Measurement of Snowfall by Radar

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ABSTRACT

Using a CPS-9 radar (wavelength 3.2 cm, beamwidth 1°) in dry snow, radar returns from a layer at an average height of 5000 ft were converted to snowfall rates (taking $Z \propto R^{2.0}$) and summed over 36 hr to obtain a map of snowfall amount. This has been compared with a "climat" map based on depth measurements of new fallen snow at 140 climatological stations within 100 mi of the radar, said amounts ranging from under 2 to over 10 inches. For the 5550 mi² area within 42 mi of the radar, the average amount by radar was set equal to the average climat amount. The radar/climat ratio was mapped, with the distribution being log normal. For ranges <42 mi, 68% of the data fell between values of 0.76 and 1.32. For ranges between 42 and 100 mi, 68% fell between 0.63 and 1.60. This scatter is about the same as other workers have found for rain. In the relation $Z = aR^b$, a value for b of 2.0 proved appropriate to this particular storm, with some evidence that a slightly higher value might have been a little better.

1. Introduction

Accumulation of snowfall over two days of stormy weather, ranging from 2 to 10 inches of snow, has been mapped 1) in terms of snow-depth measurements by climatological observers at 140 locations within 100 mi of Montreal Airport, and 2) by summation of low-level radar observations of the falling snow, over an area somewhat less, using the McGill Weather Radar, which at the time was a CPS-9 radar located at the airport.

The radar program produced CAPPI (constant altitudes PPI) maps at height intervals of 5000 ft; a brief description here of the variations with time and height, derived from these maps, may provide a helpful introduction. Fig. 1 shows, for four heights, the variation with time of the snowfall rate, averaged over an area of 10,000 mi². Radar provided maps of the reflectivity factor Z; we assumed $Z \propto R^2$ at all heights, and obtained the constant of proportionality by equating radar-derived to climatologically-observed snowfall rates, as will be described later. The assumed relation is appropriate at low levels; above 10,000 ft, where we have less confidence in the relation, values of R may be greater than those shown in the figure.

The two separate snowfalls (with an hour's respite between) were produced by a large complex extratropical cyclone. The first snowfall was associated with an occluding frontal wave that moved northeastward along the New England coast. This snowfall spread northward across the area from 1500 to 1900 (all times EST) on 6 February 1964; its continued northward motion took it out of the area over the interval 0300–0700 on 7 February. Two hours later, from 0900 to 1200, an elongated pattern of snow moved in from the southwest, associated with a NW-SE pressure trough moving northeastward along the St. Lawrence valley. The snow at a height of 15,000 ft moved out of the area at 1600; it persisted more than four hours longer at 10,000 ft, and for more than another four hours at 5000 ft.

Fig. 2 shows the height variations of the radar-derived snowfall amount, for one area of 2000 mi² in each quadrant. Values at the surface are from climatological observations, not from radar. The radar values have been normalized by equating the radar average at about 5000 ft, for a central area of 5550 mi², to the surface average for the same area. With that calibration procedure, the difference in the figure between values at 5000 ft and at the surface were bound to be small. To obtain the surface value of snowfall rate or amount from radar, it is necessary that the height interval occupied by the radar beam contain snow with nearly the same average snowfall rate, or more specifically, with nearly the same reflectivity, as the snow at the surface. In Fig. 2, if this requirement is met from the surface up to 5000 ft, as we assume, it is met to a height of 7000 ft or so in the northern quadrants and, to a height of 12,000 ft or so in the sample regions in the southern quadrants. With the radar beam extending 1° above the horizon, and the earth's curvature dropping below it, these heights indicate some dropping-off of reported target strength beyond ranges of 60 mi in the northern quadrants, and 90 in the south.

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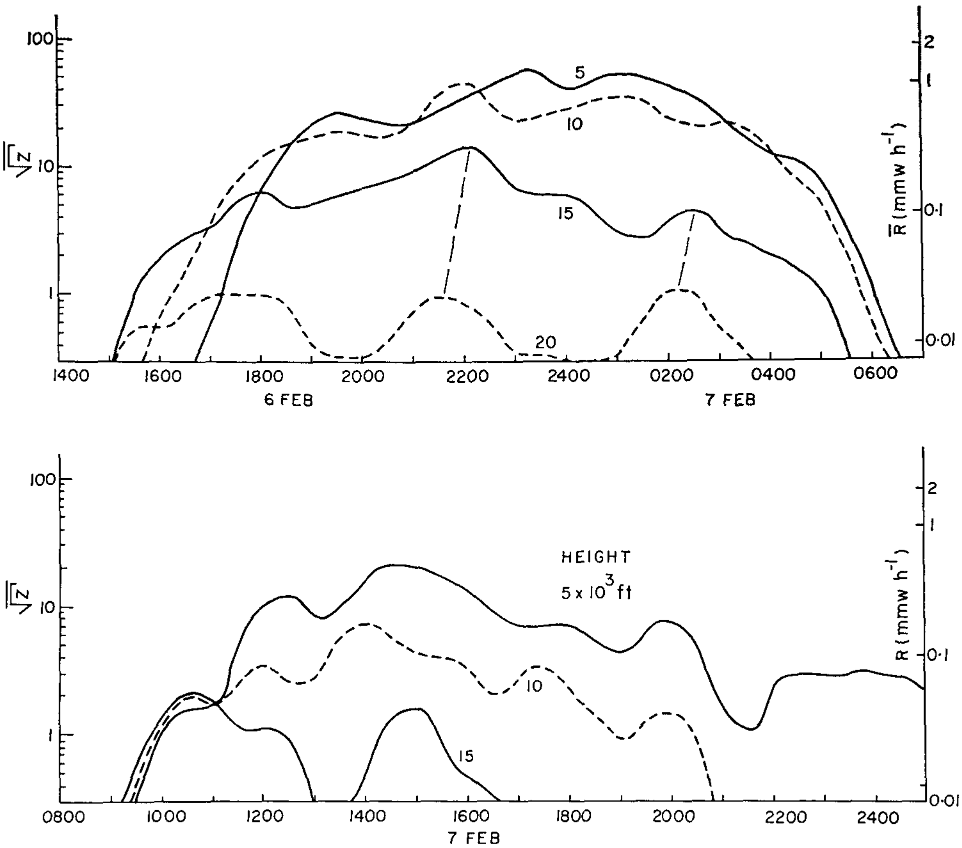

FIG. 1. Time variations on the square root of radar reflectivity Z, averaged over an area of 10 000 mi², and the snowfall rate R (millimeters of water per hour) derived from it, at height intervals of 5000 ft.

2. Climatological measurements

Most of the climatological stations, located in Fig. 3, are operated by unpaid volunteers, who record the ruler-measured amount of new-fallen snow every 24 hr, converting inches of snow to inches of water by dividing by 10. Generally, there is no information about the rate of accumulation (apart from radar) or the density of the snow (but at McGill Observatory, 9 mi east of the radar, the 3.2 and 2.6 inches of the first and second snowfalls were equivalent, respectively, to 0.30 and 0.29 inches of water). Monthly summaries of the climatological data are published by Canadian and American government agencies. Daily amounts of precipitation were recorded within 1 hr of 0800 at about 60% of the stations, within 1 hr of 1600 at another 25%, and at midnight at the rest. As it happened, these variations created no problem in this study.

A moderate degree of subjective smoothing has eliminated some small-scale irregularities from the isohyets of Fig. 3. Four stations reported more than 10 inches of snow in two small areas, while twenty stations, mostly to the southeast, reported less than 2 inches. The intermediate isohyets roughly parallel the St. Lawrence River.

a. Radar measurement of snowfall amount

The CPS-9 radar (wavelength 3.2 cm, beamwidth 1°) operated with a routine antenna program: the antenna rotated steadily in azimuth with the angle of elevation increasing by three-quarters of a degree after each rotation. For a CAPPI map at a nominal height of 5000 ft, elevation 0° was used from the farthest range (100 mi) to 73 mi, then $\frac{3}{4}$° to 42 mi, then $1\frac{1}{2}$° and progressively higher angles to zero range. For the present work,

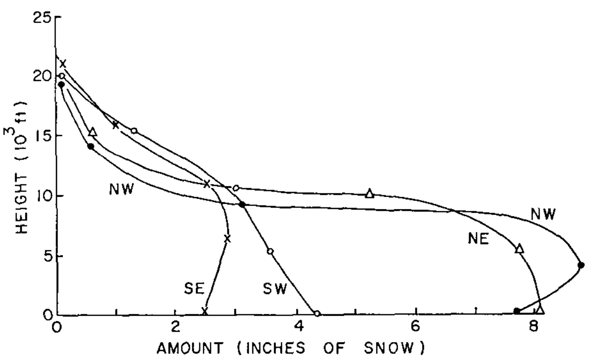

FIG. 2. Radar-derived snow amount falling through any given height, for an area of 2000 mi² at short range in each quadrant.

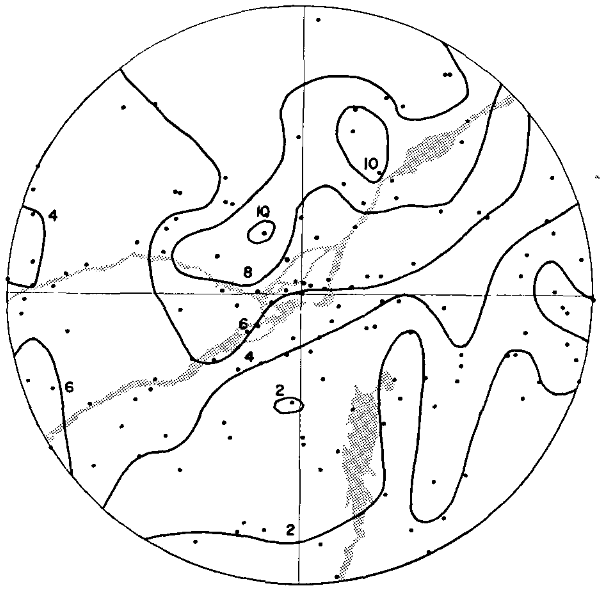

FIG. 3. "Climat" snowfall amounts, from climatological observers' measurements of depth of new-fallen snow. Numbers on isohyets give inches of snow; dots locate climatological observers. The map is centered on the radar site at Montreal Airport, and has a radius of 100 statute miles. Background map shows the St. Lawrence River flowing from southwest to northeast, the Ottawa River flowing from the west to join the St. Lawrence at Montreal, and Lake Champlain to the south-southeast.

the range intervals thus created are treated separately, and described as long, medium and short range (Fig. 4). Unfortunately, it was discovered later (from our analysis of snow data) that the vertical axis of the radar was tilted approximately $\frac{1}{2}°$ out of true; the resulting elevation angles are given in Fig. 5.

One hundred CAPPI maps, at 22.5-min intervals, covered the two periods of snowfall. Each map was reproduced in a stepped-grey scale by the flying-spot scanning of the film record, with grey-shade boundaries at nominal snowfall rates of 0.11, 0.33, 1.0 and 3.0 mmw hr^{-1} (millimeters of water per hour), based on $Z=2000\,R^{2.0}$ (Gunn and Marshall, 1958) and the calibration of the radar. From these maps, data were extracted at grid points 15 mi apart in a square array. The total amount of snow was obtained at each grid point by assuming 1) that the rate indicated on the map persisted for 22.5 min (the interval between maps), and 2) that 10 inches of snow corresponded to 1 inch of water. A correction factor was applied to the total amount, varying with the map location, to allow for non-uniformities in the analogue response of the cathode-ray tubes and lenses; this factor ranged from $\frac{2}{3}$ to $\frac{3}{2}$.

The average radar amount at the 20 grid points within a 42-mi range was found to be lower, by a factor of 3, than the average climat amount over the same area; thus, the radar amounts were increased by this factor, which corresponds to 9.5 dB in received power. The resulting map of snowfall amount appears as Fig. 6.

b. *Comparison of radar and "climat" measurements*

The pattern on the unshaded portion of the radar map (Fig. 6) resembles that on the climat map (Fig. 3), to the extent that locations of maxima, minima and strong gradients are much the same, as is the orientation of isohyets along the St. Lawrence River. Fig. 7 gives

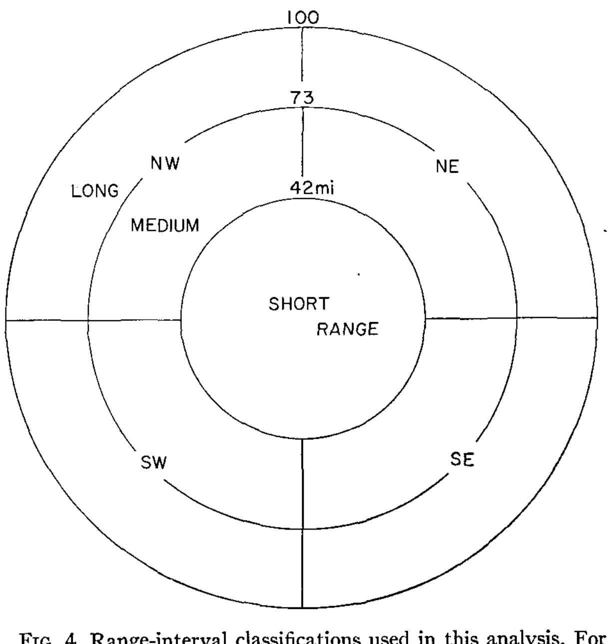

FIG. 4. Range-interval classifications used in this analysis. For "5000-ft" CAPPI maps, the elevation angle decreases at 42 and 73 mi; circles at these distances bound regions described as short, medium and long range.

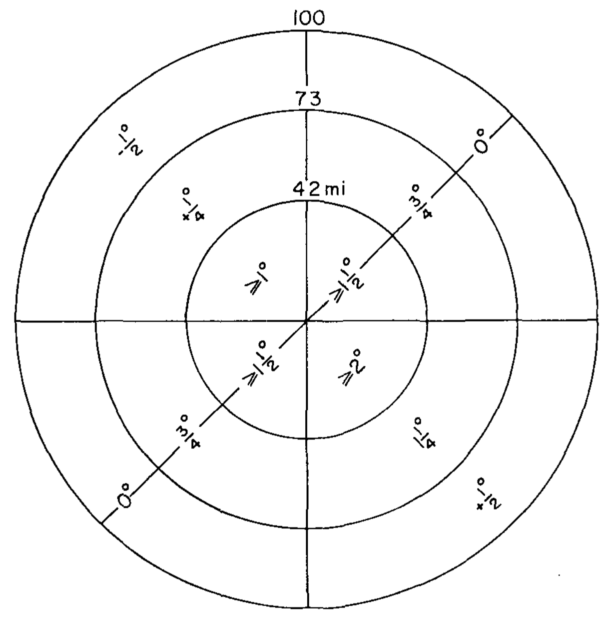

FIG. 5. Angles of elevation for the low-level map, varying with azimuth because of unintended tilt of vertical axis of antenna, by $\frac{1}{2}°$ toward the northwest.

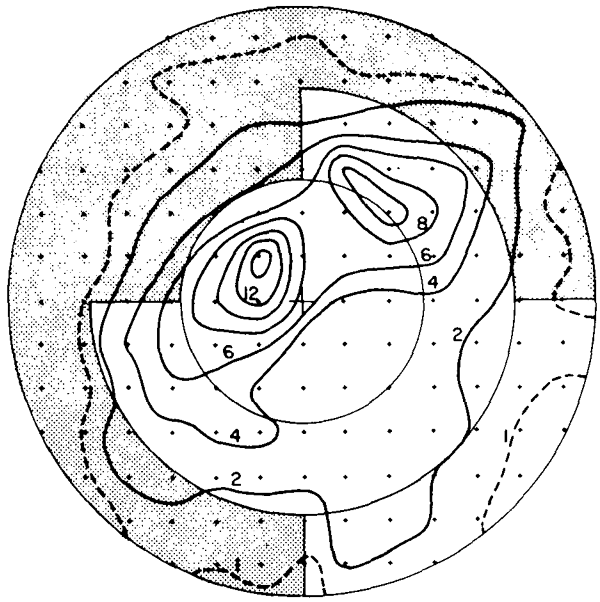

Fig. 6. Radar-derived snowfall amounts, given in inches of snow by numbers on isohyets. Dots locate grid points for which amounts read from individual radar maps were summed. Regions with low angles of antenna elevation ($\frac{1}{4}°$, $0°$, $-\frac{1}{2}°$ in Fig. 5) are shaded.

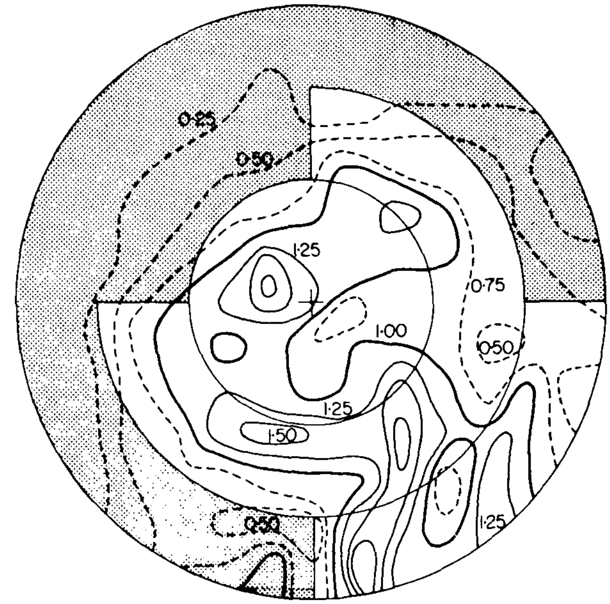

Fig. 7. Ratio of radar-derived snow amount to climat amount. Regions with low angles of antenna elevation ($\frac{1}{4}°$, $0°$, $-\frac{1}{2}°$ in Fig. 5) are shaded.

the ratio of radar-derived to "climat" amount, in terms of values taken from the radar and climat maps at the grid points. The shaded areas (Figs. 6 and 7) are those where the axis of the 1° beam had an elevation of less than $\frac{1}{2}°$ (as shown in Fig. 5). Since the low ratio values in those areas are fairly attributed to low radar values, resulting in turn from the low elevation of the beam, data from the shaded area have been rejected. The boundary of the rejected region might better have been drawn as a continuous curve (to account for the effect of the tilted antenna axis) and then have been corrected further for the varying elevation of the skyline. In the northwest quadrant, where $\frac{1}{4}°$ gave poor results, low hills provided a skyline ranging from $\frac{1}{8}°$ to $\frac{1}{4}°$.

Fig. 8 shows log-log plots of radar vs climat amounts. For the 20 grid points of the short-range interval, the range of climat amounts is from 2.3 to 9.5 inches of snow, that of the radar from 2.2 to 14.5 inches. Least-squares fitting of these data gave the straight-line locus shown, with a slope of 1.07. The slope depends on the relation used between Z and R (and we used $Z \propto R^2$) *because* in these storms the values of Z and R were known to be higher in those regions where the amount

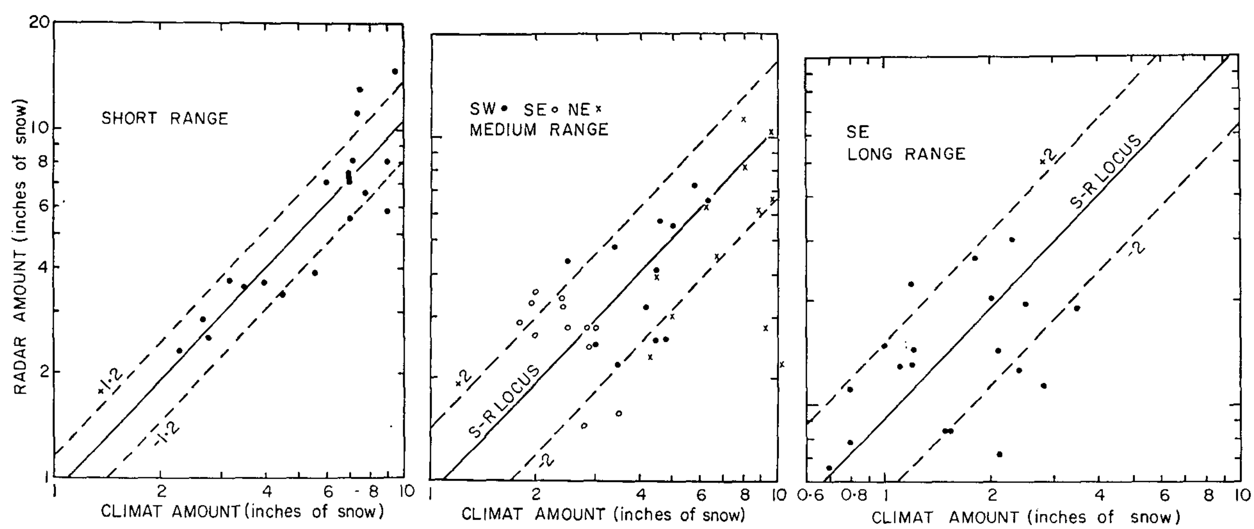

Fig. 8. Regressions of radar vs climat snow amounts at various ranges. Note that ordinate scale of 1 to 20 inches of snow applies only to short and medium ranges. The scale at long range runs from 0.6 to 10 inches of snow.

was higher. Thus, at a point with 8-inch snow, most of the snow fell at a rate of about 4 mmw hr^{-1}, while for a point with 2.8 inches, the dominant rate was about 0.5 mm hr^{-1}. The observed slope of 1.07 suggests that the square-law relationship $Z \propto R^2$ is not quite high enough to interrelate the light and heavy snows of this storm.

The broken lines of the regression plots have been drawn to bound a region, centered on the solid-line locus, containing 68% of the data, so that the vertical separation of the broken lines gives roughly 2σ, where σ is the standard deviation of the distribution with the logarithm of the radar-derived amount. (The actual distributions were found to be roughly log-normal.) The values of σ found in this way were 0.12 for the short-range region (the 5550 mi^2 within 42 mi of the radar), 0.20 for the medium-range region (8300 mi^2 in the range interval 42–73 mi), and 0.20 again for the long-range region (3700 mi^2 in the range interval 73–100 mi). The locus found for the short-range region was used for the other regions. The amounts in the long-range region are less, as it happens, lying between 0.7 and 3.7 inches. The values of 0.12 and 0.20 give corresponding coefficients of variation of 1.32 and 1.6. Thus, 68% of the values of the ratio of radar to climat amount fall in the interval −24% to +32% at the short range, and −37% to +60% at the medium and long ranges.

3. Discussion: The relation between Z and R

We have taken $Z \propto R^2$ because of Gunn and Marshall's (1958) relation $Z=2000\,R^{2.0}$. In an earlier analysis of the snowfalls considered in this paper, Carlson (1968) found that $Z \propto R^{2.67}$ reduced the variance. Shortly afterward, however, he discovered shortcomings in the recordings he had used, as well as the antenna tilt mentioned earlier. This led to the re-working reported here, with evidence for the storm or pair of storms that has been analyzed that a value of 2.0 for the index is usable, and that a slightly higher value would have been better.

Sekhon and Srivastava (1970) have studied the properties of exponential particle size distributions, particularly as they relate to snow and to the power-law relationship between Z and R. They have reanalyzed the data of Gunn and Marshall, together with those of Imai *et al.* (1955), Magono (1957) and Ohtake (1969) and have produced a consistent set of equations for snow, of which that relating Z to R is $Z=1780\,R^{2.21}$. The present study supports the finding of an index slightly greater than 2.0. It should be borne in mind, however, that in the present study there were just two snowfalls, related to one storm; further, most of the heavy snow came from the first of these snowfalls and fell north of the radar, while most of the light snow came from the second snowfall and fell south of the radar.

Ohtake and Henmi (1970) have found the relation between Z and R to vary with the crystal type dominant within the aggregate flakes. They give values of a and b in the relation $Z=aR^b$ for six dominant crystal types, as well as for the graupel and hail that might be found mixed with snow. The values of b range from 1.5 to 2.3, but are most often below 1.8. However, their findings are fairly compatible with the use of the Gunn-Marshall or Sekhon-Srivastava relationship for snow, in general, if (i) for $R>0.3$ mmw hr^{-1} the predominant crystal types are plane and spatial dendrites and stellars, and (ii) for $R<0.3$ mmw h^{-1} (and if these lower rates are relevant) the predominant types are needles, bullets, plates and columns.

Measurements of snowfall rate by radar have been reported by Kodaira and Inaba (1955) and Austin (1963). The former used a 3-cm radar to measure backscattered power from the falling snow over the snowflake sampler of Imai *et al* (1955). The power was converted to radar snowfall rates, using the Z-R relationship derived at the same time from the sampler. Snowfall rates measured by the sampler and by radar proved to be comparable, but concern was expressed about the rapid variations in the size distribution of snowflakes and in the resulting relation between Z and R. Thus, in the 2-hr period of their minute-by-minute comparisons, the index b in the relation $Z=aR^b$ was kept constant at 1.8, but the coefficient a jumped from 600 up to 2400, and later dropped to 1800.

Austin (1963) compared snowfall rates measured by gages and by radar. Because the relation $Z=2000\,R^{2.0}$ was for aggregated flakes and the "rain" relation $Z=200\,R^{1.6}$ was considered better for single crystals (private communication from K.L.S. Gunn), Austin used $Z=1000\,R^{1.6}$ as an intermediate relation. This does not appear now to have been a helpful move, but it gave quite good results for five major snowstorms with precipitation rates ranging from 4 to 30 mmw hr^{-1} and one case of showery snow. For three storms in which light snow predominated (maximum rates 0.5–4.0 mmw hr^{-1}) the radar-determined rates were low.

4. Discussion: Quality of maps of snowfall amount

Studies of the sort that we have made for one snowstorm have been made for numbers of rainstorms by Huff (1966), Leber *et al.* (1961) and by Wilson (1963, 1970a, b). The variability of our measurement of amount of snow is roughly the same as theirs for amount of rain. For example, Huff's rainstorm with the largest amount of rain gave a mean of 1.38 inches over an area of 400 mi^2. We have plotted his radar/gage ratios for his 49 gage locations on probability paper (Fig. 9), and found the same value of standard deviation as for our ratios for snow at radar ranges <42 mi. In the use of radars with a 2° beam (the 10-cm WSR-57) Leber *et al.* obtained less scatter on one occasion than Huff, out to a range of 100 mi, while Wilson and others with more cases found more scatter, particularly at ranges >60 mi. The need for target intensity invariant with height from the surface up to the top of the beam is obvious. The intelligent use of reference gages is emphasized by Wilson. For snow, there is the same opportunity for reference gages. However, the height to which the target strength is invariant is liable to be less; thus, the range to which radar measurements are relevant to surface conditions is liable to be less for snow than for rain showers. For snow melting to continuous rain, the issue is confused by the presence of the bright band.

5. Conclusions

For one major snowstorm, the amount of snow that fell, ranging from 2 to 10 inches of snow (of density 0.10 at the one point where this was measured) has been mapped, within 100 mi of the radar, in terms of climatological observations and radar records of the falling snow. The maps of snowfall amount resembled each other, to about the extent that is found in maps made by other people for summer rain. Thus, the scatter of the ratio of radar-to-depth measurements was such that 68% of the values fell between 0.76 and 1.32, for ranges <42 mi, and between 0.63 and 1.60 for ranges <100 mi. This was for a radar beamwidth of 1°. If the ratio were that of the radar measurement to the true amount of snow on the ground, the scatter would surely be less, because it is difficult if not impossible to determine the true amount by depth measurements.

To the extent that work on a single storm is indicative, then, we conclude that useful maps of snowfall amount can be prepared from radar records. This conclusion could have been reached earlier from Austin's (1963) work, but the present study provides a demonstration of actual mapping, and an upper limit for the amount of scatter in the data. Austin's work shows what can be done without the use of gage measurements on each occasion to calibrate the radar. The work of Kodaira and Inaba (1955) indicates the sort of variability in the relation between Z and R that can be expected, and supports for snow Wilson's recommendation for rain that a radar's performance in measuring precipitation can be strengthened by complementing it by a few gages.

In the relation $Z=aR^b$, the value 2.0 for snow worked well, although a slightly larger value might have worked a little better. This provides the limited support that can come from one storm for Gunn and Marshall's 2.0 and Sekhon and Srivastava's 2.21. Nothing was learned about the value of a.

When this work was undertaken, it appeared that the sensitivity of the radar to falling snow varied violently with range. Now, after much effort, we conclude that most of this variation is attributable to equipment faults in the levelling of the antenna and to cumulative errors inherent in the analogue components of the data-processing system. This demonstrates (as we see it) how shortcomings are liable to persist in the qualitative performance of a weather radar, unless that performance is subjected to searching analysis.

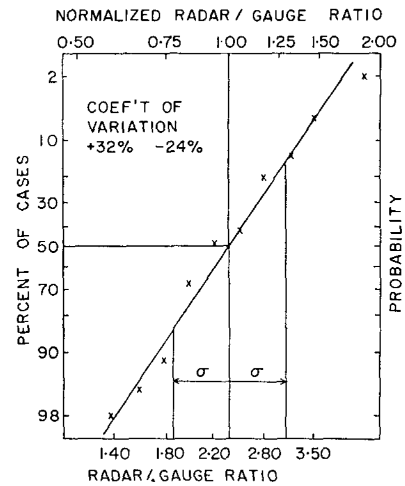

FIG. 9. Radar/gage ratios at 49 gage locations for a rainstorm with mean amount of 1.38 inches over 400 mi² [observed and reported by Huff (1966)]. Plotted on probability paper, they are shown to be distributed log normally with a standard deviation of 0.12, so that 68% of ratio values fall between −24% and +32%.

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