Spectral Transfer Functions for a Three-Component Sonic Anemometer[1]

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ABSTRACT

The transfer functions which describe the effect of line averaging and path separation on the response of a three-component sonic anemometer have been computed specifically for the Kaijo-Denki PAT 311. These computations differ from the previous work of Kaimal *et al.*, by accounting for the two parallel paths which compose each measurement axis and for the vertical separation of the two horizontal axes. There is only a minor additional attenuation of response for the vertical wind component due to the measurement axis being composed of two paths rather than one. However, the differences between the horizontal transfer functions calculated here and those presented by Kaimal *et al.* are significant and could be important for correcting sonic spectra in the inertial subrange.

1. Introduction

A sonic anemometer measures the speed of the wind by transmitting sound waves between a pair of acoustic transducers. The wind component along the sonic path, defined by the line connecting the transducer pair, is determined from the difference in transit time for sound waves transmitted in opposite directions along that path. In order to measure all three components of the wind vector, it is necessary to have three non-parallel, non-coplanar sonic paths which do not interfere with each other either in the transmission and reception of the sound waves or in exposure to the wind field. One such array is shown in Fig. 1. The W-component axis measures the vertical wind, and the A and B axes measure two independent components of the horizontal wind which are 120° apart and from which are calculated the orthogonal components of the horizontal wind.

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This design works well for gusts whose direction α, relative to the bisector of the angle between the A and B axes, is $\leqslant 45°$.

Kaimal *et al.* (1968) have calculated a set of transfer functions which describes the effect of line averaging and path separation on the response of a three-component sonic anemometer as a function of eddy size. For the purpose of the computations it was assumed that each measurement axis or component of the anemometer averaged the wind over a single path of length l and that the centers, denoted by the vectors $\mathbf{x}_A$ and $\mathbf{x}_B$, of the two horizontal axes, were in the same horizontal plane, separated by a distance d. In actuality, each measurement axis of the sonic anemometer may have two parallel paths of length l which are separated by a distance p, and the two horizontal axes may be additionally separated in the vertical by a distance h. The three-component sonic anemometer (Model PAT 311) manufactured by Kaijo-Denki Co., Tokyo, is designed with $p/l=0.2$ and $h/l=0.45$. If the experimenter desires to correct his spectral data for response errors, these features must be considered when the transfer functions are calculated. This note presents the appropriate modifications of the equations presented by Kaimal *et al.*, discusses the altered transfer functions, and tabulates the transfer functions for the Kaijo-Denki anemometer.

2. Averaging over two parallel paths

Kaimal *et al.* present a detailed derivation of the transfer functions, the ratio of the measured to the ideal one-dimensional power spectra, for the three components of the wind as measured by the sonic anemometer. Rather than repeat this derivation, the bulk of which is unaltered by the additional considerations to be presented here, only the basis of the changes and the final results will be presented. The reader can easily fill in the intermediate steps by reference to the original paper.

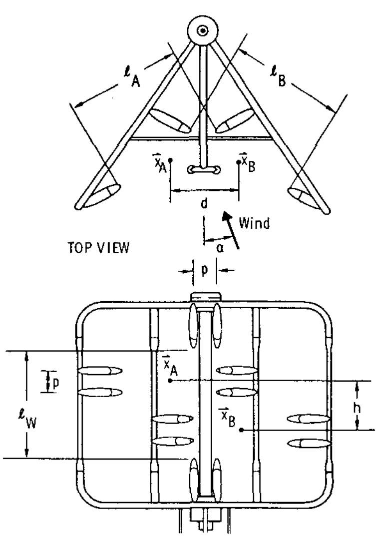

FIG. 1. Dimensions relevant to calculating the response of a three-component sonic anemometer.

The wind field measured by a single sonic axis is an average over two parallel paths of length l separated by a distance p, i.e.,

$$\tilde{u}_i(\mathbf{x},\mathbf{l},\mathbf{p})=\frac{1}{2l}\int_{-l/2}^{l/2}[u_i(\mathbf{x}+\tfrac{1}{2}\mathbf{p}+\mathbf{s})+u_i(\mathbf{x}-\tfrac{1}{2}\mathbf{p}+\mathbf{s})]ds. \quad (1)$$

Here $u_i(\mathbf{x})$ is a component of the actual wind at point $\mathbf{x}$ and $\tilde{u}_i(\mathbf{x},\mathbf{l},\mathbf{p})$ is the averaged wind at the midpoint of the measurement paths. By representing the wind field in terms of its Fourier components, it follows that the transfer function for the one-dimensional spectrum of the β-component of the wind is

$$T_\beta(\mathbf{k}_1,\mathbf{l},\mathbf{p})=\frac{\displaystyle\iint_{-\infty}^{\infty}\cos^2(\mathbf{k}\cdot\mathbf{p}/2)\left[\frac{\sin^2(\mathbf{k}\cdot\mathbf{l}/2)}{(\mathbf{k}\cdot\mathbf{l}/2)^2}\right]\Phi_{\beta\beta}(\mathbf{k})dk_2dk_3}{\displaystyle\iint_{-\infty}^{\infty}\Phi_{\beta\beta}(\mathbf{k})dk_2dk_3}, \quad (2)$$

where $\mathbf{k}$ is the wavevector with component k_1 in the direction of the mean wind and k_3 in the vertical direction, and $\Phi_{\beta\beta}$ is the spectral density tensor. A similar equation presented by Kaimal is regained by setting $p=0$. An important example of this equation is the measurement of the vertical wind component with a sonic axis which has $\mathbf{l}$ in the vertical direction and $\mathbf{p}$ in the lateral:

$$T_3(k_1l,p/l)=\frac{\displaystyle\iint_{-\infty}^{\infty}\cos^2(k_2p/2)\left[\frac{\sin^2(k_3l/2)}{(k_3l/2)^2}\right]\Phi_{33}(\mathbf{k})dk_2dk_3}{\displaystyle\iint_{-\infty}^{\infty}\Phi_{33}(\mathbf{k})dk_2dk_3}. \quad (3)$$

Evaluation of (3) requires an explicit expression for the spectral density tensor. Following Kaimal, we choose the inertial subrange expression

$$\Phi_{ij}(\mathbf{k})=A\epsilon^{\frac{2}{3}}k^{-5/3}(k^2\delta_{ij}-k_ik_j)/k^4, \quad (4)$$

where k is the magnitude of $\mathbf{k}$, ϵ the rate of turbulent energy dissipation, and A a constant which needs not be specified here. For $l=20$ cm, $p/l=0.2$, $k_1l\approx 1$, and for most applications of the sonic anemometer, this expression is correct for the wavenumber region which contributes significantly to the integrals in (3). Sub-

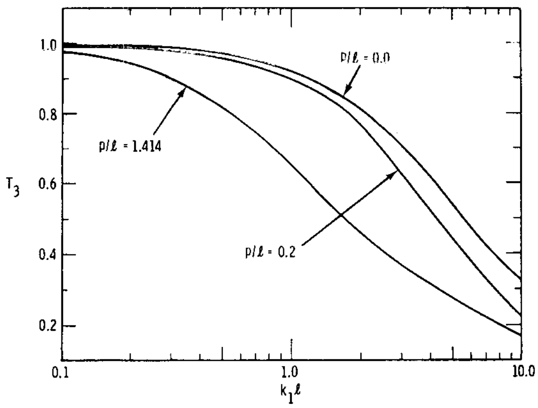

FIG. 2. Effect of averaging over two parallel paths on the transfer function for the vertical wind component.

stituting (4) into (3) and evaluating the integrals numerically produces the results shown in Fig. 2.

Qualitatively, the additional effect of a finite separation of the parallel paths is identical to increasing the path length. Within the range where spectral corrections to measured data are likely to be meaningful, i.e., $T>0.5-0.7$, there are only minor differences between T_3 for $p/l=0$ as calculated by Kaimal *et al.*, and for $p/l=0.2$ as for the Kaijo-Denki sonic anemometer. This fortunately eliminates the need for additional computation since the vertical transfer function is now also a function of α, the direction of the mean wind, i.e.,

$$\mathbf{k}\cdot\mathbf{p}=k_1p\,\sin\alpha+k_2p\,\cos\alpha. \tag{5}$$

The vertical transfer function was calculated here for $\alpha=0$, but the additional alteration in T_3 due to varying α is not significant for $|\alpha|\leqslant 30°$.

There are major changes in T_3 when p becomes comparable to l, as is seen for $p/l=1.414$. In this case T_3 must be calculated as a function of α. As shown in the following section, the effect of the small value $p/l=0.2$ also becomes significant when combined with the effect of measuring a component of the wind with two measurement axes whose centers are separated by a distance comparable to l.

3. Separation of the measurement axes

The transfer functions of the horizontal wind components are complicated by the fact that these components are measured by a combination of two separate axes. Kaimal *et al.*, has derived the appropriate expressions for these transfer functions and only two minor additions are required here. The first is the insertion of a factor $\cos^2(\mathbf{k}\cdot\mathbf{p}/2)$ in the integrals for the measured one-dimensional power spectra which appear in the numerators of the transfer functions. This accounts for the fact that each measurement axis consists of two parallel paths separated by the distance p and is identical to the modification presented in Eqs. (2)–(3), except that here $\mathbf{p}$ is in the vertical direction. The second modification accounts for the vertical separation of the two horizontal measurement axes by replacing

$$\mathbf{x}_A-\mathbf{x}_B=\mathbf{d} \tag{6}$$

with

$$\mathbf{x}_A-\mathbf{x}_B=\mathbf{d}+\mathbf{h}, \tag{7}$$

where $\mathbf{x}_A$ and $\mathbf{x}_B$ are the midpoints of the two horizontal measurement axes, $\mathbf{d}$ the horizontal component of the separation vector as defined by Kaimal *et al.*, and $\mathbf{h}$ its vertical component.

The transfer functions for the horizontal wind components were evaluated numerically to determine the individual and combined effects of averaging over two parallel paths for each axis and of separating the horizontal axes in the vertical. The results are presented in Figs. 3 and 4 for the case $\alpha=0$, where the largest deviations from the calculations of Kaimal *et al.* were found. The curves labeled a are the transfer functions as calculated by Kaimal *et al.*, for $p=h=0$. Allowing for $p/l=0.2$ (curves b) produces additional attenuation, which would be qualitatively expected, but the attenu-

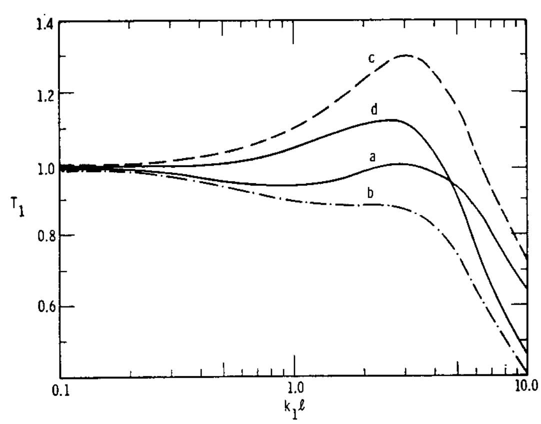

FIG. 3. Effect of averaging over two parallel paths and of path separation on the transfer function for the longitudinal wind component when $\alpha=0$ and $d/l=0.6$: a, $p/l=h/l=0$; b, $p/l=0.2$, $h/l=0$; c, $p/l=0$, $h/l=0.45$; d, $p/l=0.2$, $h/l=0.45$.

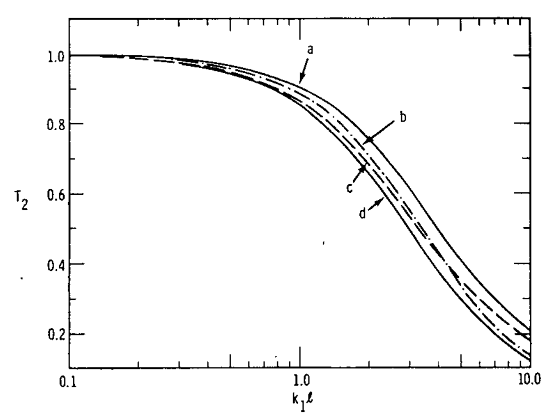

FIG. 4. As in Fig. 3 except for the lateral wind component.

ation is surprisingly large for the longitudinal wind component. Temporarily setting $p=0$ again and allowing for $h/l=0.45$ (curves c) produces an effect equivalent to increasing the horizontal separation of the axes: T_1 is raised and T_2 is lowered. Again the difference between curves a and c is quite large for T_1. Finally, the curves labeled d show the combined effect of the two additional factors considered here. The difference between curves a and d could be significant for correcting spectra in the inertial subrange. These differences would be apparent, for example, when checking for the isotropic $\frac{4}{3}$ ratio between crosswind and alongwind spectra or when using the inertial subrange form of the spectrum [Eq. (4)] to determine either the rate of dissipation or Kolmogorov's constant.

The computed transfer functions are tabulated in Table 1. Although listed to the third decimal place, the results are not quite that accurate due to the need to keep computer costs to a minimum. They are, however, accurate to $\pm 1\%$ and in most cases to ± 0.002 or better.

TABLE 1. Computed transfer functions for $d/l=0.6$, $p/l=0.2$, $h/l=0.45$.

	$T_1(k_1l)$ α (deg)				$T_2(k_1l)$ α (deg)				
k_1l	0	20	20	30	0	10	20	30	$T_3(k_1l)$
0.16	0.996	0.997	1.000	1.002	0.989	0.989	0.990	0.992	0.993
0.32	0.997	0.997	1.001	1.003	0.971	0.972	0.974	0.980	0.981
0.64	1.010	1.009	1.004	0.997	0.920	0.923	0.931	0.946	0.946
1.28	1.066	1.051	1.010	0.956	0.794	0.803	0.828	0.863	0.863
2.56	1.120	1.083	0.979	0.831	0.558	0.576	0.625	0.692	0.692
5.12	0.902	0.855	0.725	0.535	0.284	0.306	0.364	0.442	0.442
10.24	0.451	0.420	0.333	0.218	0.115	0.131	0.168	0.214	0.214
20.48	0.194	0.181	0.146	0.097	0.050	0.055	0.073	0.093	0.093

REFERENCE

Kaimal, J. C., J. C. Wyngaard and D. A. Haugen, 1968: Deriving power spectra from a three-component sonic anemometer. *J. Appl. Meteor.*, 7, 827–837.