Determination of Aerosol Height Distributions by Lidar[1]

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ABSTRACT

A new analytic solution to the lidar equation is presented, which realistically considers the scattering properties of the aerosols and the molecular atmosphere individually. With this solution, it is shown, in turbid atmospheres where the aerosols dominate the scattering properties, that accurate vertical profiles of the volume extinction cross section can be obtained with an uncalibrated lidar, provided that the total transmittance of the atmospheric layer being investigated is known. This solution is applied to data samples collected under very clear and under very dusty conditions.

1. Introduction

An obvious way to measure aerosols in the atmosphere is with lidar, for it can sample aerosols within the lowest 30 km of the atmosphere in 200 μsec, while airborne particle counters require 15 min to 1 hr at a single level before an adequate sample can be collected (Blifford, 1970). Lidar is considerably easier to use and provides much better spatial and temporal resolution. It is no simple matter, though, to obtain a reasonable description of the scattering properties of the aerosols from lidar data. Without any data from supporting experiments, the system must be well calibrated, and the physical makeup of the scatterers themselves should be known. It will be shown how these difficulties may be avoided by supplementing the lidar observations with one additional piece of information, namely, the transmittance of the layer in question.

The two scattering properties of the polydispersion of particulate scatterers which affect the amplitude of the lidar signal returned from the atmosphere are 1) the total volume extinction cross section σ which determines the attenuation of the laser pulse as it traverses its path, and 2) the backscattering cross section β which is a measure of the effectiveness of the scatterers as a target. Both parameters can vary with range. Usually, though, the composition and relative size distribution of the scatterers is assumed to remain constant so that $\sigma(Z)$ and $\beta(Z)$ at any height Z along the beam are just proportional to the total number density of the scatters at the height; and their ratio, $S=\sigma(Z)/\beta(Z)$, remains constant. Blifford's (1970) and Blifford and Ringer's (1969) measurements with airborne particle counters show that, while the aerosol size distribution will change from day to day, on any one day the relative size distribution remains reasonably constant with height. Further, the aerosol optical depth will generally be almost entirely due to scatterers located within a well-mixed surface layer; and it will be quite valid to assume throughout this layer that the aerosols can be characterized by a single value of S.

One of the initial tasks, therefore, is the determination of the ratio S. This ratio can be predicted from Mie theory for polydispersions of spherical scatterers (e.g., Deirmendjian, 1969, or McCormick *et al.*, 1968). As Deirmendjian noted, though, the calculated scattering parameters are sensitive to both the choice of refractive index and the relative size distribution selected. Further, Holland and Gagne's (1970) experimental measurements on a system of irregularly shaped particles indicate that, while the total scattering cross sections agree with those predicted for an equivalent collection of spherical particles, the backscattering cross sections can be considerably below those predicted theoretically. It is true that these were highly irregular scatterers representing an extreme unlikely to occur in nature, but these results emphasize the fact that an assumption that these particles are spherical could possibly lead to considerable error.

It will be shown below, if the lidar observations are supplemented with simultaneous measurements of the total optical depth of the atmosphere at the ruby wavelength, that the ratio S becomes one of the results of the solution of the lidar equation. In addition, the vertical distribution of the extinction and backscattering cross sections of the aerosols can be obtained without any *a priori* assumptions as to their shape, index of refraction or size distribution, other than that their

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relative size distribution remains constant with height. The solution of the lidar equation is developed specifically for this problem, but can be readily applied to the case where S is one of the predetermined inputs to the problem.

One further complication is included. To date, published analytical solutions to the lidar equation have considered only one class of scatterers (Barrett and Ben-Dov, 1967, and Davis, 1969). In mildly turbid atmospheres, certainly typical of those prevailing over Tucson, scatter from both molecules and aerosols will contribute significantly to the problem and must be considered in any solution to the lidar equation. In developing this more general solution, the simpler case with one class of scatterers is considered first. These results are identical to those of Barrett and Ben-Dov (1967), and Davis (1969), but are obtained by using a transformation variable which has direct physical interpretation in the problem. Barrett and Ben-Dov have gone through an excessive number of variable transformations before arriving at their solution. Davis, in preserving a notation that presents the return signal strengths as the number of decibels above or below some reference signal, offers a nice solution, but ends with a notation that is awkward to interpret. Actually, the solution to be developed below was suggested to the authors after a careful review of the solution presented in the Appendix to Barrett and Ben-Dov's article. In their solution, they worked through a number of transformation variables, one of which (their w) is almost twice the optical depth from the lidar to the level in question. This led to the selection of the two-way transmittance as the transformation variable, and, as noted above, this is not just a convenient collection of mathematical terms, but is a variable significant to the problem.

The primary purpose here will be to present a complete analytic solution to the lidar equation for conditions where both molecules and aerosols contribute significantly to the problem. This solution is then applied to lidar observations collected at Tucson, Ariz. A rather extensive error analysis has been incorporated into the final computer program, and these results will also be discussed.

2. Solution to the lidar equation

The basic equation for a vertically looking, monostatic lidar is

$$P(Z)=ECZ^{-2}\beta(Z)T^2(Z), \qquad (1)$$

where $P(Z)$ is the signal whose amplitude is proportional to the power received from a scattering volume at height Z; E is an output monitor pulse whose amplitude is proportional to the transmitted energy; C is a calibration constant which includes the losses in the transmitting and receiving optics, the effective receiver aperture, plus any coefficients required to make the results dimensionally correct; $\beta(Z)$ is the backscattering cross section at height Z; and $T(Z)$ is the transmittance from the lidar to height Z. If the ratio of the extinction cross section to the backscattering cross section, $S=\sigma(Z)/\beta(Z)$, remains constant over the range interval being investigated, then the two-way transmittance, $T^2(Z)$, in Eq. (1) is

$$T^2(Z)=\exp\left[-2\int_0^Z \sigma(z)dz\right], \qquad (2a)$$

$$=\exp\left[-2S\int_0^Z \beta(z)dz\right]. \qquad (2b)$$

A solution to (1) can be obtained very easily if the unknown variables are expressed in terms of $T^2(Z)$. If (2b) is differentiated with respect to range, and then solved for $\beta(Z)$, one obtains

$$\beta(Z)=\frac{-1}{2ST^2(Z)}\frac{dT^2(Z)}{dZ}. \qquad (3)$$

The lidar equation then becomes

$$P(Z)=-\frac{EC}{2SZ^2}\frac{dT^2(Z)}{dZ}. \qquad (4)$$

Separating variables and integrating from height 0 to Z then gives

$$T^2(Z)=1-\frac{2S}{C}\int_0^Z \frac{P(z)z^2}{E}dz. \qquad (5)$$

Substituting this back into (1) and solving for $\beta(Z)$ results in

$$\beta(Z)=\frac{P(Z)Z^2}{CE}\left[1-\frac{2S}{C}\int_0^Z \frac{P(z)z^2}{E}dz\right]^{-1}. \qquad (6)$$

[Eq. (6) is equivalent to Eq. (A10) of Barrett and Ben-Dov (1967) and Eq. (9) of Davis (1969).]

Eq. (5) has been skipped over in the past and is especially important to problems such as that presented by Davis (1969). For this type of problem, S is not known initially, but the total two-way transmittance, $T^2(Z^*)$, over some definite range increment 0 to Z^*, is known. The integral term can be obtained by numerically integrating the data obtained from the lidar returns; and, if the calibration constant C is known, S can be determined directly from (5) expressed as

$$S=C[1-T^2(Z^*)]\left[2\int_0^{Z^*} \frac{P(z)z^2}{E}dz\right]^{-1}. \qquad (7)$$

Thus, Davis did not have to iterate through successive guesses of S (his b), but could have obtained it directly from Eq. (7). Note also, that even if the lidar is uncalibrated, the ratio S/C can be determined from (7), and the value of $T^2(Z)$ at any intermediate range Z between 0 and Z^* can be calculated from (5).

An analogous solution can be formulated for the problem where two distinct classes of scatterers must be considered; for example, Rayleigh scatter from the molecular atmosphere (subscript R) and Mie scatter from particulates suspended in this atmosphere (subscript P). The lidar equation now becomes

$$P(Z)=ECZ^{-2}[\beta_R(Z)+\beta_P(Z)]T_R^2(Z)T_P^2(Z), \quad (8a)$$

where

$$T_R^2(Z)=\exp\left[-2\left(\frac{8\pi}{3}\right)\int_0^Z \beta_R(z)dz\right], \quad (8b)$$

$$T_P^2(Z)=\exp\left[-2S\int_0^Z \beta_P(z)dz\right]. \quad (8c)$$

In the above, S is the ratio of the extinction cross section to the backscattering cross section as defined earlier, but applies only to the particulate scatterers; the equivalent ratio for the molecular scatterers is just $8\pi/3$ if β_R has units [length steradians]$^{-1}$ and σ_R has units [length]$^{-1}$; $T_R^2(Z)$ and $\beta_R(Z)$ can be calculated within a few percent error from standard atmosphere data, or if more accuracy is required, from local radiosonde measurements; and $P(Z)$ is obtained from the lidar returns. Therefore, $\beta_P(Z)$ and $T_P^2(Z)$ are the only unknowns in (8a). If $\beta_P(Z)$ is expressed in terms of the two-way transmittance by applying Eq. (3) to the particulate scattering properties, (8a) then becomes

$$P(Z)=\frac{EC}{Z^2}\left[\beta_R(Z)-\frac{1}{2ST_P^2(Z)}\frac{dT_P^2(Z)}{dZ}\right]\times T_R^2(Z)T_P^2(Z). \quad (9)$$

This can be expressed in the form

$$\frac{dT_P^2(Z)}{dZ}-2S\beta_R(Z)T_P^2(Z)=-\frac{2SP(Z)Z^2}{CET_R^2(Z)}, \quad (10)$$

and when solved for $T_P^2(Z)$ gives

$$T_P^2(Z)=\exp\left[2S\int_0^Z \beta_R(z)dz\right]\times\left\{1-\frac{2S}{C}\int_0^Z \frac{P(z)z^2}{ET_R^2(z)}\exp\left[-2S\int_0^z \beta_R(z')dz'\right]dz\right\}. \quad (11)$$

Note, however, that since

$$\exp\left[2S\int_0^Z \beta_R(z)dz\right]=T_R(Z)^{-3S/(4\pi)}, \quad (12)$$

Eq. (11) becomes

$$T_P^2(Z)=T_R(Z)^{-3S/(4\pi)}\times\left[1-\frac{2S}{C}\int_0^Z \frac{P(z)z^2}{E}T_R(z)^{(3S/4\pi)-2}dz\right]. \quad (13)$$

Substituting this back into (8a) and solving for $\beta_P(Z)$ then yields

$$\beta_P(Z)=\frac{P(Z)Z^2}{CE}T_R(Z)^{(3S/4\pi)-2}\times\left[1-\frac{2S}{C}\int_0^Z \frac{P(z)z^2}{E}T_R(z)^{(3S/4\pi)-2}dz\right]^{-1}-\beta_R(Z). \quad (14)$$

If the ratio S is known, then $T_P^2(Z)$ and $\beta_P(Z)$ can be determined directly from (13) and (14), respectively. On the other hand, in problems where S is not known initially, if the system is calibrated and if the two-way transmittance of the aerosols from the surface to height Z^*, $T_P^2(Z^*)$, can be estimated independently from solar radiometer measurements, S can be calculated from (13) put in the form

$$S=C[1-T_P^2(Z^*)T_R(Z^*)^{3S/(4\pi)}]\times\left[2\int_0^{Z^*} \frac{P(z)z^2}{E}T_R(z)^{(3S/4\pi)-2}dz\right]^{-1}. \quad (15)$$

The height Z^* is the maximum height at which the signal can be easily read from the oscilloscope trace of the lidar return recorded on 35-mm film; and, for the data analyzed here, standard errors in the individual observations at this height have not exceeded 10%.

In general, an iterative solution of this transcendental equation will converge rapidly, as $T_R(Z^*)$ will be quite close to 1 and $T_R(Z^*)$ raised to the powers $3S/(4\pi)$ and $3S/(4\pi)-2$ will be relatively insensitive to variations in S. In very clean atmospheres, though, as the value of $T_P^2(Z^*)$ approaches 1, the term $[1-T_P^2(Z^*)T_R(Z^*)^{3S/(4\pi)}]$ becomes more sensitive to any variations in S, and the solution converges more slowly. Complications arise, though, with the analysis of real data, for both the integral expression and $T_P^2(Z^*)$ appearing in (15) can not be evaluated precisely at the outset of the iterative solution. In fact, their evaluation becomes incorporated into the iterative solution, as will be shown below.

The basic equations have been developed for a vertically looking lidar (zenith angle of zero degrees). Ideally, a lidar so positioned could collect usable data all the way from the surface to its maximum range. The lidar employed here, though, has individual transmitting and receiving optics parallel to each other and approximately 1 m apart. The beam width of the receiver (~4 mrad) is larger than that of the transmitter (~0.75 mrad), so that, at ranges beyond roughly 600 m, the entire transmitted pulse is in the field of view of the receiver, and only these data can be easily interpreted. Therefore, in order to extend the data down close to the surface, observations at larger zenith angles are required.

Before data collected at these larger zenith angles can be used in the basic equations, though, they must be corrected to approximate the returns a vertically looking lidar would have recorded. If the scattering properties of the atmosphere are assumed to be horizontally stratified, then the scattering properties away from the lidar at slant range R can be assumed identical to those directly overhead at the same height $Z=R/\sec\theta$. The range-corrected and output-normalized signal, $P(Z)Z^2/E$, that would have been returned from height Z had the lidar been looking vertically, is from simple geometric considerations

$$\frac{P(Z)Z^2}{E}=\frac{P(R)R^2}{E}[T_R^2(Z)T_P^2(Z)]^{1-\sec\theta}, \quad (16)$$

where $P(R)$ is the signal returned from slant range R with the lidar positioned at zenith angle θ. The expression $[T_R^2(Z)T_P^2(Z)]^{1-\sec\theta}$ corrects for the additional attenuation experienced by the laser pulse in traversing a slant path to height Z instead of a vertical path. Note, though, that this includes $T_P^2(Z)$, which is one of the results to be calculated. On the initial run through the data, $T_P^2(Z)$ is set equal to one at all heights except Z^*, so that the resulting values of $P(Z)Z^2/E$ will be overestimated, tending to cause the initial computations of S to be underestimated. This first estimate of S is then used in (13) to calculate new values of $T_P^2(Z)$, which in turn are used to correct the data for the second run, etc., until the solution converges.

As noted above, $T_P^2(Z^*)$ must also be continually updated. It depends on $\tau_P(0,Z^*)$, the aerosol optical depth from the surface to height Z^*, as given by

$$T_P^2(Z^*)=\exp[-2\tau_P(0,Z^*)]. \quad (17)$$

The solar radiometer, though, only measures the total optical depth of the atmosphere, τ_{tot}. The quantity $\tau_P(0,Z^*)$ is just τ_{tot} less the effects of ozone absorption, 0.007, the optical depth of the entire molecular atmosphere, 0.036, and the optical depth of the aerosols above Z^*, $\tau_P(Z^*,\infty)$. However, $\tau_P(Z^*,\infty)$ can not be measured directly. It was estimated for the initial run through the iterative solution. On subsequent runs, it was approximated by modeling the backscattering cross section of the aerosols above Z^* and then determining the optical depth of these aerosols from

$$\tau_P(Z^*,\infty)=S\int_{Z^*}^{\infty}\beta_P(z)dz. \quad (18)$$

Estimates of S and $\beta(Z^*)$ were obtained from (15) and (14), respectively. The values for $\beta_P(Z)$ at heights above Z^* were based on previously published lidar results from the upper troposphere and stratosphere, as follows.

Schuster (1970) and Goyer and Watson (1968) show the ratio of the total backscattering cross section to the molecular backscattering cross section, $(\beta_P+\beta_R)/\beta_R$, rising steadily from 15 km to a peak at approximately 18 km. At 15 km, the aerosols have a backscattering cross section, $\beta_P(15)\approx 0.3\,\beta_R(15)$. At 18 km, the aerosol backscattering cross section is shown to be anywhere from 0.5–2.5 times the backscattering cross section of the molecular atmosphere at that level. Goyer and Watson's value of $\beta_P(18)\approx 0.8\,\beta_R(18)$, collected over New Mexico, was selected as the best representation of the conditions over Tucson.

The profile of the backscattering cross section of the aerosols above Z^*, therefore, was modeled by allowing $\beta_P(Z)$ to fall exponentially from its value at Z^* to the estimated value at 15 km, then letting it rise exponentially from 15 to 18 km, and finally having it fall off exponentially to infinity with a scale height of 3.75 km (see Elterman, 1968). The value of $\tau_P(Z^*,\infty)$ was obtained by integrating under this model curve according to (18). This was subtracted from $\tau_P(0,\infty)$, obtained with the solar radiometer, to give $\tau_P(0,Z^*)$, which led to $T_P^2(Z^*)$ via (17).

Attempts were made to obtain the calibration constant C using a slightly modified version of the method outlined by Hall and Ageno (1970), in which all lidar returns from the atmosphere are referenced against returns from a standard target consisting of a flat aluminum plate coated with Eastman White Reflectance Paint. This should approximate a Lambert reflector with a reflectivity of 0.992 at the ruby wavelength. The calibration constants obtained by this scheme, however, were quite variable and appeared to be approximately 40% too high.

An alternate scheme was therefore employed, making use of the fact that Tucson experiences some exceptionally clear days on which the atmosphere in the region from 3–6 km can be assumed to be reasonably free of aerosols. On these days, the system can be calibrated against the known molecular backscattering properties of this region.

This calibration fits readily into the iterative solution. If Eq. (14) is solved for C after $\beta_P(Z)$ is set equal to zero at height Z, and after S/C is replaced by the expression obtained when Eq. (15) is solved for S/C, the following equation results:

$$C=\frac{P(Z)Z^2}{E}\frac{T_R(Z)^{(3S/4\pi)-2}}{\beta_R(Z)}\times\left[1-\left(1-T_P^2(Z^*)T_R(Z^*)^{3S/(4\pi)}\Big/\int_0^{Z^*}\frac{P(z)z^2}{E}T_R(z)^{(3S/4\pi)-2}dz\right)\times\int_0^{Z}\frac{P(z)z^2}{E}T_R(z)^{(3S/4\pi)-2}dz\right]^{-1}, \quad (19)$$

where C can be recalculated at the end of each complete iterative cycle, providing an updated value for the

TABLE 1. Measured and computed parameters from lidar data. See text for explanation.

Date	τ_{tot}	$\tau_P(0,\infty)$	$\tau_P(0,Z^*)$	Z^* (−km)	S (sr)	Error in S (per cent)
5 April	0.206	0.163	0.141	6.75	12.7	30
6 April	0.336	0.293	0.267	4.20	14.5	11
10 May	0.050	0.007	0.0055	6.00	2.5	180
18 May	0.046	0.003	0.0020	6.00	1.6	630

TABLE 2. Theoretically determined values of S vs the shape ν of the aerosol distribution function.

ν	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
S	12.5	13.3	14.5	16.0	17.7	19.6	21.3	22.7	24.5	24.5	22.6

next run through the data. This calculation can be performed at a single level where it is reasonable to suppose that only molecular scatter predominates; or a search can be made by performing the above calculation over all levels, and the level at which the minimum value is obtained will correspond to the level at which the aerosols are contributing least to the net lidar return.

3. Data collection and analysis

Data collected at Tucson, Ariz., on 5 April, 6 April, 10 May and 18 May 1971 have been analyzed. Vertical profiles of scattering parameters were derived for each day from 16 lidar observations (four observations collected at each of four elevation angles). The individual lidar observations were reduced to some 30–40 points selected at 150 m intervals.

Eq. (19) was used to compute the calibration constant on 10 and 18 May, as the atmosphere over Tucson was very clear. Solar radiometer data collected at the times of the lidar observations showed total aerosol optical depths of 0.007 and 0.003, respectively. The value of C obtained on 10 May was slightly below that of 18 May and has been used in the analysis of the lidar data presented below. Meteorologically, these two days were somewhat similar. In both cases, dry cold fronts had moved eastward through Arizona on the previous day and very clean air followed in the northwest flow behind these fronts.

On 5 and 6 April, the atmosphere over Tucson was very dusty, with total aerosol optical depths at the time of the lidar observations of 0.16 and 0.29, and with surface visibilities on the order of 50 and 20 km, respectively. Early on 4 April, a dry cold front moved down from the north through Arizona, New Mexico and western Texas, stirring up considerable dust, which was then blown toward Tucson by the easterly low-level anticyclonic flow which set in over the Southwest on 5 and 6 April.

Table 1 lists some of the data pertinent to the problem: these include the total optical depth of the atmosphere (τ_{tot}) at 690 nm determined from the solar radiometer located at the University of Arizona 6 mi south of the lidar site; the total aerosol optical depth $[\tau_P(0,\infty)]$; the estimated aerosol optical depth $[\tau_P(0,Z^*)]$ of the layer from the surface to Z^*; and $S=\sigma_P(Z)/\beta_P(Z)$, one of the major results of the calculations.

Theoretically determined values of S are presented in Table 2. The aerosols have been assumed spherical with a size distribution function that follows a Junge law given by

$$\frac{dn}{d\log r}=cr^{-\nu},$$

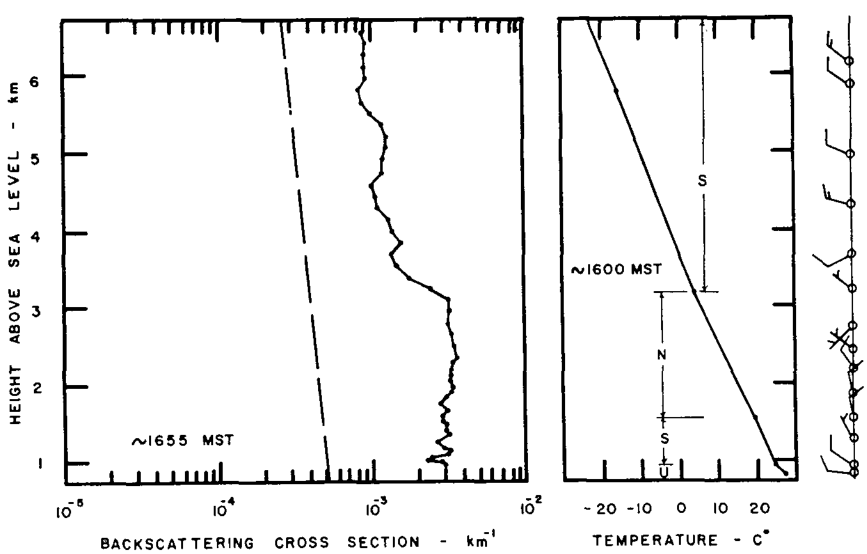

FIG. 1. Vertical profiles of the backscattering cross section, temperature and wind for 5 April 1971. (Units of backscattering cross section should be km^{-1} sr^{-1}.)

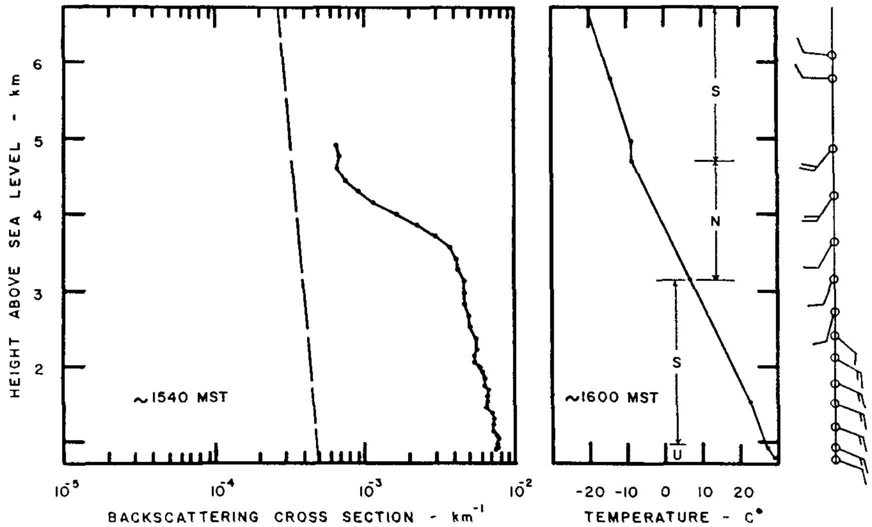

Fig. 2. Vertical profiles for the backscattering cross section, temperature and wind for 6 April 1971. (Units of backscattering cross section should be $km^{-1}\ sr^{-1}$.)

where n is the number of particles per unit volume of radius r per unit interval of r, and c and ν are constants. The value of c determines the total number density of the particles, while the value of ν determines the shape of the distribution function. Computations were carried out at the ruby wavelength for aerosols with a real index of refraction of 1.54 and for values of ν between 2 and 4 (typical for atmospheric aerosols). Note that experimentally determined values of S for the dusty skies of 5 and 6 April correspond to the smaller values of ν in Table 2 and that small values of ν indicate a preponderance of large-sized particles. Possible reasons for the large discrepancies between the theoretical and experimental values of S for 10 and 18 May are noted in the discussion of errors presented below.

Figs. 1–4 show the calculated backscattering cross sections of the aerosols, along with the temperature and wind profiles obtained from the Tucson radiosonde ascent closest to the time of lidar observations. The backscattering cross sections of the aerosols can be compared with the dashed line depicting the molecular backscattering cross section. The stability of the atmosphere is noted along the temperature profile, with N indicating a dry adiabatic lapse rate or neutral stability, S a stable lapse rate which (with the very dry atmosphere experienced here) includes any lapse rate that is less than dry adiabatic, and U a superadiabatic, absolutely unstable condition. In general, marked variations in the scattering properties of the aerosols are closely aligned with the thermal structure. Such

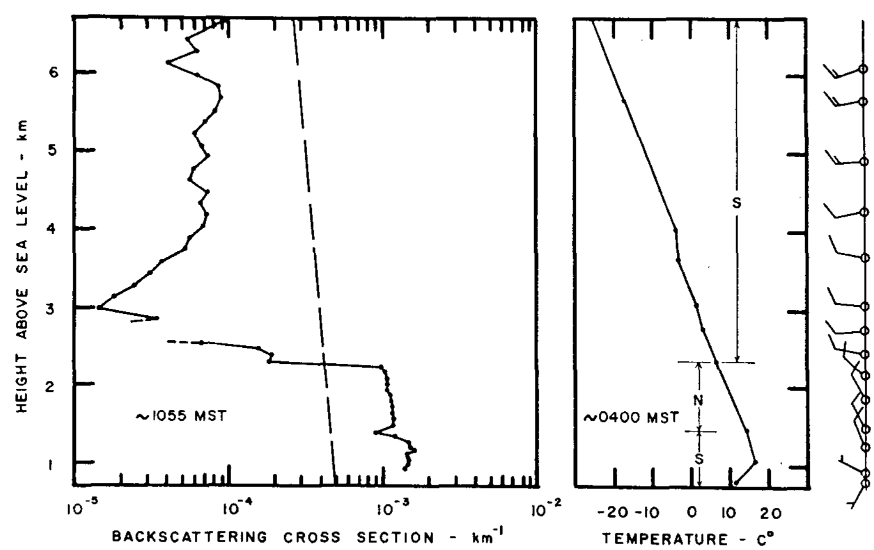

Fig. 3. Vertical profiles of the backscattering cross section, temperature and wind for 10 May 1971. (Units of backscattering cross section should be $km^{-1}\ sr^{-1}$.)

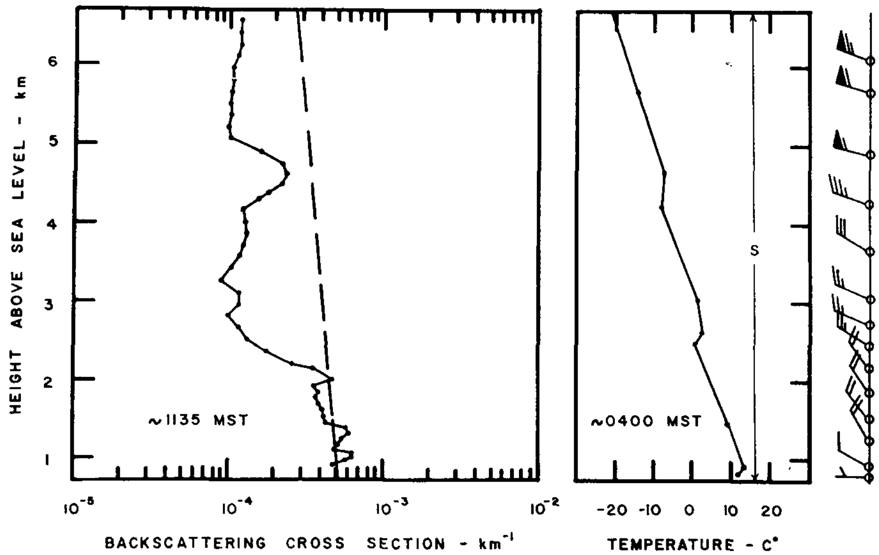

FIG. 4. Vertical profiles of the backscattering cross section, temperature and wind for 18 May 1971. (Units of backscattering cross section should be km^{-1} sr^{-1}.)

stratification of aerosols has been noted in previous investigations (e.g., Collis *et al.*, 1964, and Viezee and Oblanas, 1969).

A complete error analysis has been performed, but only the general conclusions will be discussed here. This analysis has shown that errors in $T_P^2(Z^*)$ and C will contribute greatest to the final errors in S, $\beta_P(Z)$ and $\sigma_P(Z)$, and that errors in data points, $P(Z)$, taken from the lidar returns and the output monitor signal E, can essentially be ignored.

Errors in $T_P^2(Z^*)$ will be the prime source of errors in the calculated values of $\sigma_P(Z)$ and S, but their effect will diminish as the atmosphere becomes more turbid. Therefore, on the relatively turbid days of 5 and 6 April, errors of 7 and 6% in $T_P^2(Z^*)$ resulted in $\sigma_P(Z)$ profiles and values for S with standard errors of roughly 30 and 11%, respectively. On the other hand, errors in C (estimated to be 6%) were the primary source of the resulting errors (approximately 12 and 8%) in the calculated values of $\beta_P(Z)$.

On very clear days, estimates of $T_P^2(Z^*)$ are quite accurate (standard errors of 2.2% were estimated for both the 10th and 18th of May). Errors in $T_P^2(Z^*)$, though, become greatly amplified, so that on these days the computed errors in S and $\sigma_P(Z)$ were 180 and 630%, respectively. Thus, for these very clear days, it was essentially impossible to determine S or $\sigma_P(Z)$. Further, when the aerosols are the primary source of backscattered energy, the errors in $\beta_P(Z)$ will be relatively small compared to those resulting when the net return is overwhelmed by the signal from the molecular atmosphere. Thus, on 10 May below 2.2 km, where $\beta_P(Z)\approx 2\times\beta_R(Z)$, the values of $\rho_P(Z)$ were accurate to within 11%. Above 2.2 km, however, $\beta_P(Z)\ll\beta_R(Z)$, and the errors ranged from 40 to 150%. On 18 May below 1.5 km, the backscattering cross section of the aerosols was only slightly greater than that of the molecular atmosphere, and the values of $\beta_P(Z)$ were accurate to within 15%. Above this height, $\beta_P(Z)$ fell off rapidly, and its error increased to over 100%. Thus, while the very clear days offered the best means of calibration, the lidar calculations of S and $\sigma_P(Z)$, and $\beta_P(Z)$ when $\beta_P(Z)\leqslant\beta_R(Z)$, can not be relied upon, as they were considerably more sensitive to errors in the input data than on the more turbid days.

4. Conclusions

To date, no accurate description of atmospheric aerosols has been obtained from lidar measurements, primarily because 1) lidars are inherently difficult to calibrate, and 2) because the analytic tools available for the analysis of the data have not been developed sufficiently. This paper has attacked the second problem listed above, by introducing an advanced analytic solution to the lidar equation that considers the realistic situation of two classes of scatterers, aerosols and molecules. It has also emphasized the importance of supplementing lidar experiments with additional data, such as solar radiometer measurements. When this is done, a true profile of the total extinction cross section can be obtained in turbid atmospheres, even though the lidar is uncalibrated. Under very clear skies, as might be anticipated, the aerosol scattering properties (being comparable to or less than the molecular scattering properties) tend to become submerged in the observational errors. Precise input data become especially important here. In any case, for these analytic procedures to be fully implemented, the lidar must be accurately calibrated.

REFERENCES

Barrett, Earl W., and Oden Ben-Dov, 1967: Application of the lidar to air pollution measurements. *J. Appl. Meteor.*, **6**, 500–515.

Blifford, Irving H., Jr., 1970: Tropospheric aerosols. *J. Geophys. Res.*, **75**, 3099–3103.

——, and Lynn D. Ringer, 1969: The size and number distribution of aerosols in the continental troposphere. *J. Atmos. Sci.*, **26**, 716–726.

Collis, R. T. H., F. G. Fernald and M. G. H. Ligda, 1964: Laser radar echoes from a stratified clear atmosphere. *Nature*, **203**, 1274–1275.

Davis, P. A., 1969: The analysis of lidar signatures of cirrus clouds. *Appl. Opt.*, **8**, 2044–2102.

Deirmendjian, D., 1969: *Electromagnetic Scattering on Spherical Polydispersions*. New York, Elsevier Publ. Co., 290 pp.

Elterman, L., 1968: UV, visible, and IR attenuation for altitudes to 50 km. Environ. Res. Papers No. 285, Air Force Cambridge Research Laboratories, Bedford, Mass., 49 pp.

Goyer, Guy G., and Robert D. Watson, 1968: Laser techniques for observing the upper atmosphere. *Bull. Amer. Meteor. Soc.*, **49**, 890–895.

Hall, F. F., and H. Y. Ageno, 1970: Absolute calibration of a laser system for atmospheric probing. *Appl. Opt.*, **9**, 1820–1824.

Holland, A. C., and G. Gagne, 1970: The scattering of polarized light by polydisperse systems of irregular particles. *Appl. Opt.*, **9**, 1113–1121.

McCormick, M. P., J. D. Lawrence and F. R. Crownfield, Jr., 1968: Mie total and differential backscattering cross sections at laser wavelengths for Junge aerosol models. *Appl. Opt.*, **7**, 2424–2425.

Schuster, Burton, 1970: Detection of tropospheric and stratospheric aerosol layers by optical radar (lidar). *J. Geophys. Res.*, **75**, 3123–3132.

Viezee, William, and John Oblanas, 1969: Lidar observed haze layers associated with thermal structure in the lower atmosphere. *J. Atmos. Sci.*, **8**, 369–375.