Objective Determination of the Noise Level in Doppler Spectra[1]

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(Manuscript received 4 October 1973, in revised form 14 June 1974)

ABSTRACT

A method is described for the objective determination of the noise level in Doppler spectra. The method makes use of the physical properties of white noise and is suitable for automatic computation. It is shown that the method produces reliable results when used in conjunction with the lower-bound method of estimating vertical air velocities.

1. Introduction

In recent years, Doppler radar has become an important tool for observing precipitating weather systems. The observed Doppler spectra are invariably contaminated by radar system noise. It is obviously of practical importance to know the spectral power density below which the spectrum is dominated by the instrument noise rather than by the signal received from the weather target. We shall refer to this spectral power density as the noise threshold. This contribution describes an objective method of determining the noise threshold. The method makes use of the observed Doppler spectrum and of the physical properties of white noise; it does not involve knowledge of the noise level of the radar instrument system.

The importance of identifying the noise threshold may be seen from the following two examples: (i) computation of the variance of the Doppler spectrum and (ii) estimation of vertical air velocity by the lower-bound method (Probert-Jones and Harper, 1961). The computed Doppler spectrum variance can be seriously affected by noise, if the signal-to-noise ratio is poor. To avoid this problem, spectral lines due to radar system noise should be eliminated from the variance calculation. Knowledge of the noise threshold is also important when using the lower-bound method of estimating the vertical air velocity. Use of this method requires knowledge of the fall velocities of the particles which produce the minimum discernable signal; that is, the signal with spectral density level which can just be recognized above radar system noise. There are obvious objections to the assumption of a minimum detectable particle size; however, our concern here is that of identifying the noise level.

In the past, various methods were used for identifying the noise level and eliminating spectral lines due to radar system noise. Using a "modified Battan" method, Donaldson (1967) used a noise threshold of 10 dB below the spectral peak, while Sekhon and Srivastava (1971) utilized a noise threshold of 15 dB below the peak to separate signal of meteorological origin from radar system noise. The modified Battan method is clearly unsatisfactory. It can lead to considerable errors in estimating the spectral variance and in estimating the vertical air velocity using the lower bound method (see Atlas *et al.*, 1973). Donaldson *et al.* (1972) used the mean power of the total spectrum (noise plus signal) as the noise threshold for rejecting spectral densities in variance calculations. While this method may be effective in eliminating the portions of the spectrum due to noise, it can also eliminate parts of the spectrum due to signal received from the weather target. Any rejection of meteorologically significant parts of the spectrum is clearly undesirable and is especially detrimental in determining the vertical air velocity by the lower-bound method.

By inspection of Doppler spectra, Battan (1964) determined a fixed noise threshold for separating signal from noise. This method can work successfully if the system noise remains constant and is carefully monitored. In practice, however, the radar system noise is not constant. This is because of various operations which are performed between signal received at the radar antenna and the final Doppler spectrum.

In the case of the S-band Doppler radar of the Laboratory for Atmospheric Probing, from which illustrative data are presented later, these operations involve the conventional radar electronics, a video tape recorder which is used for recording the signals after an AGC (automatic gain control) circuit, and subsequent processing. The subsequent processing involves replaying the video tape through sample and hold circuits to generate a FM tape of signals corresponding to given range gates. A digitized tape is then produced from the FM tape, and the digitized tape is processed on a general purpose computer to calculate the Fourier transforms which yield the Doppler spectra. Thus, the noise

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[1] Research supported by grants from the National Science Foundation (NSF-GA-38109) and the United Airlines Foundation.

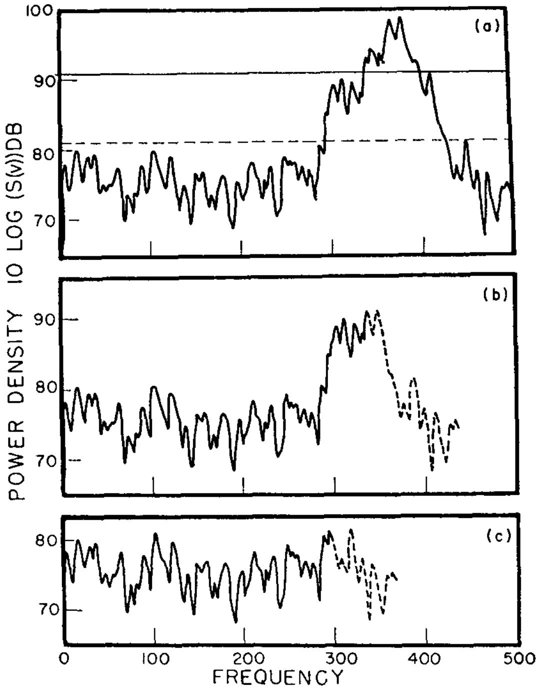


FIG. 1. (a) Schematic sketch of a Doppler spectrum and new Doppler spectra [(b), (c)] formed from (a). For details see text.

level in the final Doppler spectrum is subject to considerable uncertainties; hence the need for a method of ascertaining the noise level, using only the Doppler spectrum under investigation, rather than using prior knowledge of the system noise level.

2. Method

The signals due to weather and to radar system noise are both Gaussian. Generally, the former has a colored spectrum, while the latter has a white spectrum. We shall use the following two properties of white noise and Gaussian signals: (i) the variance of a white spectrum, which extends uniformly over a frequency span F is given by

$$\sigma_N^2 = F^2/12; \tag{1}$$

and (ii) for Gaussian signals, spectral densities $S_n = S(f_n)$ are independent, where $f_n = n/T$, n is an integer and T the record length. Furthermore,

$$\mathrm{var}(S_n) = <S_n>^2, \tag{2}$$

where the angle brackets denote ensemble average, and the variance on the left-hand side of (2) is obtained from an ensemble of realizations of the process. For white noise, the right-hand side of (2) may be estimated from one realization of the process, by averaging S_n over n. Similarly, the variance on the left-hand side of (2) may be estimated as the variance of S_n. In order to obtain stable spectra, a running average is often performed on the S_n. If a p-point running average is taken to form S_n, then Eq. (2) is replaced by

$$\mathrm{var}(S_n) = <S_n>^2/p. \tag{3}$$

In order to discuss the method of determining the noise threshold by application of the above properties, let us consider the Doppler spectrum shown schematically in Fig. 1a. From this spectrum we construct a new spectrum by rejecting spectral densities stronger than an arbitrarily assigned threshold, and by forming a continuous spectrum out of the remaining parts of the spectrum. For example, the spectra in Figs. 1b and 1c are constructed from that in Fig. 1a by applying the thresholds shown by the full and dashed lines, respectively. For any given spectrum, we can apply a series of decreasing thresholds until the "constructed" spectrum satisfies the conditions for white noise. This particular threshold would then be equal to the noise threshold. In order to test for white noise the following parameters are computed:

$$\sigma^2 = (\Sigma f_n^2 S_n/\Sigma S_n) - (\Sigma f_n S_n/\Sigma S_n)^2, \tag{4}$$

$$\sigma_N^2 = F^2/12, \tag{5}$$

$$P = \Sigma S_n/N, \tag{6}$$

$$Q = \Sigma(S_n^2/N) - P^2, \tag{7}$$

$$R_1 = \sigma_N^2/\sigma^2, \tag{8}$$

$$R_2 = P^2/Qp, \tag{9}$$

where F is the frequency spread of the spectrum, N the number of independent spectral densities, p the number of lines over which a moving average is taken, and the summation is over the N spectral densities. For white noise, the ratios R_1 and R_2 should be unity. For a spectrum containing weather signal, like the ones in Figs. 1a and 1b, it is expected that $R_1 > 1$ and $R_2 < 1$.

3. Examples and discussion

Figs. 2 and 3 give examples of the determination of the noise threshold. The Doppler spectra in Figs. 2a and 3a were obtained with a vertically pointing 10-cm Doppler radar in situations of melting band and thunderstorm rain, respectively. The pulse repetition rate is 1000 sec^{-1}, and spectral analysis was performed on only one channel of the bipolar video, which was offset by 250 Hz, so that 250 Hz corresponds to zero Doppler shift, and the unambiguous frequency range is from 0–500 Hz. Since both spectra were obtained from a $\frac{1}{2}$ sec record spectral densities separated by 2 Hz may be regarded as independent. These spectral densities are indicated by dashed lines in Figs. 2a and 3a. The spectra obtained by three-point smoothing of the single-point spectra are indicated by the solid lines. The ratios R_1 and R_2 are calculated as functions of threshold below the spectral peak and are shown in Figs. 2b and 3b, where the thick and thin lines correspond to R_1 and R_2,

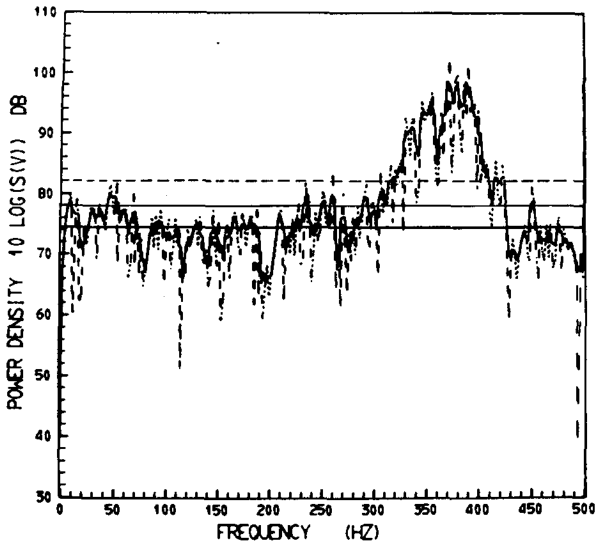


FIG. 2a. Doppler spectrum in the rain region of a melting band situation. The spectral densities are indicated by the dashed lines; the three-point averaged spectral densities are indicated by the solid lines. The heavy solid horizontal line indicates the mean noise power of the spectrum. The power threshold as determined by R_2 is indicated by the thin dashed horizontal line for the unaveraged spectrum and by the thin solid horizontal line for the three-point averaged spectrum.

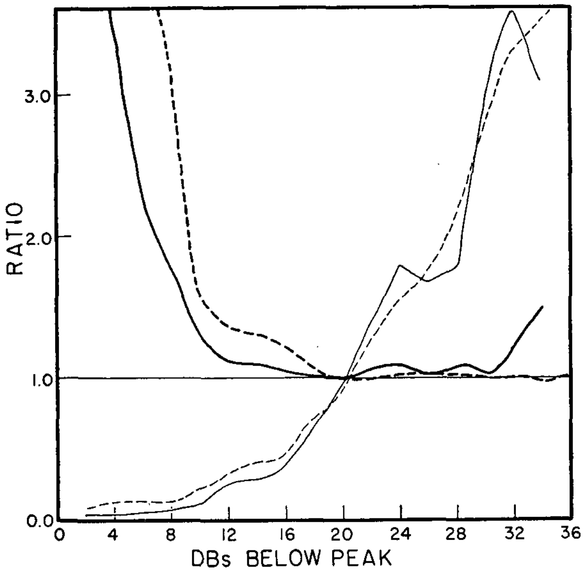
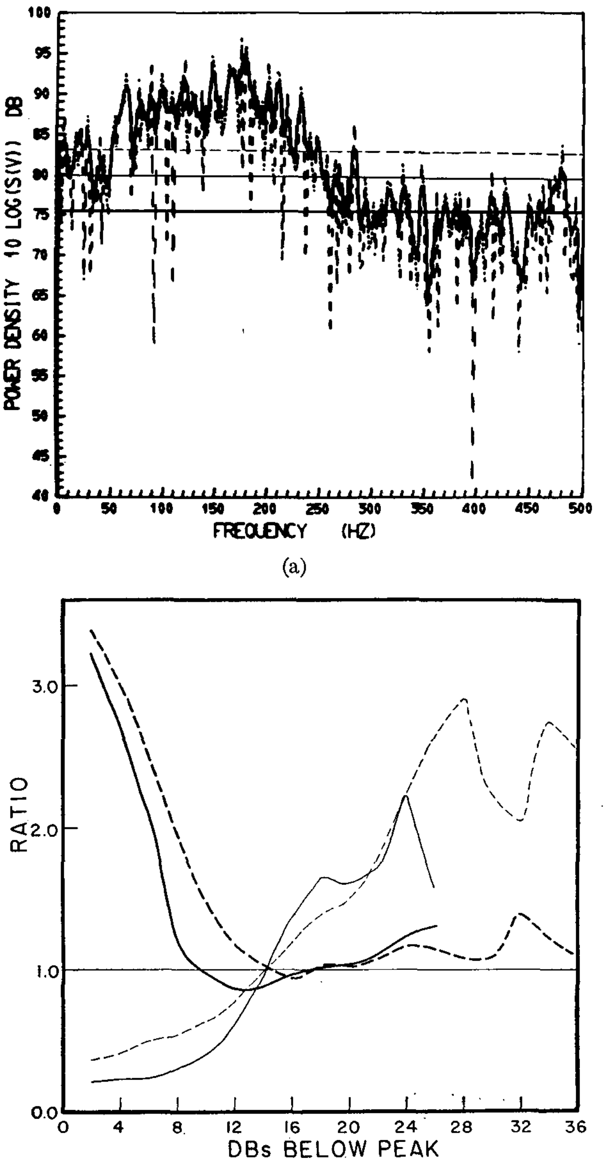


FIG. 2b. Ratio R_1 (thick lines) and R_2 (thin lines) as a function of threshold for melting band rain. The dashed lines are for an unaveraged spectrum, the full lines for a three-point smoothed spectrum.

respectively, and where the dashed and solid lines correspond to the unaveraged and three-point averaged spectra, respectively. We see that R_1 starts at large values and approaches 1 in an irregular asymptotic manner, while R_2 starts at small values and increases smoothly to values greater than 1. It therefore appears that use of the condition $R_2=1$ can provide an accurate indication of the noise threshold. The mean noise level P [Eq. (6)] is computed from that part of the spectrum which is below the noise threshold corresponding to $R_2=1$. The mean noise level is shown by the thick solid horizontal lines in Figs. 2a and 3a. As might have been anticipated, this mean noise level is virtually the same for the unaveraged and for the smoothed spectra. The noise thresholds corresponding to $R_2=1$ are shown in Figs. 2a and 3a by the thin dashed and solid lines for unaveraged and smoothed spectra, respectively. The noise threshold is lower for the smoothed spectrum because the spectral density fluctuations due to noise decrease when smoothing is performed. It is interesting to note that the noise threshold remains practically



(a)

(b)

FIG. 3. As in Fig. 2 except for thunderstorm rain.

unchanged when expressed as decibels below the spectral peak (see Figs. 2b and 3b at the point $R_2=1$). This is because the spectral peak also decreases when the smoothing is performed.

As a more extended test of the method, an analysis was performed on Doppler spectra observed during a melting band rain situation. These Doppler spectra were examined for a period of 8 min at two heights: 2.85 km in rain and 4.95 km in snow. For each of 59 Doppler spectra at the two heights, the lower (frequency) bounds were determined for fixed thresholds equal to 10 and 15 dB below the peak of the spectrum, and for a threshold equal to the mean noise level (calculated using the method described above). The lower frequency bound for any threshold was determined by finding the peak of the spectrum, then moving toward small Doppler frequencies (slower falling particles) until the spectral density first equalled the threshold. Due to the large variability of the unaveraged spectra, three-point smoothed Doppler spectra were used for this purpose. The averaging was necessary in order to preclude choosing an incorrect spectral bound which was too close to the spectral peak. Averaging over more than three points did not appreciably affect the location of the spectral bounds, and it degraded the frequency resolution of the spectra. The 8-min averages of the Doppler velocities which correspond to the spectral lower frequency bounds are shown in Table 1. (The sign convention is, positive velocities are downward.)

Table 1. Average Doppler velocity (m sec^{-1}) at lower bounds of spectra.

Threshold	Height 2.85 km	 4.95 km
10 dB below peak	5.44	1.20
15 dB below peak	3.13	0.77
Noise level	1.16	0.22

In order to interpret the above numbers, let us consider that in a melting band rain situation (i) the vertical air velocity at a fixed height may reasonably be expected to average to zero over any extended distance (or period of time, and (ii) reasonable values for fall velocities of particles of minimum detectable size in rain and snow may be taken as 1.0 and 0.25 m sec^{-1}, respectively. It would then appear that there is a meteorologically significant signal of strength less than 15 dB below the spectral peak. It also appears that a threshold equal to the mean noise level is physically the most reasonable manner of determining vertical air velocity using the lower-bound method. The noise threshold, expressed as decibels below the spectral peak, varied from spectrum to spectrum; in rain the range of the thresholds was 18–28 dB, the most frequent value being 24 dB; in snow the range was 17–30 dB and the most frequent value was 22 dB. The variability of thresholds may be anticipated to be greater in convective rain.

When the bound-to-bound width of the Doppler spectrum approaches one-half of the unambiguous frequency span, the Doppler spectrum begins to lose much of its ability to make meaningful measurements. In such a case the technique described for determining the noise threshold appears to work well in spite of the large width of the spectrum (see Fig. 3). With wider spectra, the thresholding technique would begin to produce erroneous results.

4. Conclusion

A method has been developed for estimating the noise level in a Doppler spectrum. This method makes use of the property of white Gaussian noise that the standard deviation of the spectral densities is equal to the mean spectral density. The method is suitable for automatic computation on digital computers. It has been shown that the lower bounds of Doppler spectra, observed in a situation of melting band rain and snow, yield physically reasonable estimates of vertical air velocity when the threshold for the lower-bound method is determined by the proposed method.

Acknowledgments. The authors are indebted to Prof. R. C. Srivastava for guidance and for contributions which were of primary importance to the success of this research. Acknowledgement is made to the National Center for Atmospheric Research, which is sponsored by the National Science Foundation, for computer time used in this research.

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