Chapter 16

Multiscale Modeling of the Moist-Convective Atmosphere

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ABSTRACT

One of the most important contributions of Michio Yanai to tropical meteorology is the introduction of the concepts of apparent heat source $Q_1$ and apparent moisture sink $Q_2$ in the large-scale heat and moisture budgets of the atmosphere. Through the inclusion of unresolved eddy effects, the vertical profiles of apparent sources (and sinks) are generally quite different from those of true sources taking place locally. In low-resolution models, such as the conventional general circulation models (GCMs), cumulus parameterization is supposed to determine the apparent sources for each grid cell from the explicitly predicted grid-scale processes. Because of the recent advancement of computer technology, however, increasingly higher horizontal resolutions are being used even for studying the global climate, and, therefore, the concept of apparent sources must be expanded rather drastically. Specifically, the simulated apparent sources should approach and eventually converge to the true sources as the horizontal resolution is refined. For this transition to take place, the conventional cumulus parameterization must be either generalized so that it is applicable to any horizontal resolutions or replaced with the mean effects of cloud-scale processes explicitly simulated by a cloud-resolving model (CRM). These two approaches are called ROUTE I and ROUTE II for unifying low- and high-resolution models, respectively. This chapter discusses the conceptual and technical problems in exploring these routes and reviews the authors’ recent work on these subjects.

1. Introduction: Apparent sources and sinks

Michio Yanai worked on virtually all aspects of tropical meteorology, covering a broad range of the atmospheric spectrum from the cloud scale to the planetary scale. His contributions to tropical meteorology are unique in the following ways:

1) Opened new research areas in tropical meteorology
   Examples include the analysis of the formation stage of tropical cyclones, the discovery and extensive analysis of the equatorial waves, the quantitative

analysis of heat source and moisture sink associated with cloud clusters, and the 3D heat and moisture budgets over the Tibetan plateau.

2) Developed new approaches in data analysis
   Examples include the power spectrum analysis combined with linearized equations and the analysis of the apparent heat source $Q_1$ and apparent moisture sink $Q_2$.

3) Deduced hidden reality from scarce data
   Examples include the transition of tropical disturbance from cold-core wave to warm-core vortex, the Yanai wave, and the heat, moisture, and momentum transports by cloud clusters.

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4) Made tropical meteorology more exciting to the general meteorological community
Examples include the Yanai wave and the role of moist convection in the large-scale dynamics.

5) Made unique contributions to international research programs
Examples include his contributions to GATE and the Monsoon Experiment (MONEX).

It is interesting to note that he never worked on forecast models. This does not mean that he was not interested in that subject. Although practically all of his papers are observationally oriented, they always went far beyond pure phenomenological descriptions and stimulated and inspired even those people whose primary interest was not in traditional tropical meteorology.

Except for the Yanai wave, his analyses of observed data almost always included moisture budgets, reflecting his recognition of the important role played by moist-convective processes in the tropical atmosphere. In Yanai (1961), which is based on his Ph.D. thesis, he presented a detailed analysis of typhoon formation and introduced the method of \( \frac{Q_1}{Q_2} \) analysis to investigate the transition from a cold-core wave to a warm-core vortex. Here, \( Q_1 \) and \( Q_2 \) represent the source and sink terms in the large-scale heat and moisture budget equations, respectively. In this paper, he placed the emphasis on the similarity between the horizontal distributions of \( Q_1 \) and \( Q_2 \), rather than the difference in their vertical distributions, regarding them as independent estimates of the heat of condensation released in cloud clusters. In addition to the fascinating description of the transition the paper points out, “It is an important fact that the order of magnitude of local changes in potential temperature was very small compared with each of the heat source and the expected change from the dry adiabatic relation . . . this means that the stratification of the air was nearly neutral with respect to the moist-adiabatic process” (Yanai 1961, p. 210). This last point was later emphasized by Betts (1982) and Xu and Emanuel (1989) as a characteristic of the tropical atmosphere and can be viewed as a prototype of the quasi-neutral hypothesis postulated by Arakawa (1969) and Arakawa and Schubert (1974).

During the late 1960s and early 1970s, Arakawa was struggling to formulate \( Q_1 \) and \( Q_2 \) for use in general circulation models (GCMs) in terms of the detrainment of cloud air into the environment and the cumulus-induced subsidence in the environment. This idea implicitly assumes that the gridpoint values of GCMs represent the environment of subgrid-scale clouds. The limit of this assumption will be discussed in detail in section 3. In any event, Arakawa’s hope at that time was that the use of such a formulation in the cumulus parameterization could produce realistic profiles of \( Q_1 \) and \( Q_2 \), which are needed as “feedback,” shown in Fig. 16-1a. For the loop in the figure to be closed, the model must formulate the effect of the dynamics core (and other modules, such as radiation and boundary layer processes) on the cumulus parameterization, which is called “forcing.”

As pointed out by Arakawa (2004), the procedure followed by Yanai et al. (1973) reverses the lower half of the loop, as shown in Fig. 16-1b. First, \( Q_1 \) and \( Q_2 \) are obtained as the residuals in the large-scale budget analysis of the 1956 Marshall Islands data and an estimated profile of \( Q_R \). From Yanai et al. (1973).
equations. The terminologies “apparent heat source” and “apparent moisture sink” for these quantities are then introduced, recognizing that they are different from the true heat source and moisture sink taking place locally. These differences are due to the effects of eddies smaller than the size of the observation network. Their results obtained from the 1956 Marshall Islands data are shown in Fig. 16-2. Here, $Q_R$ denotes the estimated radiation warming. Unlike in Yanai (1961), the difference between $Q_1 - Q_R$ and $Q_2$ is emphasized this time as a measure of convective activity. Note that $(Q_1 - Q_R) - Q_2$ is the apparent source of moist static energy, $c_pT + Lq + gz$, because of unresolved moist convection, where $c_pT$, $Lq$, and $gz$ are sensible heat, latent heat, and geopotential energy, respectively. Subsequent to Yanai et al. (1973), the apparent heat source and apparent moisture sink became well-accepted concepts in tropical meteorology (see Johnson et al. 2016, chapter 1).

What made the paper Yanai et al. (1973) even more unique is that the authors went farther along the curve in Fig. 16-1b to quantitatively assess the cloud environment processes, such as the detrainment and cumulus-induced subsidence, through the use of a bulk cloud ensemble model. Yanai was at first a little hesitant doing this part of the analysis since the use of a model was against his philosophy on data analysis. Yet the approach worked beautifully, and the paper was highly applauded by the community. In spite of the reversing of the loop, or rather because of the reversing, the paper gave a rationale for the use of mass-flux formulation in cumulus parameterization. In this way, a closer tie was established between observed large-scale budget and cumulus parameterization studies almost for the first time.

The purpose of this chapter is to discuss the modeling aspects of apparent sources with an emphasis on their formulation applicable to any horizontal resolution between those typically used in GCMs and cloud-resolving models (CRMs). (Here and hereinafter, sources and sinks are simply called sources.) Section 2 presents a rationale for such formulations and points out that there are two routes to achieve the objective: “ROUTE I”, following the conventional parameterization approach, and “ROUTE II”, following the coupled GCM/CRM approach. Sections 3 and 4 review the authors’ recent work on the ROUTE I and ROUTE II approaches, respectively, and section 5 presents concluding remarks.

2. Rationale for generalized formulations of apparent sources

In the conventional parameterizations of the apparent heat source $Q_1$ and apparent moisture sink $Q_2$, subgrid moist-convective processes are highly idealized inevitably. Not all of these idealizations are justifiable, and some can easily be criticized, especially from the observation side. But usually those criticisms point out only the existence of problems, not the solutions to the problems. Partly because of this, the rate of progress in representing cloud processes in climate models has been unacceptably slow, especially when it is compared to the rapid expansion of the scope of GCMs; so we may say, as Randall et al. (2003, p. 1548) do, that “the cloud parameterization problem is deadlocked.”

As pointed out by Arakawa et al. (2011) and Arakawa and Wu (2013), one of the most important sides of the deadlock is that we currently have only two ways of representing deep moist-convective processes in numerical models of the atmosphere: one with highly parameterized apparent sources and the other with explicit simulation of true sources. The former is for low-horizontal-resolution models, such as the conventional GCMs, and the latter is for high-horizontal-resolution models, such as the CRMs. Correspondingly, besides those models that explicitly simulate turbulence, there
have been two discrete families of atmospheric models, as shown in Fig. 16-3a. In this figure, the abscissa is the horizontal resolution, and the ordinate is a measure of the degree of parameterization, such as reduction in the degrees of freedom, increasing downward. These two families have been developed for applications with quite different ranges of horizontal resolution in mind. What we see here is a polarization of atmospheric models separated by a “gray zone.” The difficulties in representing moist-convective processes in this zone have long been recognized among the mesoscale modeling community (e.g., Molinari and Dudek 1992; Molinari 1993; Frank 1993). Recently, a new family of global models called the multiscale modeling framework (MMF) has been added, as shown in Fig. 16-3b. The MMF is a coupled GCM/CRM system that follows the superparameterization approach introduced and used by Grabowski and Smolarkiewicz (1999), Grabowski (2001), and Khairoutdinov and Randall (2001). In this approach, the cumulus parameterization in conventional GCMs is replaced with the mean effects of cloud-scale processes simulated by a 2D CRM embedded in each GCM grid cell. For more details of the MMF approach, including some results, see Randall et al. (2016, chapter 15).

Furthermore, because of the recent advancement of computer technology, straightforward applications of 3D CRMs to simulate true sources has begun to be feasible even for studying the global climate (e.g., Sato et al. 2009; Oouchi and Satoh 2016, chapter 14). In this way, global CRMs (GCRMs) joined the families of global models, as shown in Fig. 16-3b. This is an exciting development in the history of numerical modeling of the global atmosphere, in which the modular structure shown in Fig. 16-1a is completely abandoned as far as the scales comparable to or larger than deep moist convection are concerned.

It should be noted, however, that the gray zone still exists in the configuration shown in Fig. 16-3b. Thus, what we see now is a tripolarization of global models with differently formulated model physics for each of the three families. Consequently, both the conventional GCMs and the MMF do not converge to a GCRM as the horizontal grid size is reduced. This is not a scientifically healthy condition because the use of discretized equations is justified only when the error due to the discretization can be made arbitrarily small as the resolution is refined. This does not matter when the scales represented by the macroscopic and microscopic models are separated by many orders of magnitude, as in the case of continuum and molecular mechanics, because the refinement of the macroscopic resolution all the way down to the microscopic resolution will never be attempted. The situation in atmospheric modeling is quite different, because the spectrum is virtually continuous. The physics of atmospheric models should then produce a smooth transition of the apparent sources from those required for the conventional GCMs to those required for the mesoscale models and, eventually, to the true sources that can be simulated by the CRMs. Thus, the traditional view of apparent sources must be expanded rather drastically to include their resolution dependence, even including the true sources in the limit.

Jung and Arakawa (2004) showed convincing evidence for the resolution dependence of apparent sources through budget analyses of data simulated by a CRM with different space/time resolutions. By comparing the results of low-resolution test runs without cloud microphysics over a selected time interval with those of a high-resolution run with cloud microphysics (control), they identified the apparent microphysical source required for the low-resolution solution to be equal to the space/time averages of the high-resolution solution. This procedure is repeated over many realizations selected from the control. Figure 16-4a shows examples of the domain- and ensemble-averaged profiles of the required source of moist static energy obtained in this way. The profiles shown in red and green are obtained using the horizontal grid size and the time interval of the test run given as (2 km, 2 min), approximately representing the true sources, and (32 km, 60 min), representing apparent sources for this resolution, respectively. The red profile shows a positive source due to freezing and a negative source due to melting immediately above and below the freezing level, respectively. There are practically no other sources and sinks, because moist static energy is conserved under moist-adiabatic processes. The green profile, on the other hand, shows marked negative values in the lower troposphere and positive values in the middle-to-upper troposphere, suggesting the dominant role played by the upward transport. Figure 16-4b is the same as Fig. 16-4a, but for the required source of total (airborne) water mixing ratio. The red profile shows dominant sinks in the middle troposphere due to the generation of precipitating particles and small peaks of source near the surface due to the evaporation from rain. The green profile again suggests the dominant role played by the upward transport.

Low-resolution models, such as the conventional GCMs, are supposed to produce profiles similar to the green profiles shown in Fig. 16-4, which we call the GCM type, while high-resolution models, such as the CRMs, are supposed to produce profiles similar to the red profiles, which we call the CRM type. As Arakawa (2004) emphasized, it is important to recognize that any space/time/ensemble average of the CRM-type profiles does not give a GCM-type profile. This means that the cumulus parameterization problem is more than a statistical theory of cloud microphysics. Also, it is not a
purely physical/dynamical problem, because it is needed as a consequence of mathematical truncation. Finally, it is not a purely mathematical problem because the use of a higher resolution or an improved numerical method while using the same formulation of model physics does not automatically improve the result. A complete theory for cumulus parameterization must address all of these aspects in a consistent manner.

As we have seen in Fig. 16-4, the required apparent sources highly depend on the horizontal resolution of the model, as well as the time interval for implementing model physics. The formulation of model physics should automatically produce this dependence as it is applied to different resolutions. Conventional cumulus parameterization schemes, however, cannot do this, because they assume either explicitly or implicitly that the horizontal grid size and the time interval for implementing physics are sufficiently larger and longer than the size and lifetime of individual moist-convective systems. If the model physics of GCMs is reformulated to produce such resolution dependence, GCMs and CRMs are unified to a single family of models that can be applicable to a wide range of horizontal resolutions, including the mesoscales with the same formulation of model physics.

Arakawa et al. (2011) and Arakawa and Jung (2011) discussed two routes to achieve the unification shown as ROUTE I and ROUTE II in Fig. 16-5. The departure points of these routes are the conventional GCMs and a new generation of MMF, respectively, but they share the same destination point, a GCRM. ROUTE I breaks through the gray zone by generalizing the conventional cumulus parameterization in such a way that it can be applied to any horizontal resolution. Consequently, the concept of apparent sources is generalized to include their transition to the true sources. ROUTE II, on the other hand, bypasses the gray zone by replacing the conventional cumulus parameterization with explicit simulation of cloud-scale processes by a CRM. On this route, the transition of apparent sources to the true sources is numerically simulated. These two routes are discussed in the following two sections.

3. Route I: Unified parameterization

a. Identification of the problem

The first step to open ROUTE I is reexamination of the widely used assumption that convective updrafts...
cover only a small fraction of the area represented by a GCM grid cell. Most conventional cumulus parameterizations assume this, at least implicitly, regarding the predicted thermodynamic fields as if they represent the environment of updrafts. Let \( \sigma \) be the “fractional convective cloudiness” or “fractional updraft area,” which is the fractional area covered by convective updrafts in the grid cell. When the grid spacing is fixed, this is a measure of the population of updrafts. In terms of \( \sigma \), the above assumption means \( \sigma \ll 1 \). In the limit as the grid spacing approaches zero, however, the grid cell is occupied either by an updraft or by its environment. Then \( \sigma \) becomes either 1 or 0, and thus the circulation associated with the updraft becomes the grid-scale circulation. More generally, the total effect of cumulus convection is the sum of its grid-scale and subgrid-scale effects, and it is important to remember that cumulus parameterization is supposed to formulate only the effects, and it is important to remember that cumulus convection is the sum of its grid-scale and subgrid-scale contributions. More generally, the total effect of cumulus convection is the sum of its grid-scale and subgrid-scale effects, and it is important to remember that cumulus convection is the sum of its grid-scale and subgrid-scale contributions.

b. CRM simulations used for statistical analysis

To visualize the problem we are addressing, we have performed two numerical simulations using a CRM applied to an idealized horizontally periodic domain, one with and the other without background shear. The horizontal domain size and horizontal grid size are 512 km and 2 km, respectively. As expected, the two simulations represent quite different cloud regimes; but as far as the vertical transports of thermodynamic variables are concerned, we find only little qualitative difference between the two simulations. We therefore present only the shear case in this article.

To see the resolution dependence of the diagnosed statistics, we divide the original CRM domain into subdomains of identical size to represent a uniform GCM grid. We denote the side length of the subdomain by \( d \), which ranges from the CRM grid size to the size of the entire domain.

c. Resolution dependence of ensemble average \( \sigma \)

In the following diagnosis of the simulated data, \( \sigma \) is determined for each subdomain by the fractional number of CRM grid points that satisfy \( w \geq 0.5 \text{ m s}^{-1} \). Let angle brackets \( \langle \cdot \rangle \) denote the ensemble average over all updraft-containing subdomains (i.e., subdomains with \( \sigma > 0 \)) during the analysis period (12 h). Figure 16-6 shows the resolution dependence of \( \langle \sigma \rangle \) and associated standard deviation at \( z = 3 \text{ km} \). In the figure, we see that \( \sigma \) drastically increases as the subdomain size \( d \) decreases. Clearly, the assumption \( \sigma \ll 1 \) can be justified only for low resolutions. For high resolutions, \( \langle \sigma \rangle \) significantly deviates from 0 and becomes 1 for \( d = 2 \text{ km} \), which is the CRM grid spacing. Since there is a number of subdomains with \( \sigma = 0 \) that are not included in the ensemble average, the distribution of \( \sigma \) for high resolutions tends to be bimodal. Wu and Arakawa (2014) show that these features also appear practically at all levels in the vertical.

d. The ratio of the eddy to total vertical transports of moist static energy

We now look into the vertical transport of moist static energy diagnosed from this dataset. In view of the discussion presented in section 3a, our main interest here is the relative importance of the eddy transport \( \langle w' h' \rangle \) against the total transport \( \langle wh \rangle \). Here, \( h \) is the deviation of moist static energy from the average over the entire horizontal domain, the overbar denotes the average over all CRM grid points in the subdomain, \( \langle \cdot \rangle \) is the ensemble average over all subdomains with \( \sigma > 0 \) as defined earlier, and \( w' = w - \overline{w} \) and \( h' = h - \overline{h} \) represent the eddy components of \( w \) and \( h \). The large standard deviation shown in Fig. 16-6 indicates that there is a wide range of \( \sigma \) for each subdomain size. Figure 16-7 presents the ratio \( \langle w' h' \rangle / \langle wh \rangle \) at \( z = 1, 3, \) and 6 km with the subdomain size \( d \) and the fractional updraft area \( \sigma \). An empty box means that data are not sufficient for that combination of \( d \) and \( \sigma \). For small values of \( \sigma \), the total transport at \( z = 3 \) and 6 km is almost entirely due to the...
eddy transport regardless of the resolution. For larger values of $\sigma$, however, the total transport is primarily due to explicitly simulated grid-scale vertical velocity. These features can also be seen at $z = 1$ km, although the resolution dependence is not negligible for small values of $\sigma$. Except for this height, Fig. 16-7 clearly shows that the ratio depends primarily on $\sigma$, not on $d$, so that what matters most in generalizing the conventional cumulus parameterization is to include the dependence on the fractional updraft area, not directly on the grid spacing. This is one of the most important results of the analysis presented in this article.

e. The $\sigma$ dependence of the total, eddy, and modified eddy transports of moist static energy for $d = 8$ km

The last subsection points out that the magnitude of $\left< w^2 \right>$ relative to that of $\left< wh \right>$ crucially depends on the fractional convective area $\sigma$ even when the subdomain size $d$ is fixed. Taking the case of $d = 8$ km as an example, Figs. 16-8a and 16-8b show $\rho^*\left< wh \right>$ and $\rho^*\left< w^2 \right>$, respectively, as functions of height for each $\sigma$ bin. Here, $\rho^*$ is the density normalized by its value at $z = 3$ km. From these figures, we see that the total transport for small values of $\sigma$, say, for $\sigma \leq 0.3$, is almost entirely due to the eddy transport at practically all height even for this relatively high resolution. For larger values of $\sigma$, however, a large part of the total transport is due to the grid-scale transport, which is mesoscale for this resolution. Correspondingly, the concept of apparent source must be generalized to include its $\sigma$ dependence, converging to the true source in the high-resolution limit with $\sigma = 1$.

f. Parameterization of the $\sigma$ dependence of vertical eddy transport by homogeneous updrafts/environement

Our main problem then becomes parameterization of the $\sigma$ dependence of vertical eddy transports for use in a prognostic model. Most conventional parameterizations assume that the updrafts and the environment within each grid cell are individually horizontally homogeneous so that they can each be represented by a top-hat profile. Although this is a rather drastic idealization, we continue to use it in the first attempt of developing the unified parameterization. Then we can express properties of the updrafts and the environment by a single $z$-dependent variable for each. With this assumption, Arakawa and Wu (2013) show that the vertical eddy transport of an arbitrary variable $\psi$ is given by

$$ w^2 \psi = \sigma(1 - \sigma) \Delta w \Delta \psi, $$

(16-1)
where $D$ denotes the excess of the updraft value over the environment. Since $D_w$ and $D_c$ depend only on the updraft properties relative to the environment, it is likely that they and their product $D_w D_c$ do not significantly depend on the population of updrafts and, therefore, not on $s$. If this is the case, (16-1) shows that the $s$ dependence of the eddy transport is through the factor $s (1 - s)$. For small values of $s$, the eddy transport increases with $s$ approximately linearly but reaches its maximum at $s = 0.5$ and tends to vanish as $s$ approaches 1 because of the saturation of updrafts in the grid cell.

To see if the above reasoning with a top-hat profile is valid for the simulated dataset being used, we modify the data by replacing the prognostic variables at all updraft points with their averages over the subdomain and do the same for the environment points. Figure 16-8c shows the vertical distribution of the eddy transports of moist static energy diagnosed from the modified dataset. As anticipated, the shape of the $s$ dependence of the modified eddy transports is very close to the curve $s (1 - s)$ at all levels.

The difference between Figs. 16-8b and 16-8c represents the contribution from the inhomogeneous structure of updrafts and the environment and/or from the existence of multiple cloud types. The difference is small for small values of $s$, say, $s < 0.3$. For larger values of $s$, the difference is not small compared to the eddy transport based on the original dataset. However, since the eddy transport itself is small compared to the total transport for these values of $s$, this difference may not be as important as it appears.

g. Determination of $s$ from given grid-scale processes

From the evidence presented above, it is clear that the fractional updraft area $s$ plays a key role in the unified parameterization. Prognostic models must be able to determine $s$ for each realization of grid-scale conditions as a closure.

Most cumulus parameterizations currently being used are adjustment schemes in which a measure of convective instability is at least partially adjusted to its equilibrium value [see Arakawa (2004) for a review]. Let the vertical eddy transport of moist static energy required for the full adjustment be $(\bar{w}^* h)_F$. This can be considered as an external parameter since it represents convective forcing determined by the grid-scale destabilization.
The conventional parameterization schemes with full adjustment assume \( \overline{\psi' \psi'} = (\overline{\psi' \psi'})_E \) in addition to \( \sigma \ll 1 \). Then (16-1) with \( \psi = h \) implies

\[ \sigma \approx \frac{(\overline{\psi' \psi'})_E}{\Delta w \Delta h}. \]  

(16-2)

For (16-2) to be consistent with \( \sigma \ll 1 \), \( (\overline{\psi' \psi'})_E \ll \Delta w \Delta h \) is necessary. Thus, such a scheme can be valid if either the destabilization rate \( (\overline{\psi' \psi'})_E \) is small or the stratification is strongly unstable so that \( \Delta w \Delta h \) is large.

The unified parameterization chooses

\[ \sigma = \frac{\Delta w \Delta h}{(\overline{\psi' \psi'})_E} \]  

(16-3)

instead of (16-2). This is the simplest choice to automatically satisfy the condition \( 0 \leq \sigma \leq 1 \), as long as \( (\overline{\psi' \psi'})_E \) and \( \Delta w \Delta h \) have the same sign, while reducing to (16-2) when \( (\overline{\psi' \psi'})_E \ll \Delta w \Delta h \). When \( (\overline{\psi' \psi'})_E \geq \Delta w \Delta h \), (16-3) gives \( \sigma \approx 1 \) so that the grid cell is saturated with updrafts.

Eliminating \( \Delta w \Delta h \) between (16-3) and (16-1), we obtain

\[ \overline{\psi' \psi'} = (1 - \sigma^2)(\overline{\psi' \psi'})_E. \]  

(16-4)

More generally, the unified parameterization uses

\[ \overline{w'h'} = (1 - \sigma^2)(\overline{w'h'})_E, \]  

(16-5)

where \( (\overline{w'h'})_E \) is \( \overline{w'h'} \) associated with the full adjustment. Since \( \sigma > 0 \), (16-5) shows \( \overline{w'h'} < (\overline{w'h'})_E \). Thus, the practical application of the unified parameterization is a reduction of the eddy transports depending on the value of \( \sigma \). Note that, unlike the commonly used relaxed adjustment schemes, the reduction is only for the transport effects, not for the sources due to the diabatic effects. In other words, the reduction is for the difference between the apparent and true sources. The diabatic effects have their own dependence on \( \sigma \) as discussed in section 3.

In practical applications, however, it should be remembered that \( \Delta w \Delta \psi \) in (16-3) is unknown, because what the model gives are the gridpoint values, not the environment values. Let \( \delta \) denote the excess of the updraft value over the gridpoint values instead of the environment values. Since the relation between \( \delta \Delta \psi \) and \( \Delta w \Delta \psi \) involves \( \sigma \), the problem becomes implicit; but as Arakawa and Wu (2013) show, it ends up solving the cubic equation given by

\[ \sigma/(1 - \sigma)^3 = \lambda, \]  

(16-6)

where \( \lambda \) is a nondimensional parameter defined by

\[ \lambda \equiv \frac{\overline{w'h'}}{\delta w \delta h}. \]  

(16-7)

Recall that \( (\overline{w'h'})_E \) in (16-7) is the destabilization rate due to the grid-scale processes, while \( \delta w \delta h \) is a normalized eddy transport measuring the efficiency of the eddy transport. We see that \( \lambda \) is large either for a large destabilization rate, as is the case for mesoscale convective complex, or for small eddy transport efficiency, as is the case for stratocumulus clouds. Figure 16-9 shows the values of \( \sigma \) given by (16-6) as a function of the parameter \( \lambda \). As (16-6) shows, \( \sigma = 0 \) when \( \lambda = 0 \), and \( \sigma \to 1 \) as \( \lambda \to \infty \). The unified parameterization uses the value of \( \sigma \) determined in this way for all variables, assuming that variables other than \( h \) play only passive roles as far as the process of controlling \( \sigma \) is concerned.

h. A remark on stochastic parameterization

The unified parameterization can also provide a framework for including stochastic parameterization. Arakawa and Wu (2013) emphasized that stochastic formulation must be made under appropriate physical/dynamical/computational constraints that identify the source of uncertainty. In particular, the formulation must distinguish the uncertainty in formulating subgrid-scale processes from the irregular fluctuations of the grid-scale processes. Clearly, stochastic parameterization must deal with the former under a given grid-scale condition. Arakawa and Wu (2013) point out that different phases of cloud development are likely to be responsible for the main uncertainty.

i. Vertical transports of horizontal momentum

Up to this point we have discussed the transport of moist static energy. As shown by Arakawa and Wu...
(2013), it is expected that the partitioning between the eddy- and grid-scale transports is similar for other thermodynamic variables. The situation can be essentially different, however, for the vertical transport of horizontal momentum for the following reasons:

(i) Horizontal momentum is not a conservative variable because of the pressure-gradient force.

(ii) Horizontal momentum is a vector with two independent components, and so it is its vertical transport.

Most currently used parameterizations of the vertical transport of horizontal momentum recognize the importance of (i) but not necessarily that of (ii). These parameterizations are, therefore, effectively one-dimensional, in which the pressure gradient typically acts against the advection effect, maintaining the in-cloud horizontal velocity closer to the large-scale value (Gregory et al. 1997). Thus, the primary effect of the pressure gradient in those parameterizations is quantitative, rather than qualitative.

The point made in (ii) is well recognized in the parameterization presented by Wu and Yanai (1994). They considered a solution of the linearized diagnostic equation for pressure that can vary both in x and y with wavenumbers \( k \) and \( l \), respectively. Here \( x \) and \( y \) are the components of arbitrarily chosen horizontal Cartesian coordinates. Using this solution, they showed that the net effect of the pressure gradient and advection depends on the ratio \( k^2/l^2 \). When \( k^2 \sim l^2 \), for example, the net effect is such that both the \( x \) and \( y \) components of the transport are downgradient. When \( k^2 \ll l^2 \), on the other hand, the \( x \) component of the transport representing the line-parallel component is downdraft while the \( y \) component representing the line-normal component can be updraft. These results are generally consistent with the observational findings by LeMone (1983) and LeMone et al. (1984).

Wu and Arakawa (2014) used the shear case of the CRM-simulated dataset to analyze the vertical transports of horizontal momentum. The analysis shows that the momentum transport parameterized by analogy with the parameterizations of thermodynamic variables could work for the line-parallel component, but not for the line-normal component. For the latter, the transports due to the mesoscale organization of clouds are dominant, as has been consistently emphasized by Moncrieff (e.g., Moncrieff 1981, 1992). These results confirm the fact that the orientation of linearly organized convective systems must be known before any parameterization of the vertical transport of horizontal momentum is attempted. Unfortunately, this remains an extremely challenging task.

### j. Implementation of cloud microphysical effects

To complete the unified representation of deep moist convection, this subsection discusses the implementation of cloud microphysical processes following Wu and Arakawa (2014). As in Gerard (2007), the unified parameterization uses a single formulation of cloud microphysics that can be applied to both convective updrafts and stratiform clouds. As is done for the transports of thermodynamic variables, we assume that the updraft air is horizontally homogeneous with a top-hat profile.

For the conversions taking place within updrafts, the cloud microphysical package is expected to determine the conversion rates for a given updraft vertical mass flux. The unified parameterization calculates the mass flux through \( \rho aw_c \), where \( w_c \) is the updraft vertical velocity predicted by the cloud model, and \( \sigma \) is, as defined earlier, the fractional convective area determined by the procedure presented in section 3g. Since \( w_c \) should not significantly depend on \( \sigma \), the conversion rates should linearly increase as \( \sigma \) increases. When \( \sigma \) approaches 1, the mass flux approaches \( \rho \bar{w} \), which is the grid-scale vertical mass flux.

The package is also expected to prognostically determine the conversions taking place outside of the updrafts, assuming \( \sigma \sim 0 \) temporarily. As in the existing parameterizations, the detrainment of hydrometeors from the updrafts can play a key role in this prediction. Then, since the fractional area outside of updrafts is given by \( 1 - \sigma \), the adjustment is made through multiplying the provisional conversion rates by \( 1 - \sigma \).

The above dependencies of the conversion rates are well supported by the analysis of the simulated data presented by Wu and Arakawa (2014), except that the melting of snow/graupel does not vanish even when \( \sigma = 1 \).

### 4. Route II: The quasi-3D multiscale modeling framework

#### a. Identification of the problem

As shown in Fig. 16-5, ROUTE II for unifying low- and high-resolution models follows the MMF approach using a coupled GCM/CRM system. In applied mathematics, a number of methods have been developed to couple macroscopic and microscopic models with objectives similar to that of the MMF. For example, E et al. (2007, p. 7), who proposed the heterogeneous multiscale method (HMM), state that their objective is “to design combined macroscopic–microscopic computational methods that are much more efficient than solving the full microscopic model and at the same time give the information we need.” In the sense that the
Parameterized equations are completely replaced with explicit simulations by the microscopic model, the MMF is similar to the equation-free approach (Kevrekidis et al. 2003; Li et al. 2007). The grid system used in these approaches may look like Fig. 16-10a. Typically, the embedded microscopic models are restarted and run until they reach equilibrium at each time step of the macroscopic model. In addition, the spatial domain of the microscopic model is very small for computing efficiency. Thus, the existence of large spectral gaps both in time and space is crucial in these methods.

In the MMF, on the other hand, the GCM and CRMs are run simultaneously. In this particular sense, the MMF is similar to the seamless multiscale methods (E et al. 2009). What makes the MMF unique is that the computing efficiency is gained through sacrificing three-dimensional representation of cloud-scale processes. This is achieved using 2D CRM grids such as those illustrated by Fig. 16-10b. The motivation for using 2D grids comes from the fact that 2D CRMs are reasonably successful in simulating the thermodynamic effects of deep moist convection (e.g., Tao et al. 1987; Grabowski et al. 1998; Xu et al. 2002).

The ROUTE II approach is an attempt to broaden the applicability of the prototype MMF without necessarily using a fully three-dimensional GCRM. As is the case for the prototype MMF, ROUTE II replaces the conventional cumulus parameterization with explicit simulation of cloud-scale processes. Correspondingly, Fig. 16-1a is replaced with Fig. 16-11. Forcing and feedback are now the GCM’s effect on the CRM and the CRM’s effect on the GCM, respectively. Through forcing, the CRM recognizes

1) the vertical structure predicted by the GCM, including the GCM-scale convective instability, and
2) the horizontal inhomogeneity and anisotropy predicted by the GCM so that the CRM can respond to the GCM-scale three-dimensional processes.

Through feedback, the GCM recognizes

3) the mean “eddy” effects of cloud and associated processes simulated by CRM.

The precise definition of “eddy” is given later. Items 1 and 3 are parallel to the parameterization approach including the unified parameterization. What makes ROUTE II unique is item 2, which is difficult to achieve by the parameterization approach. Because of this feature, we call the ROUTE II approach the quasi-3D (Q3D) MMF.

Since the publication of Jung and Arakawa (2010), Arakawa et al. (2011), and Arakawa and Jung (2011), we have made considerable revisions to the Q3D algorithm, which is described in detail in Jung and Arakawa (2014). Here we present an outline of the revised algorithm.
b. **Q3D grid system**

Since CRM is used to simulate cloud-scale processes, the Q3D MMF is naturally more expensive than GCMs with conventional or unified parameterizations. Yet, when a standard resolution is used for the GCM component, the Q3D MMF is expected to be much less expensive than fully three-dimensional GCRMs. The Q3D MMF achieves this objective through the use of a grid system that has gaps in the horizontal space. Figure 16-12 illustrates an example of the Q3D grid system. It consists of a GCM grid and two perpendicular sets of CRM gridpoint channels. Unlike the grid shown in Fig. 16-10b for the prototype MMF, each channel is extended beyond a GCM grid cell. In this way, the error due to the artificial confinement of cloud clusters within a single GCM grid cell is eliminated, as shown by Jung and Arakawa (2005). The essential difference between the two grids becomes apparent in the limit as the grid interval of GCM approaches that of the CRM. In this limit, each of the CRM grids shown in Fig. 16-10b shrinks to a single point. The gridpoint channels shown in Fig. 16-12, on the other hand, formally remain the same as the GCM resolution becomes higher, except that the interval between the parallel channels becomes smaller. Thus, interaction of clouds with their environment is still simulated in this limit. The parallel channels are, however, not directly communicating with each other so that the convergence of the Q3D MMF to a 3D GCRM is not automatic, as discussed in section 4e.

The number of gridpoint arrays within each channel is optional. In practical applications, however, only a limited number of arrays are used for computing efficiency. In the example shown in Fig. 16-12, there are three gridpoint arrays in each channel so that there are three grid points in the lateral direction. The use of such a few grid points across the channel inevitably constrains simulation of cloud-scale three-dimensional processes. Interactions with larger-scale three-dimensionality, however, are maintained, as discussed in the following subsections.

c. **The use of background fields**

In the formulations of forcing and feedback shown in Fig. 16-11, the CRM component of the Q3D MMF uses background fields obtained through interpolations from the GCM grid points. Using these fields, an arbitrary CRM variable \( q \) is decomposed as

\[
q = \overline{q} + q',
\]

where \( \overline{q} \) is the background value of \( q \), and \( q' \) is the deviation of \( q \) from its background value. The background fields are used in the following three ways:

1) To specify lateral boundary conditions for the channel domains through which the CRM recognizes three-dimensional fields predicted by the GCM;
2) To provide reference fields to which CRM variables are relaxed to maintain compatibility of the two models;
3) To represent the subgrid components of CRM variables by the deviations from the background.

More details of these procedures are given in the next three subsections.

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**FIG. 16-11.** As in Fig. 16-1a, but for ROUTE II.

**FIG. 16-12.** Illustration of grid points used by the Q3D MMF.
d. Forcing: Lateral boundary condition

If the number of gridpoint arrays across the channel is not so few, our problem is basically that of limited-area modeling applied to the channel domain. Ideally, the lateral boundary condition should be nonreflecting for outflow and outward-propagating gravity waves, but designing a boundary condition that fully satisfies these requirements is a notoriously difficult problem. The problem we are facing is even more demanding because the two lateral boundaries are so close to each other. Under such a condition, Jung and Arakawa (2010) found that the cyclic boundary condition is the only one that is stable without excessive damping among many possibilities they examined. Based on this result, the CRM assumes that \( q' \) is cyclic across the channel, while its prediction recognizes the lateral as well as longitudinal gradients of \( \bar{q} \). The use of the cyclic condition for \( q' \) means that the background field effectively determines the boundary condition. Thus, the parallel channels communicate only through the GCM component. We also let the perpendicular channels communicate with each other only through the GCM component so that they intersect only virtually with no singularity at their formal intersection points.

e. Forcing: Relaxation

Recall that \( \bar{q} \) represents an interpolated GCM field, not a space-averaged CRM field. In the Q3D algorithm, however, these two are made sufficiently close through relaxation of \( q \) running averaged over a channel segment to \( \bar{q} \) at the center of the segment. Here, the length of the channel segment is chosen to be the GCM grid spacing.

Jung and Arakawa (2014) discuss the problem of choosing the time scale for the relaxation. From a series of sensitivity experiments, they derive the following conclusions:

(i) If the relaxation time scale is very short, the CRM solution is too strongly constrained by the GCM solution, and, therefore, CRM loses its self-stabilization effect;

(ii) If the relaxation time scale is not very short, the self-stabilization effect can be maintained. But if it is shorter than a critical time scale \( \tau_{\text{crit}} \), the development of intermediate-scale cloud organization in CRM is still too strongly constrained by the GCM solution;

(iii) If the relaxation time scale is longer than \( \tau_{\text{crit}} \), the intermediate-scale cloud organization can take place in the CRM. But if the time scale is too long, compatibility between GCM and CRM solutions is not maintained.

Here, the intermediate scale refers to the scales between the GCM-resolvable scale and the cloud scale. If the GCM-resolvable scale is the synoptic scale, the intermediate scale is the mesoscale. Since the time scale associated with horizontal advection plays a crucial role in the development of mesoscale organization, the advective time scale given by \( d/V \) can be a good choice for \( \tau_{\text{crit}} \), where \( d \) and \( V \) denote the GCM horizontal grid spacing and the characteristic magnitude of horizontal velocity in the CRM solution, respectively. If \( d = 96 \) km, as is currently used, and \( V \sim 15 \text{ m s}^{-1} \), as is representative in these simulations, \( \tau_{\text{advec}} \) is about 1.8 h, while the sensitivity experiments described by Jung and Arakawa (2014) suggest \( \tau_{\text{crit}} \sim 2 \text{ hr} \).

If the relaxation time scale is chosen near the advective time scale \( d/V \), it becomes shorter as the GCM horizontal grid spacing decreases as long as the characteristic magnitude of \( V \) does not significantly change. Then the GCM and CRM components of the Q3D MMF tend to produce nearly identical solutions. This is crucially important for the convergence of the Q3D MMF to a GCRM, which is one of the most important objectives of the Q3D MMF. Although this assumption of nearly constant \( V \) may hold for most cases, it is likely that we need to adaptively determine the characteristic magnitude from the predicted fields in future applications.

f. Feedback

As stated in section 4a, feedback consists of the implementation of the mean effects of cloud and associated processes simulated by the CRM to the GCM component. For advective and dynamic processes, the CRM is supposed to implement only the eddy effects so that the CRM component does not overdo its job as a replacement for subgrid-scale parameterizations. This is in parallel to what the unified parameterization does for feedback. The Q3D MMF, however, uses its own definition of eddy: that is, the deviation from the background value rather than from a space-averaged value. Through the relaxation described in section 4e, however, this difference is made small at least for high resolutions. These eddy effects and the full diabatic effects calculated at each grid point are then averaged over all grid points in the channel segment centered at each GCM point. The length of the channel segment is again chosen to be equal to the GCM grid spacing. Finally, the effects of formally intersecting channel segments are averaged and then implemented into the GCM.

g. The base model

Although the strategy for the Q3D algorithm outlined above should be applicable to any base model, the model we have used for development of the Q3D MMF...
is based on the anelastic three-dimensional vorticity equation model developed by Jung and Arakawa (2008). In the model, the vertical component of vorticity is predicted at the top level, and the horizontal components of vorticity, potential temperature, and the mixing ratios of various phases of water are predicted at all levels. Vertical velocity is determined from the predicted horizontal components of vorticity by solving an elliptic equation. The horizontal components of velocity are then diagnostically determined from the known distributions of the horizontal components of vorticity and the vertical velocity. For more details, see Jung and Arakawa (2010).

h. Highlight of experimental results

Jung and Arakawa (2014) present a detailed evaluation of the Q3D MMF developed along the line described above. In this evaluation, an idealized horizontally periodic domain is used to simulate a wave-to-vortex transition in the tropics. In such a transition, the dynamics–convection interaction plays an essential role, as pointed out by Yanai (1961). A benchmark simulation (BM) is performed first with a fully three-dimensional CRM to provide a reference for Q3D MMF simulations and their initial conditions. The horizontal and vertical domain sizes are 3072 km × 3072 km and 30 km, respectively, and the horizontal grid spacing is 3 km. There are 34 layers in the vertical, using a stretched grid with the size ranging from about 0.1 km near the surface to about 2 km near the model top.

Using the horizontal grid network illustrated in Fig. 16-12 with the GCM and CRM grid spacing of 96 km and 3 km, respectively, several Q3D simulations are performed from selected dates of BM. Only one grid-point array is used in each channel; therefore, only one grid point is independent across the channel. Because of the use of 96-km GCM grid spacing, Q3D solutions cannot be directly compared with BM solutions. Yet if the Q3D simulation produces the essential large-scale features of the BM, it should be considered as highly successful because the ratio of the number of CRM grid points used by the Q3D MMF and that by the BM is only 6% for this configuration of grid points. This ratio becomes even smaller if the GCM resolution is coarser, as is normally the case, or the CRM resolution is finer.

In the rest of this subsection, selected results of a 13-day Q3D simulation are shown. For the comparison with the Q3D simulation, the BM fields are averaged over the horizontal area represented by each GCM grid point. Figure 16-13a shows the time sequence of the vertical component of vorticity at \( z = 3 \) km taken from the BM. In this time sequence, we see that this period is characterized by development and subsequent persistence of two intense vortices. Figure 16-13b shows the corresponding time sequence taken from the Q3D simulation. As in the BM, two intense vortices are developed and maintained, only with slight differences in the location and intensity of the vortices toward the end of the period.

The results from the Q3D simulation shown in Fig. 16-13b can be compared with those from a run shown in Fig. 16-13d in which the feedback from the CRM to the GCM is not included. In this simulation, the two vortices do not intensify and tend to merge into one vortex. These results confirm that the dynamics–convection interaction is crucial for the development and persistence of the vortices and the Q3D simulation is quite successful in representing the interaction.

To see the importance of using the background fields in the Q3D MMF, a test simulation is performed with a “2D MMF” in which 2D CRMs are used instead of 3D CRMs in the Q3D MMF. The CRM component of the 2D MMF still forms two perpendicular sets of gridpoint arrays virtually intersecting at the center of each GCM grid cell. Because of their two-dimensionality, however, the CRMs recognize the large-scale horizontal inhomogeneity only along the channel, not across the channel. Each CRM predicts only one horizontal vorticity component, the component normal to the grid-point array. Otherwise, the prediction algorithm for this simulation is the same as that for the standard Q3D MMF. Figure 16-13c presents the evolution of the vertical component of vorticity simulated by the 2D MMF. (Note that this 2D MMF is not equivalent to the prototype MMF because of the use of different grid structures.) Comparing this sequence to that shown in Fig. 16-13b, we see that the recognition of large-scale inhomogeneity across the channel makes a drastic difference in the vortex development.

Figure 16-14 shows the time series of the precipitation and evaporation rates at the surface taken from the BM, Q3D MMF, and 2D MMF simulations averaged over their respective horizontal domains. Figure 16-14a shows that, after the initial adjustment period of a day or so, the surface precipitation rate of the Q3D simulation becomes very close to that of the BM, while that of the 2D MMF simulation is considerably underpredicted. Figure 16-14b presents similar results for the evaporation rate, showing that the 2D MMF predicts a significantly less-active hydrological balance.

The preliminary tests of the Q3D MMF presented above are quite encouraging, showing the big potential of the Q3D MMF as the basic framework for future NWP and climate models. More details on the Q3D MMF are presented by Jung and Arakawa (2014).
5. Concluding remarks

Michio Yanai worked on virtually all aspects of tropical meteorology, covering a broad range of the spectrum from the cloud scale to the planetary scale. One of his most important contributions is the introduction of the concepts of apparent heat source $Q_1$ and apparent moisture sink $Q_2$ in the large-scale heat and moisture budgets of the atmosphere. Because of the inclusion of eddy effects, $Q_1$ and $Q_2$ are generally different from true heat sources and true moisture sinks taking place locally. In low-resolution models, such as the conventional general circulation models (GCMs), cumulus parameterization determines $Q_1$ and $Q_2$ from the gridpoint values of the model using a closure assumption. In the multiscale modeling framework (MMF), on the other hand, $Q_1$ and $Q_2$ are numerically simulated by a cloud-resolving model (CRM) coupled with the GCM.

FIG. 16-13. Time sequences of the horizontal maps of the vertical component of vorticity at $z = 3$ km. See text for explanations of the sequences. Rearranged from Jung and Arakawa (2014).
The main theme of this article is to build a bridge, or bridges, between apparent sources and true sources. Since the difference between them is the inclusion of eddy effects in the former, a generalized representation of those effects is crucial in building such a bridge. It is shown that there are two routes to do so: ROUTE I and ROUTE II. ROUTE I follows the parameterization approach, while ROUTE II follows the MMF approach.

We realize that more extensive and intensive work is essential to complete well-built bridges between various ways of representing moist-convective processes. Such work may well become a central issue in the coming era in the history of numerical modeling of the atmosphere. If Michio Yanai were still with us, he would certainly make valuable contributions to this era in his own way.

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**REFERENCES**


![FIG. 16-14. Time series of the surface precipitation and evaporation rates. Rearranged from Jung and Arakawa (2014).](image-url)


