Calculations Related to Formation of a Rain Shower by Salt Seeding[1]

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1. Possible role of salt in shower formation

The points made by Havens (1972) and by Blanchard (1972) are well taken. We especially regret the error in reporting the cloud top temperature as -2C, which apparently originated as a copying error, and which further confounded the already confusing sentence on page 782. The sentence would better convey the sense of the situation if it read, ". . . but AgI seeding was discontinued when it was concluded (perhaps erroneously) that the clouds were not rising far enough above the 0C level for AgI seeding to be effective." Earlier, the project meteorologist had declined to declare an "official" randomized test case as the clouds appeared to have little potential for artificial modification should the random decision be "silver iodide."

When the radiosonde data became available, it appeared that the temperature of the coldest cloud summits had been close to -10C. Despite this, we concluded that salt seeding had initiated the shower. This conclusion was based upon 1) the absence of any visible glaciation at the top of the raining clouds, and 2) the fact that the dimensions of the rain area were roughly those of the area seeded with salt, and larger than the area where AgI was dispensed. With regard to point 1), there may have been ice particles of natural origin or

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[1] *Editors' note:* This is the reply to the letters by Duncan C. Blanchard and A. Vaughn Havens which were published in the April issue (*J. Appl. Meteor.*, **11**, 556–557).

produced by AgI in the cloud top despite our visual impressions.

Although the experiment was convincing to the persons directly involved, it was perhaps too much to expect our report of it to convince the larger community of scientific readers that salt seeding initiated the shower. The reader is referred to the color photograph in the original paper (Fig. 5 of Biswas and Dennis, 1971) and invited to draw his own conclusions.

Blanchard (1972) and others[2] have questioned our conclusion that the 23 July experiment "clearly demonstrated" that salt seeding initiated a Langmuir chain reaction. This point is also valid; we cannot be absolutely certain that salt seeding caused the shower and, on the other hand, some kind of a shower might have been produced by salt seeding without a chain reaction occurring.

The calculations started in the winter of 1969–70 and mentioned briefly in our paper have now been extended considerably in line with Dr. Blanchard's suggestion. The results are summarized in the remainder of this note. They are not presented as proof that salt seeding initiated the shower, but merely to show how salt seeds could have acted in the 23 July cloud.

Section 2 presents elementary calculations which indicate that the number of raindrops in the shower exceeded the number of salt seeds released below cloud base. Section 3 presents results of calculations with a steady-state cloud model to deduce probable updraft and liquid water content profiles in the cloud. In Section 4 a precipitation embryo growth model is used to trace the trajectories and growth rates of the salt seeds within the model cloud. It shows that the larger salt seeds should have reached breakup size within 15 min of their arrival at cloud base. This renders plausible the concept of a chain reaction leading to a significant rain shower as opposed to the sprinkles noted in some previous hygroscopic seeding experiments (e.g., Mason, 1971, 377–380).

2. How many raindrops?

Apart from visual observations from the ground and from the seeding aircraft, the only information available on the rain produced in the shower is the taped X-band radar reflectivity data. The radar data can only be analyzed by making some assumption regarding the radar reflectivity factor as a function of rainfall rate. We have assumed the familiar

$$Z=200R^{1.6}, \tag{1}$$

where Z is the radar reflectivity factor (mm^6 m^{-3}) and R is rainfall rate (mm hr^{-1}). Computer analysis of the taped radar data from the 10,000-ft level[3] using (1) suggests that the shower yielded 350 kT of water.[4]

If one assumes that the 160 kg of salt had been ground to yield uniform particles 10μ in diameter, 350 kT of water could be produced if every salt particle grew to yield a raindrop of about 1.7 mm diameter. However, raindrops never come in a single size; indeed, the calculation of the amount of rainwater from the radar data assumes a particular type of raindrop size distribution, namely, a Marshall-Palmer (1948) distribution.

We make the reasonable assumption that the raindrop size distribution below a seeded cloud does not differ greatly from that in rain below an unseeded cloud. The distribution evolves during fall with some drop collisions leading to coalescence and others leading to drop breakup (e.g., Spengler, 1972), and probably becomes independent of that in the region of formation after the rain has fallen a few thousand feet.

The Marshall-Palmer raindrop size distribution is given by

$$N_D dD=N_0 \exp(-\lambda D)dD, \tag{2}$$

where $N_D dD$ is the number of raindrops per unit volume with diameters between D and $(D+dD)$, N_0 is a constant, and λ is the parameter controlling the size distribution. Marshall and Palmer suggest further that

$$\lambda=41R^{-0.2}, \tag{3}$$

where λ is in cm^{-1} and R is in mm hr^{-1}. This is in line with the common observation that heavy rains tend to contain larger drops than do light rains, but the dependence of λ upon R is not strong.

Kessler (1969) has calculated relationships between rainwater mass per unit volume and raindrop concentration for Marshall-Palmer distributions extending to infinitesimally small drops. Under the simplifying assumption that the 23 July shower had a uniform raindrop size distribution, we can consider Kessler's equations as applying to the total shower volume and rainwater mass, rather than to a unit volume. Reproducing his Eqs. (8.4) and (8.5), we have

$$N_t=\frac{N_0}{\lambda}, \tag{4}$$

$$M=\frac{\pi}{6}\rho N_0\frac{\Gamma(4)}{\lambda^4}, \tag{5}$$

where N_t is the total number of raindrops, M is the total mass of rainwater in the shower, ρ is the density of water, and $\Gamma(4)=6$. Combining these equations we can write

$$N_t=\frac{N_0}{\lambda}=\frac{6M\lambda^3}{\pi\rho\Gamma(4)}. \tag{6}$$

Indicated rainfall rates in the 23 July shower ranged up to 18 mm hr^{-1}. Accepting 10 mm hr^{-1} as typical, we find using (3) that λ was likely near 25 cm^{-1}. Utilizing

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[2] Private communications.

[3] All heights above MSL.

[4] One (metric) ton$=10^3$ kg; 1 kT$=10^9$ gm.

(6) would indicate that the 350 kT of water consisted of roughly 2×10^{15} raindrops; in fact, some downward adjustment is necessary to allow for truncation of the raindrop size distribution. Truncating the distribution at a diameter of 500μ, which agrees with some drop size distributions observed at the ground, involves a reduction in the total number of drops by a factor of 3, to 7×10^{14}.

In the absence of drop breakup and subsequent growth, and *under the most favorable assumption that no two salt seeds ended up in the same raindrop*, the calculations so far pose a requirement for roughly 7×10^{14} salt seeds.

Dividing 160 kg of salt into 7×10^{14} salt seeds yields a mass-weighted mean diameter of 6μ. The salt actually used was a 50-50 mixture by weight of fine and coarse salt. The fine salt had a median diameter near 5μ but a *median mass* diameter of 15μ, while the coarse grind median mass diameter was 150μ. In this connection it should be noted that any salt dust particles less than 5μ in diameter would not qualify as raindrop embryos, as the cloud droplets forming around them would differ little from the larger droplets of the natural cloud droplet population. Therefore, we conclude that the number of effective salt seeds available was considerably less than 7×10^{14}, so that some form of chain reaction must have been operative if salt seeding did produce the shower.

3. Results of cloud model calculations

In order to determine if a chain reaction was possible on 23 July, it is necessary to consider the updrafts and liquid water contents within the clouds and compute the growth rates and trajectories of salt seeds introduced at cloud base. Unfortunately, the instrumented aircraft used to collect cloud physics data on the project was released for the day just prior to the start of the seeding run, as all clouds in the area appeared to be weakening and none of the project personnel expected any unusual developements to follow the final seeding run of the day. In the absence of data from within the cloud, it is necessary to rely upon environmental data to infer the in-cloud conditions. Fortunately, the radiosonde observation for 0000 GMT of 24 July at the Rapid City Regional Airport was taken during the experiment while the cloud line was about 10–20 mi north-northeast of the airport.

The radiosonde data (Fig. 1) have been used as input data for a steady-state one-dimensional cloud model developed by Hirsch (1971) to predict updraft speeds, liquid water concentrations, and other variables within convective clouds in the Northern Great Plains. The model was developed from earlier cloud models (Weinstein and Davis, 1968; Simpson and Wiggert, 1969; Wisner, 1970) to provide better agreement with cloud characteristics as recorded by instrumented aircraft and radar in the Northern Plains region.

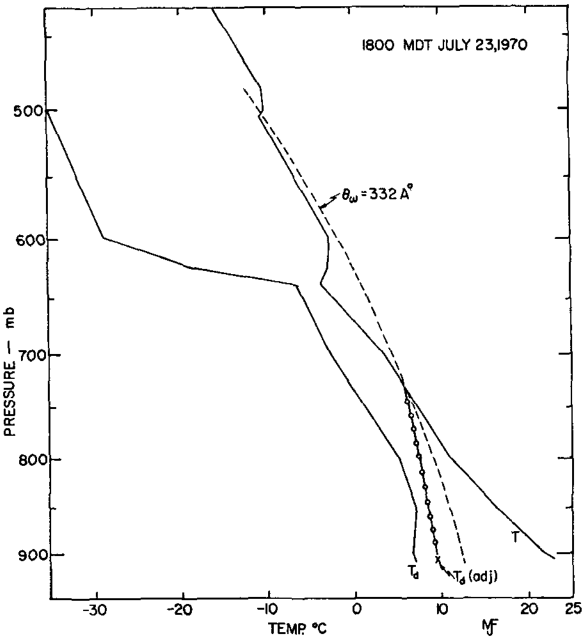

Fig. 1. Pseudo-adiabatic diagram for the evening (1800 MDT) sounding at Rapid City. The dew point as adjusted for conditions prevailing about 1700 MDT and that yields the observed cloud base at 2780 m is also shown.

The modeling studies have shown that the air feeding the cloud line must have been more moist than that sampled by the radiosonde; otherwise, the clouds could not have penetrated the lower inversion based at 640 mb (Fig. 1). The 1700 MDT observation at Ellsworth Air Force Base, almost directly under the cloud line, gives further support for this view; it showed a surface temperature and dew point of 23C and 11C, respectively, compared to 23C and 7C indicated by the radiosonde. The mixing ratio line finally accepted as the best estimate of actual conditions near the surface is indicated on Fig. 1. It places the cloud base at 9000 ft (2780 m) in agreement with observations from the seeding aircraft. The aircraft was operating about 8000 ft and the crew estimated cloud base to be 1000 ft above them.

A further requirement in the use of the Hirsch cloud model is the assignment of an updraft radius. The model has been run several times with different updraft radii. The run with an updraft radius of 2 km yields a cloud top of 18,250 ft (5580 m), in good agreement with radar and photographic observations, and has been accepted as appropriate for the most active parts of the cloud line.

Hirsch (1971) has developed a version of his model with modifications to the parameterization of microphysical processes to represent salt seeding effects. This version has not been used in the present case, where salt seeding effects are investigated in terms of individual raindrop embryos.

The profiles of updraft and liquid water content (LWC) for a cloud with 2 km updraft radius are shown in Fig. 2. The peak updraft (14 m sec^{-1}) is greater than we had anticipated for such a shallow cloud. [Note: The maximum updraft of 3 m sec^{-1} mentioned by Biswas and Dennis (1971) referred to the maximum updraft experienced by the seeding aircraft, which was operating under cloud base.] The fact that the maximum indicated updraft exceeds the terminal fallspeed of 5-mm raindrops shows that a rain accumulation zone could have developed in the upper part of the cloud. The LWC profile for this model does not take into account mixing from the cloud top, so the values near the cloud top are too high. However, we have reasonable confidence in the values of LWC ($\sim$2 gm m^{-3}) around the 5-km level.

4. Growth and breakup of raindrop embryos

In order to check the possibility of drop breakup in the case of 23 July 1970, we turn to the precipitation embryo growth model developed by Musil (1970) to study hailstone formation. The model uses collection efficiencies based on experimental and theoretical data available up to 1969. Condensational growth and competition among embryos are neglected. The model assumes that any embryo or drop moving to within a specified "eject distance" of the cloud top is lost by evaporation or carried off by the wind shear. The model also assumes that any raindrop reaching 5 mm diameter breaks up, yielding drops having one-half the diameter of the original drop, and follows the subsequent history of one such drop. While this arrangement in effect "builds in" the possibility of a chain reaction, it appears conservative to us in that breakup is always postponed until a drop reaches 5 mm diameter. Spengler (1972) reports some fragmentation due to collisions for 2 mm drops and raindrop observations at the ground show 5 mm drops to be very rare. Brazier-Smith *et al.* (1971) have called attention to the possible role of satellite drops produced by collisions of smaller raindrops in hastening the conversion of cloud water to rainwater.

In the present case, the Musil model has been run with the steady-state updraft profile predicted by the Hirsch model. The steps for calculation for the Hirsch model (LWC and updraft) have been set at 200 m starting from cloud base (2780 m). For calculations with the drop growth model, the LWC profile of the Hirsch model has been adjusted by setting it to zero at the cloud top (5580 m), so in effect we have "tapered off" the LWC for the last 200 m of the vertical profile (dotted line, Fig. 2). The LWC 200–300 m below cloud top may still be too high, but the profile appears reasonable on the whole for clouds of the observed dimensions. The fact that the clouds formed a continuous line protected them to some extent against entrainment of dry air.

Time-height trajectories for embryo sizes 60, 80, 120 and 160μ (corresponding to salt seed diameters of 15, 25, 35 and 50μ diameter, respectively) are shown in Fig. 2. With the "eject distance" set at 10 m, the results (Fig. 2 and Table 1) indicate that embryos $\leqslant 60\mu$ in diameter do not grow to breakup size and are thrown out and evaporated at the cloud top. Consideration of the trajectories of Fig. 2 suggests that only embryos

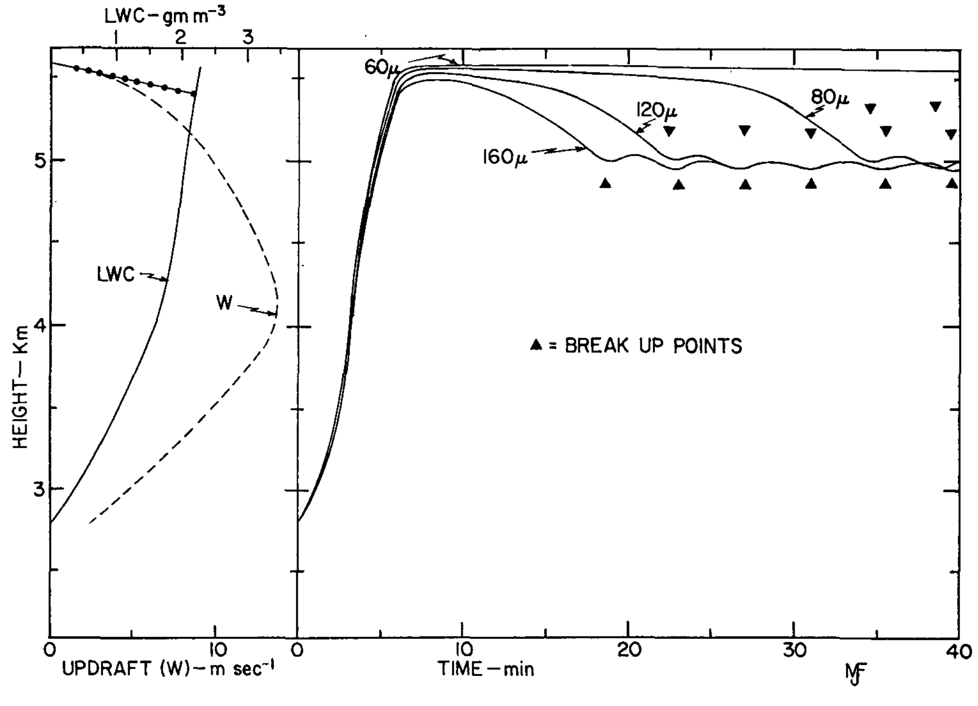

FIG. 2. Model predictions of liquid water content, updraft profile and time-height trajectories for growth of 60, 80, 120 and 160μ embryos in the seeded cloud.

TABLE 1. Summary history of drop growth with computer model for salt seeding case of 23 July 1970.

Diameter of dry salt particle yielding embryo at 0.1% supersaturation (μ)	Embryo diameter at cloud base (μ)	Diameter after 11 min (μ)	Time at which diameter reaches 1 mm (min)	Time at which diameter reaches 5 mm (breakup point) (min)	Number of breakups in 20 min	Number of breakups in 40 min	Remarks
10	40	—	—	—	0	0	Drop only 49μ and near cloud top in 7 min
12	50	—	—	—	0	0	Drop only 70μ and near cloud top in 7 min
15	60	110	—	—	0	0	Drop only 240μ and near cloud top in 40 min
20	70	160	35.5	—	0	0	Drop 3.1 mm and near top in 40 min
25	80	210	27.5	34.5	0	2	Breakup continues every 4 min
35	120	490	55.5	22.5	0	5	Breakup continues every 4 min
50	160	940	11.5	18.5	1	6	Breakup continues every 4 min
75	200	1600	9.5	16.5	1	6	Breakup continues every 4 min

of 100μ or more at cloud base have a good chance of surviving and falling back through the cloud.

Interpretation of the cloud model results can be extended through a comparison with the timing and location of the first radar echoes. Salt was released starting at 1643 MDT about 250 m below cloud base in updrafts of 1.5–3 m sec^{-1} and therefore should have reached cloud base by 1645. The first echoes on 10-cm radar appeared at 1656 ($t=11$ min on Fig. 2). The diameters at $t=11$ min of the drops resulting from the various sizes of salt seeds are shown in Column 3 of Table 1.

The S-band radar system, while not closely calibrated for the 1970 season, has been shown since to be capable of detecting echoes at the ranges in question in regions where Z_e, the equivalent radar reflectivity factor, is as low as 15 dBz. A 15-dBz value of Z_e could be realized from 1 mm drops in a concentration of only 30 m^{-3} if they extended throughout a contributing region. Drops of 250μ diameter (resulting in 11 min from salt seeds of $\sim$25μ diameter) would have to be present in concentrations near 10^5 m^{-3} to produce the same value of Z_e. We suspect that the initial radar echo was due to a relatively few drops of about 1 mm diameter, formed around salt seeds of about 50μ dia.

The location of the first echo at 17,000 ft (5190 m) does not conflict with the hypothesis that the large salt seeds were responsible for first echo, but the first echo height does not provide a sensitive way to distinguish the effects of embryos of different sizes (Fig. 2). Furthermore, it is in a region where radar echoes formed by natural processes would be likely to appear.

A few of the largest drops (formed around salt seeds over 100μ in diameter) might have already reached breakup size at $t=11$ min, but their concentrations would be very low. Once breakups began for the 50μ salt seeds around 1703 ($t=18$ min), the number of raindrops would increase rapidly.

The manner in which the rain accumulations affected the cloud dynamics, how the rain began to fall out of the cloud around 1705, and the organization of the shower into several distinct cells are beyond the scope of the cloud models used here. The use of time-dependent or two-dimensional models might provide more insight into the cloud's history.

5. Conclusions

In view of the points raised by Havens and Blanchard, and of the results of our model calculations, we restate our two basic conclusions as follows:

1) There is a high probability that the shower was formed by salt seeding.

2) If the observed rainfall was produced by salt seeding, a chain reaction was likely involved in its initiation.

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