Polar Boundary Conditions in Zonally Averaged Global Climate Models

MICHAEL HANTEL

Meteorologisches Institut der Universität Bonn, 53 Bonn, Federal Republic of Germany

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ABSTRACT

We consider global climate models based on zonally averaged balance relations. Inherent boundary conditions require the meridional fluxes of non-negative properties (temperature, humidity, energy, etc.), as well as the flux of zonal momentum, to vanish at both poles. On the other hand, the meridional divergence of these fluxes does not vanish at either pole. An important exception from this general non-zero polar divergence condition of meridional fluxes is the transport of zonal momentum; the meridional divergence of zonal momentum flux vanishes at the pole because there is neither zonal surface stress nor horizontal wind. These conditions are derived from the balance equations for energy and momentum. Furthermore, they are tested with observed flux data for specific humidity and zonal wind. The closure problem in such models is often overcome by a diffusive parameterization of the fluxes in terms of meridional gradients. It is shown that, due to the above conditions, the exchange coefficient for the energy transport may not vanish at the poles. This has implications for semi-empirical models designed to test climate's stability and transitivity.

1. Introduction

The question whether the climate of the global atmosphere is stable or unstable is currently a matter of considerable concern to climatologists. The most straightforward method of studying this problem seems to be the continued integration of high-resolution, four-dimensional circulation models. It turns out, however, that there are several limiting effects against this approach; for instance, the predictability which breaks down after some weeks (Lorenz, 1964), and the long averaging time necessary for reducing the rms noise in the model climate. Concerning the latter effect, Leith (1973) estimates that a climate change experiment designed to resolve a shift in the climatic mean of one-eighth of the standard deviation of day-to-day fluctuations would require a record length of at least one year.

We consider in this study a simplified approach: the class of statistical-dynamical models in the sense of SMIC (1971). The concept of these models is that the physical conservation laws are applied to statistics of the atmospheric variables—mass, momentum, heat and water content. We are concerned here only with conservation laws averaged with respect to longitude and time. The unknown quantities (wind, temperature, etc.) are then functions of latitude and height. The condition of stationarity is, by the way, not essential for the points made here and is chosen only for simplicity; the main conclusions are also valid for time-dependent conservation equations.

The physical consistency of these models depends on proper polar boundary conditions. If, for example, the vertical mean horizontal divergence of the meridional energy flux would vanish at a pole, internal consistency would also require that the net radiation flux across the upper surface of the atmosphere vanish at this pole. Such a condition, however, would be contradictory to observation.

Another point concerning zonally averaged atmospheric models is the closure or parameterization problem of the horizontal and vertical eddy fluxes. We discuss here only the parameterization of the horizontal eddies with aid of a Fickian-type diffusive expression in terms of the meridional gradient of the respective mean property (Defant, 1921). Specifically, we shall show that the meridional exchange coefficient for the energy flux generally does not vanish at the pole.

2. Polar boundary conditions

We shall assume that all meteorological field quantities or fluxes considered are continuous and differentiable functions of the geographic coordinates λ (longitude) and ϕ (latitude). It follows that the zonal average of non-negative properties A like (potential) temperature, specific humidity, mixing ratio of trace substances, geopotential, kinetic energy, etc., must have the meridional gradient zero at the poles:

$$\phi=\pm\pi/2: \quad \{A\}'=0, \quad A=\theta, q, \Phi, V^2/2, \tag{2.1}$$

where the braces denote averaging with respect to λ, and the prime denotes partial differentiation with respect to ϕ. These intuitively evident boundary conditions can be proved by elementary differential calculus reasoning.

Similarly, the zonal average of meridional transports of non-negative properties must vanish at the poles:

$$\phi=\pm\pi/2: \qquad \{vA\}=0, \tag{2.2}$$

where v is the meridional wind component, and A is as in (2.1). We now show that (2.2) is valid for any property A subject to an equation in pressure (p) coordinates of the form

$$\frac{1}{\cos\phi}\frac{\partial\{vA\}\cos\phi}{\partial y}=\frac{\partial\{r_A\}}{\partial p}. \tag{2.3}$$

This conservation relation for A requires the meridional divergence of the horizontal flux of A to be balanced by the vertical divergence of a generalized vertical flux r_A. We shall specify r_A for the various transported properties later. Eq. (2.3) is stationary. We thus interpret the braces from now on as an averaging operator with respect to λ and t (time).

Splitting the horizontal divergence into two parts we have at the poles

$$\lim\frac{\partial\{vA\}}{\partial y}-a^{-1}\lim[\{vA\}\tan\phi]=\frac{\partial r_A(\phi=\pm\pi/2)}{\partial p}. \tag{2.4}$$

Here and in the following, lim is an abbreviation for $\lim_{\phi\to\pm\pi/2}$; a denotes the earth's radius. Since the right-hand side is finite, the left-hand side must also be finite. But $\lim\tan\phi=\pm\infty$. It follows that $\lim\{vA\}=0$ as stated in (2.2). In arguing this way we have tacitly assumed that $\lim\{vA\}'$ must be finite, an assumption which seems self-evident *a priori.*

We note that the previous argument leans critically on the validity of (2.3). By choosing specifically $A\equiv 1$, Eq. (2.3) becomes the continuity equation: $r_A\equiv r_1\equiv -\omega$, where ω is the vertical velocity in pressure coordinates. This yields the obvious condition $\lim\{v\}=0$ [Eq. (2.5)]. Stationarity of (2.3) is not crucial for (2.2) as can be easily seen. In other words, the zonal mean polar transport of a non-negative property A must vanish at any instant. This condition applies even for A equal to the zonal momentum u, although u does not obey a relation of the form (2.3); this case is discussed in Section 4.

Not only the mean meridional wind but also the mean zonal wind must vanish at the poles:

$$\phi=\pm\pi/2: \qquad \{u\}=0,\quad \{v\}=0. \tag{2.5}$$

This is obvious because both horizontal wind components change sign at $\phi=\pm\pi/2$. Notice that condition (2.5) is even met by an actual wind field blowing across the pole.

No *a priori* polar condition like (2.1), (2.2) is available for $\{vA\}'$ except that it must be finite. Obviously, the meridional divergence $(\cos\phi)^{-1}\partial\{vA\}\cos\phi/\partial y$ does generally not vanish for $\phi=\pm\pi/2$ since it must balance the right-hand side of Eq. (2.3). We shall discuss this basic non-zero polar divergence condition in terms of the moist static energy and the latent heat equation. An important exception from this rule is the meridional transport of zonal wind which is divergence-free at either pole.

3. Polar energy flux convergence

The thermodynamic energy equation in pressure coordinates may be written

$$\frac{d}{dt}(gz+c_pT+Lq)=g\frac{\partial}{\partial p}(H_R+H_T+LH_q), \tag{3.1}$$

where $gz+c_pT+Lq$ is the moist static energy (Kreitzberg, 1964). Because this quantity is closely related to the equivalent potential temperature (Riehl and Malkus, 1958) we shall denote it θ_e, with physical dimensions of joules per gram. The diabatic heating is supplied by the divergence of three vertical fluxes: H_R is the radiation flux, H_T and H_q the microturbulent fluxes of temperature and specific humidity, respectively (L being the latent heat of condensation). The sign of fluxes H_R, H_T, LH_q (J gm^{-1} sec^{-1}) is such that positive values are upward. No horizontal components of the flux divergences have been taken into account because their omission does not affect the reasoning to follow; further, these components are small anyway. Concerning the physical content of Eq. (3.1) we note (i) the hydrostatic assumption is the main approximation made; (ii) the equation is not affected by condensation of water vapor or evaporation of rain or ice within the control volume; and (iii) the diffusive fluxes H_T and H_q are not of importance except within the planetary boundary layer.

We average the energy equation with respect to λ and t over a long enough time period such that $\partial\{\theta_e\}/\partial t=0$. The result is a conservation relation of the form (2.3) with $A\equiv\theta_e$ and

$$r_A\equiv r_{\theta_e}\equiv g\{H_R+H_T+LH_q\}-\{\omega\theta_e\}. \tag{3.2}$$

It follows that condition (2.2) applies to the moist static energy: the meridional energy flux $\{v\theta_e\}$ must vanish at the poles.

We consider next the horizontal divergence of the energy flux:

$$\frac{1}{\cos\phi}\frac{\partial\{v\theta_e\}\cos\phi}{\partial y}=\frac{\partial\{v\theta_e\}}{\partial y}-a^{-1}\{v\theta_e\}\tan\phi. \tag{3.3}$$

For $\phi\to\pm\pi/2$, the second term on the right-hand side is indefinite. We write it in the form

$$\{v\theta_e\}\tan\phi\equiv\frac{\{v\theta_e\}\sin\phi}{\cos\phi}. \tag{3.4}$$

Employing l'Hôpital's rule, we find

$$\left.\begin{aligned}\lim[\{v\theta_e\}\sin\phi]' &= \lim[\{v\theta_e\}'\sin\phi+\{v\theta_e\}\cos\phi] \\ &= \pm\lim\{v\theta_e\}' \\ \lim(\cos\phi)' &= -\lim\sin\phi = \mp 1\end{aligned}\right\}. \quad (3.5)$$

It follows that

$$\lim\{v\theta_e\}\tan\phi = -\lim\{v\theta_e\}', \quad (3.6)$$

which yields the polar divergence:

$$\lim\frac{1}{\cos\phi}\frac{\partial\{v\theta_e\}\cos\phi}{\partial y} = 2\lim\frac{\partial\{v\theta_e\}}{\partial y}. \quad (3.7)$$

We see that the meridional derivative of the flux $\{v\theta_e\}$ at the poles is directly proportional to its meridional divergence. This applies not only to $\{v\theta e\}$ but to any meridional flux divergence. Keeping this point in mind we need not make a difference in the subsequent discussion between the meridional divergence and the meridional derivative of an arbitrary property flux.

It is now obvious that the divergence of $\{v\theta_e\}$ is generally not zero at the poles. Using the balance relation (2.3) for $A\equiv\theta_e$—which was not used in deriving condition (3.7)—we find

$$\lim\frac{\partial\{v\theta_e\}}{\partial y} = \frac{1}{2}\frac{\partial r_{\theta_e}(\pm\pi/2)}{\partial p}. \quad (3.8)$$

We consider this polar energy balance first in terms of the vertical mass average (denoted by the caret):

$\phi=\pm\pi/2$:

$$\frac{\partial\{\widehat{v\theta_e}\}}{\partial y} = \frac{g}{2\{p_s\}}\{H_{R,s}+H_{T,s}+LH_{q,s}-H_{R,0}\}. \quad (3.9)$$

The subscripts s, 0 denote the pressure levels surface and zero, respectively, which are both horizontal at the poles due to condition (2.1). The sum of the first three terms on the right is the surface energy balance and supposedly small at the pole. In other words, the radiation loss $-H_{R,0}$ across the atmosphere's upper surface must be balanced at the pole by the convergence $-\partial\{\widehat{v\theta_e}\}/\partial y$ of the mean horizontal energy flux.

Second, we like to speculate about the implications of Eq. (3.8) if not vertically averaged. The vertical energy flux r_{θ_e} in its generalized form (3.2) comprises mean and mesoscale eddy fluxes of moist static energy $(\omega\theta_e)$ in addition to the radiation flux (H_R) and the small-scale diffusive fluxes of sensible and latent heat (H_T, H_q). Since the latter are hardly of influence in most of the atmospheric vertical column, we should have the approximate polar balance

$$\phi=\pm\pi/2: \qquad \underset{\text{(I)}}{\frac{\partial\{v\theta_e\}}{\partial y}} \approx \frac{1}{2}\frac{\partial}{\partial p}\{\underset{\text{(II)}}{gH_R}-\underset{\text{(III)}}{\omega\theta_e}\}. \quad (3.10)$$

In order to account for the relative influence of these three terms we assume a polar net radiation loss of the order $\{H_R\}\approx 100$ W m^{-2}. Taking this figure as representative for a 500-mb layer we find the radiation flux divergence to be of the order (II)$\approx 10^{-2}$ W kg^{-1}. Concerning the vertical energy flux we note that θ_e has values between 260 and 360 J gm^{-1}, the vertical variation being about 50 J gm^{-1} by 500 mb and the meridional about 10 J gm^{-1} by 10° latitude (Palmén and Newton, 1969, Fig. 17.4). The zonal mean vertical velocity in higher latitudes is of the order of at most 0.2×10^{-4} mb sec^{-1} [as compared to 1–3×10^{-4} mb sec^{-1} in Hadley cell latitudes (see Oort and Rasmusson, 1971, Table A3)]. It follows that the vertical energy flux convergence due to the mean circulation, representative for a 500 mb-layer, is of the order (III)$\approx 0.1\times10^{-2}$ W kg^{-1} in polar latitudes. Thus, the polar vertical energy flux convergence cannot balance the radiation loss. Rather, the horizontal convergence (I) must be responsible for the balance. In Hadley cell latitudes, on the other hand, terms (II) and (III) are generally of the same order.

It is easy to show that a realistic horizontal convergence is indeed sufficient for balancing the polar radiation loss. Employing the above figure of 10 J gm^{-1} for the variation of θ_e over a 10° latitude strip, we find with a meridional wind of the order 1 m sec^{-1} the required horizontal energy flux convergence (I)$\approx 10^{-2}$ W kg^{-1}.

The above estimates employ only the mean contribution to $\{\omega\theta_e\}$ and $\{v\theta_e\}$. This does not, however, affect the result just obtained since in polar latitudes the eddy contribution to the vertical flux $\{\omega\theta_e\}$ can hardly be of any influence under current climatic conditions. An ice-free Arctic Ocean, on the other hand, with sizeable convective activity aloft, could change the eddy part of $\{\omega\theta_e\}$ considerably. Consequently, an arctic energy budget very different from that of today would require a different polar divergence of $\{v\theta_e\}$ which would also have consequences for lower latitudes. For present-day conditions we conclude that *the vertical radiation flux divergence of Eq. (3.10) must be balanced at the poles by horizontal energy flux convergence.* Clearly, this condition depends on the season: for annual mean conditions and in winter, the energy flux converges, whereas in summer it diverges. In the transition seasons it vanishes for a short period. Generally, however, the polar divergence of the horizontal energy flux should be significantly different from zero.

4. Polar latent heat flux convergence

The qualitative conclusions of the previous section should be corroborated by measurements. Unfortunately, no observed data of the zonal mean horizontal flux of moist static energy were conveniently available. Horizontal latent heat transport data, however, were available over the north polar cap, with the necessary horizontal resolution of at least 2.5° latitude.

We shall briefly discuss the polar conditions of the water transport equation which are very similar to the energy flux conditions.

Like the static energy which stands in continuous exchange with space across the upper surface of the atmosphere, the water vapor field stands in exchange with the hydrosphere across the earth's surface through evaporation and precipitation. The specific humidity follows a conservation relation of the form (2.3), with $A\equiv q$. Thus, all general properties discussed above apply to the q transport. The polar divergence is

$$\phi=\pm\pi/2:\qquad \frac{\partial\{vq\}}{\partial y}=\frac{1}{2}\frac{\partial}{\partial p}\{gH_q+gH_C-\omega q\},\tag{4.1}$$

where H_q is the microturbulent diffusive water vapor flux as defined above, and H_C the vertical flux of water in condensed form (water or ice). The vertical average of (4.1) equals evaporation minus precipitation. Though this difference is small at the poles (order 0.1 mm day^{-1}) as compared with Hadley cell latitudes (1–2 mm day^{-1}), it is not zero and primarily negative at the North Pole (see, e.g., Newton, 1972), requiring a net water vapor flux convergence there.

These considerations are confirmed by Fig. 1 which shows the vertical mean meridional water vapor transport pattern between the equator and the North Pole for 6-year averaged January conditions. The transport converges at the pole with an approximate gradient of -0.4(gm kg^{-1}) (m sec^{-1}) (556 km)$^{-1}$. This value corresponds to 1.7 mm day^{-1} for precipitation minus evaporation which is larger than the mean climatological winter conditions [0.2 mm day^{-1} according to Newell *et al.* (1969) for December–February]. Since actual precipitation minus evaporation data for thc 1967–72 January conditions are not available, no direet balance check with the horizontal water vapor flux convergence is possible. It suffices to say that the polar flux convergence of latent energy is indeed different from zero. Within the limits of accuracy, this difference is significant as further discussed in the next section.

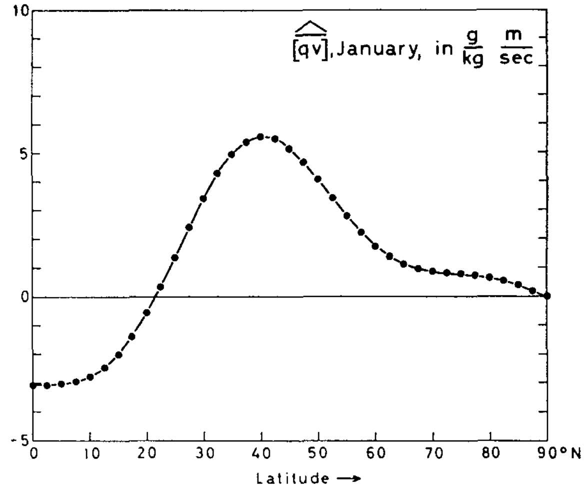

FIG. 1. Meridional transport of water vapor mixing ratio, for January, averaged with respect to time (1967–72), longitude and height (1000 to 70 mb). Unpublished data of P. Speth.

5. Polar momentum flux convergence

Most of the arguments of the previous sections can be applied to the polar boundary condition for the flux of zonal momentum although there are a few distinct differences. The balance equation for atmospheric angular momentum in pressure coordinates is

$$\frac{dI}{dt}=-a\cos\phi\left(\frac{\partial\Phi}{\partial x}+g\frac{\partial\tau_x}{\partial p}\right),\tag{5.1}$$

with $\Phi\equiv gz$ the geopotential, τ_x the zonal component of horizontal stress, and

$$I=a\cos\phi(u+a\Omega\cos\phi).\tag{5.2}$$

Following Newton (1972) we have neglected the height dependence of angular momentum; mountain torques (Mintz, 1955) are included in τ_x. We average the momentum equation with respect to longitude and time and obtain

$$\frac{1}{\cos^2\phi}\frac{\partial\{vu\}\cos^2\phi}{\partial y}=f\{v\}-g\frac{\partial\{\tau_x\}}{\partial p}-\frac{\partial\{\omega u\}}{\partial p}.\tag{5.3}$$

With the aid of the continuity equation the meridional wind can be put in the form

$$\{v\}=\frac{g}{2\pi a\cos\phi}\frac{\partial\Psi}{\partial p},\tag{5.4}$$

where Ψ is the mass-transport streamfunction (gm sec^{-1}). It follows that the right-hand side of Eq. (5.3) can be written as $\partial r_u/\partial p$, with

$$r_u\equiv\frac{fg\Psi}{2\pi a\cos\phi}-g\{\tau_x\}-\{\omega u\}.\tag{5.5}$$

This generalized vertical momentum flux is analogous to the vertical energy flux r_{θ_e} as defined in (3.2). The principal differences between r_u, r_{θ_e} and the pertinent balance relations (5.3), (2.3) are: (i) Eq. (5.3) depends upon $\cos^2\phi$, (2.3) only upon $\cos\phi$; (ii) the momentum flux r_u has no contribution across the atmosphere's upper surface whereas r_{θ_e} contains the radiation flux H_R which does cross the surface $p=0$; and (iii) at the pole, the flux (5.5) vanishes at all pressure levels whereas the vertical energy flux (3.2) does not; this point is further discussed below.

We ask for the polar boundary conditions of $\{vu\}$ and $\{vu\}'$. Splitting the left-hand side of (5.3) into two parts we have

$$\frac{\partial\{vu\}}{\partial y}-2a^{-1}\{vu\}\tan\phi=\frac{\partial\{r_u\}}{\partial p}.\tag{5.6}$$

In order for $\{vu\}'$ to be finite, $\{vu\}$ must vanish at the poles since $\partial\{r_u\}/\partial p$ is finite there but $\tan\phi$ is not. Further, as in the derivation of Eq. (3.7), we find by again employing l'Hôpital's rule that

$$\lim \frac{1}{\cos^2\phi} \frac{\partial\{vu\} \cos^2\phi}{\partial y} = 3 \lim \frac{\partial\{vu\}}{\partial y}, \tag{5.7}$$

yielding the polar condition

$$\phi = \pm\pi/2: \qquad \frac{\partial\{vu\}}{\partial y} = \frac{1}{3}\frac{\partial r_u}{\partial p}. \tag{5.8}$$

We see that the horizontal derivative of the momentum transport is balanced close to either pole by 33% of the vertical divergence of the flux (5.5). But the first term in the vertical divergence is proportional to $\{v\}$ which is zero at the pole. In other words, the streamfunction Ψ vanishes in all levels at the pole because the earth's axis is a streamline for the mass transport; the derivative $\partial\Psi/\partial y$ vanishes due to $\{v\}=0$. Further, if $\{\tau_x\}$ is parameterized, as usual, by the vertical gradient of $\{u\}$, it also vanishes due to condition (2.5). Therefore, the polar momentum flux divergence could only be balanced by a vertical eddy flux divergence of the form $0.33\ \partial\{\omega^*u^*\}/\partial p$, where the asterisk denotes the deviations from the mean { }. If u is to be continuous, however, u^* cannot be different from zero close to and at the pole. We thus conclude that the meridional momentum flux divergence vanishes at either pole, i.e.,

$$\phi = \pm\pi/2: \qquad \frac{\partial\{vu\}}{\partial y} = 0. \tag{5.9}$$

This result is confirmed by Fig. 2 which shows the meridional momentum transport pattern between equator and the North Pole for 6-year average January conditions. Within the limits of drawing accuracy, $\{vu\}'$ vanishes exactly at the North Pole.

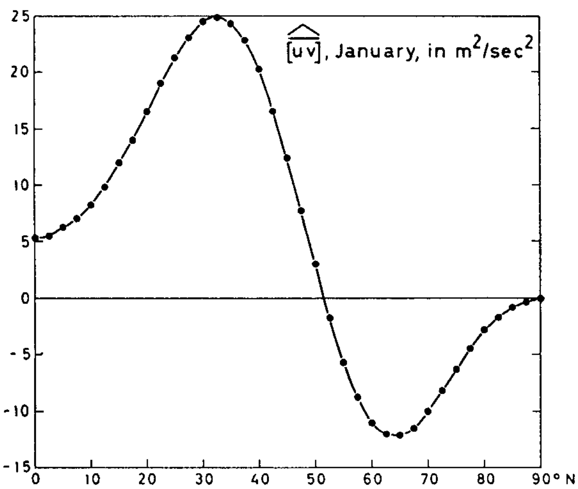

FIG. 2. As in Fig. 1, except for the transport of zonal momentum.

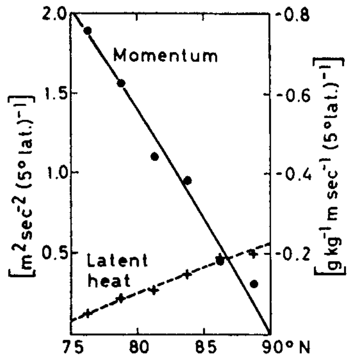

FIG. 3. Vertically averaged divergence of latent heat flux $\{qv\}'$ (data from Fig. 1) and of momentum flux $\{uv\}'$ (data from Fig. 2) for the north polar cap, January 1967–72. Unpublished data of P. Speth.

This is an interesting result, particularly when confronted with the energy flux. In order to show the significant difference between the polar divergence of momentum and latent heat flux, the profiles of both flux derivatives close to the North Pole are plotted in Fig. 3. The ratio of the respective ordinates is the same as the ordinate ratio of Figs. 1 and 2. Despite the minor scattering of the individual values it should be obvious from Fig. 3 that *the momentum flux divergence indeed vanishes at the pole whereas the latent heat flux converges there.*

6. Diffusive parameterization of eddy heat fluxes

Defant (1921) first applied the Austausch concept to the diffusive parameterization of large-scale energy fluxes in a remarkable early study concerning global climatic modelling. He indicated that the meridional flux of sensible heat can be expressed by the meridional temperature gradient, the constant of proportionality being a positive-valued smooth function of ϕ, of the order 10^6 m² sec⁻¹, with a maximum in mid-latitudes. These results have been confirmed by Wiin-Nielsen and Sela (1971) for several pressure surfaces in terms of a quasi-geostrophic model. We apply Defant's Fickian-like diffusive law to the meridional energy transport to obtain

$$\{v\theta_e\} = -k\frac{\partial\{\theta_e\}}{\partial y}. \tag{6.1}$$

This formula has been used recently by several authors for climate modelling (e.g., Sellers, 1969; Saltzman and Vernekar, 1971; Faegre, 1972). Zonal mean meridional transports have been explicitly taken into account by Sellers but are sometimes included in the parameterization (6.1) [see Faegre (1972)]. As noted in Section

3, the horizontal divergence of the mean fluxes can be of some influence.

If Eq. (6.1) is applied with constant diffusion coefficient k, its value is determined by global requirements (Defant, 1921; Saltzman and Vernekar, 1971). There are objections, however, against a k constant with latitude. It has been argued recently by Stone (1973) in terms of a model for the eddy and radiative fluxes that there is no response of the global mean horizontal temperature gradient to changes in the rotation rate with constant k. Stone concludes that a horizontal diffusion coefficient constant with latitude is a poor assumption. In the models of Sellers, Sela and Wiin-Nielsen, and Faegre, k varies with latitude.

We are concerned here only with the polar aspect. In two of the models just mentioned (Faegre, Sela and Wiin-Nielsen) k vanishes at a pole. Let us look for the consequences. Obviously, (6.1) vanishes at the poles due to condition (2.1). The meridional derivative of (6.1) is

$$\{v\theta_e\}' = -\frac{k'}{a}\{\theta_e\}' - \frac{k}{a}\{\theta_e\}''. \tag{6.2}$$

Since the left side generally does not vanish at either pole as shown in Section 3, whereas the first term on the right does vanish due to (2.1), we conclude (note that in general $\{\theta_e\}'' \neq 0$):

$$k(\pm\pi/2) \neq 0. \tag{6.3}$$

We shall discuss the implications, if condition (6.3) is violated, in terms of Faegre's (1971) model in which a meridional k profile with vanishing k was used at the South Pole. It follows that the total radiation balance must vanish there. Following Faegre's parameterization of the radiative fluxes, this leads implicitly to a relation for the southern boundary temperature T:

$$\phi = -\pi/2: \qquad K\sigma T^4 - \beta T + \alpha = 0, \tag{6.4}$$

where σ is the Stefan-Boltzman constant, and α, β, K are (for variable ϕ) empirical functions of latitude adjusted to fit the conditions of the present atmospheric circulation; in (6.4) the latter are specified constants.

Eq. (6.4) was not observed by Faegre, apparently due to the fact that the climate equation was solved by numerical techniques. Beyond the implication of vanishing polar radiation balance, which does not seem to be very realistic, the condition (6.4) causes the model to collapse at the South Pole. It is easily seen that there are at most two discrete temperatures T obeying (6.4). Faegre, however, reported five different temperature profiles.

We conclude that the far-reaching consequences drawn by Faegre (1972) concerning the existence of but five climate solutions for the earth-atmosphere-ocean system, of which only one is stable, are to be considered with caution (see also Schneider and Gal-

TABLE 1. Polar boundary conditions for atmospheric field quantities and fluxes: $\{\ \}$ denotes zonal and time averaging, primes denote $\partial/\partial\phi$, ϕ is latitude, Φ the geopotential, Ψ the mass transport streamfunction, and $\theta_e = \Phi + c_pT + Lq$ the moist static energy.

Field quantities	Meridional fluxes	Meridional gradients	Meridional divergences
$\{u\}=0$	$\{vu\}=0$	$\{\Phi\}'=0$	$\{vu\}'=0$
$\{v\}=0$	$\{v\Phi\}=0$	$\{T\}'=0$	$\{v\theta_e\}'\neq 0$
$\{\Psi\}=0$	$\{vT\}=0$	$\{q\}'=0$	$\{vq\}'\neq 0$
$\{\tau_x\}=0$	$\{vq\}=0$	$\{\theta_e\}'=0$	
$\{\tau_y\}=0$	$\{v\theta_e\}=0$	$\{V^2/2\}'=0$	
	$\{vV^2/2\}=0$		

Chen, 1973). More consistent proofs for the instability and intransitivity of model climates seem necessary.

7. Concluding remarks

The arguments of this study concerning moist static energy, latent heat and momentum balance can be transferred to other intensive properties like kinetic energy or trace constituents. The results are summarized in Table 1. The braces can be interpreted as a zonal average or as a zonal-plus-time average. The meridional derivative of field quantities is denoted as gradient, the derivative of fluxes as divergence. As has been shown in Section 3, the zonal mean divergence of a flux at the poles is proportional to the derivative of the flux; the constant of proportionality is 2 for all relevant transports except for zonal momentum transport for which it is 3. We note that no general polar condition for $\{\omega\}$ is available which indicates that sizeable convective activity even in zonally averaged form would, in principle, be possible over the polar caps; apparently, however, this is not the case for today's climate.

The zonal momentum seems to be the only intensive field quantity in the atmosphere whose meridional flux divergence is required to vanish at the poles. The fluxes of all other properties seem in general to diverge or converge at the pole in order to balance the pertinent vertical transports across the horizontal boundaries of the atmosphere. These results have been gained by theoretical consideration of the pertinent balance relation. Further, the significant difference between the polar divergences of momentum and latent heat flux have been demonstrated in Fig. 3. Since water vapor is an important part of the moist static energy we consider Figs. 1 and 3 as a preliminary proof for the conclusions of Section 3 concerning the energy transport.

The implications for climate models based upon the zonally (and vertically-and-time) averaged energy equation are obvious. If diffusive parameterization is used for the meridional transports—which mainly comprise the eddy contributions—the energy exchange coefficient must not vanish at either pole in order to keep the problem properly posed. If boundary conditions as discussed here are violated this is not immediately detected, particularly if the resulting nonlinear equation is solved numerically.

The present discussion has shown the interdependence between the polar energy balance and the parameterization assumptions. More generally, it has shown the relevance of the polar boundary conditions for a variety of meteorological quantities. It might be argued that the results are only due to the particular coordinate system used and thus merely of mathematical relevance; even the term boundary condition might be questionable if applied to the poles because a pole does not look like a "boundary". These arguments, however, would only apply if the earth did not rotate. For a rotating earth the familiar polar coordinate system clearly bears some physical relevance. The earth's angular momentum significantly affects the zonal wind budget and thus, via the meridional circulation coupling, the budgets of all other quantities.

Thus we are tempted to speculate about the interdependence between the polar and lower latitudes in the light of the particular sensitivity of the world climate to the conditions in high latitudes. This speculation is supported both by model results and by observations. The model experiments of Schneider and Gal-Chen (1973) showed a relatively insensitive behavior of the tropical latitudes to parameter perturbations. Likewise, observations suggest that there has been no ice age in the tropics in the last 600 million years (Flohn, 1969) whereas ice ages have appeared in middle and higher latitudes since then. Even in the most recent history there are observations of sizeable temperature fluctuations of the order of 1C per 10 years but only outside the tropics (SMIC, 1971, Fig. 3.8).

A more general viewpoint concerning the class of semiempirical climate models considered here can be traced back to Defant (1921). His work is most important both for introducing the first global climate model based on a quantitative balance relation for sensible energy as well as for applying the Austausch concept to the large-scale energy transport. In his, as well as in the subsequent investigations of this type (notably Budyko, 1969; Sellers, 1969; Saltzman and Vernekar, 1971; Sela and Wiin-Nielsen, 1971; Faegre, 1972; Schneider and Gal-Chen, 1973; and others), the proper parameterization of both the meridional energy transport and the radiation transport properties of the atmosphere plays the key role. Defant discussed qualitatively the complex interrelations between the parameters governing the solar constant, and the radiative and circulation characteristics of the atmosphere; he recognized the existence of mechanisms which tend to balance perturbations of one of the parameters by opposite perturbations of the others.

It might be stressed here that these model parameters, specifically the actual exchange coefficients, are valid only for today's climate conditions. Consequently, one cannot study large deviations from today's climate with today's parameters. All one can do is to ask for instabilities of the models tending to leave the delicate balance of the present climate. This line of thinking should be similar to the linearization approach in the study of nonlinear systems. Only small deviations from the mean may be admitted whereas large deviations are not properly described by linear approximations. Similarly, semi-empirical climate models are only valid within limits fairly close to today's conditions since the coefficients would certainly be entirely different for entirely different climates. In particular, far-reaching conclusions indicating, for example, the possibility of a completely ice-covered earth, are to be considered with strong reservations.

Despite these basic limitations, we finally like to support Budyko's quotation at the end of Schneider's and Gal-Chen's paper. Whereas semi-empirical climate models can hardly solve the exorbitant problem of climate stability and transitivity, they are promising toys and even tools toward posing the right questions and should thus be continued and eventually understood.

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