NOTES AND CORRESPONDENCE

Effect of Moisture on the Hydrostatic Pressure

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ABSTRACT

This Note describes a method of deriving a modified form of the hydrostatic equation by making allowance for the presence of moisture in an atmospheric column.

The atmospheric pressure varies in accordance with two factors: 1) the mass of overlying atmosphere and 2) the vertical component of acceleration. Under hydrostatic conditions the changes in pressure at any point are determined by changes in the mass of the overlying atmospheric column. Whereas the mass variations, in any atmospheric column, may be affected by advective processes and/or by horizontal convergence, there is ample evidence to indicate that condensation of water vapor also contributes significantly to pressure variations. Thus, if we want to simulate real-atmosphere pressure distributions by integrating hydrostatic equation, it seems desirable to modify the conventional form of the equation by making allowance for the presence of moisture within the column.

This note describes a method of deriving the modified hydrostatic equation. In this derivation it is tacitly assumed that condensed water is not accelerating.

Under dry conditions the hydrostatic equation, in potential temperature form, is

$$\frac{\partial}{\partial z}\left(\frac{p}{p_0}\right)^{\kappa}=-\frac{g}{c_p\theta},\quad \text{where } \kappa=R/c_p. \tag{1}$$

Let us define

$$\rho=\rho_d+\rho_v+\rho_{cl}+\rho_R,$$

where the subscripts d, v, cl and R refer to densities of dry air, water vapor, cloud water and liquid precipitation, respectively. Then the hydrostatic equation can be written as

$$\frac{\partial p}{\partial z}=-(\rho_d+\rho_v+\rho_{cl}+\rho_R)g,$$

or

$$\frac{\partial p}{\partial z}=-(\rho_d+\rho_v)g-(\rho_{cl}+\rho_R)g. \tag{2}$$

From the equation of state and the definition of virtual temperature,

$$(\rho_v+\rho_d)=\frac{p}{RT^*}, \tag{3}$$

where $T^*=T(1+0.6Q_v)$ is the virtual temperature. Also by definition,

$$Q_v=\frac{\rho_v}{\rho_d},\quad Q_{cl}=\frac{\rho_{cl}}{\rho_d},\quad Q_R=\frac{\rho_R}{\rho_d}. \tag{4}$$

Substituting (3) and (4) in (2), we get

$$\frac{\partial p}{\partial z}=-\frac{pg}{RT^*}-\rho_d(Q_{cl}+Q_R)g,$$

or substituting for T^*

$$\frac{\partial p}{\partial z}=-\frac{pg}{RT(1+0.6Q_v)}-\rho_d(Q_{cl}+Q_R)g. \tag{5}$$

From Poisson's equation

$$T=\theta\left(\frac{p}{p_0}\right)^{\kappa}. \tag{6}$$

Then Eq. (5) becomes

$$\frac{\partial p}{\partial z}=-\frac{pg}{R\theta(p/p_0)^{\kappa}(1+0.6Q_v)}-\rho_d(Q_R+Q_{cl})g.$$

Multiplying both sides by $(p/p_0)^{\kappa}$ and using (6), we obtain

$$\left(\frac{p}{p_0}\right)^{\kappa}\frac{\partial p}{\partial z}=-\frac{pg}{R\theta(1+0.6Q_v)}-\frac{T}{\theta}\rho_d(Q_R+Q_{cl})g.$$

If we now divide both sides by p and use the definition

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$p=\rho_d RT$, we have

$$\frac{1}{p}\left(\frac{p}{p_0}\right)^{\kappa}\frac{\partial p}{\partial z}=-\frac{g}{R\theta}\left(\frac{1}{1+0.6Q_v}+Q_R+Q_{cl}\right),$$

or

$$\frac{\partial}{\partial z}\left(\frac{p}{p_0}\right)^{\kappa}=-\frac{g\kappa}{R\theta}\left(\frac{1}{1+0.6Q_v}+Q_R+Q_{cl}\right),$$

or

$$\frac{\partial}{\partial z}\left(\frac{p}{p_0}\right)^{\kappa}=-\frac{g}{c_p\theta}\left(\frac{1}{1+0.6Q_v}+Q_R+Q_{cl}\right),\ \text{since}\ \kappa=\frac{R}{c_p}.\quad (7)$$

This is the same as Eq. (1) *except* for the quantity in parentheses which arises due to the presence of moisture (both in vapor and condensed form) in the atmosphere.

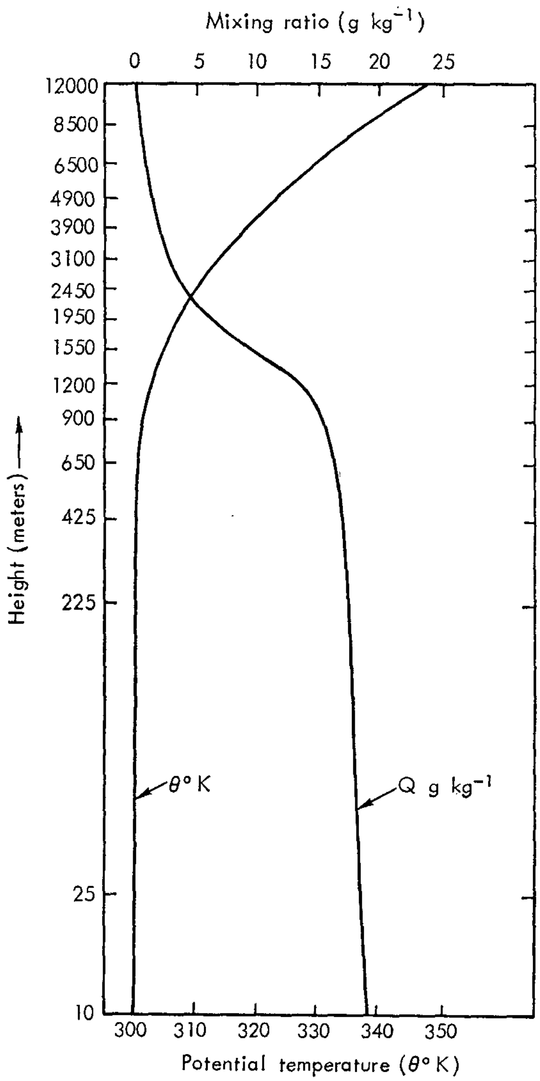

FIG. 1. Vertical profiles for potential temperature (θ) and mixing ratio (Q_v).

TABLE 1. Surface pressure values (mb).

Case	Surface pressure
(i)	1009.1
(ii)	1006.9
(iii)	1008.0

On an approximate basis we can write Eq. (7) as

$$\frac{\partial}{\partial z}\left(\frac{p}{p_0}\right)^{\kappa}=-\frac{g}{c_p\theta}(1-0.6Q_v+Q_R+Q_{cl}),\qquad (8)$$

or

$$\frac{\partial}{\partial z}\left(\frac{p}{p_0}\right)^{\kappa}=-\frac{g}{c_p\Theta},$$

where

$$\Theta=\theta[1+(0.6Q_v-Q_{cl}-Q_R)]$$

defines "virtual" potential temperature.

The importance of Eq. (8) *vis-à-vis* Eq. (1) has been evaluated by numerical integration of these equations for three atmospheric columns, i.e., (i) completely dry, (ii) moist but *without* condensed water, and (iii) moist *with* condensed water. This, in effect, involves integrating Eq. (1) for the case (i), Eq. (8) with $Q_R=Q_{cl}=0$ for case (ii), and Eq. (8) with both Q_R and $Q_{cl}\neq 0$ for case (iii).

Fig. 1 shows vertical profiles of θ and Q_v which have been used in cases (i) and (ii). These profiles are based on an observed sounding at Grand Bahama Island in the Caribbean region. Integration for case (iii) has been performed, basically, to demonstrate the effect of the presence of liquid water, Q_R, on the hydrostatic pressure; thus, Q_R has been specified to be 1 gm kg^{-1}. In this particular instance, the presence of cloud water amount (Q_{cl}) has not been considered. It may be remarked here that a non-zero value for Q_R may be used even when the conditions are not saturated. In the case of saturated or supersaturated conditions, Q_{cl} has also to be accounted for.

Table 1 shows the computed surface pressure for the three cases.

It can be seen that Eq. (1) overestimates the surface pressure by 2.2 mb for "normal" moist conditions and by 1.1 mb when condensed water is also present. A comparison of cases (ii) and (iii) shows that the presence of condensed water in the column *increases* the surface pressure by 1.1 mb. This result is physically consistent since the weight of liquid water can cause significant changes at the surface.

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