Relating Rainfall Rate to the Slope of Raindrop Size Spectra

FRANCIS J. MERCERET

National Hurricane Research Laboratory, NOAA, Coral Gables, Fla. 33124

30 September 1974 and 4 November 1974

1. Introduction

The classic paper on raindrop spectra from which the exponential "Marshall-Palmer" distribution takes its name not only specified the form

$$dN(D)=N_0e^{-\lambda D}dD \tag{1}$$

for the raindrop size spectrum $N(D)$ (drops cm^{-3}), but also suggested the relation

$$\lambda=41R^{-0.21} \tag{2}$$

holds between the slope λ (cm^{-1}) and the rainfall rate R (mm h^{-1}) (Marshall and Palmer, 1948). Both relations were empirical and applied to ground-based measurements of temperate climate rain. The National Hurricane Research Laboratory (NHRL) has collected tropical raindrop samples from airborne foil impactors for several years in tropical cloud-lines, storms and hurricanes. These data confirm the applicability of Eq. (1) above ground in the tropics (Merceret, 1974a, b). They also confirm the validity of Eq. (2) under certain conditions which this note describes.

The data are raindrop counts in 0.25 mm size categories. Samples simultaneously satisfied the criteria of Cornford (1967, 1968a, b) for statistical reliability and our own requirement that the sample volume be $\leqslant 1$ m^3 to represent a relative homogeneous meteorological environment. Four least-squares fits of the logarithm of (1) to the data were created. Table 1 presents their properties.

The slopes from each of the four fits for 398 samples collected in tropical cloud lines, storms and hurricanes were related to the rainfall rates computed from the samples (using the sample distributions and a table of terminal velocities) by a pair of least-squares fits

TABLE 1.

Fit	Property N_0 constrained to 0.08 cm^{-4}	N_0 free	$N(D)$ Independent variable	D Independent variable
1	x			x
2	x		x	
3		x		x
4		x	x	

TABLE 2.

Independent variable	Fit 1			Fit 2			Fit 3			Fit 4		
	A	B	r	A	B	r	A	B	r	A	B	r
R	41.87	0.204	0.984	44.77	0.203	0.971	35.29	0.276	0.777	42.33	0.242	0.789
λ	42.43	0.210	0.984	45.87	0.215	0.971	49.89	0.462	0.777	58.68	0.393	0.789

of the form

$$\lambda = AR^{-B}. \tag{3}$$

One fit treated λ as the independent variable, the other treated R as the independent variable. Neither A nor B was constrained.

2. The results

Table 2 shows the results of the regression analysis including the correlation coefficient r of the data to Eq. (3). Not only are the constrained fit slopes better correlated with the rainfall rate, but the values of the coefficients are more consistent. An average of the values for fits 1 and 2 leads to a form

$$\lambda = 44R^{-0.21}, \tag{4}$$

which differs by 5% at the most from any value for fits 1 and 2 from the table and is within 7.3% of the classical value.

3. Conclusion

Examination of 398 tropical raindrop spectra having the approximate form $dN(D) = N_0 e^{-\lambda D} dD$ supports the validity of a slope-rainfall relation of the form $\lambda = AR^{-B}$, but only if the λ are computed from an intercept-constrained, least-squares fit to the drop-size distribution data. For spectra constraining N_0 to the classical value of 0.08 cm^{-4}, the relation between λ and R is close to the classical one.

REFERENCES

Cornford, S. G., 1967: Sampling errors in measurements of raindrops and cloud droplet concentrations. *Meteor. Mag.*, **96**, 271–282.

——, 1968a: Sampling errors in measurements of particle size distributions. *Meteor. Mag.*, **97**, 12–16.

——, 1968b: The measurement of shower rainfall using an airborne foil impactor. *J. Appl. Meteor.*, **7**, 956–957.

Marshall, J. S., and W. McK. Palmer, 1948: The distribution of raindrops with size. *J. Meteor.*, **5**, 165–166.

Merceret, F. J., 1974a: A note on the vertical distribution of raindrop size spectra in Tropical Storm Felice. *Meteor. Mag.* (in press).

——, 1974b: On the size distribution of raindrops in Hurricane Ginger. *Mon. Wea. Rev.*, **102**, 714–716.