A New Hot-Wire Liquid Cloud Water Meter

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ABSTRACT

The National Hurricane Research Laboratory has developed and flight tested a new airborne liquid water meter for cloud physics measurements. The sensor is maintained at constant temperature rather than at constant current, and the operating temperature is held well below the *in-situ* boiling point. These two changes from previous instruments, such as the popular Johnson-Williams meter, permit accurate response over a wider range of drop sizes and finer spatial resolution. Flight tests on NOAA Research Flight Facility aircraft showed the new unit to be more sensitive, more stable, and more rapidly responding than the J-W and Levine instruments presently on board.

1. Introduction

The ease of processing a continuous electrical signal which may be easily digitized gives the hot-wire liquid water meter a great advantage over replicating devices for measuring the spatial and temporal distribution of liquid precipitation and cloud water, but there are problems associated with the hot-wire systems currently in widespread use. The most common hot-wire system is that manufactured by Johnson-Williams[1]; the problems arising from its use are typical. Its spectral response is narrow, it responds fully only to drops $<30\ \mu\text{m}$, and its baseline tends to drift (Knollenberg, 1972). Errors introduced by its time constant, a second or more, can be significant as well (Spyers-Duran, 1968). High power consumption per unit sampling volume, difficulty of calibration, and sensitivity to environmental parameters are also disadvantageous.

At the National Hurricane Research Laboratory (NHRL) we have developed a hot-wire instrument which retains all of the advantages of previous units and which is improved in the following ways:

1) It responds to a wider range of drop sizes.
2) After initial calibration in a wet facility it requires calibration only in a dry wind tunnel.
3) It is stable on both long and short terms.
4) The power required per sampling volume is much reduced.
5) The dependence of the calibration on flight parameters and environmental conditions is known exactly and in a practically usable form.
6) The response is faster.
7) It allows for the simple construction of matched sets of instruments.
8) The sensitivity may be adjusted in flight to meet any conditions without loss of calibration accuracy.

These improvements have been accomplished by two changes from previous practice. The most important change is from constant current to constant temperature operation of the sensor. The other change is from operating the sensor above the boiling point of water to operating it below the boiling point. The methods and consequences of these changes will be explained in this paper.

2. The general operating principles of hot-wire instruments

The hot-wire liquid water meter is an adaptation of the hot-wire anemometer. When an object is placed in a fluid flow it loses heat to the fluid if the object is warmer than the fluid. For small cylinders the general heat transfer law is of the form

$$P=(A+BU^N)(T_W-T_A), \tag{1}$$

where P is the rate of cooling (dimension of power), U the flow velocity component normal to wire axis, T_A the fluid temperature, T_W the wire temperature, and A, B, N are constants depending on the fluid properties, wire material and wire geometry. This is often called King's law after L. V. King who derived a more specific form for wires of circular cross section (King, 1914). The parameters A, B, N are usually called the King's law constants.

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[1] Johnson-Williams, Inc., Mountain View, Calif. Mention of a proprietary product does not constitute an endorsement thereof by either the authors or the U. S. Government.

If the wire is used in a two-phase flow such as water in air, the cooling due to air and that due to water must be separately computed and added. Further, if the wire is hot enough to change the phase of any part of the flow such as boiling the water, the loss of heat vaporization must be considered in the heat balance.

The variation of the King's law constants with the flow and wire parameters are complex but known [see Hinze (1959), Chap. 2, for example]. An excellent summary of the field may be found in Corrsin (1961). For our purposes, it is sufficient to note the following dependencies:

N is a function of geometry only. For wires of circular cross section, $N=\frac{1}{2}$.
$A\approx Lk$
$B\approx Lk/\nu^{\frac{1}{2}}$

Here L is the wire length, k the heat conductivity of the fluid, and ν the kinematic viscosity of the fluid. These are sufficient to correct for changes in wire length and flight altitude using wires of a given diameter and material. Changing wire material will also change the coefficients, and a calibration is needed for each wire material and diameter.

In hot-wire anemometry and nimbiometry,[2] the wire sensor is heated electrically and the electrical power supplied to the sensor is measured. Thus, if the wire current I and voltage E are known as a function of time, we have at each instant

$$IE=(A+BU^N)(T_W-T_A), \tag{2}$$

assuming quasi-equilibrium between the wire and the flow. The effects of deviations from equilibrium are discussed in Hinze (1959, Chap. 2). Where T_W, T_A and N are known we may do either of the following:

Given A and B, find U (anemometry); or
Given U and a relation between A and B for air and water, find the proportion of water to air (nimbiometry).

Even without knowledge of T_W, A, B and N we can generate empirical curves of water content versus power out for fixed values of T_A and U.

The remarks to this point apply equally to constant current and constant temperature systems. It is now appropriate to discuss these modes of operation separately in detail.

3. Constant current nimbiometry

The Johnson-Williams instruments mentioned in the Introduction operate in the constant current mode above the boiling point of water as do all other hot-wire nimbiometers used previously. For a given wire length the heat transfer equation at constant current in this condition becomes

$$E\approx(A_a+B_aU)(T_{W_a}-T_A)(1-\epsilon)+(A_w+B_wU^N)(T_{W_w}-T_A)\epsilon+l\frac{dM}{dt}, \tag{3}$$

where ϵ is the fraction of wire hit by water, l the heat of vaporization of water, and M the water mass impinged on wire and evaporated. Subscript w refers to conditions in water and subscript a to air.

Because of the large value of l, this is usually simplified to

$$E=l\frac{dM}{dt}, \tag{3a}$$

and often attempts are electronically made to make this form more accurate by subtracting the King's law terms obtained from a dry wire. The latter is futile since the "wet" King's law terms exceed the dry even in very low water content clouds.

The constant current approach has the advantage of electrical simplicity and convenience. The form of the simplified heat transfer equation is ideal. Unfortunately, the method has had limited success. Because it is necessary to boil the water to measure it, larger drops which are sliced by the wires are not fully evaporated; thus, the total liquid water content is seriously underestimated when a significant portion of the water occurs in droplets of raindrop size (Levine, 1965). In heavy rain, portions of the wire cool excessively while others become red hot and the effective length of the wire changes unpredictably, leading to random changes in calibration. The time constant of the constant current method is also quite long, being limited by the thermal inertia of the wire.

The alternative of using the constant current mode at lower currents (hence lower temperatures) has been wisely ignored. The dependence of T_W on E introduces such nonlinear coupling between the terms of the heat balance equation that the technique is no longer used even for single-phase flows. Before the advent of suitable constant temperature circuitry, the constant current technique was the standard one for anemometry in turbulent flows. The mass of research reported during those years makes clear the empirical nature of constant current calibrations and the restriction of their validity to a narrow range of environments and operating conditions (see Hinze, 1959).

4. Constant temperature nimbiometry

This mode of operation utilizes the monotonic relation between the resistivity and temperature of most metals. A feedback circuit such as will be described later is used to hold the resistance, hence temperature, of the sensor wire constant. The heat transfer equa-

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[2] From the latin *Nimbus*: rainstorm or cloud.

tion then becomes

$$P=\frac{E^2}{R_{\text{wire}}}\approx(A_a+B_aU^N)(T_{W_a}-T_A)(1-\epsilon)$$
$$+(A_w+B_wU^N)(T_{W_w}-T_A)\epsilon+l\frac{dM}{dt},\quad (4)$$

which is quite similar to the constant current formula (3) but here T_W, being held electronically constant, is *not* a function of E, except for local cooling differentials to be discussed next. The option of operating above the boiling point is available but the preferred mode is $T_W<T_{\text{vap}}$ in which case the $l(dM/dt)$ term is negligible.

In order to discuss the effect of local cooling differentials we will make some definitions and introduce the relationship between wire resistance and temperature.

Let the subscript 0 denote reference conditions, R the wire resistance, and an overbar an average over the whole wire. We define $\bar{T}_W-T_0\equiv\bar{\theta}$, $T_{W_w}-T_0\equiv\theta_w$, $T_{W_a}-T_0\equiv\theta_a$, where subscripts w and a are as defined earlier. Let

$$K_w=A_w+B_wU^{\frac{1}{2}},\quad K_a=A_a+B_aU^{\frac{1}{2}}.$$

Then the basic heat transfer law is

$$P=\epsilon K_w\theta_w+(1-\epsilon)K_a\theta_a\quad (5)$$

if the reference values are taken to be the ambient ones and where, as before, ϵ is the wet fraction of the wire.

The wire temperature-resistance formula is

$$R=R_0[1+\alpha(T_w-T_0)]\text{ to first order,}$$

where α is the thermal coefficient of resistivity which may be found in tables such as those of Weast (1965).

From these we obtain

$$\left.\begin{array}{l}R_w=\epsilon R_0(1+\alpha\theta_w)\\ R_a=(1-\epsilon)R_0(1+\alpha\theta_a)\\ \bar{R}=R_0(1+\alpha\bar{\theta})=\text{constant by design}\\ \epsilon\theta_w+(1-\epsilon)\theta_a=\bar{\theta}\end{array}\right\}.$$

Further, we note that the current through the wet portion of the wire, though varying, equals that through the dry, and since $P_w=I^2R_w$, $P_a=I^2R_a$, we have

$$I^2=\frac{P_w}{R_w}=\frac{P_a}{R_a}.$$

Thus,

$$\frac{K_w\theta_w}{(1+\alpha\theta_w)}=\frac{K_a\theta_a}{(1+\alpha\theta_a)}$$

for known values of α, $\bar{\theta}$, K_w and K_a. The above can be solved parametrically for $\theta_a(\theta_w;\epsilon)$ or any permutation thereof subject to the limitation $\theta_a\geqslant\bar{\theta}\geqslant\theta_w\geqslant0$ which the physically meaningful roots must satisfy.

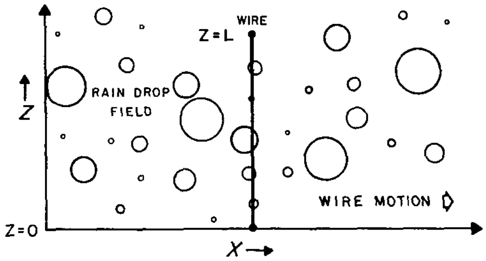


FIG. 1. Coordinate system for analysis of raindrop interceptions by the hot-wire sensor.

The calculations were performed numerically for $K_w/K_a=50$ based on wind and water tunnel calibrations of hot-sensor anemometers, with values of ϵ from 0 to 1.0 in steps of 0.1 for assorted values of $\bar{\theta}$; the net result was that

$$\left.\begin{array}{l}P_w\approx\epsilon^2K_w\bar{\theta}\\ P_a\approx K_a\bar{\theta}\end{array}\right\}.$$

Thus the selective heating of the portion in air just about cancels the reduced length in air, while the effect of the extra cooling power of the water compounds the effect due to increasing length in water.

Numerical values for the effect of altitude on A and B have been computed using the ICAO Standard Atmosphere and formulas presented above. The effect amounts to a decrease of 12% in A and 30% in B from the surface to 470 mb. Correction for these changes can be inserted in data analysis routines.

The analysis given by Hinze (1959), demonstrating the higher frequency response of the constant temperature mode compared to that of the constant current mode, is only partially applicable here due to the effect of local cooling, but the field results presented later show considerable improvement still occurs.

It remains to relate the above formulas to cloud liquid-water content. Since K_a, K_w and $\bar{\theta}$ are known for the instrument, the output can be trivally processed to yield

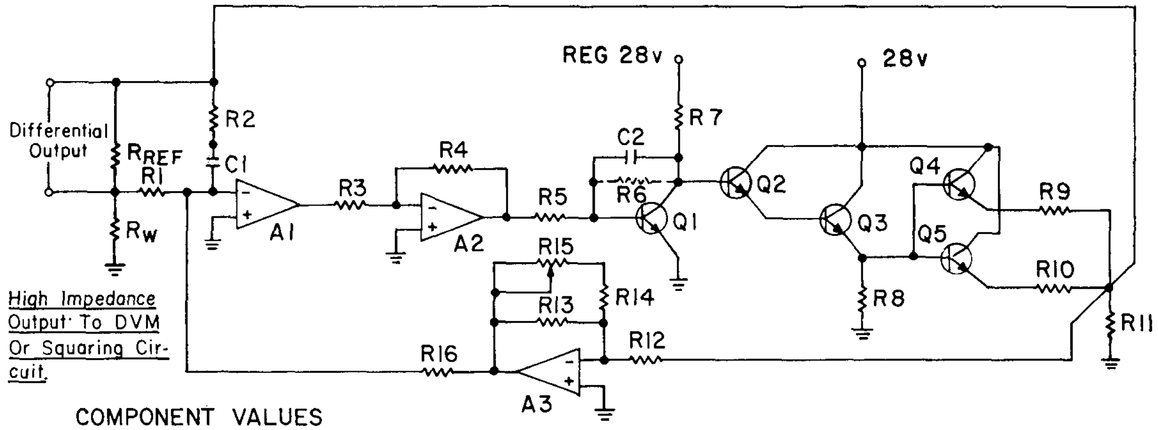
$$\frac{P_w}{K_w\theta}=\frac{P-P_a}{K_w\theta}\approx\epsilon^2.\quad (6)$$

But ϵ is the sum of all the fractional lengths occupied by droplet contact as the wire moves along with the aircraft (see Fig. 1), i.e.,

$$\epsilon(x)=\frac{1}{L}\int_0^L L(x,z)dz,$$

where L is the wire length and $L(x,z)$ is an operator such that

$$L(x,z)=\begin{cases}0 & \text{in air}\\ 1 & \text{in water}\end{cases}$$



COMPONENT VALUES

R-1	4.7K, 1%	RREF	1.0Ω 250 Watt
R-2	1K	Rw	12" 177 Micron Dia. Tungsten Wire
R-3	680Ω		
R-4	680Ω	C-1	.01μf
R-5	4.7K	C-2	.05μf
R-6	10K (if req'd)		
R-7	2.7K	A-1	741 C
R-8	1K	A-2	741 C
R-9	.1Ω, 10 Watt	A-3	741 C
R-10	.1Ω, 10 Watt		
R-11	1K	Q-1	2N1613
R-12	10K	Q-2	2N1613
R-13	10K	Q-3	MJE340 Requires Heat Sink
R-14	10K (see note 2)	Q-4	HEP704 } Require Heat Sink
R-15	10K, 10 Turn	Q-5	HEP704 } Require Heat Sink
R-16	4.7K, 1%		

NOTE: 1. All Resistors 1/4 Watt, Except As Noted.
2. Value May Change Due To Feedback Factor Or As Determined By Required Temp. of Rw.

FIG. 2. Circuit diagram for the NHRL nimbiometer.

Then

$$\epsilon^2(x)=\frac{1}{L^2}\int_0^L\int_0^L L(x,z)L(x,y)dydz.$$

If we assume local isotropy of the liquid water drop distribution normal to the flight path and let y be normal to z and x, then $dydz=dA$, the differential of area in a plane normal to the flight path. The operator $H(x,y,z)=L(x,z)\,L(x,y)$ has the same properties in three dimensions as $L(x,y)$ has in two, i.e.,

$$H=\begin{cases}0 & \text{in air}\\ 1 & \text{in water}\end{cases}$$

Hence

$$P(x)\approx\epsilon^2(x)\approx\int_{\substack{\text{water}\\ \text{only}}} dA, \tag{7}$$

or the power is proportional to the area of water intercepted.

Since the time-integrated value

$$\int Pdt=(1/U)\int Pdx$$

if U is nearly constant,

$$\int Pdt\approx\int\int dAdx=\int dV.$$

So the time-integrated power delivered to the probe is proportional to the volume of water intercepted over the interval or, equivalently,

$$P\approx\frac{dV}{dt}$$

If we multiply by the density of water we have

$$P\approx\frac{dM}{dt}, \tag{8}$$

just as for the constant current system except that here the water mass need not be evaporated. At constant resistance, $P\approx E^2$, a simple matter for a digital or analog computer to handle.

Since we need only cut drops rather than evaporate them, there should be little spectral filtering over a wide range of sizes. Since the system need not supply heat of vaporization, power consumption should be much lower. By setting the wire temperature to 0°C, we make a supercooled water detector. By matching A and B in an ordinary wind tunnel, we can make a matched pair of instruments without need for a wet calibration facility.

5. The electronics for the NHRL nimbiometer

The basic circuit for the instrument is derived from a design by Wyngaard and Lumley (1967).

Fig. 2 is a circuit diagram of the electronic servo system. For simplicity, some components have been omitted. Each unit may require stabilizing capacitors, for instance, which depend on the details of the individual builder's wiring for their value and placement. These are not included in the diagrams or discussions following.

The wire is shown as a resistance R_w in a voltage divider consisting of R_{ref} and R_w. The voltage E_0 at the top of the divider is supplied by the power amplifier controlled by amplifier A_1.

The voltage across the probe is given by

$$E_w=\frac{E_0 R_w}{R_w+R_{\mathrm{ref}}},\quad \text{when } R_w \ll R_1.$$

The amplifier A_1 acts as a summing amplifier and integrator. It detects and amplifies any voltage difference between E_w and the reference signal E_w' from the wire temperature control amplifier A_3.

Amplifier A_3 is configured so that

$$E_w'=\frac{R_{13}}{R_{12}}\left(\frac{R_{14}+R_{15}}{R_{13}+R_{14}+R_{15}}\right).$$

Thus, A_1 will drive the power amplifier in such a manner that the wire will be heated (or allowed to cool) until $E_w=E_w'$. If R_{13} is selected equal to R_{12} then

$$\frac{R_w}{R_w+R_{\mathrm{ref}}}=\frac{R_{14}+R_{15}}{R_{13}+R_{14}+R_{15}},$$

and by adjusting $R_{14}+R_{15}$ we may control R_w so that

$$R_w=\frac{(R_{14}+R_{15})R_{\mathrm{ref}}}{R_{13}}.$$

The linear relation between R_{15} and R_w coupled with that between R_w and T_w allows R_{15} to be calibrated linearly in terms of wire temperature for any given probe.

Capacitor C_1 integrates the bias current of A_1 to insure initial turn-on and prevents the unit from latching up in operation. Its value should be small so as not to degrade the instrument's frequency response.

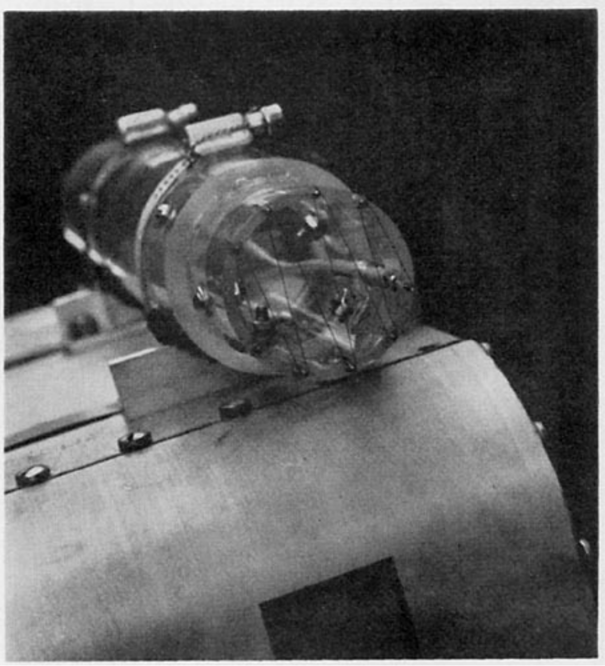


Fig. 3. NHRL nimbiometer sensor mounted on foil impactor. Probe diameter is 5.08 cm.

The power amplifier is designed to handle as much as 15 A with a 1 Ω probe. R_{ref} must be capable of dissipating an appropriate amount of power.

The control amplifiers A_1, A_2, A_3 are type 741 integrated current operational amplifiers requiring regulated ±15 V power supplies. Military grade amplifiers should be used.

With careful bypassing the circuit works safely and reliably with probes from a few tenths of an ohm to several ohms at probe temperatures from ambient to incandescent.

6. The NHRL nimbiometer probes

A hot-wire probe is in essence a wire strung between two terminals with no obstructions to bar free passage of the flow over it. For convenience, the wire may be folded so long as no section becomes shorter than about 100 wire diameters. Sections shorter than this deviate from King's law due to end effects from support-induced cooling, and for shorter wires the cooling may deviate considerably from that due purely to the normal component (Champagne *et al.*, 1967).

In order to calibrate our system we mounted the wire on a cylindrical probe on the side of the primary reference instrument, a foil impactor, installed on the NOAA Research Flight Facility (RFF) C-130 as shown in Fig. 3. The close proximity of the instruments assured like flow fields for droplets large enough to be measured by the impactor despite the disturbing influence of the aircraft. Tungsten wire 0.177 mm in diameter was used for the sensor. Tungsten was

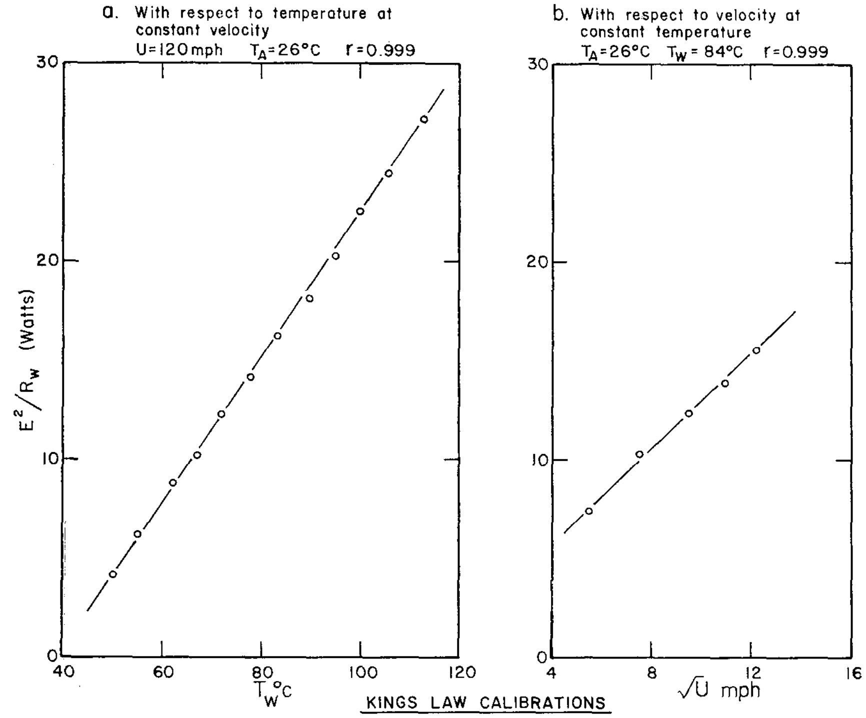


FIG. 4. Typical dry Kings law calibration for the nimbiometer.

selected for its strength in small sizes despite certain difficulties in working with it (Merceret and Schricker, 1973). Extra strength mitigates against probe breakage upon ice impacts. Many other materials and configurations may be appropriate depending on the user's particular requirements.

7. Laboratory and flight tests

The instrument has been flown aboard both DC-6 (39C) and C-130 (41C) aircraft of the Research Flight Facility. It has been laboratory tested in the subsonic aerodynamic wind tunnel at the School of Engineering of the University of Miami.

The flight tests demonstrated that we could set the operating temperature of the probe within 0.5°C of that desired with the first breadboard circuit without precision components.

The values of the King's law constants measured in flight and in the lab agreed within 10% and King's law was obeyed with a correlation coefficient greater than 0.99 over speeds from 20 to 150 m s^{-1}.

The power dissipation per unit sampling volume measured but 5% of that of the J-W meter and the sensitivity of the NHRL instrument was much greater than that of the J-W. Except where we deliberately damped the response with large capacitances on the amplifiers, our instrument was two orders of magnitude or more faster than the J-W. A response time of 1 ms is not unrealistic at higher operating temperatures like 50°C.

The rapid response may in part account for the extreme sensitivity of this instrument despite its cooler operating temperature. Characteristic times for small droplet collisions with the probe at flight speed of 100 m s^{-1} are measured in microseconds. The extended high-frequency response permits a more complete rise of the output signal due to short-period interactions.

8. Calibration of the nimbiometer

In order to determine the King's law constants, it is necessary to know the wire temperature. Since the circuitry sets the wire's operating resistance, the temperature was determined by measuring the resistance of the wire as a function of its temperature in a constant temperature bath. The measured value of the thermal coefficient of resistence of our probe agreed within 3% with that reported for tungsten by Weast (1965).

Thereafter, the King's law constants were determined at several different temperatures by operating the unit in the low-speed aerodynamic wind tunnel at the School of Engineering of the University of Miami. Measurements were taken of the wire voltage with wind speeds from 9 to 67 m s^{-1}. An example is shown in Fig. 4. The values for A and B were independent of the wire temperature from 40 to 100°C and were

$$\left.\begin{aligned} A &= 0.023 \pm 0.001 \text{ W °C}^{-1} \\ B &= 0.025 \pm 0.0045 \text{ W °C}^{-1} (\text{m s}^{-1})^{\frac{1}{2}} \end{aligned}\right\}.$$

Flights at roughly 300 mph yielded results consistent with these values in dry air. English units are used where necessary in the figures to coincide with the output from the data systems.

The wet calibrations were of necessity all airborne since we have no wet laboratory facility available to us. The foil impactor was used as the primary reference while a Johnson-Williams unit as well as an ultraviolet absorption hygrometer-nimbiometer (UVH) were used to supplement the foil data. The J-W and UVH were not located near the hot-wire nimbiometer and there may be some effect due to the flow field around the aircraft. These data are indicated separately so that they may be considered with appropriate caution. A sample of the results is shown in Fig. 5. Each point represents an average over a period of 10–30 s of flight time as required to get an adequate sample from the foil impactor. Wet calibrations at different temperatures and airspeeds overlapped within 15% when adjusted according to King's law. The calibration remained stable from day to day and week to week as one would expect so long as the sensor remained undamaged. The sensitivity of the instrument depends on the operating and ambient temperatures and the airspeed. For $U=134$ m s^{-1}, $T_w=82°$C and $T_a=10°$C, we obtained a sensitivity of 0.62 g m^{-3} W^{-1} at an altitude of 500 m.

Since the form of the calibration presented in (8) is predicated on the droplets being larger than the wire of the probe, it was a pleasant surprise to find that the form seems to remain valid for cloud droplets as well. We don't know how small the droplets must be before the relation fails, but empirically the relation seems to hold within 20% for non-precipitating tropical maritime cumulus and, hence, for droplets on the order of 30 μm diameter or larger. The ideal thing, of course, would be to calibrate the instrument against a cloud droplet spectrometer, but none was available for the measurements reported here.

9. Conclusions

The constant temperature hot-wire nimbiometer is superior both to replicating devices and to constant current hot-wire instruments for the general measurement of liquid water content in and below precipitating clouds. Because of its unique characteristics it is easily adapted to the measurement of the supercooled component of cloud water. The use of King's law in the calibration enables dry wind tunnel facilities to be used after an initial wet calibration, and matched pairs can be readily constructed. The numerous potential errors mentioned in the discussion, including equilibrium approximations and the effects of the aircraft on the exposure of the instrumentation, remained cumulatively small in our tests.

The use of constant temperature hot-wire techniques should significantly improve the state of the art of nimbiometry.

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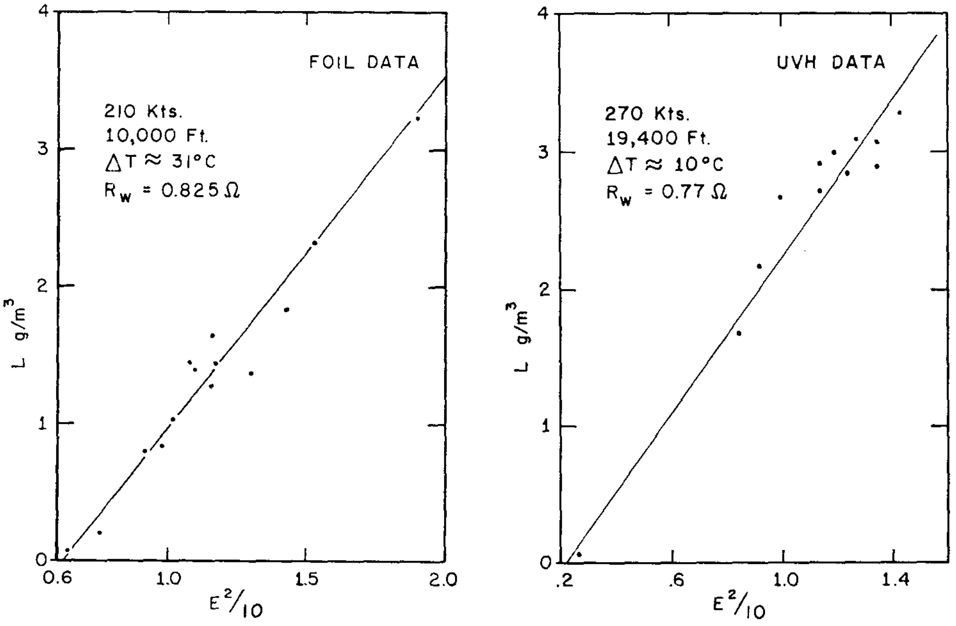

Fig. 5. Typical wet calibrations of the nimbiometer. The foil impactor data are restricted to regions where 90% of the liquid water is in raindrop sizes. The UVH measurements are from a variety of environments including non-precipitating cumulus.

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