A Comparison of Wet and Dry Bent-Over Plumes

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ABSTRACT

The theory of moist bent-over plume behavior given by Csanady and by Wigley and Slawson is expanded and clarified to illustrate the differences between moist and dry plume behavior under various atmospheric stability conditions associated with linear gradients of temperature and humidity. If plume types are defined according to the behavior of a dry plume in stable, neutral and unstable conditions, then it is found, for example, that a condensed (or 'wet') plume rising in an atmosphere with lapse rate equal to the saturated adiabatic lapse rate will behave as a 'neutral' plume, while a dry plume in the same atmosphere will behave as a 'stable' plume. Also, while the condensed portion of a given plume rises according to one stability criterion, the re-evaporated portion may rise according to another.

1. Introduction

Considerable efforts have been made by a number of authors toward elucidating the very complicated nature of dry buoyant plume behavior under various atmospheric stability conditions. The atmosphere is generally defined to be stable, unstable or neutral, depending on whether the lapse rate of temperature is less than, greater than or equal to the dry adiabatic lapse rate. Plume behavior is found to differ markedly under these three conditions. Thus, dry plumes can be said to be "stable," "unstable" or "neutral" according to the plume paths which are characteristic of these stability classes.

Recently there has been additional effort directed toward the problem of wet or moist plume behavior (Csanady, 1971; Wigley and Slawson, 1971). This work arises out of the increased awareness of the problems associated with cooling tower plumes and with the moist plumes which result from the use of stack gas scrubbing techniques. In this short paper attention is drawn to the fact that the characteristics of wet plume rise correspond to those of dry plumes provided that the atmospheric stability classes are defined using the *saturated* adiabatic lapse rate as a reference. Thus, the qualitative aspects of wet plume behavior may easily be compared with those of a dry plume.

The most detailed treatment of dry plumes available at present is that given by Slawson and Csanady (1971). These authors divide the plume path into three phases, "initial," "intermediate" and "final." In the initial phase the self-generated turbulence of the plume determines its motion. In the intermediate and final phases atmospheric turbulence, in the inertial sub-range and larger eddy sizes, respectively, dominates in determining plume growth and rise. In each of these phases the plume is supposed to be one of finite, well-defined size whose growth is caused by entrainment of ambient air from the environment. Different "entrainment velocities" are used in each phase. The parallel between dry and wet plumes enables the latter to be discussed in terms of such a three-phase theory.

The mathematical framework linking dry and moist plume theory has been given by Csanady (1971). Using the entrainment approach, and closely following the development of Morton (1957), he derives a set of equations which expresses the conservation of mass, moisture, momentum and energy of the identifiable plume in terms of the fluxes of these properties. For the initial phase of plume rise the governing equations become

$$U\frac{dR^3}{dx}=3\alpha\frac{M}{U}, \tag{1}$$

$$U\frac{dM}{dx}=F-S, \tag{2}$$

$$U\frac{d}{dx}(F-\lambda S)=-MN^2, \tag{3}$$

$$U\frac{d}{dx}\left(H+\frac{1}{g}S\right)=-MG, \tag{4}$$

where

$$M=UR^2W \qquad \text{[momentum flux]}$$

$$F=UR^2g\frac{(\rho_a-\rho)}{\rho} \qquad \text{[flux of buoyancy]}$$

$$S=UR^2g\sigma \qquad \text{[flux of suspended liquid]}$$

$$H=UR^2(q-q_a) \qquad \text{[flux of humidity excess]}$$

$$N^2\approx\frac{g}{T_a}\frac{\partial\theta_a}{\partial z}$$

$$G=\frac{\partial q_a}{\partial z}$$

$$\lambda=\frac{L}{C_pT_a}$$

In these relations U is the mean horizontal wind speed (supposed constant), R the plume radius, α an entrainment parameter applicable to the initial phase, W the vertical velocity component of the plume, ρ the density, σ the mass fraction of liquid water in the plume, q the specific humidity, T the absolute temperature, θ the potential temperature, L the latent heat of vaporization of water, and C_p the specific heat capacity at constant pressure. The dependent variables without suffix apply to the plume and those with suffix a to the atmosphere; the independent variables x and z are the horizontal downwind distance and vertical distance, respectively. More precisely, virtual temperatures should be used when discussing moist plumes (see, for example, Wigley and Slawson, 1971). This modification does not affect the following arguments and it is liable to be important only when the moisture gradient G is large.

The inhomogeneous term on the right-hand side of (3), the energy equation, involves the lapse rate of potential temperature in order to account for the changes in the heat content of a parcel of air due to adiabatic expansion or compression. It is important to realize that, for the condensed portion of a moist plume, this term must be modified because of the differences between dry adiabatic and saturated adiabatic processes. For an *uncondensed* moist plume, then

$$N^2=N_d{}^2\approx\frac{g}{T_a}(\Gamma_{ad}-\Gamma), \tag{5}$$

where $\Gamma=-\partial T_a/\partial z$ and Γ_{ad} is the *dry* adiabatic lapse rate. However, for the *condensed* portion of a moist plume, we must use

$$N^2=N_s{}^2\approx\frac{g}{T_a}(\Gamma_{as}-\Gamma), \tag{5a}$$

where Γ_{as} is the *saturated* adiabatic lapse rate.

This modification of the basic equations has some interesting and important consequences. In dry plume theory the equations simplify greatly in a neutral atmosphere (i.e., $\Gamma=\Gamma_{ad}$). This simplification does not carry over to wet plume theory. Thus, in a neutral atmosphere, for the condensed portion of a plume, Eq. (3) still contains a nonhomogeneous term on the right-hand side equal to

$$-M\frac{g}{T_a}(\Gamma_{ad}-\Gamma_{as}).$$

One would therefore expect that a wet plume in a neutral atmosphere would behave much like a dry plume in an unstable atmosphere in which the lapse rate was given by

$$\Gamma=\Gamma^*=\Gamma_{ad}+(\Gamma_{ad}-\Gamma_{as}).$$

This becomes evident when Γ^* is substituted into the energy equation (3) for a dry (or uncondensed) plume, and the resulting equation is compared with the energy equation for a wet plume in a neutral atmosphere.

A minor additional complication arises in the treatment of wet plumes because the saturated adiabatic lapse rate increases with decreasing temperature. Thus, $N_s{}^2$ is not a constant in an atmosphere of constant lapse rate and, in such an atmosphere, a wet plume would tend to become "less unstable" as it rises. Constancy of the Väisäla frequency is thus a more fundamental simplication of plume rise theory than is the assumption of constant lapse rate, although the difference would only be noticeable for plumes rising several kilometers above ground.

2. Qualitative aspects

A re-evaluation of the relationship between environmental stability classes and plume behavior must be made for moist plumes. Rather than considering conditions as "stable" and "unstable," we now have the possibility of different plume behavior in: 1) a stable situation (now defined by $\Gamma<\Gamma_{as}$), 2) a conditionally unstable situation ($\Gamma_{as}<\Gamma<\Gamma_{ad}$), and 3) an unstable situation ($\Gamma>\Gamma_{ad}$). This terminology is common in meteorological literature.

In order to extend the qualitative descriptions of dry plume behavior to moist plumes, we first note that there are two factors which differentiate wet and dry plumes. First, as has been stated above, under the same environmental conditions, different values of N^2 must be used in describing wet and dry plumes; these are $N_s{}^2$ and $N_d{}^2$, respectively. This might be expected to be the most important factor, since it can cause wet and dry plumes to have paths of completely different characters. Also, as a consequence of this factor alone, the heights of rise of wet and dry plumes under the same conditions can differ appreciably (see Table 2). Second, even if this point is ignored, wet and dry plumes are described mathematically by slightly different equations [due to the terms involving S in Eqs. (2)–(4)] and thus will follow different paths. However, one would not expect this factor to alter the qualitative aspects of plume rise. Indeed, Csanady (1971) has shown that, even quantitatively, wet and dry plume paths differ by only a few percent provided that they have the same value of N^2.

We can suppose, then, that it is the difference between N_s^2 and N_d^2 which is the dominating factor in controlling the difference between the behavior of wet and dry plumes under the same environmental conditions. The condensed part of a moist plume can therefore behave in three essentially different ways which are exactly analogous to the behavior of a dry plume under stable, neutral or unstable atmospheric conditions except that the saturated adiabatic lapse rate is the reference lapse rate. A wet plume can be termed "stable" (if $\Gamma<\Gamma_{as}$), "unstable" (if $\Gamma>\Gamma_{as}$) or "neutral" (if $\Gamma=\Gamma_{as}$). This result can be expressed conveniently in tabular form (see Table 1) and is illustrated diagramatically in Fig. 1.

Table 1 and Fig. 1 (the parenthetical numbers in the table identify the corresponding plumes of Fig. 1) demonstrate the basic differences between wet and dry plumes. In the figure the plumes compared may be considered either to have the same stack parameters (efflux velocity and initial temperature excess) or to be measured using appropriate dimensionless coordinates. In Fig. 1a, both plumes exhibit "unstable" behavior in an unstable atmosphere, but the dimensionless rise is greater for the wet plume.

Fig. 1b illustrates what is perhaps the most important case; that which occurs when the environment is conditionally unstable. Here the wet plume is unstable and tends to rise exponentially, while the dry plume exhibits characteristic stable plume behavior and has a finite maximum height of rise.

Fig. 2 shows wet and dry plumes observed at the Paradise Steam Plant (T.V.A.). The behavior of the two plumes appears to correspond to Fig. 1b (the wet, unstable-dry, stable situation) although no atmospheric data are available to confirm this interpretation. The dry-flue gas plumes from the two 600-ft and one 800-ft chimneys are stable in appearance while the adjacent cooling tower (wet) plumes are typically unstable. In the light of the present theory this otherwise disparate behavior of the two plumes is not difficult to understand. In fact, since the cooling tower plumes are likely to have had lower exit velocity and initial buoyancy than the dry plumes, the difference in behavior between them might have been even more pronounced if they had had the same initial characteristics.

Table 1. Wet and dry plume types under various atmospheric conditions. Figures in parentheses refer to Fig. 1.

Environmental lapse rate	$\Gamma<\Gamma_{as}$	$\Gamma=\Gamma_{as}$	$\Gamma_{as}<\Gamma<\Gamma_{ad}$	$\Gamma=\Gamma_{ad}$	$\Gamma>\Gamma_{ad}$
Dry plume type*	stable(6)	stable	stable(4)	neutral	unstable(2)
Wet plume type	stable(5)	neutral	unstable(3)	unstable	unstable(1)

* Following Slawson and Csanady (1971).

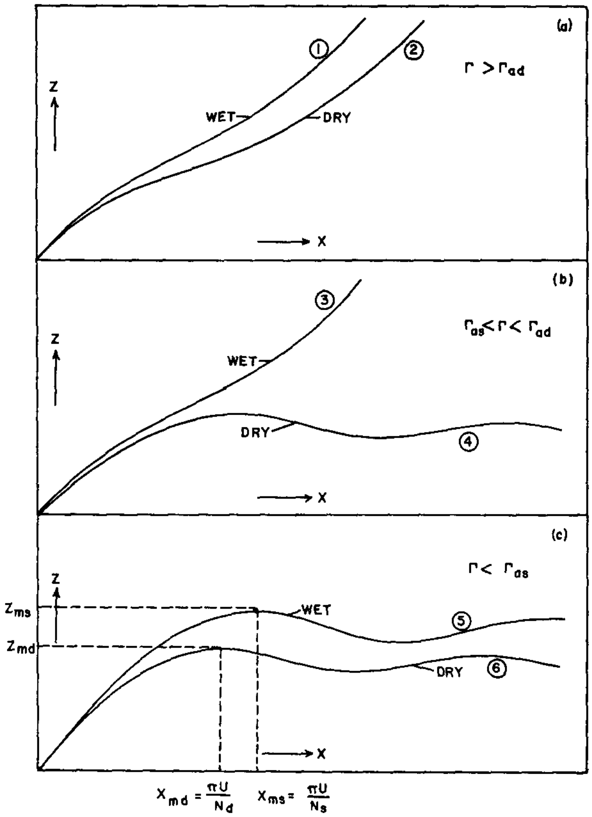

Fig. 1. A comparison of wet and dry plumes under (a) unstable ($\Gamma>\Gamma_{ad}$), (b) conditionally unstable ($\Gamma_{as}<\Gamma<\Gamma_{ad}$), and (c) stable ($\Gamma<\Gamma_{as}$) conditions. The circled numbers refer to the figures in Table 1. The coordinates z and x are non-dimensional.

Fig. 2. Wet cooling tower and dry flue-gas plumes from the Paradise Steam Plant (T.V.A.). Although this enlarged photograph lacks background definition, it is useful in illustrating the difference between wet and dry plume behavior in a conditionally unstable environment.

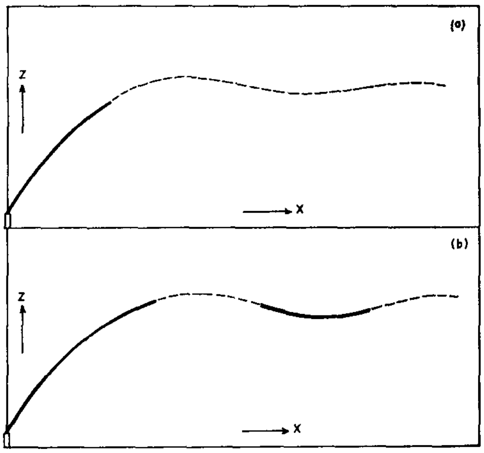

FIG. 3. Typical moist plume paths under stable conditions ($\Gamma < \Gamma_{as}$). The condensed section is shown by a bold line and the re-evaporated section by a dashed line. In (b) the condensed section extends to above the asymptotic height of rise and so admits the possibility of re-condensation further downwind. Both condensed and uncondensed sections are stable in appearance.

In the stable atmosphere case ($\Gamma < \Gamma_{as}$) shown in Fig. 1c both wet and dry plumes are of the typical stable plume shape, both reaching a maximum height-of-rise and levelling off downwind in the usual oscillating manner (e.g., Slawson and Csanady, 1971). Compared with the dry plume the condensed plume reaches a greater maximum rise (z_{ms} compared with z_{md} for the corresponding dry plume) at a point further downwind from the source (x_{ms} compared with x_{md} for the dry plume). Also, since N^2 determines the oscillation frequency for stable plumes, the oscillations of the wet plume will be of a lower frequency (and a greater amplitude) than those of a dry plume.

Although only the initial phase of plume growth has been mentioned above, Table 1 applies to *all* stages of growth and enables the more general results given by Slawson and Csanady to be extended to moist plumes. Also, although the lapse rate of specific humidity occurs in the moist plume equations [G in Eq. (4)], the prime effect of this term will be in determining the length of the condensed phase; G, therefore, acts more as a scaling factor in the plume profile than in fundamentally changing the type of rise as given in Table 1 and the present discussion can be applied to most situations, even when G is non-zero.

As an additional point of interest it can now be seen that there are two conditions under which a plume may be called neutral; i.e., when the environmental lapse rate $\Gamma = \Gamma_{as}$ if the plume is wet, and when $\Gamma = \Gamma_{ad}$ if the plume is dry. A more detailed comparison of these two neutral plumes, including behavior beyond the initial phase, reveals an important difference between them. The neutral wet plume occurs in an environment where the lapse rate is less than the dry adiabatic lapse rate. The degree of atmospheric turbulence is then considerably less than that which would be associated with a neutral *dry* plume. Thus, the initial or self-induced turbulence-dominated phase of plume rise may be expected to exist for a greater distance downwind for a neutral wet plume than for a neutral dry plume. Any levelling off of the plume associated with the intermediate and final (atmospheric turbulence dominated) phases would therefore be displaced further downwind in the case of a neutral wet plume.

The simple comparisons made in Fig. 1 show that wet plumes "see" the atmosphere as being more unstable than do dry plumes, and, as a consequence, will rise higher given identical stack parameters (initial temperature excess and efflux velocity) and environment conditions. Because of this a number of interesting situations may arise in the behavior of *moist* plumes which, in general, contain both condensed (wet) and uncondensed (dry) portions. These are illustrated in Figs. 3–5 which show typical moist plume paths in stable, conditionally unstable and unstable environments. In producing these diagrams a useful simplification has been employed. This is the fact that, if a moist plume does condense, it will do so close to the stack and generally less than a few stack diameters downwind (Wigley and Slawson). Thus, we may suppose a moist plume to consist of a *very short* "dry" section, followed by a condensed or "wet" section, followed by another "dry" section (except under unusual atmospheric conditions, primarily related to the specific humidity profile in the atmosphere). In the diagramatic representation the initial dry section will

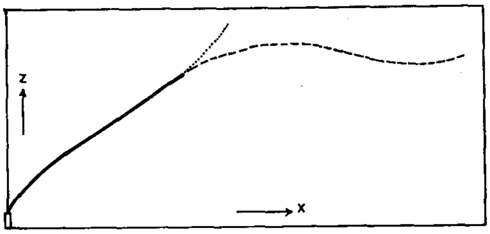

FIG. 4. Moist plume path in a conditionally unstable environment ($\Gamma_{as} < \Gamma < \Gamma_{ad}$). The bold and dashed lines represent the condensed and re-evaporated portions, respectively; the condensed portion is unstable and the re-evaporated portion stable in appearance. The dotted line is the (unstable) path the plume would have followed if it had remained "wet".

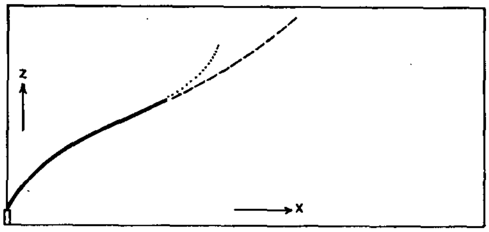

FIG. 5. Moist plume path in an unstable environment ($\Gamma > \Gamma_{ad}$). The condensed section (bold line) is unstable, as is the re-evaporated section (dashed line). However, the rise after re-evaporation is less than would be the case if the plume had remained "wet" (dotted line).

generally be too short to be noticeable and so has been neglected.

Figs. 3–5 are only representative plume types. More detail, in particular a more detailed discussion of the different phases of plume growth under various atmospheric conditions for *dry* plumes, can be found in the paper by Slawson and Csanady.

3. Quantitative aspects

So far we have discussed the qualitative effects of various linear (or near-linear) temperature and humidity gradients on the rise of moist plumes based on existing dry plume theory. The question now arises: just what is the magnitude of the extra rise due to the difference between Γ_{as} and Γ_{ad}? The result naturally depends on the length of the condensed section, since the longer it is the greater the difference. An answer can be found quite easily in two special cases, $\Gamma=\Gamma_{ad}$ and $\Gamma=\Gamma_{as}$, which mark the transitions between Figs. 3 and 4 and between 4 and 5.

In the initial phase for conditions of constant stability the plume rise is given by (Slawson and Csanady)

$$z=\left[\frac{3U^2l}{\alpha^2N^2}\left(1-\cos\frac{Nx}{U}\right)\right]^{\frac{1}{3}}. \tag{6}$$

Here N depends on the difference $\Gamma_{ad}-\Gamma$ for an uncondensed moist plume and on the difference $\Gamma_{as}-\Gamma$ for a condensed moist plume [see Eqs. (5)], and l is a buoyancy length scale related to the initial value of the flux of buoyancy (and hence the initial value of the temperature excess of plume over environment).

It is convenient now to non-dimensionalize the plume rise by measuring it in relation to the dry plume neutral stability rise,

$$z_0=\left(\frac{3lx^2}{2\alpha^2}\right)^{\frac{1}{3}},$$

which is the limiting form of (6) for $N=0$. Thus, if z^* is the non-dimensional rise defined by

$$z^*=z/z_0,$$

we have

$$z^*=\left[\frac{2U^2}{N^2x^2}\left(1-\cos\frac{Nx}{U}\right)\right]^{\frac{1}{3}}. \tag{7}$$

If we write z_s^* for the height of rise of a condensed plume and z_d^* for a dry plume under the same environment conditions, then, for $\Gamma=\Gamma_{ad}$ (neutral atmosphere), and since $\cos i\phi=\cosh\phi$,

$$z_s^*=\left\{\frac{2U^2}{N_0{}^2x^2}\left[\cosh\left(\frac{N_0x}{U}\right)-1\right]\right\}^{\frac{1}{3}}=h_1,$$

$$z_d^*=1,$$

where

$$N_0{}^2=\frac{g}{T_a}(\Gamma_{ad}-\Gamma_{as})$$

is a positive quantity. For $\Gamma=\Gamma_{as}$ we have

$$z_s^*=1,$$

$$z_d^*=\left[\frac{2U^2}{N_0{}^2x^2}\left(1-\cos\frac{N_0x}{U}\right)\right]^{\frac{1}{3}}=h_2.$$

The term h_1 is therefore the relative height of a wet plume compared with an equivalent dry plume in a neutral atmosphere, and h_2 the relative height of a dry plume compared with an equivalent wet plume in an atmosphere with lapse rate $\Gamma=\Gamma_{as}$.

The quantitative effect of the difference between Γ_{as} and Γ_{ad} can be estimated by evaluating h_1 and h_2 as functions of the dimensionless downwind distance N_0x/U. These values are given in Table 2 where it should be noted that h_2 for $N_0x/U=5$ has been calculated using the relation

TABLE 2. Relative heights of dry and wet plumes.

N_0x/U	0.5	1.0	2.0	3.0	5.0
h_1	1.007	1.028	1.114	1.263	1.803
h_2	0.993	0.972	0.912	0.762	0.641

$$z_d^*=\left[\frac{2U^2}{N_0{}^2x^2}\left(3+\cos\frac{N_0x}{U}\right)\right]^{\frac{1}{3}},$$

which is appropriate to the region $\pi\leqslant(Nx/U)\leqslant2\pi$ [see Slawson and Csanady, Eq. (8)].

It can be seen that for distances downwind from the stack of order $2U/N_0$ or larger, the condensed plume rises appreciably higher than the uncondensed plume. Since $N_0\approx1.3\ 10^{-2}\ \mathrm{sec}^{-1}$ (for $T_a\approx10\mathrm{C}$, N_0 increases as T_a increases), this means that differences become appreciable at distance of $\sim150U$ [m] (for U in m sec^{-1}) downwind. The differences become more pronounced if the environment is warmer and will thus be more important in warm climates. Also, the differences become greater the more unstable the environment, although the dependence on environment stability is relatively slight.

It is of interest to compare the maximum heights of rise under conditions when both condensed and uncondensed plumes behave in a stable manner (i.e., $\Gamma<\Gamma_{as}$). If z_{ms} is the maximum height for a condensed plume and z_{md} that for a dry plume (see Fig. 1c), then

$$\frac{z_{ms}}{z_{md}}=\left(\frac{\Gamma_{ad}-\Gamma}{\Gamma_{as}-\Gamma}\right)^{\frac{1}{3}}.$$

The asymptotic heights of rise will be in the same ratio.

For the case of an isothermal atmosphere we find

$$\frac{z_{ms}}{z_{md}} \approx 1.26$$

which is a significant difference.

Finally, it is important to note that these comparisons apply to wet and dry plumes having identical initial conditions of buoyancy and efflux velocity. Scrubbing of a plume will produce a considerable loss of buoyancy and consequent reduction in plume rise. This effect is to some degree offset by the effectively enhanced buoyancy of a wet plume. In fact, if the wet plume is sufficiently long, this enhanced buoyancy may completely overcome the loss of initial buoyancy due to scrubbing. For example, typical temperature excesses for the Lakeview Generating Station, Toronto, before and after scrubbing are 84 and 42C (ambient temperature 10C). Scrubbing therefore causes a 50% reduction in the initial flux of buoyancy and hence in the buoyancy length-scale l. Since height of rise depends on $l^{\frac{1}{3}}$ [see Eq. (6)], the expected reduction in plume rise due to scrubbing is of order 20%. From Table 2 it can be seen that a wet plume length of order $2U/N_0$ to $3U/N_0$ would be sufficient to compensate for this loss of initial buoyancy and allow greater overall plume rise. This consideration places additional importance on the problem of predicting the lengths of wet plumes.

4. Conclusions

Dry plume and wet plume behavior have been shown to be very similar provided that the reference lapse rate for discussing wet plumes is taken to be the saturated adiabatic lapse rate. This enables us to predict the characteristic shapes of wet plume paths which will be observed under any atmospheric stability condition. Perhaps the most important result of these predictions (which are listed in Table 1 and illustrated in Fig. 1) is that, when the environment is conditionally unstable, a wet plume will behave in an "unstable" manner while a dry plume will behave in a "stable" manner.

Since a general moist plume may be divided into dry (uncondensed) and wet (condensed) parts, characteristic moist plume paths may be constructed for various atmospheric stability conditions. These are illustrated in Figs. 3–5. It is shown here, for instance, that an apparently unstable wet plume may become stable after re-evaporating. Since the re-evaporated portion may well be invisible, the appearance of the condensed portion may lead one to make drastically incorrect conclusions regarding the ultimate plume rise and the resulting downwind distribution of pollutants.

These effects, and the others discussed in the preceding sections, may be summarized by the statement that, to a wet plume, the environment appears to be more unstable than it does to a dry plume. As a consequence, given the same stack conditions and the same environmental lapse rate, the height of rise of a wet plume will generally be greater than that for a dry plume.

For any particular plume, scrubbing causes a considerable loss of initial buoyancy. However, if the plume length is sufficient, it is possible that the atmospheric stability effect may more than compensate for this initial loss of buoyancy and produce greater overall plume rise.

It should be noted that, in the practical application of these conclusions, the possible effects of drift and of spill-over, of downwash (this being particularly important in cooling tower applications), and of inhomogeneities in the humidity profile of the atmosphere should be given due consideration.

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REFERENCES

Csanady, G. T., 1971: Bent-over vapor plumes. *J. Appl. Meteor.*, **10**, 36–42.

Morton, B. R., 1957: Buoyant plumes in a moist atmosphere. *J. Fluid Mech.*, **2**, 127–144.

Slawson, P. R., and G. T. Csanady, 1971: The effect of atmospheric conditions on plume rise. *J. Fluid Mech.*, **47**, 33–50.

Wigley, T. M. L., and P. R. Slawson, 1971: On the condensation of buoyant, moist, bent-over plumes. *J. Appl. Meteor.*, **10**, 253–259.