A Method of Removing Lamb Waves from Initial Data for Primitive Equation Models

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ABSTRACT

A simple method of reducing the amplitude of Lamb waves in primitive equation model forecasts has been proposed and tested. This method makes use of a Boussinesq-type approximation in which the vertical mean mass divergence is set equal to zero. It effectively reduces the Lamb waves by a factor of 3 in the example shown here and does not degrade the forecast accuracy. The largest reduction in Lamb wave amplitude is found in the tropical regions.

1. Introduction

A large-amplitude pressure oscillation can be generated by a primitive equation numerical prediction model when the initial velocity fields are incorrectly specified for large-scale flow, as pointed out by Benwell and Bretherton (1968). In fact, Richardson (1922) in his famous attempt at numerical weather forecasting encountered this problem, but he did not pursue its exact cause. Curiously, our suggestion for eliminating the waves closely follows what Richardson suggested in Chapter 10 of his book, but did not try. In later review papers by Platzman (1967) and Phillips (1970), an analysis of Richardson's difficulty was diagnosed. Phillips, in particular, used a linear analysis of the primitive equations to show how Lamb waves are generated. Briefly, the excitation of the Lamb or external wave mode in numerical models is caused by allowing a relatively large unbalanced divergence in the initial state. Benwell and Bretherton solved this problem for the Bushby and Timpson (1967) model by modifying the linearized balance equation such that the initial divergence was small. We should point out that over the limited area of the Bushby and Timpson model the Lamb wave acted like a standing rather than a propagating wave.

With the advent of asynoptic observing systems such as satellites, constant level balloons, etc., there is a need to turn to four-dimensional assimilation techniques. These methods do not necessarily make use of the traditional balance equation relationships (Kasahara, 1972). As a continuation of our work on initialization by Houghton and Washington (1969) and Houghton *et al.* (1971), we suggest here a simple method of suppressing the Lamb wave oscillations which is not tied to any particular balance equation. In fact, it may be imposed on the observed momentum field directly without modification of the initial pressure field.

2. Description of method

The pressure oscillation we suppress by this method has little vertical dependence, identifying it as the Lamb wave, or external gravity wave, mode. Benwell and Bretherton pointed out that the use of a particular balance relationship can develop systematic errors in the wind field which, in turn, lead to systematic divergence and convergence patterns in the vertical.

Since the Lamb wave is a horizontally propagating acoustic wave generated by a mean convergence or divergence throughout the atmosphere, we can eliminate it by invoking a Boussinesq-type approximation to the mass continuity equation. In the z vertical coordinate system, we require that the time change of density in the continuity equation be zero initially. As a result, the continuity equation becomes

$$\nabla\cdot\rho\mathbf{V}+\frac{\partial}{\partial z}(\rho w)=0, \qquad (1)$$

where $\mathbf{V}=\mathbf{i}u+\mathbf{j}v$ is the horizontal wind vector, ∇ the horizontal gradient operator, ρ density and w the vertical velocity.

If we integrate (1) vertically from the height of the terrain H and to the top of the model atmosphere z_T, then (1) becomes

$$\int_H^{z_T}\nabla\cdot\rho\mathbf{V}+\rho w|_{z_T}-\rho w|_H=0. \qquad (2)$$

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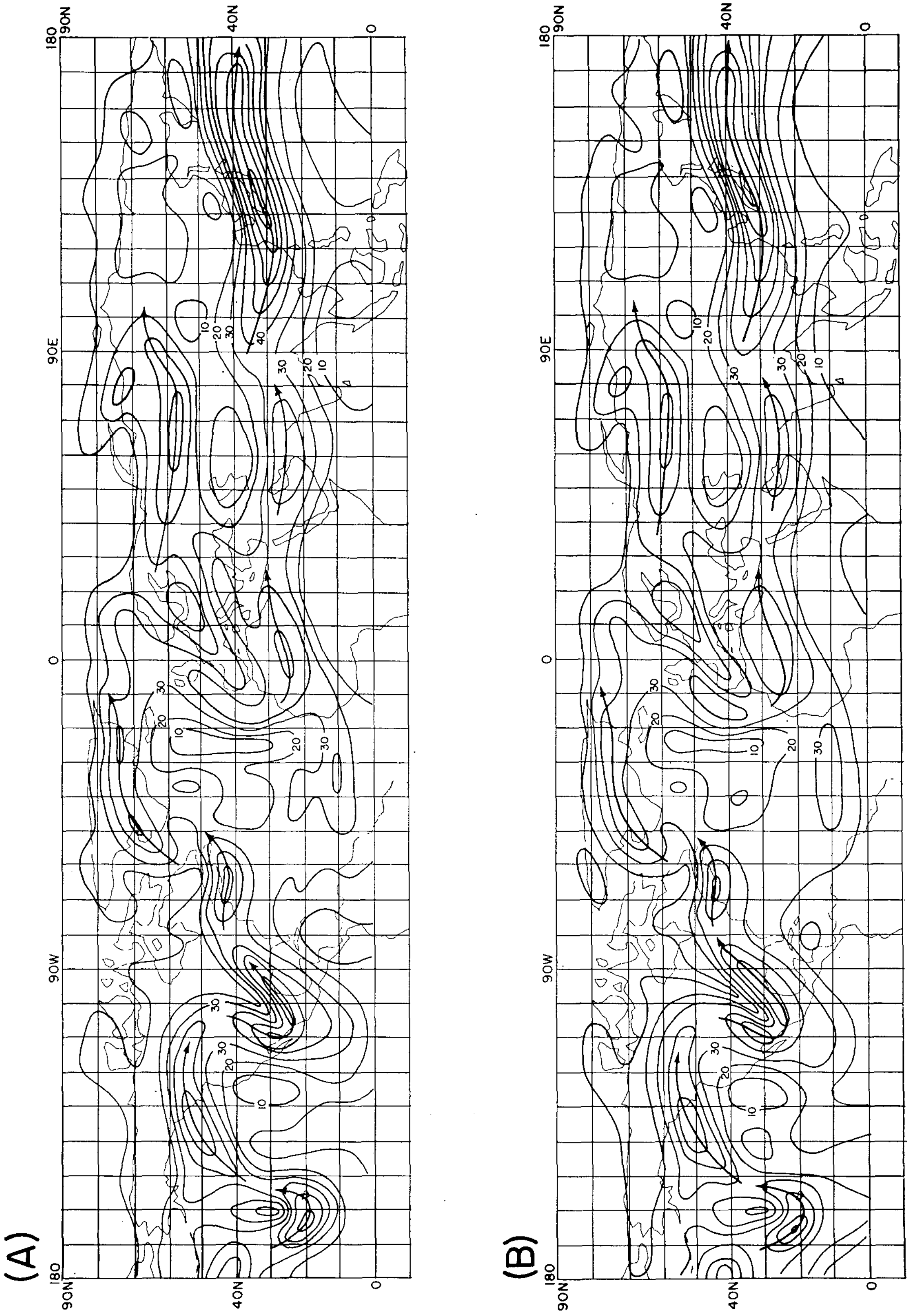

FIG. 1. Geostrophic wind speed at 10.5 km for 1200 GMT 10 December 1967 (A) and modified wind speed at 10.5 km (B). Contour interval is 10 m s^{-1}. The axis and direction of the speed maxima are marked with arrows.

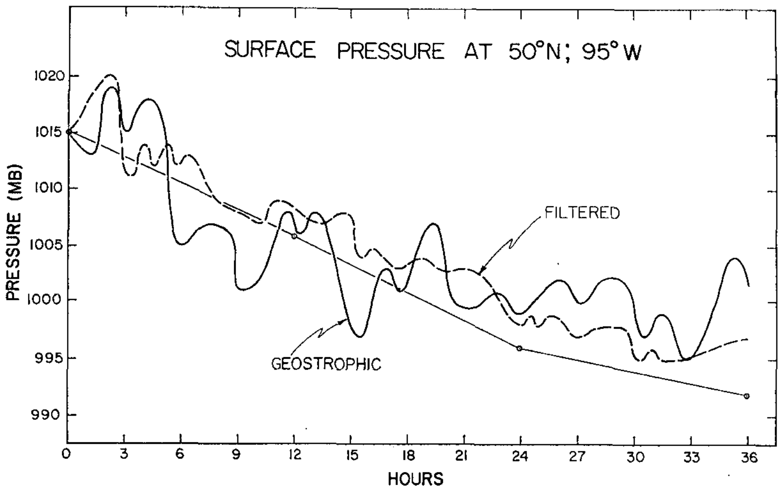

FIG. 2. Plot of surface pressure traces at 50°N, 95°W as a function of time. Heavy solid line is forecast from geostrophic initial state. Heavy dashed line is forecast from modified initial state. Light solid line is observed pressure trace.

Using Leibnitz's rule and the boundary conditions $w=0$ at $z=z_T$ and $w=\mathbf{V}\cdot\nabla H$ at $z=H$, we can write (2) as

$$\nabla\cdot\int_H^{z_T}\rho\mathbf{V}dz=0. \tag{3}$$

We can separate the momenta $\rho\mathbf{V}$ into their rotational and divergent components by employing the Helmholtz theorem where ψ is the streamfunction and χ the velocity potential:

$$\rho\mathbf{V}=\mathbf{k}\times\nabla\psi+\nabla\chi. \tag{4}$$

By performing the curl and divergence operations on (4), we obtain

$$\nabla^2\psi=\zeta, \tag{5}$$

$$\nabla^2\chi=D, \tag{6}$$

where ζ is vorticity, D divergence, and

$$\nabla^2=\frac{1}{a^2\cos^2\varphi}\left[\frac{\partial^2}{\partial\lambda^2}+\cos\varphi\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial}{\partial\varphi}\right)\right]. \tag{7}$$

Note that the vertical integral of (6) yields Eq. (3). Since a Lamb wave has divergence of essentially one sign throughout the atmosphere, we suppress most of its energy by requiring that the vertical integral of divergence be zero. We should point out that large-scale synoptic features usually have a change in sign in divergence in the vertical and such a procedure tends to preserve this characteristic. We modify the divergence field such that the vertical mean is subtracted out so that we obtain a modified divergence, i.e.,

$$D'=D-\left(\int_H^{z} Ddz\Big/\int_H^{z_T}dz\right). \tag{8}$$

The modified divergence is used to define a slightly modified wind field where D is replaced by D' in (6). The modified divergence departs only slightly from the original with the advantage that the Lamb waves are almost completely removed. We want to contrast this modification of the wind field with that of Benwell and Bretherton in which they suggested a change of the balance equation resulting in a smaller initial divergence. As mentioned earlier, the advantage of this scheme is that it does not require the solution of the balance equation; however, it does require that vorticity be much larger than divergence which is usually well satisfied for large-scale atmospheric motions.

We should mention that this method is easily adapted for use with other vertical coordinate systems by making the same assumptions we have used here. Also, Smith (1974) has used a similar approximation for energy balance studies.

3. Results

Our suggestions for suppressing Lamb waves were tested in the Northern Hemisphere version of the NCAR General Circulation Model (Baumhefner, 1972; Wellck *et al.*, 1971), with horizontal resolution of 2.5° in latitude and longitude and six layers, each 3 km thick. The initial pressure analyses for this case (1200 GMT 10 December 1967) were subjectively analyzed by hand. The initial wind field was obtained through geostrophic equations from the pressure fields. The latitudinally averaged wind at 10°N was extrapolated to the equator and then linear interpolation was used to obtain the wind between. An example of the original geostrophic speed is shown in Fig. 1A.

Even though geostrophic winds were used, there is non-zero mass divergence. The results in terms of the

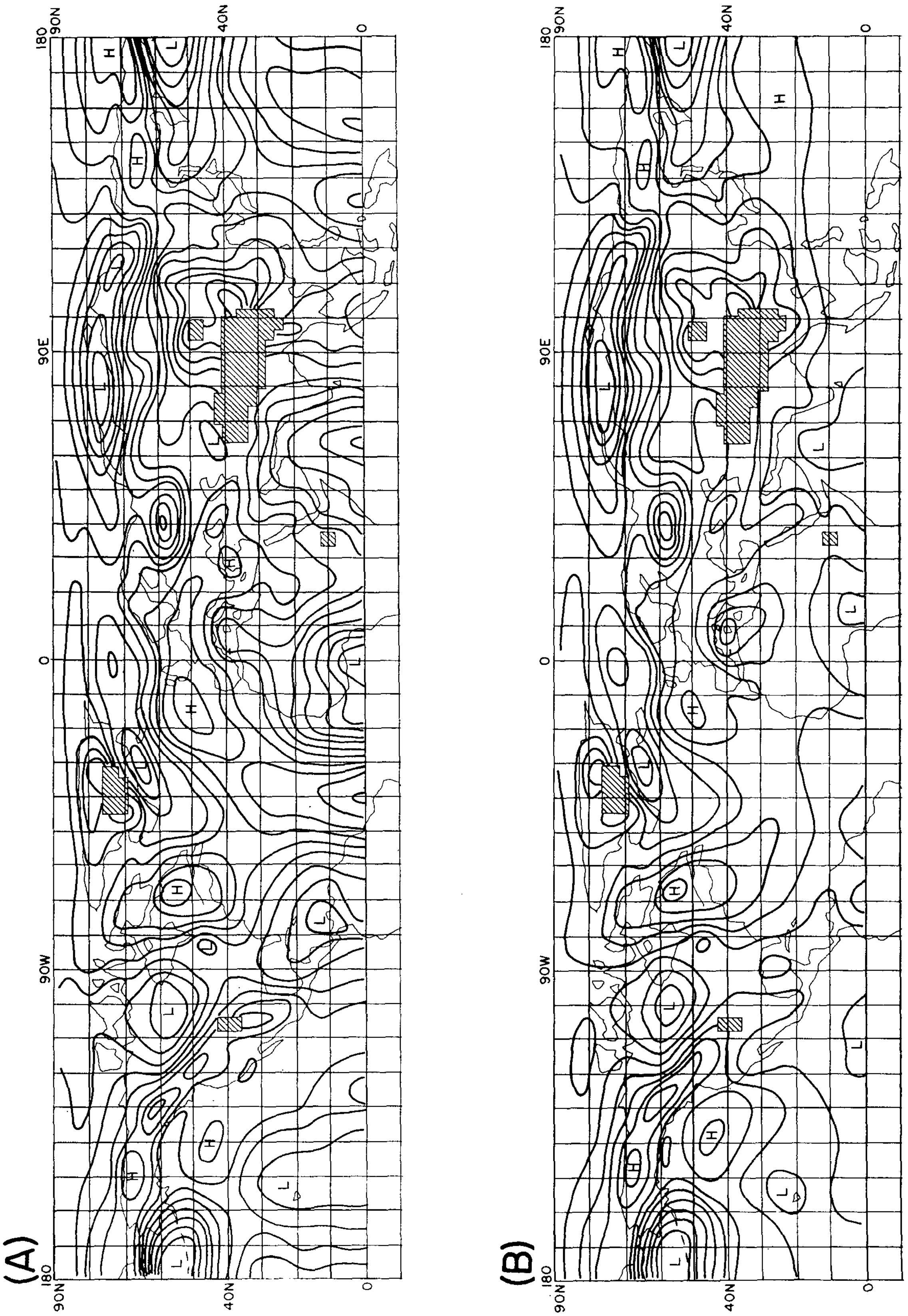

FIG. 3. Forecast of 24 h sea-level pressure from geostrophic initial conditions (A) and from modified initial conditions (B). Contour interval is 5 mb.

TABLE 1. Forecast error (30–70°N) 1200 GMT 10 December 1967: initial data.

		Surface pressure				6 km pressure			
		Geostrophic	Filtered	Normalized	Persistence	Geostrophic	Filtered	Normalized	Persistence
Day 1	S_1	64	64			39	40		
	rms*	50	43	0.41	34	73	66	0.34	64
Day 2	S_1	73	72			53	52		
	rms*	65	59	0.56	74	102	95	0.50	135
Day 3	S_1	81	81			60	60		
	rms*	90	79	0.75	89	126	111	0.58	159
Day 4	S_1	81	80			62	61		
	rms*	98	87	0.83	93	143	127	0.66	177

* rms error in meters.

modified wind speed are shown in Fig. 1B. The pattern of wind speed has been altered only slightly; however, the peak magnitudes have been reduced as a result of the solution of the Poisson equations (5) and (6) for the rotational and divergent parts of the wind. The maximum deviations were 13 m s^{-1} in the southern part of the troughs over the western United States and the east central Pacific. The average reduction in each speed maxima was 6 m s^{-1}.

The velocity field now initialized by the Lamb wave filter and the original geostrophic field were used as initial conditions for two 7-day forecasts. The physics incorporated in this model are similar to the six-layer, 5° model described by Kasahara and Washington (1971). Fig. 2 illustrates the behavior of the surface pressure at 50°N, 95°W during the first 36 h of integration for both initial states. The amplitudes of the short-period waves in the geostrophic case are of the order of 5 mb or even higher. With the filtered initial state, the gravity wave amplitudes were reduced by as much as a factor of 3 in this case. Comparison with the observed pressure tendency at this point indicates roughly the same forecast accuracy for the synoptic-scale oscillations in both cases.

A look at the 24 h geographical distribution of the sea-level pressure forecasts in Fig. 3 reveals a large-amplitude Lamb wave most prominent in the tropics with a horizontal scale the order of several thousand kilometers. The forecast in which the Lamb waves have been filtered shows a considerable reduction in the amplitudes. The relative change between the two forecasts, with and without the Lamb wave removal, in the mid-latitudes is quite small for the synoptic-scale flow. Table 1 shows the skill of the two forecasts compared to NMC-analyzed data for four days. Both the rms error and the National Weather Service S_1 error were calculated, and these scores were contrasted with a persistence and a normalized rms. The normalization is achieved by dividing the actual rms error by the average rms error of two randomly chosen states of the atmosphere. The normalizing values used are 104 m for the surface and 194 m for 6 km pressure derived from data during December 1967. It is clear that the difference in S_1 score for both levels is minimal for the entire forecast period. However, the rms error shows a distinct drop for the filtered case at both levels. The apparent increase in skill appears more at the latter half of the 4-day period when compared to the persistence values. There is still considerable noise at Day 1 in both forecasts as indicated by the slightly better persistence forecast. If a value of 0.7 is accepted at the upper limit of usable skill for the normalized rms error, then even with the Lamb wave filtering technique, the skill at the surface is minimal after 2.5 days. The mid-tropospheric forecast has skill to about 4.5 days for this case. Although this skill value is only for one case, several other runs with different initial conditions produced roughly the same skill.

4. Conclusions

The method described in Section 2 effectively reduces unwanted Lamb-wave amplitudes in a primitive equation forecast. The reduction in rms error achieved by this method is mainly due to the elimination of gravity wave components that are not strongly interacting with the synoptic waves. (However, the overall forecast skill of the large-scale synoptic patterns is essentially unaltered with the filtered technique.) This method can be extended by placing different constraints upon the divergence field other than that used in this paper. For example, one may be able to remove some of the internal gravity wave modes from the initial data by specifying the vertical dependence of initial divergence pattern. We should also point out that Dickinson and Williamson (1972) developed a method of separating the normal modes of primitive equation models about a mean state with a mean wind which is a function of latitude. For a linear atmosphere, this procedure allows for a clear separation of gravity and Rossby modes. However, it has yet to be applied with real atmospheric data to more realistic spatially varying atmosphere.

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