}, \quad 10^2 < \text{Ra} < 10^4 \\ &= 0.48 \frac{\kappa_m}{D} \left(\text{Ra} \frac{\text{Sc}}{\text{Pr}} \right)^{0.250}, \quad 10^4 < \text{Ra} < 10^7.\end{aligned}\quad (12)$$

f. Longwave radiative cooling

I assume that the cylinder with ice accreting is radiating at 0°C and the clouds and air are at T . Radiative cooling of the impinging water is then

$$Q_l = -\pi\sigma[(T + T_0)^4 - T_0^4] \approx -2\pi\sigma T_0^3 T \left(2 + 3 \frac{T}{T_0} \right), \quad (13)$$

where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-1}$ is the Stefan-Boltzmann constant. If the clouds are colder than T , Q_l will be larger, but this term is small compared to Q_c and Q_e . The polynomial in (13) is obtained by expanding the quartic and keeping the first three terms.

g. Shortwave radiative heating

During daylight hours there is diffuse shortwave radiation from the sun even in the total overcast that occurs with freezing rain:

$$Q_s = -\pi \text{Sr}/2, \quad (14)$$

where $\text{Sr} (\text{W m}^{-2})$ is solar radiation flux at ground level. I had included shortwave heating in my heat balance calculations in

the CRREL model only at weather stations where solar radiation is reported. Here an estimate is provided to use in the absence of solar radiation data. The value of Sr depends on cloud cover, latitude of the site, day of year, and time of day. For a completely overcast sky Sr can be estimated as

$$\text{Sr} = C_f S_0 \cos \theta, \quad (15)$$

where the solar constant $S_0 = 1361 \text{ W m}^{-2}$ (Kopp and Lean 2011), C_f is the fraction of solar radiation transmitted by the overcast, and θ is solar zenith angle. Haurwitz (1948) provides values for the fraction of the solar radiation transmitted by an overcast sky by cloud type from eight years of measurements at Blue Hill Observatory south of Boston. Kasten and Czeplak (1980) provide results from 10 years of measurements in Hamburg, Germany. I use $C_f = 0.18$, which is the mean of the values for nimbostratus from these references weighted by the number of measurements. The calculation of θ for specific latitude, longitude, day of year, and time of day is provided in appendix B, following Woolf (1968).

h. Icicles

The basic freezing fraction formulation presented here ignores the more complicated icicle calculation in the CRREL model (Jones 1996) so that it can be provided as an Excel tool. Icicles are formed in the CRREL model in an hour with $f < 1$ when unfrozen water begins to drip off the cylinder and there is sufficient cooling to freeze the outside of the drip, thus keeping it attached to the cylinder. Additional unfrozen water may then freeze to the sides of this protoicicle or as an additional ice-water drip at its bottom. The icicles that form are assumed to be parallel to the raindrop trajectories so that they do not themselves intercept precipitation. The icicle diameter and length may increase over the course of the freezing rain event in subsequent hours with $f < 1$, creating short, fat icicles in relatively cold conditions and longer, thinner icicles when freezing is slower. At the end of the freezing rain event, the mass of icicles is added to the R_{eq} of the ice that freezes directly on the cylinder. The contribution of the icicle mass to the basic freezing fraction R_{eq} depends on the conditions. Combinations of $T \geq 0^\circ\text{C}$, small dewpoint depressions, and significant solar radiation result in little to no icicle formation. With high precipitation rates, even large cooling fluxes may be insufficient to freeze all of the impinging water quickly, but much of what begins to drip off will freeze as icicles, possibly resulting in freezing fraction R_{eq} values nearly as large as simple model R_{eq} values.

i. Other

In the CRREL model heat balance calculation I also include the loss of water by evaporation. This small term is ignored in this basic freezing fraction formulation along with viscous heating and heating from the kinetic energy of the rain drops. Here I am also ignoring freezing of the impinging rain by conductive heat loss into a cold cylinder. The mass of water that can be frozen by this mechanism depends on how cold it is and the mass, heat capacity, and heat conduction in the cylinder, which depend on the cylinder material.

FIG. 5. (top) Variation of solar radiative heating Q_s at two latitudes for three dates and (bottom) the resulting variation in freezing fraction f for the nominal values of the four ASOS weather parameters.

j. Contributions to freezing fraction

The effect on the freezing fraction (2) of the supercooled rain, convective, evaporative, and longwave cooling as well as solar heating relative to the cooling needed to freeze the freezing rain is best shown graphically. I chose nominal conditions for each ASOS weather parameter: $T = -1.1^\circ\text{C}$, $T - T_d = 0.6^\circ\text{C}$, $U = 2 \text{ m s}^{-1}$, and $P = 2.5 \text{ mm h}^{-1}$ to illustrate how freezing fraction changes as these parameters change for a cylinder diameter $D = 0.03 \text{ m}$. Figure 5 shows the daily variation in Q_s and f at 90°W and latitudes 30° and 48°N on 10 November, 20 December, and 10 March, with a nimbostratus overcast and the four ASOS parameters at their nominal values. At night, with no solar heating, $f = 0.90$. In daylight hours, the decrease in solar heating and corresponding increase in f with increasing latitude and proximity to the winter solstice is apparent.

The six panels in Fig. 6 show f as a function of each pair of the ASOS parameters with the other two parameters at their nominal values and a nominal $Q_s = 50 \text{ W m}^{-2}$. For all parameters at their nominal values, $f = 0.65$. All the impinging rain-water freezes when $f = 1$ and for larger f the cylinder also cools. None freezes when $f = 0$ and for negative f the cylinder warms. Figures 6a–c show f as a function of precipitation rate and one of each of the other parameters. In almost all cases f increases as P or T decreases and as U or dewpoint depression increases. The exception is for $T = 0^\circ\text{C}$ for the nominal 0.6°C dewpoint depression. In this case the only cooling comes from evaporation, which is smaller than the assumed shortwave heating, so as P decreases the ratio of the net heating flux to

the flux of water increases and f becomes more negative. Figures 6d and 6e show increasing cooling with increasing wind speed for air temperature decreasing and dewpoint depression increasing. In Fig. 6d there is a warming flux for $T = 0^\circ\text{C}$ and wind speeds below about 3.5 m s^{-1} . Figure 6f shows that f is linear in both T and $T - T_d$, so for this range of temperatures the quadratic terms in (10b) and (13) are insignificant.

4. Results

I applied this freezing fraction formulation to the 20 freezing rain events that Sanders and Barjenbruch (2016, hereafter SB16) randomly select to demonstrate their ILR function and FRAM model. The ASOS station location and the event date, along with the duration, ice thickness, and precipitation that are shown in the three “SB study” columns in Table 2 are from the SB16 table. The latitude, longitude, and service level for each station are also included in Table 2. For each event, I downloaded the METAR data for the date specified and the previous and following day UTC and extracted the regular hourly observations, including the hourly precipitation amount, type, and intensity, wind speed, temperature, dewpoint temperature, and ice thickness from the mandatory section and remarks. The number of hours with freezing rain, the amount of precipitation in those hours and the total ice thickness from the icing sensor for each event are shown in the first three columns in the “This study” section of Table 2. Every event in this study has more hours with freezing rain and more precipitation than in SB16. Nine of the freezing rain

FIG. 6. Freezing fraction as a function of the ASOS parameters. In each panel two of the four parameters are varied while the other two and $S_r = 50 \text{ W m}^{-2}$ are at their nominal values: $P = 2.5 \text{ mm h}^{-1}$, $U = 2 \text{ m s}^{-1}$, $T = -1.1^\circ\text{C}$, $T - T_d = 0.6^\circ\text{C}$: (a) P and U , (b) P and $T - T_d$, (c) P and T , (d) U and T , (e) U and $T - T_d$, and (f) $T - T_d$ and T . The freezing fraction for the nominal conditions is shown by the black-filled symbol.

events either started before or continued after the single day limit in SB16. The number of hours before (after) the SB16 date is shown in parentheses before (after) the total number of hours with freezing rain in the Nhr column. The SB16 freezing rain events are also limited by the 0.51-mm hourly precipitation minimum and the 0.25-mm hourly ice thickness minimum. The ice thickness totals for this study include the reported values in the hours with freezing rain and in the

hour immediately before and the hour immediately after an hour with freezing rain as long as no other precipitation type was reported then. This assumes that the ice measured in those hours is freezing rain or freezing drizzle either before precipitation was detected by the LEDWI or as a result of precipitation in the previous hour freezing slowly. The number of hours in these 20 events for specific precipitation amount and ice thickness combinations is shown in Table 3.

TABLE 2. Freezing rain events from the SB16 table with that duration (Nhr), precipitation (Pep), and ice thickness, along with this study's duration, precipitation, and ice thickness and calculated simple model (Simple), freezing fraction (Fr Fra), and icing sensor (Ice Sen) R_{eq} values. The number of hours before (after) the SB16 date is shown in parentheses before (after) the total number of hours with freezing rain in the Nhr column.

Station	Lat (°)	Lon (°)	Service level	Date	SB16				This study				
					Nhr	Pep (cm)	Ice (cm)	Nhr	Pcp (cm)	Ice (cm)	Simple R_{eq} (cm)	Fr Fra R_{eq} (cm)	Ice Sen R_{eq} (cm)
Altoona, PA	40.296	-78.320	C	26 Nov 2013	6	2.21	0.74	14 (1)	3.15	1.47	1.30	0.14	0.58
Holland, MI	42.743	-86.105	D	21 Dec 2013	5	0.30	0.46	15 (10)	0.76	0.91	0.43	0.37	0.29
Matoon, IL	39.478	-88.279	D	17 Feb 2014	3	0.46	0.33	6	0.66	0.61	0.41	0.32	0.24
Joplin, MO	37.152	-94.498	C	21 Dec 2013	11	1.40	0.86	(3) 25	2.29	1.22	1.04	0.90	0.48
Augusta, GA	33.467	-82.039	D	12 Feb 2014	4	0.58	0.38	12	0.86	0.61	0.54	0.37	0.24
Augusta, ME	44.321	-69.797	D	1 Dec 2013	3	0.25	0.20	14 (7)	0.69	0.64	0.35	0.22	0.25
Lumberton, NC	34.610	-79.059	D	12 Feb 2014	6	1.30	0.61	13	2.08	1.07	1.38	1.32	0.42
Mount Holly, NJ	39.943	-74.846	D	14 Dec 2013	3	0.58	0.41	4	0.71	0.43	0.31	0.08	0.17
Meachem, OR	45.510	-118.420	—	24 Feb 2014	3	0.23	0.13	11 (6)	0.43	0.31	0.16	0.15	0.12
Mitchell, SD	43.775	-98.039	D	15 Dec 2014	4	0.56	0.43	6	0.84	0.51	0.65	0.13	0.20
Indianapolis, IN	39.733	-86.267	A	11 Jan 2015	3	0.41	0.41 ^a	17 (9)	1.55	0.23	0.67	0.53	0.10
Pittsfield, MA	42.428	-73.290	D	4 Jan 2015	3	0.23	0.36	6	0.30	0.43	0.13	0.11	0.17
Massena, NY	44.933	-74.850	D	4 Jan 2015	9	1.63	1.45	(1) 15	2.21	2.03	1.02	0.61	0.80
Clearfield, PA	41.050	-78.415	D	2 Feb 2015	4	0.58	0.41	5	0.81	0.41	0.34	0.30	0.16
York, PA	39.920	-76.873	D	12 Jan 2015	4	0.28	0.20	7	0.43	0.33	0.17	0.04	0.13
Hagerstown, MD	39.710	-77.726	C	9 Dec 2014	6	1.47	1.09	(2) 11	1.88	1.45	0.95	0.48	0.57
Sussex, NJ	41.200	-74.623	D	9 Dec 2014	3	0.84	0.38	4	0.91	0.43	0.42	0.04	0.17
Wink, TX	31.783	-103.200	D	2 Jan 2015	6	1.37	0.61	9	1.63	0.81	0.70	0.41	0.32
Fort Smith, AR	35.333	-94.367	—	28 Feb 2015	5	0.46	0.38	18 (5)	0.94	0.91	0.43	0.37	0.36
Lewiston, ID	46.383	-117.017	C	4 Dec 2014	3	0.25	0.20	9	0.56	0.31	0.23	0.15	0.12

^a This should be 0.23 cm according to the METAR data.

TABLE 3. Number of hours with various precipitation and ice thickness combinations for the 221 h in the freezing rain events in Table 2. English units are used to match the METAR reports. SB16 uses only 94 h, all of which belong to the combination in the first row.

Precip (in.)	Ice (in.)	No. of hours
≥ 0.02	≥ 0.01	137
≥ 0.02	0	15
0.01	0	9
0.01	≥ 0.01	24
0	0	10
0	≥ 0.01	26

The events in SB16 include only the 94 h in the first row that are on the specified day.

For each event I use the METAR data to calculate the increment in R_{eq} in each hour with freezing rain using both the simple model (1) and the freezing fraction formulation in section 3, estimating the shortwave heating flux for that location, date, and time. The 0.0254-m diameter cylinder intercepting the freezing rain is assumed horizontal and perpendicular to the wind at the height of the anemometer. The R_{eq} is also calculated from the ASOS ice thickness remarks using the calibration in Ryerson and Ramsay (2007). The last three columns show the simple, freezing fraction, and icing sensor R_{eq} values.

The three R_{eq} values for the freezing rain events in Table 2 are plotted in Fig. 7. For eight of the events the freezing fraction R_{eq} is within 0.1 cm of the simple model R_{eq} (close enough that the difference is not important). The largest

FIG. 7. Equivalent radial ice thicknesses from the ASOS icing sensor, simple model, and freezing fraction for the events in Table 2. The events with simple model $R_{eq} > 0.5$ cm, along with some of the smaller R_{eq} events, are labeled with the station name.

difference is 1.15 cm at Altoona, for which the air temperature is at or above 0°C and dewpoint depression is only 0.6°C for nine of the 13 h. For twelve of the events the freezing fraction and icing sensor R_{eq} values are within 0.1 cm. For three additional events the freezing fraction R_{eq} is less than the icing sensor R_{eq} , and for five events the opposite is true. In the following I pick one event from each of these three categories to gain insight into their differences. But first, Fig. 8 shows the anomalous Indianapolis event. The ASOS 1-min page 2 icing sensor frequency time series (NCDC 2006; <ftp://ftp.ncdc.noaa.gov/pub/data/asos-onemin/6406-2015/>) in the top panel shows a single icing sensor cycle. The next three panels show ASOS parameter time series and the last three panels are calculated parameters. There are 17 h of freezing rain at Indianapolis, but only three of those hours show ice accreting on the icing sensor probe, which means the observers at this service level A station generated the freezing rain reports in the following 14 h. The icing sensor was working, and it is likely that ice did not accrete on it because of the small evaporative cooling (zero dewpoint depression) during those hours (Indianapolis NWS observers 2021, personal communication). In contrast, the freezing fraction calculation shows enough convective and evaporative cooling to freeze

FIG. 8. Icing sensor, METAR data (Z = freezing rain, S = snow, R = rain, and U = unidentified precipitation), calculated Q_c , Q_e , Q_s , and Q_f freezing fraction, and equivalent radial ice thicknesses at Indianapolis, IN. The reports of freezing rain and drizzle for 0200–1500 UTC 12 Jan do not come from the icing sensor.

FIG. 9. Icing sensor, METAR data (Z = freezing rain, S = snow, R = rain, and U = unidentified precipitation), calculated Q_c , Q_e , Q_s , and Q_f , freezing fraction, and equivalent radial ice thicknesses at Fort Smith, AR. The ice thickness reports for 2 h at the end of the event with no reported precipitation are included in the icing sensor R_{eq} .

FIG. 10. Icing sensor, METAR data (Z = freezing rain, S = snow, R = rain, and U = unidentified precipitation), calculated Q_c , Q_e , Q_s , and Q_f , freezing fraction, and equivalent radial ice thicknesses at Lumberton, NC. The high precipitation rates may have resulted in water flowing or blowing off the icing sensor probe before it could freeze.

most of the precipitation impinging on the horizontal cylinder in most of the freezing rain hours.

Of the twelve events with nearly the same freezing fraction and icing sensor R_{eq} values, for three of them (Fort Smith, Arkansas; Pittsfield, Massachusetts; and Meachem, Oregon) those values are essentially the same as the simple model R_{eq} values. At Pittsfield the icing sensor thickness is greater than the precipitation and the icing sensor R_{eq} exceeds the simple model value. There are a couple of possible explanations: 1) the icing sensor at that location is one with a 20% greater response to ice accretion, or 2) a significant fraction of the freezing rain is made up of drizzle drops that are small enough that they do not contribute to the measured precipitation but are intercepted by the icing sensor probe. The Fort Smith event is shown in Fig. 9. The precipitation rates at Fort Smith are small, and in five of the freezing rain hours the ice thickness is greater than the precipitation. Temperatures are warming with decreasing dewpoint depression throughout the event, but in most hours there is enough cooling to freeze all the impinging precipitation, even in hours with shortwave heating.

An example of a small icing sensor R_{eq} compared to the freezing fraction value at a service level D station is provided by the Lumberton, North Carolina, event (Fig. 10).

Precipitation rates are high but icing sensor thicknesses are small indicating that little of that precipitation froze on the probe. In contrast, the cold and windy conditions result in $f = 1$ for most hours and a freezing fraction R_{eq} almost as high as the simple model R_{eq} . Perhaps a significant portion of the impinging precipitation ran down off or was blown off the small icing sensor probe before it could freeze because of the high precipitation rate. This possibility was pointed out in NOAA (2013). The small icing sensor thickness would also occur if most of the precipitation was ice pellets rather than freezing rain.

The Massena event (Fig. 11) is an example of a freezing fraction R_{eq} significantly less than the icing sensor R_{eq} . For most of the event, the low precipitation rates, moderate winds, and cold temperatures result in $f = 1$ and similar values for all three R_{eq} values with the icing sensor reporting about the same amount of ice as precipitation. During the last five hours, the increased precipitation rate, warmer temperatures, lower winds, and some shortwave heating result in freezing fractions significantly less than one and down to zero in the last two hours. In contrast, the icing sensor probe still has ice accreting even when the freezing fraction is zero. A possible explanation for the ice continuing to accrete on the probe in

FIG. 11. Icing sensor, METAR data (Z = freezing rain, S = snow, R = rain, and U = unidentified precipitation), calculated Q_c , Q_e , Q_s , and Q_f , freezing fraction, and equivalent radial ice thicknesses at Massena, NY. The freezing fraction is zero in the last 2 h while the icing sensor is reporting additional ice accretion.

these marginal conditions is cooling from the heat sink. The preceding comparisons are between the freezing fraction R_{eq} , the simple model R_{eq} , and R_{eq} calculated from the icing sensor thickness using the [Ryerson and Ramsay \(2007\)](#) calibration. Two columns in the [SB16](#) table indicate a comparison between the simple model and the icing sensor thickness on a horizontal surface. What is used, however, is the simple model R_{eq} multiplied by π , which represents the ice thickness on a flat surface that is always perpendicular to the rain drop trajectory. That flat surface is horizontal only when there is no wind, which occurs rarely in the 20 freezing rain events. Thus, the simple model comparison in the [SB16](#) table is invalid. [NOAA's \(2013\)](#) appendix C section 3.0 makes the same error.

5. Discussion

In some of the events in [Table 2](#), the icing sensor reports freezing rain and ice even in hours when a combination of $T \geq 0^\circ\text{C}$, small dewpoint depression, and shortwave heating result in no calculated freezing: $f = 0$. Even in those conditions, precipitation on the icing probe can freeze by conduction of the heat of fusion to the colder heat sink. Compared to the small probe, the finned heat sink cools more rapidly

because of its large surface area and stays cold because of its large mass. The tendency of the icing sensor to report icing when it is not observed elsewhere is noted by [NOAA \(2013\)](#), which attributes it to the micrometeorology of the local environment, and by [Reeves \(2016\)](#), which attributes it to warmer ground temperatures at the mPING observer locations compared to the ASOS location. A finite element model of the icing sensor could be used to investigate the effect of the heat sink on the probe in marginal freezing rain conditions.

A check of the effect of solar radiation on the freezing fraction shows that if shortwave radiation is ignored there is no change in the freezing fraction R_{eq} for half of the events in [Table 2](#) (not shown). For seven events R_{eq} increases by less than 0.1 cm, and for the remaining three events by less than 0.2 cm.

The occurrence of mixed precipitation (freezing rain and snow or freezing rain and ice pellets) can complicate the interpretation of R_{eq} . In their analysis of 15 years of data, [Cortinas et al. \(2004\)](#) find that freezing rain occurs alone 69% of the time, so for the majority of freezing rain hours the freezing fraction R_{eq} is not complicated by other types of precipitation. The occurrence with ice pellets 18% of the time and with snow 14% of the time are different. Service level C and D stations do not report ice pellets or more than one precipitation type, and when there is freezing rain and snow only snow is reported. While service level A and B stations are augmented by observers who can report more than one precipitation type and can report ice pellets, no information is provided on the fraction of the precipitation that is freezing rain. Snow falling with freezing rain is likely to stick to cylindrical elements, so will be incorporated into the ice accretion. However, that unknown fraction of the precipitation is already frozen, so the actual R_{eq} may be greater than the freezing fraction R_{eq} if $f < 1$. Ice pellets, although they accumulate on horizontal surfaces, are likely to bounce off cylinders, so when ice pellets and freezing rain are mixed, the actual R_{eq} will be smaller than the freezing fraction R_{eq} .

The occurrence of mixed phase (water and ice) drops can also complicate the interpretation of R_{eq} . As mixed phase drops are not reported at weather stations, how often they occur and the relative fractions of ice and water are not known. [Thériault et al. \(2006\)](#) refers to the mixed phase particles that can occur with freezing rain as slush. In contrast to wet snow, these slush particles are mainly water with a little ice. They are created for only a narrow range of melting layer temperatures, so would be expected to occur with freezing rain only occasionally. When slush drops are present, they are likely to accumulate on cylindrical components just as liquid drops do because of the low ice fraction. In that case, as with mixed snow and freezing rain, the actual R_{eq} may be larger than the freezing fraction R_{eq} when $f < 1$. The difference between the situations of mixed snow/freezing rain and freezing rain that includes mixed phase drops is that freezing rain is not reported at unaugmented ASOS stations when it occurs with snow. If employees at those airports join the mPING network, their reports of precipitation type would provide valuable information for calculating R_{eq} at those ASOS locations.

The mPING information is especially valuable for spatial and temporal mapping of the occurrence of freezing rain, ice pellets, snow, and rain between ASOS stations as shown in Reeves et al. (2014). Where mPING reports freezing rain, the simple and freezing fraction R_{eq} values could be estimated using precipitation, wind speed, temperature, and dewpoint data from the nearest ASOS station. These estimates could be improved by using data from stations in other networks [e.g., Remote Automatic Weather Stations (RAWS), state, highway, or personal] closer to the mPING report, as long as the station equipment works in freezing rain.

Precipitation, wind speed, temperature and dewpoint data from short term forecast model output can be combined with mPING reports to estimate R_{eq} for the model grid cells where freezing rain is currently reported, assuming that the precipitation remains freezing rain in hours with $T \leq 0^\circ\text{C}$ over the duration of the forecast.

Simple model and freezing fraction R_{eq} values can be tested by comparing them to measurements on cylinders at a weather station, similar to the calibration experiments described in Ryerson and Ramsay (2007). They mounted horizontal cylinders at about the height of the temperature and dewpoint sensors on racks they could rotate so the cylinder axis was perpendicular to the wind direction. Because the wind speed is likely to be higher at the standard 10-m anemometer height, the wind speed at the cylinder height should be measured using an anemometer that is kept ice-free. The weather parameters (precipitation amount, wind speed and direction, air temperature, dewpoint temperature, and shortwave radiation) may be recorded automatically but observers are needed to document the precipitation types and the mass of ice on the cylinders throughout the freezing rain event. The time series of R_{eq} calculated from the ice mass on the cylinder and the bare cylinder diameter and length can then be compared to the simple model and freezing fraction R_{eq} time series.

6. Conclusions

The equivalent radial ice thickness is the parameter that describes the ice load on cylinders from freezing rain. The simple model provides R_{eq} calculated from the impinging precipitation, given by P and U . With additional data, the fraction f that freezes can be calculated using the physically based equations for heat transfer in section 3. The primary freezing mechanisms are convective cooling and evaporative cooling with magnitudes determined by weather parameters U , T , and T_d . Shortwave radiation in daytime hours, which can be estimated where it is not measured, decreases the freezing fraction. When $f < 1$ the freezing fraction hourly increment in R_{eq} is f multiplied by the simple model R_{eq} increment. This physically based freezing fraction is in contrast to Sanders and Barjenbruch (2016) ILR formulation, which has no physical basis and is biased to low freezing fractions by the data they choose to fit.

Users can apply the freezing fraction R_{eq} formulation at locations with freezing rain and measurements or forecasts of the relevant weather data to quantify the spatially varying ice load and associated damage severity. Both the mPING

reports and the CRREL freezing rain team observations indicate that ice accretion can vary significantly over the terrain between weather stations because of variations in precipitation type, P , U , T , and T_d . An Excel tool that calculates the freezing fraction R_{eq} for a user specified cylinder diameter and hourly weather data along with Julian day and time is provided in the supplemental material. The location is specified by latitude, longitude, and elevation. The tool calculates the R_{eq} increment for the hour, with which the user updates the cylinder diameter for the ice accretion calculation using the weather data for the next hour. This Excel format is useful for investigating the effects of various weather conditions on the freezing fraction or for investigating individual freezing rain events. For larger quantities of data, the simple model and the freezing fraction equations in section 3 can be programmed for bulk analysis.

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Data availability statement. ASOS METAR data are available from <https://www.ncei.noaa.gov/data/global-hourly/>. ASOS 1-minute page 2 data can be downloaded from <ftp://ftp.ncdc.noaa.gov/pub/data/asos-onemin/>.

APPENDIX A

Properties of Air that Depend on Atmospheric Pressure Pa or Temperature T

Saturation vapor pressure of air over water at 0°C :

$$e_0 = 611.21(1.0007 + 3.46 \times 10^{-6} \text{ Pa}) \text{ N m}^{-2}. \quad (\text{A1})$$

Thermal conductivity of air:

$$k = 4.186 \times 10^{-5}(573 + 1.8T) \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}. \quad (\text{A2})$$

Molecular diffusivity of water vapor in air:

$$\kappa_m = 0.211 \left(\frac{T_0 + T}{T_0} \right)^{1.94} \left(\frac{1013}{\text{Pa}} \right) \text{ cm}^2 \text{ s}^{-1}. \quad (\text{A3})$$

Density of air:

$$\rho_a = \frac{0.348 \times 10^{-3} \text{ Pa}}{T_0 + T} \text{ g cm}^{-3}. \quad (\text{A4})$$

Dynamic viscosity of air:

$$\mu = 1.71 \times 10^{-4} + 5.2 \times 10^{-7} T \text{ g cm}^{-1} \text{ s}^{-1}. \quad (\text{A5})$$

Kinematic viscosity of air:

$$\nu = \mu / \rho_a \text{ cm}^2 \text{ s}^{-1}. \quad (\text{A6})$$

APPENDIX B

Solar Zenith Angle

The solar zenith angle θ depends on latitude φ , declination of the sun D , and the hour angle H :

$$\cos\theta = \sin\varphi \sin D + \cos\varphi \cos D \cos H, \quad (\text{B1})$$

with

$$\begin{aligned} \sin D &= \sin(23^\circ 26' 37.8'') \sin \delta, \\ \cos D &= \sqrt{1 - \sin^2 D}, \\ \delta &= 279.9348 + d + 1.914827 \sin d - 0.079525 \cos d \\ &\quad + 0.019938 \sin 2d - 0.001620 \cos 2d. \end{aligned} \quad (\text{B2})$$

Here d is the angle of the Julian day jd :

$$d = \frac{360(jd - 1)}{365.242}. \quad (\text{B3})$$

The hour angle depends on the UTC time t_{UTC} , longitude L (negative west of Greenwich), and true solar noon M :

$$\begin{aligned} H &= 15(t_{\text{UTC}} - M) + L, \\ M &= 12 + 0.123750 \sin d - 0.004289 \cos d \\ &\quad + 0.153809 \sin 2d + 0.060783 \cos 2d. \end{aligned} \quad (\text{B4})$$

In the absence of local solar radiation data, (B1)–(B4) can be used to calculate $\cos\theta$ for (12) to estimate solar flux Q_s at ground level for specified latitude, longitude, day of year, and time of day.

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