Probability Distribution of Vertical Longitudinal Shear Fluctuations

GEORGE H. FICHTL

Aerospace Environment Division, NASA-George C. Marshall Space Flight Center, Huntsville, Ala. 35812

(Manuscript received 13 March 1972, in revised form 2 June 1972)

ABSTRACT

In order to properly design aerospace systems (conventional airplanes, V/STOL vehicles, space vehicles, etc.) the engineer must consider the vertical structure of the horizontal wind during the launch and landing phases of flight. One way to do this is with vertical two-point wind differences (wind shear). In turbulent flows like those found near the ground, wind shear is composed of a steady-state part associated with the mean wind profile and a fluctuating part produced by atmospheric turbulence. Mean wind profile theory can be used to specify the steady-state wind shear; however, the fluctuating part is a stochastic process which can only be specified statistically. This paper discusses some recent measurements of third and fourth moments of vertical differences (shears) of longitudinal velocity fluctuations obtained in unstable air at the NASA 150 m meteorological tower site at Cape Kennedy, Fla. Each set of measurements consisted of longitudinal velocity fluctuation time histories obtained at the 18, 30, 60, 90, 120 and 150 m levels, so that 15 wind-shear time histories were obtained from each set of measurements.

It appears that standardized third and fourth moments S and K of wind shear are universal functions of $\Delta z/\bar{z}$ and $\bar{z}/L_0$, where $\Delta \bar{z}$ is the vertical distance between the two points over which the wind difference is calculated, $\bar{z}$ the height of the mid-point of Δz above natural grade, and L_0 the surface Monin-Obukhov stability length. As $\Delta z/\bar{z} \rightarrow 2, K \rightarrow 3$, $S \rightarrow 0$, and $S>0$, $K>3$ for $\Delta z/\bar{z}<2$. Thus, it appears that the joint distribution function of the longitudinal wind fluctuations at two levels is not bivariate Gaussian and that it can only be approximated with a Gaussian distribution for sufficiently large value of $\Delta z/\bar{z}$. The kurtosis K appears to be independent of $\bar{z}/L_0$. However, the skewness S seems to experience a rather abrupt transition at $\bar{z}/L_0 \sim O(-1)$. The implications of these and other results relative to the design and operation of aerospace vehicles are discussed.

1. Introduction

Vertical variation of the horizontal wind (wind shear) can be of paramount importance during the launch of a space vehicle or the takeoff flight phase of a VTOL vehicle. This results from the fact that wind shear can induce torques on the vehicle which in turn could result in yaw- and pitch-plane responses. For engineering purposes wind shear can consist of two parts, a mean or steady-state part and a superimposed fluctuation. The mean wind shear can be calculated with currently available wind profile models (Blackadar, 1965; Lumley and Panofsky, 1964). However, the fluctuating part is a direct result of atmospheric turbulence and it must therefore be treated as a stochastic process. Furthermore, in order to specify the fluctuating part of wind shear in an engineering design context the distribution function of the wind shear must be available. This is best accomplished with a functional form of the associated probability density function which contains a number of parameters which are specified by no more than three or four central moments, say. Once these moments are known the distribution function would thus be completely specified. The purpose of this paper is to present recent measurements of the distribution function of wind shear and the associated third and fourth standardized moments. In particular, we will be concerned with the vertical variation of the longitudinal component of turbulence (the component of turbulence along the mean wind vector).

2. The data source

The data source consists of eleven sets of longitudinal turbulent velocity fluctuation time histories digitized at 0.2-sec intervals with approximately 18,000 data points per history. The longitudinal velocity fluctuations were calculated with horizontal wind speed and direction data collected from the 18, 30, 60, 90, 120 and 150 m levels at the NASA meteorological tower site at the Kennedy Space Center with Climet (Model C1-14) wind sensors. The measurements were taken during the daytime in unstable air. The computation procedure used to calculate the longitudinal turbulent velocity fluctuations has been discussed by Fichtl and McVehil (1970). Supporting temperature measurements were made at the 18 and 30 m levels with Climet (Model C1-016) aspirated thermocouples. The mean wind and temperature data were used to calculate surface values of the Monin-Obukhov stability length and the friction velocity. Details concerning the instrumentation at the NASA tower site can be found in a report by Kaufman and Keene (1968). The surface roughness lengths associated with the site have been calculated by Fichtl and McVehil (1970).

3. Statistical computations

In a horizontal boundary layer which is statistically homogeneous and stationary, the instantaneous longitudinal component of velocity is given by

$$u(x,y,z,t)=\bar{u}(z)+u'(x,y,z,t), \qquad (1)$$

where $\bar{u}(z)$ is the mean wind at height z above natural grade and $u'(x,y,z,t)$ is a longitudinal turbulent velocity fluctuation at horizontal position (x,y) and time t. To calculate wind shear (two-point wind difference along the vertical of the longitudinal wind) we evaluate (1) at levels z_2 and z_1 $(z_2>z_1)$ and calculate the difference given by

$$\Delta u(z_1,z_2,x,y,t)=\Delta\bar{u}(z_1,z_2)+\Delta u'(z_1,z_2,x,y,t), \qquad (2)$$

where

$$\Delta u(z_1,z_2,x,y,t)=u(x,y,z_2,t)-u(x,y,z_1,t), \qquad (3)$$

etc. The quantity $\Delta\bar{u}$ is the mean wind shear and as pointed out in the Introduction it can be calculated with available mean wind profile models. The quantity $\Delta u'$ is the fluctuating part of the shear which must be treated statistically. To examine the statistical properties of the fluctuating shear, the second, third and fourth central moments were calculated for the 15 time histories of $\Delta u'$ for each of the 11 cases. This resulted in 165 values of each of the above-stated statistics. The corresponding distribution functions of $\Delta u'$ were computed for each shear time history. The idea here is to find a distribution function for $\Delta u'$ which is completely specified by the central moments

$$\mu_n=\frac{1}{T}\int_0^T(\Delta u'(t))^n dt, \quad n=2,3,4, \qquad (4)$$

where T is the length of the time history. It should be noted that $\mu_1=0$.

4. Distribution functions

Fig. 1 contains experimental estimates of the distribution functions $F(\Delta u')$ for one of the 11 cases under consideration. These distributions, plotted on normal probability paper, are typical of the 11 cases and correspond to the 15 different shear time histories obtained by differencing the longitudinal wind time histories between the various levels on the Cape Kennedy tower. A Gaussian process corresponds to a straight line in Fig. 1. The fact that the data points in Fig. 1 do not tend to form straight lines means that the process $\Delta u'(t)$ is non-Gaussian. Furthermore, we may conclude from these results that $u'(z_1,t)$ and $u'(z_2,t)$ are not joint Gaussian. If $\Delta u'$ is a non-Gaussian process then what distribution does it possess? The Pearson system (Elderton, 1963) was used in an attempt to answer this question. This system consists of 13 probability density functions (p.d.f.). Each p.d.f. is characterized by certain conditions on the nondimensional quantities

$$\beta_1=\frac{\mu_3^2}{\sigma^6}, \qquad (5)$$

$$\beta_2=\frac{\mu_4}{\sigma^4}. \qquad (6)$$

The procedure for determining the appropriate p.d.f. for graduating data consists of determining which of these conditions are satisfied by the experimental estimates of β_1 and β_2. All of the 165 available estimates of β_1 and β_2 revealed that the Pearson type IV p.d.f. should be the appropriate one for representing the p.d.f. of $\Delta u'$. In addition, the type IV p.d.f. could not be rejected at the 5% level for both the Chi-square and Kolmogorov-Smirnov tests in all cases. Each solid line in Fig. 1 corresponds to a "fit" of the type IV distribution function to the data in which the parameters of the distribution are estimated with σ, β_1 and β_2. Except for the departure of curves 4, 5 and 9 from the experimental data in the tails, the type IV distribution function appears to be a very good representation of $F(\Delta u')$.

The type IV p.d.f. of a random variable ξ with $\mu_1=0$ is given by

$$p(\xi)=\frac{1}{aQ(r,\nu)}\left[1+\left(\frac{\xi}{a}-\frac{\nu}{r}\right)^2\right]^{-m}\exp\left[-\nu\tan^{-1}\left(\frac{\xi}{a}-\frac{\nu}{r}\right)\right],$$

$$-\infty<\xi<\infty, \qquad (7)$$

where

$$r=\frac{6(\beta_2-\beta_1-1)}{2\beta_2-3\beta_1-6}, \qquad (8)$$

$$m=\tfrac{1}{2}(r+2), \qquad (9)$$

$$\nu=\frac{-r(r-2)\beta_1^{\frac{1}{2}}}{[16(r-1)-\beta_1(r-2)^2]^{\frac{1}{2}}}, \qquad (10)$$

$$a=\frac{\sigma}{4}[16(r-1)-\beta_1(r-2)^2]^{\frac{1}{2}}, \qquad (11)$$

$$Q(r,\nu)=e^{-\nu\pi/2}\int_0^\pi \sin^r\Phi e^{\nu\Phi}d\Phi. \qquad (12)$$

The reader is referred to Elderton (1953) for the details concerning this Pearson type IV p.d.f. It can be readily shown that as $\mu_3/\sigma^3\to 0$ and $\mu_4/\sigma^4\to 3$, the type IV distribution reduces to the normal one. The normal distribution is a transition distribution of the Pearsonian system.

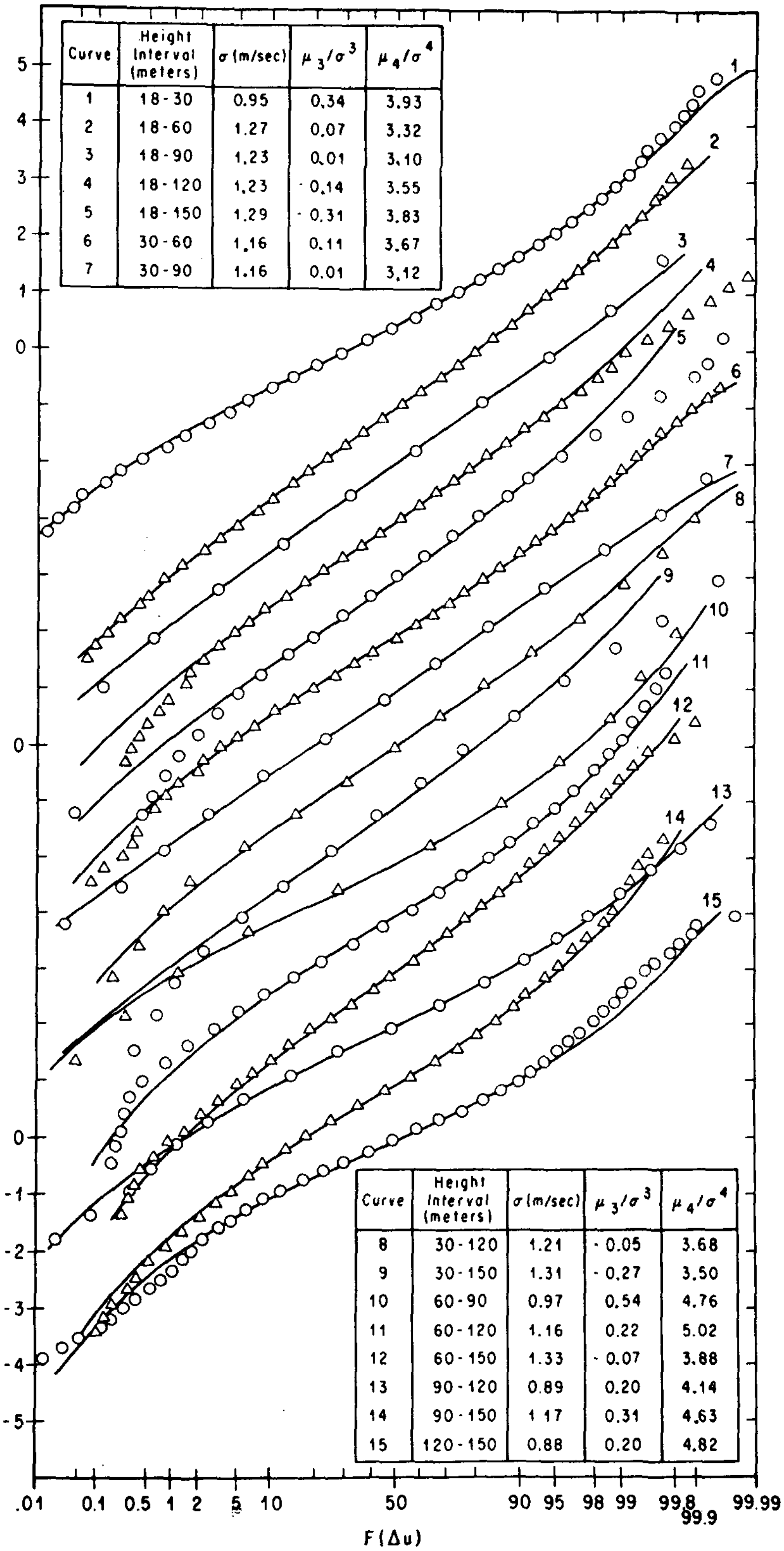

FIG. 1. Experimental estimates of the distribution functions $F(\Delta u')$ for various height intervals. The solid lines correspond to the Pearson type IV distribution predicted with the standard deviation σ, and the third and fourth moments μ_3 and μ_4. A straight line corresponds to a Gaussian distribution.

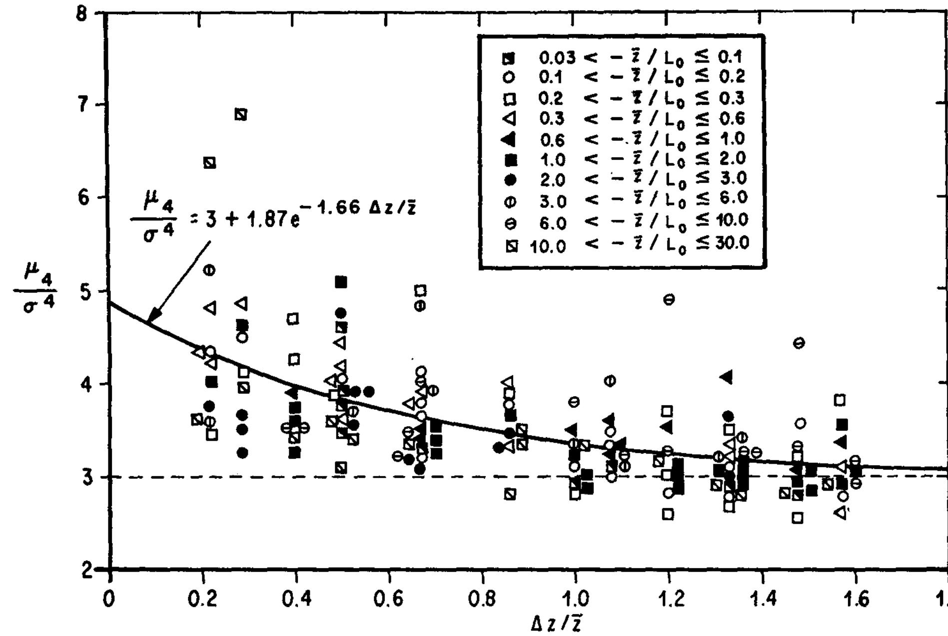

Fig. 2. Experimental estimates of the standardized fourth moment μ_4/σ^4 as a function of $\Delta z/\bar{z}$ for various categories of $\bar{z}/L_0$.

5. Scaling considerations

It is clear from the above that in order to specify a Pearson type IV density function of $\Delta u'$ for a given meteorological condition one requires σ, μ_3 and μ_4. A model for σ has been developed in a previous study (Fichtl, 1971). To develop models for μ_3 and μ_4 it is hypothesized that

$$\frac{\mu_3}{\sigma^3}=G\left(\frac{\Delta z}{\bar{z}}, \frac{\bar{z}}{L_0}\right), \tag{13}$$

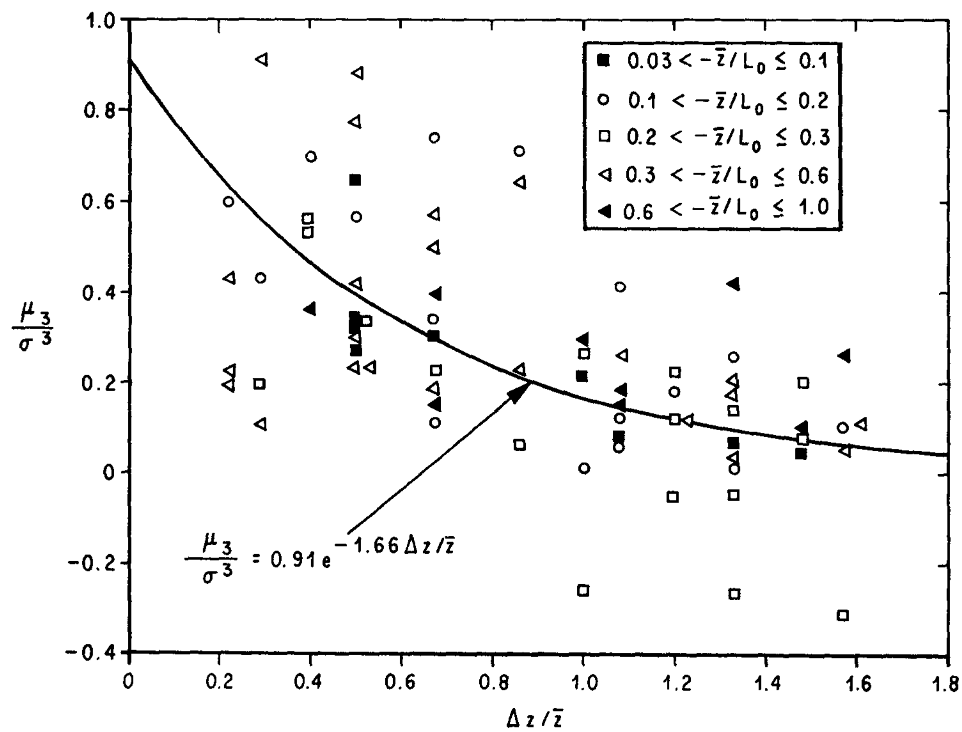

Fig. 3. Experimental estimates of the standardized third moment μ_3/σ^3 as a function of $\Delta z/\bar{z}$ for $0.03<-\bar{z}/L_0\leqslant 1.0$.

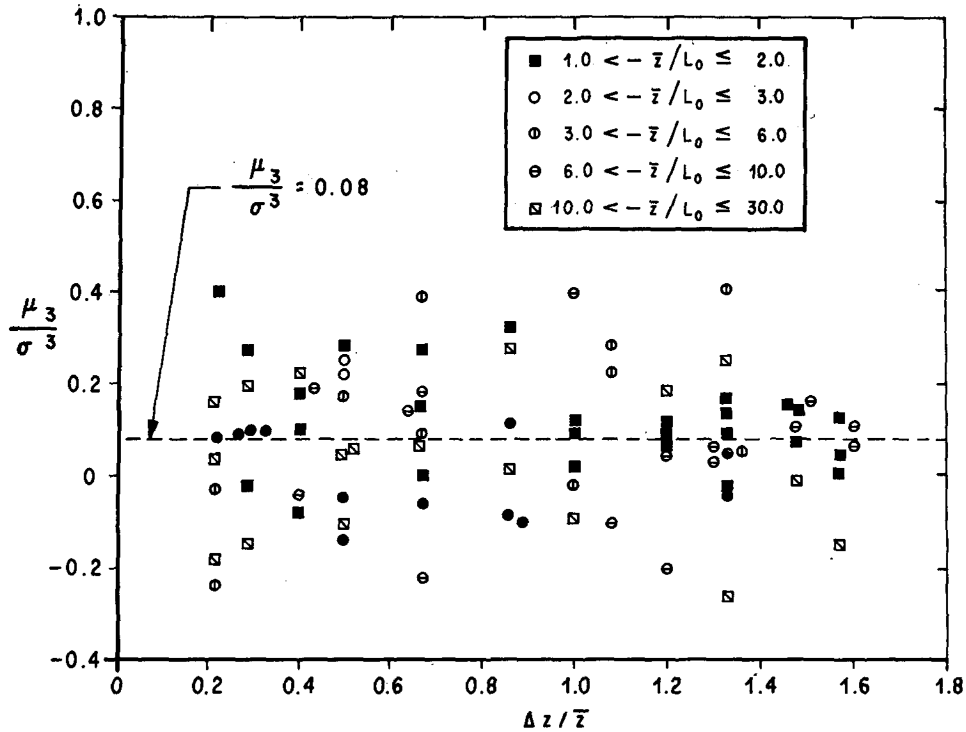

FIG. 4. Experimental estimates of the standardized third moment μ_3/σ^3 as a function of $\Delta z/\bar{z}$ for $1<-\bar{z}/L_0\leqslant 30$.

$$\frac{\mu_4}{\sigma^4}=H\left(\frac{\Delta z}{\bar{z}},\frac{\bar{z}}{L_0}\right), \qquad (14)$$

where $\sigma=\mu_2^{\frac{1}{2}}$ and G and H are universal functions of $\Delta z/\bar{z}$ and $\bar{z}/L_0$. The quantities Δz and $\bar{z}$ are defined as

$$\Delta z=z_2-z_1, \qquad (15)$$

$$\bar{z}=\frac{z_2+z_1}{2}, \qquad (16)$$

and L_0 is the surface Monin-Obukhov stability length. The quantity $\Delta z/\bar{z}$ can only take on values in the interval $0\leqslant\Delta z/\bar{z}\leqslant 2$. The dependence of the functions G and H upon the nondimensional quantities $\Delta z/\bar{z}$ and $\bar{z}/L_0$ follows from the hypothesis that μ_3/σ^3 and μ_4/σ^4 are uniquely determined by z_1, z_2 and L_0. This hypothesis at first glance appears to be consistent with the Monin-Obukhov similarity hypothesis. However, because of the depth of the layer ($z\leqslant 150$ m) we are considering in this paper, one might ask why we have not included the effect of the Ekman layer via the length u_{*0}/f, where u_{*0} and f denote the surface friction velocity and the Coriolis parameter. In a previous analysis (Fichtl, 1971) of the standard deviation $\Delta u'$ for the 11 cases in this paper it appeared that the experimental data could functionally be represented as

$$\frac{\sigma}{u_{*0}}=M\left(\frac{f\Delta z}{u_{*0}},\frac{f\bar{z}}{u_{*0}},\frac{u_{*0}}{fL_0}\right), \qquad (17)$$

where M is a universal function. The dependence of the nondimensional standard deviation σ/u_{*0} on $f\Delta z/u_{*0}$, $f\bar{z}/u_{*0}$ and u_{*0}/fL_0 follows from the hypothesis that σ scaled with u_{*0} is uniquely determined by z_1, z_2, L_0 and u_{*0}/f. This hypothesis includes the dominant effects of the Ekman layer, namely, atmospheric stability, and the action of Coriolis forces in the horizontal mean flow momentum conservation equations. It does not include baroclinic effects resulting from a height-dependent horizontal pressure gradient force. When the present study was initiated it was hypothesized that the μ_3/σ^3 and μ_4/σ^4 were universal functions of $f\Delta z/u_{*0}$, $f\bar{z}/u_{*0}$ and u_{*0}/L_0f because of the results concerning σ/u_{*0}. However, to give the results mathematical expression required rather complicated formulae. In an attempt to simplify the results for engineering use, the u_{*0}/f dependence was neglected and the functional forms given by (13) and (14) were used. It turned out that the scatter in μ_3/σ^3 and μ_4/σ^4 about their expected values was essentially the same for both scaling hypotheses. In other words, the parameter u_{*0}/f did not eliminate or explain the experimental scatter in the plots of μ_3/σ^3 and μ_4/σ^4 as functions of $\Delta z/\bar{z}$ and $\bar{z}/L_0$. This might be interpreted to mean that scaling μ_3 and μ_4 with σ eliminates the u_{*0}/f effect, in view of the fact that σ/u_{*0} does depend on u_{*0}/f [see Eq. (17)].

6. Experimental estimates of μ_3/σ^3 and μ_4/σ^4

Figs. 2–4 contain the experimental estimates of μ_3/σ^3 and μ_4/σ^4 as functions of $\Delta z/\bar{z}$ for various categories of $\bar{z}/L_0$. We conclude from Fig. 2 that μ_4/σ^4 is a function of $\Delta z/\bar{z}$. In addition, we conclude from the mix of the data points in the same figure that μ_4/σ^4 does not depend on $\bar{z}/L_0$ in the interval $0.03<-\bar{z}/L_0\leqslant 30$. The function

$$\frac{\mu_4}{\sigma^4}=3+1.87e^{-1.66\Delta z/\bar{z}} \tag{18}$$

summarizes the observations in Fig. 2.

The results concerning the third moment data are not as simple as those of the fourth moment data. We conclude from Figs. 3 and 4 that the quantity μ_3/σ^3 depends on z/L_0 insofar as μ_3/σ^3 is a monotonically decreasing function of $\Delta z/\bar{z}$ for $-\bar{z}/L_0\leqslant 1$ and essentially a constant for $-\bar{z}/L_0>1$. The function

$$\left.\begin{array}{ll}\dfrac{\mu_3}{\sigma^3}=0.91e^{-1.66\Delta z/\bar{z}}, & -\bar{z}/L_0\leqslant 1\\[2ex] \dfrac{\mu_3}{\sigma^3}=0.08, & -z/L_0>1\end{array}\right\} \tag{19}$$

summarizes the results for μ_3/σ^3.

To understand the nature of the transition in μ_3/σ^3 at $-\bar{z}/L_0\sim O(1)$, we note that to a first approximation $\bar{z}/L_0$ is the ratio of the buoyant to mechanical production rates of turbulent kinetic energy at height $\bar{z}$ in the horizontally homogeneous boundary layer. If $-\bar{z}/L_0<1$ the mechanical production rate exceeds the buoyant production rate and *vice versa* for $-\bar{z}/L_0>1$. Thus, the dominant turbulent kinetic energy production mechanism controls the third-order shear statistics. However, it does not appear to control the fourth-order moments in view of the absence of a dependence of μ_4/σ^4 on $\bar{z}/L_0$.

If the process $\Delta u'$ were Gaussian then we would have $\mu_3/\sigma^3=0$ and $\mu_4/\sigma^4=3$. It is clear from Figs. 2–4 that these conditions are not satisfied over the experimental range of $\Delta z/\bar{z}$, so that $\Delta u'$ is a non-Gaussian process in the unstable boundary layer. Furthermore, it follows that the joint distribution of $u'(z_1,t)$ and $u'(z_2,t)$ is not a bivariate Gaussian distribution. As $\Delta z/\bar{z}\rightarrow 2$ (maximum value of $\Delta z/\bar{z}$) μ_4/σ^4 tends to have values near 3; however, μ_3/σ^3 tends to have significant nonzero values, approximately equal to 0.1 for large values of $\Delta z/\bar{z}\approx 2$ for the experimental range of $\bar{z}/L_0$. As $\Delta z/\bar{z}\rightarrow 0$ the departures of μ_4/σ^4 and μ_3/σ^3 from the values appropriate for a Gaussian distribution are relatively large. Thus, if one desires to approximate $F(\Delta u')$ with a Gaussian distribution then the approximation would be best applied at $\Delta z/\bar{z}\approx 2$. The approximation would be extremely poor for values of $\Delta z/\bar{z}\lesssim 1$.

The turbulent velocity fluctuations should be continuous functions of position, so that as $\Delta z\rightarrow 0$ we should have $\Delta u'/\Delta z\rightarrow \partial u'/\partial z$. In the limit as $\Delta z\rightarrow 0$ we conclude from (18) and (19) that

$$\left.\begin{array}{l}\overline{(\partial u'/\partial z)^3}/\overline{(\partial u'/\partial z)^2}^{\frac{3}{2}}=\begin{cases}0.91, & 0.03<-\bar{z}/L_0\leqslant 1\\ 0.08, & -\bar{z}/L_0>1\end{cases}\\ \overline{(\partial u'/\partial z)^4}/\overline{(\partial u'/\partial z)^2}^{2}=4.87, \quad 0.03<-\bar{z}/L_0.\end{array}\right\}. \tag{20}$$

We conclude from the results that $\partial u'/\partial z$ is a positively skewed process and possesses a distribution function which is peaked more than the Gaussian distribution.

The analysis in this section is analogous to a study by Stewart (cf. Batchelor, 1956) of third- and fourth-order moments of two-point velocity differences in a wind tunnel turbulent flow. In his analysis he analyzed the statistics of the two-point difference $u'(x+\Delta x,y,z,t)-u'(x,y,z,t)$. Thus, Stewart examined a longitudinal difference while the two-point difference in this paper is a lateral one. Stewart found results similar to those reported in this paper, namely, that large non-Gaussian behavior occurs at small values of Δx with $\mu_4/\sigma^4\approx 3.4$; however, the third moments were negative. At large values of Δx the fourth-order statistic μ_4/σ^4 took on values approximately equal to 2.9 and $\mu_3/\sigma^3\approx -0.1$. In addition, his results imply that

$$\left.\begin{array}{r}-0.34\lesssim\overline{(\partial u'/\partial x)^3}/\overline{(\partial u'/\partial x)^2}^{\frac{3}{2}}\lesssim -0.48\\ \overline{(\partial u'/\partial x)^4}/\overline{(\partial u'/\partial x)^2}^{2}\approx \quad 3.6\end{array}\right\}, \tag{21}$$

for the experimental range of Reynolds number (based on mean tunnel velocity and the spacing of the grid used to generate the turbulence) $5300\leqslant \mathrm{Re}\leqslant 42{,}200$.

7. Applications

The results in this paper can be used in the operation and design of aeronautical and aerospace systems. In this paper we shall examine an operational application to give the reader a "feel" for how the shear statistics might be applied to aeronautical and aerospace programs. To do this we must recognize the fact that the p.d.f. of $\Delta u'$ associated with a given height interval Δz at height $\bar{z}$ in a specified boundary layer flow is a conditional p.d.f., namely, $p(\Delta u'|u_{*0},L_0;f,\Delta z,\bar{z})$. This function is a conditional p.d.f. of $\Delta u'$ given u_{*0} and L_0 and the parameters f, Δz and $\bar{z}$. We distinguish between the two sets (u_{*0},L_0) and $(f,\Delta z,\bar{z})$ because the quantities u_{*0} and L_0 are random variables in a climatological context, while f, Δz and $\bar{z}$ are merely parameters. Multiplication of this conditional p.d.f. by the climatological p.d.f. of L_0 and u_{*0} yields the joint p.d.f. of $\Delta u'$, u_{*0} and L_0. This distribution could be extremely useful in design problems.

In the operational situation one would use the above-stated conditional p.d.f. because one would be interested in making a statistical inference about $\Delta u'$ given a particular boundary layer flow condition. The boundary layer is completely specified by u_{*0}, L_0 and f. The idea here is that at the time of the operation one would have precalculated values of a desired conditional statistic of $\Delta u'$ which is crucial to the operation for various values of u_{*0} and L_0. These quantities could be depicted in the form of nomograms or any other convenient representation. At the time of the operation (launch of a space vehicle, takeoff of a VTOL aeronautical system, etc.) one would monitor u_{*0} and L_0. These parameters would then be used to estimate the corresponding statistics of $\Delta u'$ of operational interest from the above stated precalculated values of $\Delta u'$. This statistic of $\Delta u'$ would then be compared with a predetermined operational critical value of $\Delta u'$ and a decision would then be made based upon this comparison. For example, let us suppose that an aeronautical VTOL system could not takeoff in a shear condition in which $\Delta u'$ exceeded a critical value $(\Delta u')_c$. Since $\Delta u'$ is a stochastic quantity one must specify the maximum value of risk R_m one is willing to accept of $\Delta u'$ exceeding $(\Delta u)_c$ during the takeoff flight phase. The conditional risk R of $\Delta u'$ exceeding the value of $(\Delta u')_0$ is given by

$$R(\Delta u' > (\Delta u')_0 | u_{*0}, L_0; f, \Delta z, \bar{z})$$

$$= \int_{(\Delta u)_0}^{\infty} p(\Delta u' | u_{*0}, L_0; f, \Delta z, \bar{z}) d(\Delta u'). \quad (22)$$

We now set $R=R_m$ and calculate $(\Delta u')_0$ as a function of u_{*0} and L_0 for a prescribed set of values for f, Δz and $\bar{z}$. This permits one to obtain a plot of $(\Delta u')_0$ as a function of u_{*0} and L_0. Prior to takeoff one would "measure L_0 and u_{*0}" and then determine $(\Delta u')_0$ from this plot. If $(\Delta u')_0 < (\Delta u')_c$, then $R_c < R_m$, where R_c is the risk of $\Delta u'$ exceeding $(\Delta u')_c$ given the current values of u_{*0} and L_0. In this case the VTOL vehicle could takeoff since there would not be a violation of the takeoff risk constraint on $\Delta u'$. If $(\Delta u')_0 \geqslant (\Delta u')_c$ then $R_c \geqslant R_m$. In this case appropriate action would be taken to abort the mission or advance the takeoff time.

A word about "measuring L_0 and u_{*0}" is in order at this point. The quantities L_0 and u_{*0} can be 1) computed directly with turbulent temperature and longitudinal and vertical velocity fluctuation data, or 2) computed with mean wind and temperature measurements acquired at two levels near the ground ($z \lesssim 30$ m) and wind profile models consistent with the Monin-Obukhov similarity hypothesis (Monin and Yaglom, 1971). The latter measurements and associated computations would be the easier of the two and would be the best approach for estimating u_{*0} and L_0 in an operational situation.

The above discussion was based on the use of the expected values of μ_3/σ^3 and μ_4/σ^4 as given by Eqs. (18) and (19) and σ/u_{*0} as given by Fichtl (1971). However, Figs. 2–4 indicate a relatively large spread in the observations and this reflects a source of uncertainty in the estimates of $(\Delta u)_0$. One way to handle this problem is to establish envelopes on the parameters σ/u_{*0}, μ_3/σ^3 and μ_4/σ^4 for given values of the parameters $\bar{z}/L_0$, $\Delta z/\bar{z}$ and $f\bar{z}/u_{*0}$. These envelopes could correspond to confidence bands for specified acceptance levels (95% bands, say). One could then calculate $(\Delta u)_0$ for various combinations of σ/u_{*0}, μ_3/σ^3 and μ_4/σ^4 within these bands and then select the maximum value of $(\Delta u)_0$ in the case of the operational problem. However, more data than that used in this study would be needed to establish these bands. In addition, other procedures could be established depending on the accepted operational or design philosophies.

8. Concluding comments

The analysis in this paper is valid for the unstable boundary layer in which $0.03 < -\bar{z}/L_0 \leqslant 30$. This analysis should be extended to the neutral ($\bar{z}/L_0=0$) and stable ($\bar{z}/L_0>0$) boundary layers. This extension would then yield a model which would cover the full range of possibilities and thus would enhance the engineering utility of the model herein. It should also be noted that the model herein is strictly valid for the horizontal and statistically stationary and homogeneous boundary layer. This means that caution should be exercised in the application of the model. If a change of surface roughness, or buildings, trees, or other obstacles, are present upstream of the point of potential application, then the model herein could yield incorrect statistical estimates of shear. The applicability of the model will then depend upon the distance of the site from the inhomogeneity and the engineering application of the shear estimate.

In closing this discussion, it is worthwhile to point out the recent results of research conducted by Deardorff (1972). The results of his calculations imply that u_{*0}/f is not the relevant length scale in the unstable boundary layer, but rather this length should be replaced with the height z_i of the inversion below which the convective mixing is confined. If this is true then the following hypotheses should be examined:

$$\frac{\mu_3}{\sigma^3} = G_1\left(\frac{\Delta z}{\bar{z}}, \frac{\bar{z}}{L_0}, \frac{z_i}{L_0}\right),$$

$$\frac{\mu_4}{\sigma^4} = H_1\left(\frac{\Delta z}{\bar{z}}, \frac{\bar{z}}{L_0}, \frac{z_i}{L_0}\right),$$

where G_1 and H_1 are universal functions of the indicated parameters. Perhaps these hypotheses might explain some of the scatter in Figs. 2–4.

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