Hailstone Size Distributions and Equivalent Radar Reflectivity Factors Computed from Hailstone Momentum Records

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ABSTRACT

An electronic hailstone momentum sensor has been developed which records hailstone impacts on magnetic tape. The instrument and the programs for analyzing the recorded data are described. Sensors were operated during six hailstorms in 1969 and recorded 624 hailstone impacts. Derived frequency distributions of hailstone size show the median diameter for all stones recorded to be near 0.9 cm. Significant variations exist among storms. Values of the equivalent radar reflectivity factor Z_e have been computed for the hailshafts sampled, for X-band and S-band radar, and for both wet and dry hailstones. The values range up to 70 dBz ($10^{7.0}$mm^6 m^{-3}) and show good agreement with radar-measured values. The results suggest that dual-wavelength radar systems are not much better than S-band sets alone for estimating hailstone size.

1. Introduction

Methods of observing hailstones at the ground have included the collection of hailstones and storage in freezers (Douglas, 1963), passive sheets of aluminum foil on which falling hailstones leave permanent indentations (Schleusener and Jennings, 1960), and the photography of freshly fallen hailstones. All of these methods suffer from one or both of two flaws; namely, an inability to determine the time of fall of individual hailstones and laboriousness in reducing the observations. Participants in Project Hailswath (Goyer *et al.*, 1966) were unanimous on the requirement for improved hail recording instruments. This paper contains a brief description of one such instrument and an analysis of preliminary observations.

2. The hailstone momentum sensor

The instrument is an electronic hailstone momentum sensor. For each hailstone impact, the sensor encodes the momentum of the hailstone in a pulse-frequency format. This information is then recorded on magnetic tape, and the tapes are subsequently analyzed on a computer.

The sensor configuration used in 1969 is shown in Fig. 1. The basic principle of the momentum sensor is illustrated by Fig. 2 (taken from McNeil *et al.*, 1969). A hailstone of mass m falling with terminal velocity V_t strikes the much larger mass M of the sensor, initially at rest. The impact imparts an initial velocity u_2 to the sensor mass, and the hailstone rebounds with velocity V_2. Conservation of linear momentum shows that the initial velocity of the sensor mass after impact is given by

$$u_2 = mV_t(1+r)/(m+M) \approx p(1+r)/M, \qquad (1)$$

where $p \equiv mV_t$ is the hailstone momentum prior to impact and r the coefficient of restitution. The hailstone momentum p is determined by measuring the initial velocity u_2 of the known mass M, and estimating the value of r.

The lack of accurate knowledge of the coefficient of restitution represents a source of error in the momentum measurement. However, the value of r for hailstone impacts is unlikely to exceed 0.2, even for hard stones. Thus, even if it is assumed that $r=0$, the error in p will seldom be greater than 20%. The errors in the estimates of hailstone diameters are even smaller, as shown in Section 3.

In the actual sensor, the hailstone strikes a circular plate 15 cm in diameter. The impact transfers momentum to a large mass (approximately 4.5 kg) of which the plate is one part. A geophone (or seismometer) attached to the mass serves as the velocity transducer, from which the initial velocity u_2 can be determined.

The output signal from the geophone is processed by the electronic circuit shown in block diagram form in Fig. 3. A peak detector derives from the geophone output a pulse whose amplitude is proportional to the initial velocity of the sensor mass, and hence to the hailstone momentum. This pulse is passed through the compressor, a circuit with a power-law response characteristic used to compress the dynamic range of the signal; this step is necessary to permit using low-cost audio magnetic tape recorders to record the data. The output of the compressor is used to modulate a voltage-

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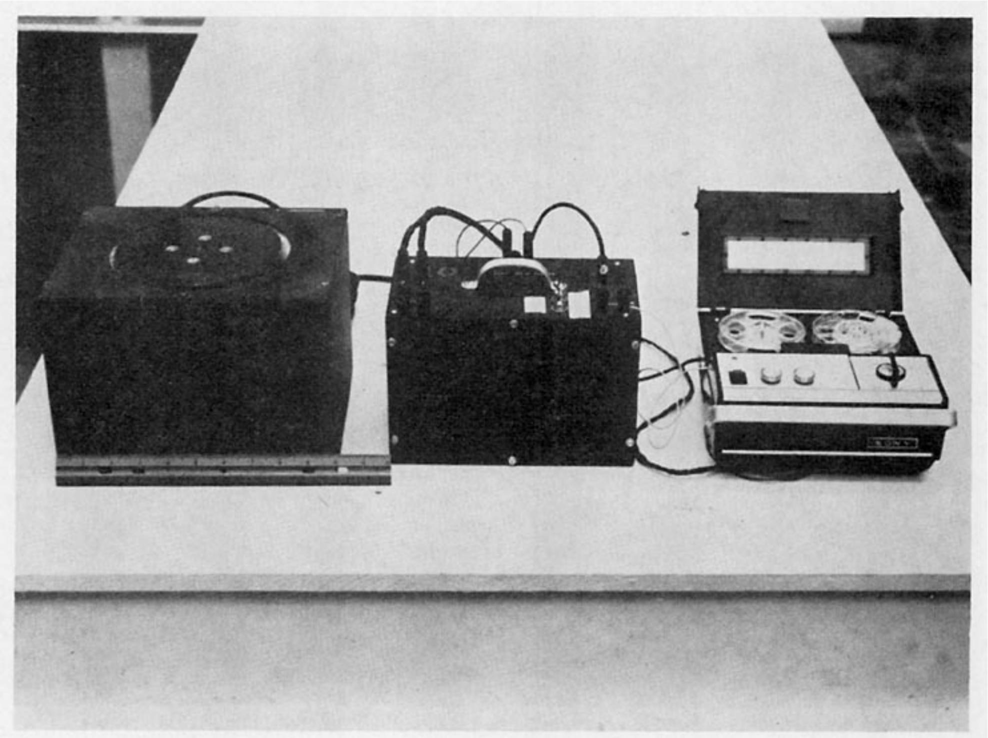

FIG. 1. Hailstone momentum sensor and recorder.

controlled oscillator, which generates a train of pulses with a pulse repetition frequency proportional to the input amplitude. A timing circuit allows this oscillator to operate for a period of 100 msec. The output pulses are uniform in amplitude and duration, and the total number of pulses in the 100-msec interval is a function of the hailstone momentum. These pulses are recorded on audio magnetic tape, using low-cost portable cassette-type recorders. Details of the electronic circuitry are given by Fremstad (1968) and Gjelsvik (1969).

The sensor is capable of recording one hailstone impact each 140 msec, the extra 40 msec being required during playback to recognize the separation between consecutive impacts. It responds to hailstone momenta in the range from about 0.001 up to 2 kg-m sec^{-1}. The sensor is calibrated by dropping steel balls from known heights onto the sensor plate. Independent testing by the Illinois State Water Survey has shown that the device produces consistent result and covers the useful range of hailstone momenta (Changnon and Staggs, 1969).

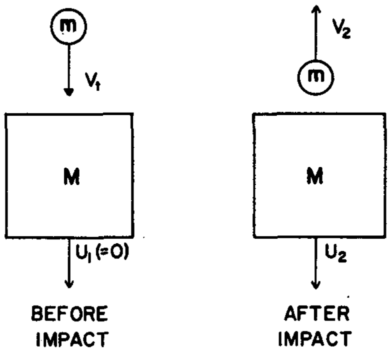

FIG. 2. Basic principle of momentum sensing, where m and M are hailstone and sensor mass, respectively, and V_t, u_1 and V_2, u_2 are the corresponding velocities before and after impact.

3. Data analysis

The magnetic tapes are played back into a PDP-8 computer, which determines the time interval between successive hailstone impacts and counts the number of pulses in the 100-msec interval for each hailstone. This information is punched onto paper tape, which is then fed into an IBM 1130 computer. This second computer (used because it is more easily programmed in Fortran than the PDP-8) calculates for each hailstone the time of impact (relative to the beginning of the hailfall), the momentum, and the other parameters discussed below.

To translate hailstone momentum into more useful parameters such as mass or diameter, it is necessary to make several simplifying assumptions. We have assumed that the hailstones are spheres of density 0.9 gm cm^{-3}, and following a suggestion by Macklin and Ludlam (1961), that they have a constant drag coefficient of 0.6. The assumption of this drag coefficient compensates for minor surface roughness variations and shape irregularities, but would be incorrect for highly ellipsoidal or very irregularly shaped hailstones.

Under the above assumptions the hailstone momentum can be used to determine both the diameter and the fallspeed of the stone. The mass of a spherical hailstone of diameter D is given by

$$m=\pi\rho D^3/6, \tag{2}$$

where ρ is the hailstone density. Its terminal fallspeed is

$$V_t=\left(\frac{4g\rho D}{3\rho_a C_D}\right)^{\frac{1}{2}}, \tag{3}$$

where g is the acceleration due to gravity, ρ_a the air density, and C_D the drag coefficient. Thus, its diameter is

$$D=\left(\frac{27\rho_a C_D p^2}{\pi^2 g\rho^3}\right)^{1/7}. \tag{4}$$

The fact that D varies as the two-sevenths power of the measured momentum p makes the calculation of D rather insensitive to calibration errors and to uncertainties in the value of the coefficient of restitution. Errors in the value assigned to C_D are of even less importance, as it enters (4) only as the one-seventh power.

For each impacting hailstone the 1130 computer program prints estimates of diameter, fallspeed, and four radar cross sections. The radar cross sections are obtained by reference to a table of normalized backscattering cross sections read into the computer as part of the program. The sources of the values in the table are as follows:

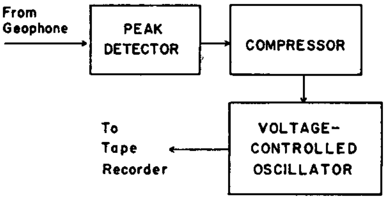

FIG. 3. Block diagram of electronic circuitry.

1) Ice at 3.2-cm wavelength (X band) [table 4 of Stephens (1961)].
2) Ice at 10-cm wavelength (S band) [table 1 of Herman and Battan (1961a)].
3) Ice with 0.01-cm water film at 3.2-cm wavelength (X band) [Figs. 3 and 7 of Herman and Battan (1961b)].
4) Ice with 0.01-cm water film at 10-cm wavelength (S band) [Fig. 5 of Herman and Battan (1961b)].

Results of the various analyses are given in the following sections.

4. Hailstone size distributions

Although the hailstone momentum sensor is designed for unattended operation, one unit was operated in 1969 by a mobile observer to increase the number of hail

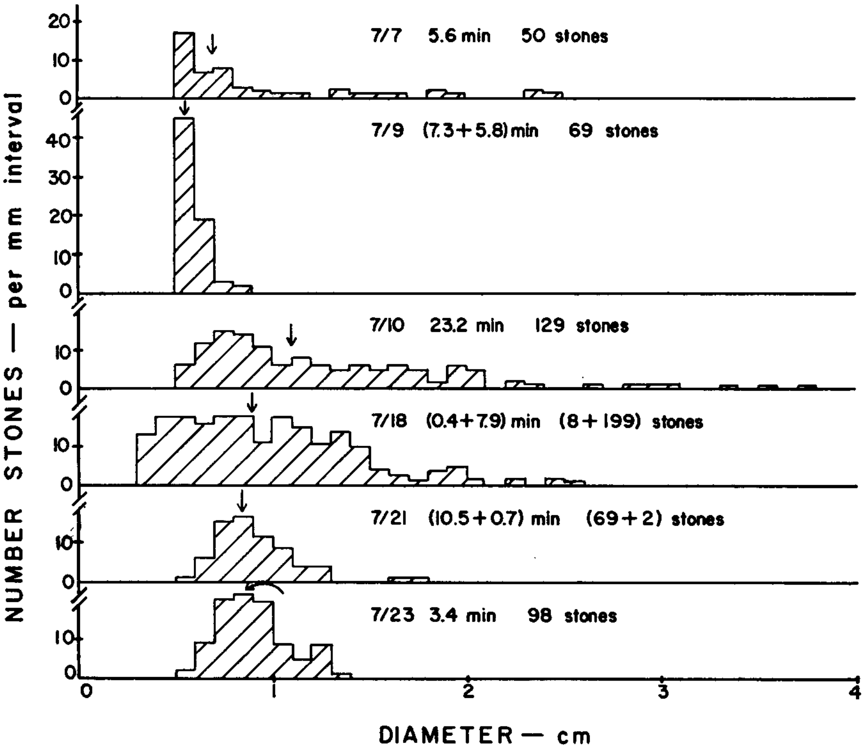

FIG. 4. Size distributions of hailstones recorded in six storms in 1969. Arrows denote medians for storms. Median for season is near 0.9 cm.

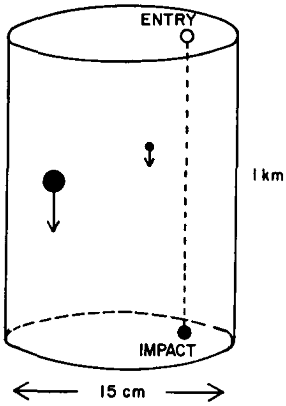

FIG. 5. Model for calculating Z_e. Conditions in a cylinder extending 1 km above sensor plate of 15 cm diameter are assumed to extend far enough horizontally to provide a filled radar beam. Small hailstones have longer residence time in the cylinder than large ones. The computer program works backward from impact time to find the entry time for each hailstone. About 10 hailstones are usually present in the cylinder.

incidents recorded and to provide a check on its output. The observer was directed by radio into the path of storms suspected on the basis of radar observations of being hail producers. In this way useful records were obtained from five hailstorms. Records were also obtained from one storm using an instrument at a fixed site. The total record from six storms comprises 65 min of operation during actual hailfalls and shows the impacts of 624 hailstones.

The hailstone diameters computed from the momentum records are given in Fig. 4. They agree with the operator's visual observations. For example, in the storm of 9 July he observed small scattered hailstones, while on 10 July he noted the occurrence of "a few hailstones up to 2 inches diameter." As the probability of a very large stone striking the 15-cm sensor plate is small, one would not expect the instrument to be very useful in determining the maximum hailstone size for a storm. However, the maximum computed diameter of 3.8 cm on 10 July agrees fairly well with the 2-inch (5-cm) maximum diameter noted by the observer, whose sampling area probably extended 20–30 ft from the sensor location.

The median diameter of all the hailstones recorded during the season is near 0.9 cm. The cutoff at 0.5 cm in the data from the first two storms (7 and 9 July) is believed due to the threshold setting in the peak detector used to eliminate raindrops as well as very small and/or soft particles such as snow pellets. Subsequent adjustments permitted the recording of smaller hailstones, as happened on 18 July. The cutoff at 0.5 cm on 10, 21 and 23 July is believed due to an absence of smaller stones.

The size distributions agree with earlier observations in South Dakota using passive hail indicators (Schleusener and Jennings, 1960). For example, the median diameter of all hailstones recorded at some 60 fixed sites in Perkins County, S. Dak., during the summer of 1965 was 0.8 cm.

It is apparent from Fig. 4 that the size distributions vary from storm to storm. However, the median diameter does not move very far upward in the severe storms. While the 10 July storm was the most severe to strike Rapid City in several years and produced many large hailstones, the median diameter was still only 1.1 cm. This finding differs somewhat from the report of Douglas (1963), who found median diameters ranging up to 1.6 cm in hailstorms in Alberta.

5. Calculation of equivalent radar reflectivity factors

Several authors (e.g., Atlas, 1963) have published values of the equivalent radar reflectivity factor [Z_e(mm^6 m^{-3})] for various hailstorms. Each measured value represents an average over a contributing region whose exact dimensions depend upon the beam width and pulse duration of the radar and the range to the hailstorm. If the contributing region is significantly larger than the hailshaft, the measured value of Z_e will not accurately represent the peak concentration of hailstones in the shaft. At X-band the measured value of Z_e is reduced by attenuation due to rain and hail between the hailshaft and the radar. On the other hand, rain often contributes to the measured value of Z_e in a hailshaft. It is unlikely that one would measure[3] Z_e at less than 40 dBz in a hailstorm, as hail is usually associated with rain of sufficient intensity (approximately 10 mm hr^{-1}) to produce such a value.

In using the radar backscattering cross sections (mentioned in Section 3) to compute Z_e one must consider a hypothetical contribution region with dimensions comparable to those of a hailshaft and compatible with the characteristics of actual radar sets. Obviously, if one considered a contributing region of 1 m^3, absurdly high values of Z_e would occur as the result of a single hailstone passing through it. We have computed Z_e for a hypothetical contributing region extending to 1 km above the ground under the assumption that the conditions above the hail sensor are representative of the entire contributing region. This is equivalent to assuming that the sensor samples a hailshaft which extends uniformly across the radar beam. The computed values of Z_e correspond to those that would be measured by a radar observing the hailshafts in question provided that 1) the hailshaft fills the contributing region, 2) there is no attenuation, and 3) the rain accompanying the hail does not increase the received power by a significant amount.

The program for computing Z_e involves working backward in time from the impact of each hailstone, using its estimated terminal speed to determine at what time it came within 1 km of the ground. The computed values of Z_e vary every few seconds, increasing each time a new hailstone enters the cylinder 1 km above the

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[3] Reflectivity factor in dBz = 10 log (Z_e/1 mm^6 m^{-3}).

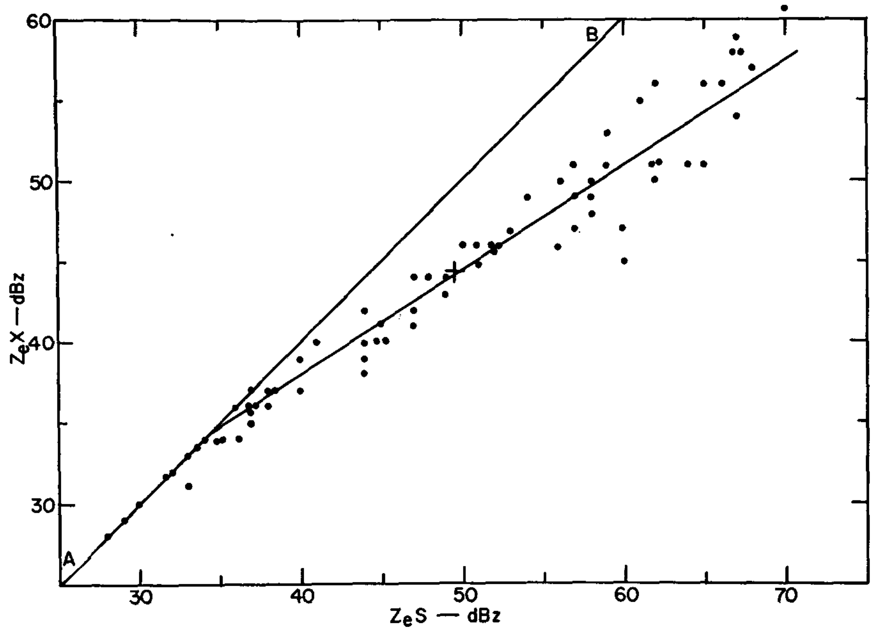

FIG. 6. Scatter diagram showing computed equivalent radar reflectivity factors at 10-cm and 3.2-cm wavelengths (Z_eS and Z_eX, respectively) for hypothetical contributing region of Fig. 5 assuming wet hailstones. Cross denotes median value.

ground and decreasing each time a hailstone falling through the cylinder strikes the ground (Fig. 5). As the cylinder typically contains about 10 hailstones, the change produced by the entry or exit of one stone is around 1 dB. A typical hailstone falls 1 km in ~1 min, so each hail episode, measured from entry of the first stone into the contributing region to the impact of the last stone on the ground, is about a minute longer than the hail duration at the ground. The reflectivity factors existing at the start of each minute of the various records provide a data sample of 73 independent observations, each involving a different set of stones.

Fig. 6 is a scatter diagram comparing the computed values of Z_e at 10- and 3.2-cm wavelengths, denoted henceforth as Z_eS and Z_eX, for wet hailstones. (We shall concentrate upon wet hailstones in the remainder of the discussion as most hailstones are wet on their arrival at the ground.) If the hailstones were Rayleigh scatterers at both wavelengths, the reflectivity factors would be the same and all the points would be scattered along the line AB. In fact, the larger hailstones cease to behave as Rayleigh scatterers at 3.2 cm when Z_e approaches 35 dBz ($10^{3.5}$mm^6 m^{-3}).

Our median value of Z_eX (45 dBz) cannot be compared directly with values previously reported in the literature. First it is necessary to derive from our data statistics appropriate to the methods of measurement and analysis employed by previous authors.

Douglas (1964) has reported X-band reflectivity factors calculated on the basis of several years of observations of hailstones at the ground in Alberta. Lacking detailed information on the distribution of the hailstones with time, he computed average reflectivity factors under the assumption that they were distributed rather uniformly over their period of fall. Table 1 presents similar average X-band reflectivity factors for the six storms in our sample for both wet and dry hailstones. (Although expressed in dBz, these are arithmetic rather than geometric averages.) The medians of the average values are 50.5 and 50 dBz for wet and dry hail, respectively. The corresponding medians for 67 samples read from Fig. 2 of Douglas (1964) are 50 and 46 dBz, respectively. This suggests little difference in the size distributions and concentrations of hailstones in typical hailshafts in Alberta and South Dakota.

In comparing our computed reflectivity factors with those measured by radar, it is probably most appropriate to consider the maximum computed value in each storm. These values are also given (for X band) in Table 1; the median value of the maximum Z_eX for both wet and

TABLE 1. Arithmetic time averages and maxima of X-band reflectivity factors (dBz) for storms sampled in 1969 assuming wet and dry hailstones.

	Average		Maximum	
Date	Wet	Dry	Wet	Dry
7 July	50	49	56	55
9 July	38	35	44	42
10 July	52	54	61	65
18 July	53	51	58	56
21 July	46	45	50	49
23 July	51	50	55	54
Median	50.5	50	54.5	54.5

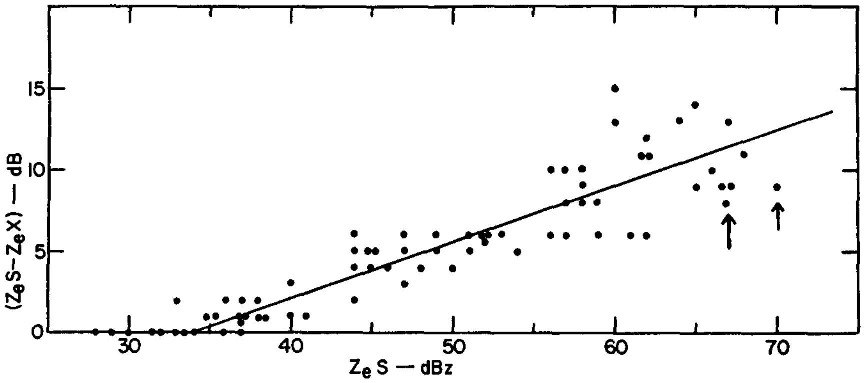

FIG. 7. Scatter diagram of (Z_eS−Z_eX) as a function of Z_eS for wet hailstones. The standard error of estimate about the regression line fitted by eye is 1.8 dB. Arrows refer to very large hail at the start of the 10 July storm.

dry hail is 54.5 dBz. Fig. 2 of Donaldson (1961) shows that the median Z_eX measured at 5000 ft in the cores of New England hailstorms is near 52 dBz. Dennis *et al.* (1970) report a median value of 48 dBz for the maximum Z_eX in a sample of South Dakota hailstorms. The maximum usually occurred some distance above the ground in both the New England and South Dakota hailstorms so measured values of Z_eX at the ground would be slightly lower in both cases. Comparing measured and computed values suggests that the contribution to Z_eX by raindrops is more than offset by attenuation and beam filling effects. Further evidence to support this conclusion was obtained from the hailstorm of 7 July, for which it was possible to compare the computed Z_eX directly with measurements by an X-band radar system. The measured values of Z_e in the region where the hailstone momentum sensor was located stayed below 50 dBz, while the computed value of Z_eX reached 56 dBz briefly.

The computed values of Z_eS range up to 70 dBz in agreement with actual measurements in South Dakota and elsewhere. Attenuation is less significant at S-band than at X-band but beam filling considerations are important unless very large antennas are employed to produce narrow beams. Values of Z_eS in excess of 70 dBz must be quite rare and limited to the most severe hailstorms (Atlas, 1963).

6. Dual-wavelength systems

Dual-wavelength radar systems have been used in the Soviet Union for determining hailstone sizes (Sulakvelidze *et al.*, 1967, pp. 134–146). Their use is based upon the facts that hailstones of substantial size cease to function as Rayleigh scatters at X-band wavelengths, and that a definite ratio of Z_eS to Z_eX exists for hailstones of a given size, shape and structure. However, in assessing the value of dual-wavelength systems it should be noted that hailstones in a single shaft usually extend over a range of sizes.

Some insight into the potential of dual-wavelength systems can be obtained from Fig. 7, which shows the difference (Z_eS−Z_eX) as a function of Z_eS for wet hailstones. As expected, the two functions are highly correlated; the standard error of estimate about the fitted regression line of Fig. 7 is only 1.8 dB. An actual measurement of (Z_eS−Z_eX) involves the sum of the calibration errors of the two radar sets, each of which would certainly exceed 1 dB, as well as attenuation errors. It appears, therefore, that an estimate of (Z_eS−Z_eX) based on Fig. 7 and a measurement of Z_eS would be as useful as one based on actual dual-wavelength measurements.

In spite of the high correlation between (Z_eS−Z_eX) and Z_eS, the dual-wavelength system would still have possibilities if large hail were associated with points consistently above the regression line of Fig. 7. In fact, the largest hail of the 1969 season was associated with points well *below* the regression line (arrows).

In practice, attenuation of the X-band signal by rain and the hail itself would contribute significantly to any measured value of the difference (Z_eS−Z_eX). One might detect hail with a dual-wavelength system through a more complicated approach combining attenuation and Mie scattering effects, as proposed by Eccles and Atlas (1969), but it appears that the simple dual-wavelength concept offers little improvement over the use of an S-band set alone.

7. Summary and conclusions

The electronic hailstone momentum sensor has proven to be satisfactory for determining the times of arrival and the sizes of hailstones striking it. The overall size distribution of hailstones recorded in western South Dakota in 1969 agrees with those previously recorded in the same area by a different technique. Differences among storms are apparent but the median hailstone diameter remains near 1 cm even in severe storms. Equivalent radar reflectivity factors computed from the derived size distributions agree reasonably well with measured values. The computed equivalent radar reflectivity factors at S band and X band are closely correlated, which casts doubt upon the value of a simple dual-wavelength radar system (as opposed to an S-band set alone) for hail detection.

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