On the Inclusion of a Snowfall Term in the Relationship between the Energy and Water Balances over Land

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From the earlier work of Budyko (1958), Lettau (1969) has produced the interesting and very useful nondimensional relationship between the energy and water balances over land:

$$D^* = (1+B^*)(1-C^*), \qquad (1)$$

where B is the Bowen ratio, C the runoff ratio, and D the dryness ratio (the ratio of net radiation to the heat equivalent of the precipitation), and the asterisk indicates the ratio is formed as the quotient of annual means. This relationship, however, assumes that the precipitation is all rainfall, and must be expanded to include the effects of snowfall in the annual precipitation.

Using energy flux and hydrometric notations, (1) becomes

$$Q_N/L_eP = (1+Q_H/Q_E)(1-N/P), \qquad (2)$$

where Q_N, Q_H and Q_E are the fluxes of net radiation, sensible heat and latent heat of vaporization, respectively, N and P are the mean annual streamflow and precipitation, respectively, and L_e is the latent heat of evaporation. The relationship is only valid for mean annual conditions because it is derived from the mean annual water balance and energy budget equations:

$$P = N+E, \qquad (3)$$

$$Q_N = Q_H + Q_E. \qquad (4)$$

To include the effect of snowfall in the annual precipitation, (4) must be modified to

$$Q_N - Q_M = Q_H + Q_E, \qquad (5)$$

where Q_M is that portion of the net radiation used directly in the melting of the snow. The heat equivalent of the precipitation in (2) must also be modified to L_eP+L_fS, where L_f is the latent heat of freezing, and S is the water equivalent of the mean annual snowfall. Therefore,

$$\begin{aligned}(Q_N-Q_M)/(L_eP+L_fS) &= (Q_H+Q_E)/(L_eP+L_fS)\\ &= (1+Q_H/Q_E)EL_e/(L_eP+L_fS)\\ &= (1+B^*)E/(P+L_fS/L_e)\\ &= (1+B^*)(P-N)/(P+0.134S), \qquad (6)\end{aligned}$$

where E is evapotranspiration.

It will be seen that (6) is analogous to (1) and (2) but takes account of radiational snowmelt and also the heat equivalent of the snowfall portion of the precipitation. Eq. (6), therefore, might be used to define modified dryness and runoff ratios for regions of significant snowfall.

The melt process is very complex, since the required energy may come from several sources such as warm air (i.e., negative Q_H), condensation, rainfall, soil heat, and solar radiation (Q_M). The proportion of these various factors on an annual basis probably varies areally with such climatic factors as cloudiness, temperature, humidity, wind and net radiation. Dewalle and Meiman (1971) calculated that net radiation supplied an average of 58% of the melt energy in a two-day study in a forest environment in Colorado with mean air temperatures of 9–12C. Treidl (1970), however, in a one-day energy budget study of snowmelt in Michigan, calculated that sensible and latent heat fluxes within the boundary layer of a warm, moist air mass under a strong wind flow provided 82% of the snowmelt energy. Little can be found in the literature about annual fluxes of Q_M.

Alternately, (6) may be written

$$Q_N/(L_eP+L_fS) = (1+B^*+M^*)(P-N)/(P+0.134S), \qquad (7)$$

which may be shown is equivalent to

$$D^* = (1+B^*+M^*)(1-C^*), \qquad (8)$$

where $M^* = Q_M/Q_E$ might be called the "melt ratio," and treated in a manner similar to B^*.

Some tentative estimates of the magnitude of M^* have been obtained from an energy budget study being prepared for Marmot Creek Experimental Watershed (50°57′N, 115°10′W) which produces estimates of evapotranspiration which fit the water budget quite well. If all the snowmelt was caused by the nonradiative factors, the lower limit of Q_M and M^* is fixed at zero. At the other extreme, if all the snowmelt is attributed to radiation, 4296 langleys would be required to melt the average snowfall of 53.7 cm (water equivalent). The mean annual Q_E was estimated at 26,355 langleys, so the maximum M^* at Marmot Creek is 0.163. In the energy budget study, to take a realistic approach, the mean annual hydrograph and snow survey data indicated that approximately 5% of the snow melted in March, 10% in April, 40% in May and June, and 5% in July. From a consideration of mean temperatures, humidities, wind and rainfall, it was rather arbitrarily assumed that all the melt in March, 90% in April, 75% in May, and 50% in June and July were attributable to Q_M. Based on these assumptions, M^* was 0.13 with a variation between 0.10 and 0.17 in the eight years of the study.

It might be assumed that annual values of Q_M/Q_E could be a linear function of $S/(P-N)$, but the correlation of these at Marmot Creek was not good, probably because the denominator should also include the change in storage (ΔW). To date, no method of quantifying ΔW in the water balance is available.

Because (6), (7) and (8) were developed for mean annual conditions (assuming ΔW and soil heat flux are zero), they must not be used for specific years or any shorter periods when these assumptions are not valid. For some periods or some purposes, it may be necessary to include the minor energy terms of photosynthesis, respiration, storage in vegetation, and the gain or loss of heat from advection and precipitation. The magnitudes of most of these are generally less than the uncertainties of measurement or estimation of the major terms. Following the reasoning used in the development of (6), and neglecting the minor terms, the relationship between the energy and water balances over land for any period is

$$(Q_N-Q_G)/(L_eP+L_fS) = (1+B+Q_M/Q_E)(P-N\pm\Delta W)/(P+0.134S), \quad (9)$$

where Q_G is the energy flux through the ground.

If there is no precipitation in the period, (9) reduces to

$$(Q_N-Q_G)/L_e = (1+B+Q_M/Q_E)(\Delta W-N). \quad (10)$$

Much has been written about the difficulties of calculating B from precise measurements of temperature and vapor pressure gradients (Williams, 1961; Fritschen, 1965; Mukammal, 1971). These difficulties are increased in below-freezing temperatures. If methods could be developed for quantifying ΔW and Q_M/Q_E, (9) would provide a means of estimating B for annual or shorter periods from more commonly measured climatic and hydrometric parameters. Even without precise calculations of ΔW and Q_M/Q_E, in many cases they may be estimated with sufficient accuracy to provide adequate estimates of B for the calculation of evapotranspiration for the water balance.

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