On Some Characteristics of the S_1 Score

J. C. THOMPSON AND GARY M. CARTER
Dept. of Meteorology, San José State College, San José, Calif. 95114

5 April 1972

For many years, prognostic charts have been verified by computation of the "S_1 score," developed by Teweles and Wobus (1954). This score is defined as

$$S_1 = 100\frac{\Sigma|e_G|}{\Sigma|G_L|}, \qquad (1)$$

where:

e_G error in the forecast pressure difference (gradient) between selected stations or geographic locations

G_L observed or forecast pressure difference (gradient), whichever is larger. (A distinction between the terms "difference" and "gradient" being immaterial to the discussion, they are used interchangeably hereafter.)

It is the purpose of this note to examine some aspects of the following question: Recognizing that forecasters have shown considerable skill in estimating the predictive error distribution when making probability forecasts for a number of weather elements, which of the values in a predictive error distribution of pressure gradients should be selected in order to obtain the best (lowest) S_1 score?

For this purpose, consider the problem of optimizing the long-run score for a single predicted pressure gradient between two stations or locations. The numerator of the score, denoted here as S, may then be written as the first absolute moment of a predictive error (probability) distribution

$$S = \Sigma|p - p'|, \qquad (2)$$

where p is a variable representing the values of pressure gradients prescribed by the probability distribution, and p' a single value of p selected from the distribution as the predicted gradient.

Since, as pointed out by Cramer (1951), "The first absolute moment ... becomes a minimum when [p'] is equal to the median," S will be minimized by predicting the median of the probability distribution.

Now, considering the denominator of the score, the forecaster is free to choose only the forecast pressure gradient, since the observed gradient is beyond his control. In order to obtain the best score, therefore, his choice should be to make the denominator as large as possible, consistent with keeping the numerator as small as possible. This can be accomplished by *predicting a pressure gradient somewhat larger than the median of the probability distribution.*

A quantitative illustration of the preceding qualitative statement may be obtained by examining symmetric, and positively- and negatively-skewed distributions. For the first of these, it is appropriate to assume that the predicted error distribution is Gaussian; thus, the entire distribution can be specified by its mean (equal to both the median and the mode) and standard deviation. For the skewed forms, convenience suggests the Poisson distribution, which can be specified by its standard deviation (equal to the square root of the mean); both forms of skewness[1] may be illustrated.

For a single pressure difference between selected stations, it is then a simple matter to write a computer program which will provide an array of S_1 scores resulting, in the long run, from each of the alternative gradients which might be chosen from the error, or probability, distribution. From a series of such arrays, the values of the pressure gradients which will produce the minimum S_1 score can be determined. Table 1 shows such values for a limited range of pressure gradients predicted as the median value of the appropriate probability distribution, and the standard deviations of those gradients. For the sake of brevity, the precision of the tables has been restricted to whole number values.

For very "sharp" predictive error distributions (small standard deviations) no corrections, or only small additions, are required to produce the best S_1 score regardless of the magnitude of the median value. However, for distributions with larger standard deviations, small median values should be increased by successively larger, and clearly non-trivial, amounts.

The values labeled "indeterminate" arise as a consequence of the denominator in Eq. (1), which can take on a value of zero. This problem normally does not arise in the usual verification procedure, since the numerator and denominator of the score are summed independently over an entire weather chart, and the likelihood of a completely "flat" map (zero pressure gradient), both predicted and observed, is very small indeed. Clearly, however, a logical decision to optimize the score cannot be made where a prediction of zero is one alternative and a like value may occur.

The independent summation of numerator and denominator, together with a probable lack of independence between error distributions for adjacent pressure gradients, introduces other problems. For example, although it may seem desirable to the local forecaster that each predicted pressure gradient, at least in his own region, should represent an optimum decision on the part of the prognostic analyst, a better S_1 score for the whole map may actually be produced if a non-optimum individual pressure gradient, i.e., a non-optimum individual S_1 score, is predicted in certain areas.

During the years the S_1 score has been in use, it seems likely that practicing prognostic analysts should have arrived, qualitatively at least, at results similar to these. For example, a rule-of-thumb attributed to some prognostic analysts suggests that "one should not predict a weak pressure gradient unless quite sure it will occur." This rule is obviously confirmed by the preceding table.

[1] The moment coefficient of skewness is the inverse of the standard deviation.

Table 1. Figures in the body of the table are the number of millibars which, when added algebraically to the median of the indicated error (probability) distribution of a predicted pressure gradient, will produce a long-run minimum S_1 score. Values in blank portion of table are all zero.

Standard deviation of predicted errors [mb]	Predicted median pressure difference [mb], or pressure gradient [mb (unit distance)$^{-1}$] 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	Gaussian distribution																	
0	*	0																
1	*	0																
2	*	2	1	0														
3	*	3	2	1	1	1	0											
4	*	4	3	2	2	1	1	1	1	0								
5	*	5	4	4	3	2	2	1	1	1	1	1	1	0				
	Poisson distribution (positive skewness)																	
0	*	0																
1	*	0																
2	*	2	1	0														
3	*	4	2	1	1	0												
4	*	5	3	2	2	1	1	1	0									
5	*	7	5	4	3	2	2	1	1	1	0							
	Poisson distribution (negative skewness)																	
0	*	0																
1	*	1	0															
2	*	1	1	1	1	0												
3	*	2	2	1	1	1	1	1	0									
4	*	4	3	2	2	2	1	1	1	1	1	1	0					
5	*	5	4	4	3	2	2	2	1	1	1	1	1	1	1	1	1	0

* Indeterminate values.

In this connection, it is interesting to speculate on how much of the improvement in prognostic charts, as measured by the S_1 score, may have been due to such experience in its use. One may also speculate along similar lines concerning possible "learning" processes by the computer in producing Numerical Weather Prediction prognoses. It should be noted, however, that the authors have no desire to make value judgments concerning the score, but rather wish to suggest that not only analysts who are making the prognoses, but also research workers who are concerned with their improvement and field forecasters who are using them, may find it useful to examine the consequences of these, and perhaps other, characteristics of the score.

REFERENCES

Cramer, Harald, 1951: *Mathematical Methods of Statistics.* Princeton University Press, p. 178.

Teweles, Sidney, Jr., and Hermann B. Wobus, 1954: Verification of prognostic charts. *Bull. Amer. Meteor. Soc.*, **35**, 455–463.