Penetration of Solar Irradiances Through the Atmosphere and Plant Canopies

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ABSTRACT

The equation of radiative transfer is applied to the analysis of solar irradiances penetrating into a plant canopy covered by a turbid atmosphere. The method of discrete coordinates is applied to vertically inhomogeneous atmospheres and plant canopies. It is shown that four-point quadrature yields results with an accuracy which is consistent with irradiance measurements.

1. Introduction

Modern agricultural practice seeks to maximize the density of plants growing in a given area. However, as the plant density is increased, the solar irradiance which can penetrate to the lower leaves is reduced. It is therefore desirable to determine the extent of such penetration within a plant canopy in order to provide some guidance for optimizing the plant density.

Considerable effort has been expended to compute solar irradiances within homogeneous plant canopies. For example, Allen *et al.* (1970) recently applied the theory of radiative transfer of Duntley (1942) to the computation of solar irradiances within a plant canopy. Ross and Nilson (1968) also computed the solar irradiance within a plant canopy. These analyses have not considered the effect of the atmosphere above the canopy; we will present an analysis which includes atmospheric effects.

It is desirable to characterize the optical properties of the atmosphere and of plant canopies by a minimum number of parameters consistent with accuracy. This must be achieved because these parameters will be a function of depth within the atmosphere and/or heterogeneous plant canopy and they will also vary with wavelength of the incident solar radiation. The computation of solar irradiances in real situations can become very time-consuming if a simple analysis is not employed. Allen *et al.* recently analyzed the irradiances measured in a corn canopy by Allen and Brown (1965). Although the fit was very good, they required six parameters to achieve their fit. The following analysis reduces the number of parameters required to describe the penetration of solar irradiance through a plant canopy to only three. The analysis also renders it feasible to include the effect of a horizontally homogeneous turbid atmosphere above the plant canopy.

2. Analysis

It will be assumed that the transfer of solar radiation through a plant canopy can be described by the same equation of radiative transfer that describes the transfer of solar radiation through other multiply scattering media, such as clouds. The equation of transfer defining the diffuse radiance $I(\tau,\mu,\phi)$ is

$$\mu\frac{dI}{d\tau}(\tau,\mu,\phi)=-I(\tau,\mu,\phi)+\tfrac{1}{4}aF_0P(\mu,\phi;\mu_0,\phi_0)\exp(-\tau/\mu_0)$$

$$+\frac{a}{4\pi}\int_0^{2\pi}\int_{-1}^{+1}P(\mu,\phi;\mu',\phi')I(\tau,\mu',\phi')d\mu'd\phi',\quad(1)$$

where:

- μ — $\cos\theta$
- θ — zenith angle
- ϕ — azimuth angle
- $s(z)$ — scattering coefficient
- $\alpha(z)$ — absorbing coefficient
- $a(z)=s/(\alpha+s)$, the albedo for single scattering

$$\tau=\int_0^z[s(z')+\alpha(z')]dz',$$

the optical depth or scaled leaf area index

- πF_0 — solar irradiance perpendicular to the direction of incidence
- μ_0 — cosine of solar zenith angle
- $P(\mu,\phi;\mu',\phi')$ — phase function defining the light incident at μ',ϕ' which is scattered in the direction μ,ϕ

We assume that the phase function may be represented by a Henyey-Greenstein function. van de Hulst (1971) showed that the Henyey-Greenstein function could be expanded as a series of Legendre polynomials

$$P(\Theta)\approx\sum_{l=0}^{L}(2l+1)g^{l}P_{l}(\cos\Theta), \tag{2}$$

where Θ is the angle between incident and scattered radiances, $P_l(\cos\Theta)$ are the Legendre polynomials of order l, and the asymmetry factor is defined by

$$g=\frac{1}{2}\int_{-1}^{+1}P(\Theta)\cos\Theta d(\cos\Theta).$$

Chandrasekhar's (1950) method of discrete coordinates may now be applied to derive the radiances which are solutions to (1). These solutions will be subjected to the following conditions:

1) The diffuse radiances incident on the upper boundary of the atmosphere are zero; thus,

$$I^{1}(0,\mu_k)=0 \quad (1\geqslant\mu_k>0). \tag{3}$$

2) The diffuse radiances are continuous at boundaries between homogeneous, albeit optically dissimilar layers. Radiances $I^{j}(\tau_j,\mu_k)$ in each layer designated by j are characterized by different a_j, g_j, and $(\tau_{j+1}-\tau_j)$; thus,

$$I^{j}(\tau_j,\mu_k)=I^{j+1}(\tau_j,\mu_k)\begin{cases} j=1,\cdots,J-1. \\ -1<\mu_k<1\end{cases} \tag{4}$$

3) The upward directed radiances at the ground are assumed to be isotropic and proportional to the product of the ground albedo A and the total downward irradiance incident on the ground; thus,

$$\begin{aligned} I^{J}&(\tau_J,\mu_k) \\ &=A\left[2\sum_{k=1}^{K/2}w_{k'}\mu_{k'}I^{J}(\tau_J,\mu_{k'})+\mu_0F_0\exp(-\tau_J/\mu_0)\right] \\ &\qquad(-1<\mu_k<0;\quad 0<\mu_{k'}<1)\end{aligned} \tag{5}$$

where $w_{k'}$ are quadrature coefficients which are summed over the integrand evaluated at $\mu_{k'}$. The total optical thickness τ_J is produced by atmospheric layers, which may contain aerosols or clouds, and vegetative layers, which may vary as the canopy is penetrated.

The solution of the equation of radiative transfer in a homogeneous medium by the method of discrete coordinates has been described by Lenoble (1956). While conditions 2) and 3) differ from those considered by Lenoble, they do not markedly affect the method of solution. Diffuse irradiances are computed from

$$|F(\tau)|=2\pi\sum_{k=1}^{K/2}w_k\mu_kI^{j}(\tau,\mu_k), \tag{6}$$

where $\mu_k>0$ is used to compute $F\downarrow(\tau)$ and $\mu_k<0$ for $F\uparrow(\tau)$.

The number of quadrature points to be used in the solution to (1) is a trade-off between accuracy, which is improved by using many points, and the time required to solve the problem, which is reduced by using few points. Shettle and Weinman (1970) showed that effective two-point quadrature yielded irradiances which were accurate to within a few percent in optically thick media. In order to improve the accuracy of solutions for optically thinner media, four-point quadrature is used in the subsequent analysis: *viz*, $K=4$ in (6), and $L=3$ in (2).

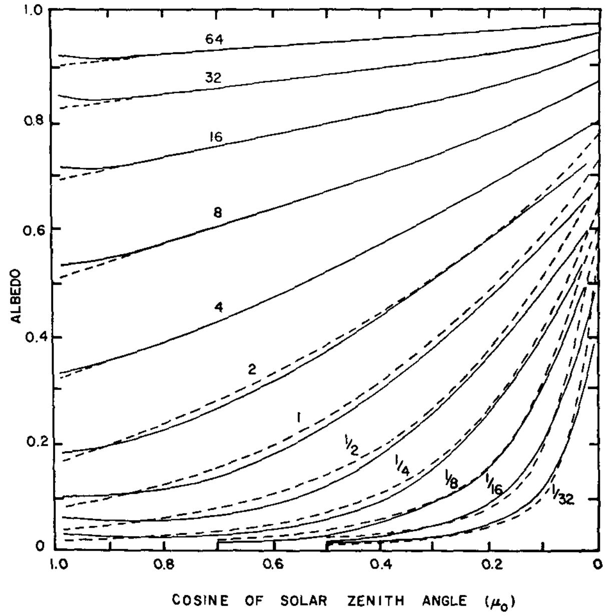

FIG. 1. Albedo of a conservatively scattering haze as a function of total optical thickness, designated above each curve, and solar zenith angle. The phase function is characterized by $g=0.75$. Results of the present analysis are represented by solid lines and the results of van de Hulst and Grossman (1968) by dashed lines.

The effective albedo $\tilde{A}(\tau)$, measured at depth τ, is the ratio of the upward directed diffuse irradiance to the incident total downward irradiance, i.e.,

$$\tilde{A}(\tau)=F\uparrow(\tau)/[F\downarrow(\tau)+\pi\mu_0F_0\exp(-\tau/\mu_0)]. \tag{7}$$

The solar energy that is absorbed by leaves within a unit depth inside a plant canopy is proportional to the divergence of the net irradiance; thus,

$$Q(z)=\frac{\partial}{\partial\tau}[F\downarrow(z)-F\uparrow(z)+\pi\mu_0F_0\exp(-\tau/\mu_0)]\frac{d\tau}{dz}. \tag{8}$$

3. Results

a. Comparison with previous theoretical computation

Fig. 1 provides a comparison between the albedo $\tilde{A}(0)$ of a haze, as a function of solar zenith angle, computed by the present four-point discrete coordinate method and the albedo computed rigorously by van de Hulst and Grossman (1968). The turbid atmosphere considered in this case is assumed to be represented by a single layer $(J=1)$ with $a_1=1$, $g_1=0.75$ and $A=0$.

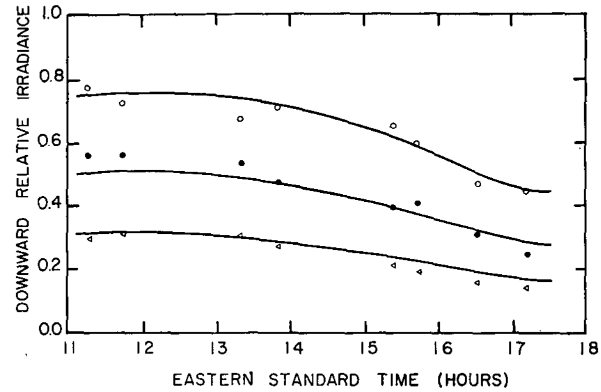

FIG. 2. Relative irradiance of 1μ radiation within the 250 cm high corn canopy measured in Ithaca, N. Y., on September 1963. The top, middle, and bottom curves represent canopy heights 150, 100 and 50 cm, respectively. The curves are theoretical predictions based upon Eq. (16); the data points are experimental values.

The solid line represents results obtained from the present analysis, whereas the dashed lines represent the results computed rigorously.

b. Measurements within a corn canopy

We apply the method of discrete coordinates to analyze the solar irradiances measured by Allen and Brown (1965) within a corn canopy on 13 September 1963 in Ithaca, N. Y. This problem may be characterized by a two-layer model, where the upper layer $j=1$ describes the atmosphere, and the lower layer $j=2$ the corn canopy.

1. Optical characteristics of the atmosphere

Since the sky was cloud free on this date, only the ambient turbidity of the atmosphere probably affected the incident solar radiation. Flowers *et al.* (1969) cited values for turbidity in western Pennsylvania in September. The optical thickness of the atmosphere at 1μ is derived from this turbidity by scaling the extinction coefficient measured at 0.5μ wavelength by the $\lambda^{-1.3}$ factor cited by Flowers *et al.* Elterman (1965) showed that the optical thickness at 1.06μ is primarily attributable to aerosols confined to a layer below $h=3$ km. Because a Canadian high pressure region was in the area, we will assume that the optical thickness is less than the mean value for the region, namely that $\tau_1=0.08$.

Deirmendjian (1969) computed the asymmetry factor for water droplets with a size distribution that is characteristic of hazes and found that $g_1=0.75$. The haze particles are assumed to be non-absorbing.

2. Optical characteristics of corn leaves

It was shown by Shettle and Weinman (1970) that if the diffuse radiance is approximated by

$$I(\tau,\mu)=I_0(\tau)+\mu I_1(\tau), \qquad (9)$$

then the diffuse irradiances are

$$F(\tau)=\pi[I_0(\tau)\pm\tfrac{2}{3}I_1(\tau)], \qquad (10)$$

where a positive coefficient corresponds to $F{\downarrow}(\tau)$ and a negative coefficient to $F{\uparrow}(\tau)$. Combining equations (6) and (7) of Shettle and Weinman (1970) yields the equations of transfer which determine these irradiances; namely,

$$\frac{dF{\downarrow}(\tau)}{d\tau}=-2(1-a)F{\downarrow}(\tau)+[\tfrac{3}{4}(1-ag)-(1-a)][F{\uparrow}(\tau)-F{\downarrow}(\tau)]+a\pi F_0(\tfrac{1}{2}+\tfrac{3}{4}g\mu_0)\exp(-\tau/\mu_0), \qquad (11a)$$

$$\frac{dF{\uparrow}(\tau)}{d\tau}=2(1-a)F{\uparrow}(\tau)+[\tfrac{3}{4}(1-ag)-(1-a)][F{\uparrow}(\tau)-F{\downarrow}(\tau)]-a\pi F_0(\tfrac{1}{2}-\tfrac{3}{4}g\mu_0)\exp(-\tau/\mu_0), \qquad (11b)$$

where a, g and τ are defined in (1) of the present paper.

Allen *et al.* (1970) showed that diffuse irradiances measured within a corn canopy should satisfy Dutley's equations:

$$\frac{dF{\downarrow}}{d\tilde{n}}=-\tilde{\mu}F{\downarrow}+\tilde{B}(F{\uparrow}-F{\downarrow})+\frac{\tilde{F}_0' I_n}{\mu_0}\exp[-(\tilde{\mu}_0'+\tilde{B}_0'+\tilde{F}_0')\tilde{n}/\mu_0], \qquad (12a)$$

$$\frac{dF{\uparrow}}{d\tilde{n}}=\tilde{\mu}F{\uparrow}+\tilde{B}(F{\uparrow}-F{\downarrow})-\frac{\tilde{B}_0' I_n}{\mu_0}\exp[-(\tilde{\mu}_0'+\tilde{B}_0'+\tilde{F}_0')\tilde{n}/\mu_0], \qquad (12b)$$

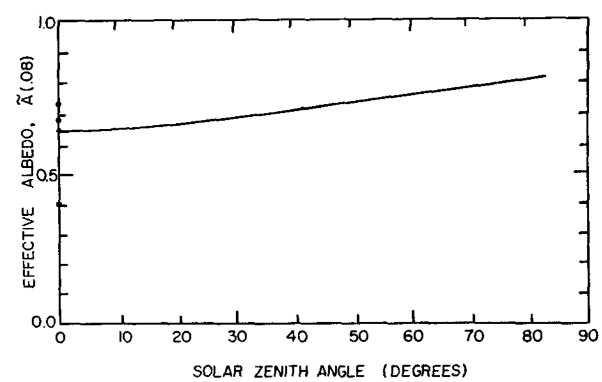

FIG. 3. Effective albedo of a cornfield for 1μ radiation as a function of solar zenith angle. The open and solid circles are based on laboratory measurements at 1μ for corn leaves. The triangle and square are field values obtained for corn canopies in North Carolina and New York, respectively. No solar zenith angle data were available.

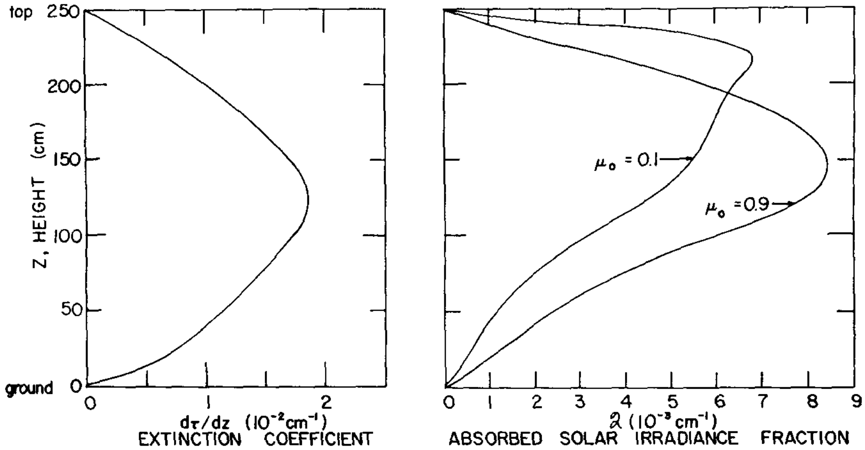

FIG. 4. Extinction coefficient ($d\tau/dz$) for 1μ radiation as a function of height within a corn field derived from the parameters given by Allen *et al.* (1970); and fraction of incident 1μ solar irradiances absorbed per centimeter in a corn canopy as a function of height and the cosine of the solar zenith angle.

where:

- $\tilde{\mu}$ absorption coefficient for diffuse light
- $\tilde{B}$ backscattering coefficient for diffuse light
- $\tilde{\mu}_0'$ absorption coefficient for specular light from the zenith
- $\tilde{B}_0'$ backscattering coefficient for specular light from the zenith
- $\tilde{F}_0'$ forward scattering coefficient for specular light from the zenith
- $\tilde{n}$ leaf area index
- μ_0 cosine of solar zenith angle
- I_n incident solar irradiance

Comparison of Eqs. (11) and (12) shows that the diffuse absorption coefficient is related to the albedo for single scattering by

$$\tilde{\mu}/(\tilde{\mu}_0'+\tilde{B}_0'+\tilde{F}_0')=2(1-a), \quad (13)$$

while the diffuse backscattering coefficient yields

$$\tilde{B}/(\tilde{\mu}_0'+\tilde{B}_0'+\tilde{F}_0')=\tfrac{3}{4}(1-ag)-(1-a), \quad (14)$$

and the solar extinction coefficient yields an optical thickness

$$(\tilde{\mu}_0'+\tilde{B}_0'+\tilde{F}_0')dn=d\tau. \quad (15)$$

The optical depth is thus proportional to the leaf area index scaled down to include effects of the orientation or curling of the leaves.

Allen *et al.* found that mature corn leaves have $\tilde{\mu}=0.035\pm0.007$, $\tilde{B}=0.736\pm0.048$, and that $\tilde{\mu}_0'+\tilde{B}_0'+\tilde{F}_0'=0.70$. No data were available to differentiate the characteristics of immature, mature, and dead leaves which might be found at different heights within the canopy.

The parameters which characterize each layer in the model are summarized in Table 1.

3. Computational results

The relative downward irradiances, $\mathfrak{F}\downarrow(\tau)$, measured by Allen and Brown (1965) are shown in Fig. 2, where

$$\mathfrak{F}\downarrow(\tau)=[F\downarrow(\tau)+\mu_0\pi F_0\exp(-\tau/\mu_0)]/[F\downarrow(\tau_1)+\mu_0\pi F_0\exp(-\tau_1/\mu_0)]. \quad (16)$$

The effective albedo measured above the corn field, $\tilde{A}(\tau_1)$, computed from (7), is compared with the values cited bv Allen et al. in Fig. 3. [Note that the solar zenith angles μ_0 at which $\tilde{A}(\tau_1)$ were measured were not cited by Allen *et al.* (1971). They presented measured values of $\tilde{A}(\tau_1)$ on the left ordinate of their figures as we have done.]

The fraction of incident solar irradiance absorbed per centimeter of depth at a wavelength of 1μ is given by

$$\mathcal{Q}(z)=Q(z)/[F\downarrow(\tau_1)+\mu_0\pi F\exp(-\tau_1/\mu_0)], \quad (17)$$

where $Q(z)$ is defined in Eq. (8). The fraction of absorbed solar irradiance, $\mathcal{Q}(z)$, is plotted as a function of height within the corn canopy and as a function of solar zenith angle in Fig. 4. The magnitude of $\mathcal{Q}(z)$ is relatively insensitive to μ_0 over a wide range of values. Most solar energy is absorbed at a height of 150 cm, where the leaves are most dense. The insert on the left side of Fig. 4 shows the extinction coefficient, $d\tau/dz$, as a function of height within the corn canopy. Allen and

TABLE 1. Parameters characterizing the transfer of 1μ sunlight through a corn canopy beneath a cloud-free turbid atmosphere. The value of A was taken to be 0.25.

j	layer	Height (m)	τ	a	g
1	atmosphere	$2.5<z<3000$	$0.08>\tau>0.0$	1.00	0.75
2	corn canopy	$0.0\leqslant z\leqslant 2.5$	$3.17>\tau\leqslant 0.08$	0.98	−0.44

Brown (1965) measured the leaf area index n at heights of 50, 100 and 150 cm in a 250 cm high corn canopy. The leaf area index is related to the optical depth by (15). The optical depth data were plotted as a function of height. This curve was graphically differentiated to yield the extinction coefficient, $d\tau/dz$.

4. Conclusion

We have shown that solar irradiances within turbid media can be computed with an accuracy of a few percent by Chandrasekhar's method of discrete coordinates which utilizes four-point Gauss' quadrature. Accuracy, such as that illustrated in Fig. 1, is consistent with the accuracy of irradiances measured by pyranometers.

Fig. 2 shows that it is possible to compute solar irradiances within vertically inhomogeneous turbid layers. Such layers can represent the atmosphere as well as plant canopies. Although we computed irradiances during clear sky conditions, the present method may also be used to compute irradiances under an overcast sky [see, e.g., Shettle and Weinman, 1971].

It is shown that the three parameters, a, g and τ, derived from laboratory measurements of the optical properties of leaves, are sufficient to characterize the solar irradiances reflected from a plant canopy. Fig. 3 shows that remotely sensed plant canopies can yield data on these parameters which may be used to characterize a particular crop.

Fig. 4 shows the solar energy absorbed by corn leaves as a function of height within the canopy. The present model permits one to compute the effects of particular agricultural practices on the solar energy absorbed by a given thickness of plant leaves.

Computation of the present two-layer model required 0.7 sec on a Univac 1108 to produce the data shown in Figs. 2–4 for each sun angle. It is thus economical to utilize the present scheme as a fast subroutine to compute solar irradiances in complex agricultural models. We believe that the present computational scheme may have considerable agricultural applications.

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