The Combined Effect of Temperature and Humidity Fluctuations on Refractive Index

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ABSTRACT

Depending on whether the radiation under consideration is acoustic, visible or microwave, either temperature or humidity fluctuations are ordinarily assumed to be an insignificant source of refractive index fluctuations. For applications in the atmospheric surface layer or in a free convection layer, the value of the local Bowen ratio β, which is the ratio of sensible to latent heat flux densities, can be used to determine when variations in both temperature T and water vapor pressure e are important considerations. When $|\beta|<0.3$ for applications involving visible radiation, $|\beta|<0.6$ for acoustic radiation, and $|\beta|>0.32$ for microwave radiation, both T and e fluctuations have at least a 10% effect on the amplitudes of refractive index fluctuations, provided T and e are highly correlated. If T and e are uncorrelated, this 10% level is $|\beta|<0.067$, $|\beta|<0.13$ and $|\beta|>1.45$ for acoustic, visible and microwave radiation, respectively. With knowledge of β and the extent of the T and e correlation, refractive index "fluxes" and structure function coefficients can be calculated from (or inversely, can be used to calculate) the corresponding parameters for temperature and humidity.

1. Introduction

Measurements of refractive index fluctuations of air are employed to study the turbulent structure of temperature and water vapor in the atmosphere. Instruments that sense refractive index effects have the advantages of being fast responding, e.g., sonic anemometers and microwave refractometers, and of being suitable for remote deployment, e.g., acoustic sounders and radars. The increased use of such instruments justifies a reexamination of the causes of refractive index fluctuations.

In the case of acoustic remote sensors, monostatic sounding systems are used to detect sound backscattered from the atmosphere; regions of strong echoes are frequently associated with elevated inversions that trap pollutants beneath, or they can be coincident with thermal plumes in well-mixed regions near the surface of the earth. The strip-chart records produced by acoustic sounders are maps of the relative strength of C_n, the square root of the acoustic refractive index structure function coefficient. This coefficient is related to the temperature parameter C_T^2 (e.g., Little, 1969) and, less strongly, to the water vapor pressure parameter C_e^2. These coefficients are a measure of the intensity of the high-frequency (small spatial scale lengths) variations of temperature and humidity that scatter acoustic and electromagnetic energy; they derive mathematically from Kolmogoroff's description of isotropic turbulence. The role of C_e^2 in acoustic backscattering will be closely examined in this paper.

Electromagnetic propagation through the atmosphere is also affected by both temperature and humidity fluctuations. These effects interfere with communication systems but, beneficially, allow remote sensing of turbulence. For line-of-sight effects on optical radiation, such as scintillation of laser beams and quivering or blurring of images, fluctuations of moisture content are usually ignored and a measured quantity, optical C_n^2, is assumed proportional only to C_T^2 (e.g., Tatarski, 1971). However, Wesely and Derzko (1975) have shown that C_e^2 should also be included to determine C_n^2 in optical paths in the atmospheric surface layer above wet surfaces. At radio frequencies, water vapor pressure fluctuations are usually assumed to be the primary source of scattering by the clear atmosphere; in radar studies the contributions of temperature fluctuations to scattering cross sections are ordinarily believed to be relatively small. Additionally, although microwave refractometers are known to be sensitive to both specific humidity and temperature (e.g., Martin, 1972), temperature effects are frequently neglected above wet surfaces.

In this paper, the combined effects, as opposed to separate effects, of temperature and humidity fluctuations on the refractive indices of air for acoustic, microwave and optical radiation will be determined.

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To do so, the importance of the extent of correlation between temperature and humidity changes will be considered. Their relative amplitudes, when properly normalized for the atmospheric surface layer or free convection layers, can be expressed in terms of the Bowen ratio β, which is the ratio of the sensible to latent heat flux density.

2. Relative importance of C_e^2 and C_T^2

Table 1 summarizes the separate effects of the fluctuations of temperature, humidity, pressure and wind on various refractive index structure function coefficients. The formulas for the tabulated dependencies are well known and will be apparent later in this paper in expressions derived for more general cases. When examining this table, the reader should keep in mind that the coefficients are directly proportional to atmospheric scattering cross sections. (The attenuation of acoustic and electromagnetic signals by molecular absorption and turbulent scattering within propagation paths is not discussed in this paper.) Also, as will be shown later, the ratio C_T/C_e is directly proportional to β found locally in either forced or free convection above the earth's surface. The behavior of β in the atmospheric surface layer is quite well understood; values over water are given by Priestley and Taylor (1972) and typical determinations above land are summarized by Wesely and Alcaraz (1973).

As shown in Table 1, the individual effects of the fluctuations of temperature and water vapor content on values of various refractive index structure function coefficients are easily computed. However, the combined effect of high-frequency temperature and moisture content variations depends on how well they are correlated. If they are perfectly correlated, either positively or negatively, the refractive indices can be treated as conservative quantities as was done by Wesely and Alcaraz (1973), but if the fluctuations are partially uncorrelated, then this treatment is invalid.

It was first shown by Gossard (1960) for radio waves and later by Friehe *et al.* (1972) for visible radiation that the cospectrum or covariance of T and e should be considered when computing the spectral density or variance of fluctuations in refractive index. Indeed, strong positive correlation has been found in the atmospheric surface layer for flux-carrying eddies above land (Swinbank and Dyer, 1967) and above ocean (Phelps and Pond, 1971; McBean and Miyake, 1972). Further, G. W. Thurtell (private communication) has found spectral correlation coefficients greater than 0.95 at about 0.5 Hz within a few meters above the top of a rather open corn canopy, and recent measurements at Argonne National Laboratory have found nearly identical results above industrial cooling ponds and grassy surfaces. Other recent measurements at Argonne have shown that for very small eddy sizes in the atmospheric surface layer, the extent of correlation between T and e is negligible. At elevations of several hundred meters, where turbulence becomes partially decoupled from the surface layer, temperature and humidity appear to be uncorrelated, as indicated in measurements by Gjessing *et al.* (1973). It seems likely that the detailed nature of T and e correlation within 1–2 km above the surface depends on the height and structure of the planetary boundary layer. In fact, more information about T and e correlation in the lower atmosphere would contribute to the understanding of the vertical transport processes for sensible and latent heat.

3. Effects of T and e correlation on C_n^2

The structure function coefficient for any refractive index n can be expressed as (e.g., see Tatarski, 1971)

$$C_n^2=\overline{(m_1-m_2)^2}/r^{2/3},$$

where $m\equiv n-\bar{n}$ and the overbar indicates a time average. In homogeneous turbulence, we can assume $\overline{m_1^2}+\overline{m_2^2}=2\overline{m^2}$ which allows us to write

$$C_n^2=(2\overline{m^2}-2\overline{m_1m_2})/r^{2/3}, \tag{1}$$

in which r is the separation distance between the measurements of m_1 and m_2 and corresponds to an effective scattering distance for the radiation under

TABLE 1. Magnitudes of the structure function coefficient for various types of radiation passing through the atmosphere, expressed per unit of temperature, humidity, pressure and wind structure function coefficients. We assume that temperature, humidity, pressure and wind fluctuations are all uncorrelated and that $\bar{T}=300$ K and $\bar{p}=1000$ mb. The effect of wind fluctuations on sound propagation is determined by the speed of sound and the angle of scattering with respect to wavefront movement (e.g. Tatarskii, 1971).

	C_n^2/C_T^2 (K^{-2})	C_n^2/C_e^2 (mb^{-2})	C_n^2/C_p^2 (mb^{-2})	C_n^2/C_u^2 (m s^{-1})$^{-2}$
Acoustic	$(4\bar{T}^2)^{-1}$	$0.17(4\bar{p}^2)^{-1}$	small	s^{-2}
	2.8×10^{-6}	0.043×10^{-6}		0–8.3×10^{-6}
Visible	$A_1^2\bar{p}^2\bar{T}^{-4}$	$(A_1-A_2)^2\bar{T}^{-2}$	$A_1^2\bar{T}^{-2}$	0
	0.76×10^{-12}	0.0017×10^{-12}	0.067×10^{-12}	
Microwave	$A^2\bar{p}^2\bar{T}^{-4}$	$(A-B-C/\bar{T})^2\bar{T}^{-2}$	$A^2\bar{T}^{-2}$	0
	0.74×10^{-12}	17×10^{-12}	0.067×10^{-12}	

study. For (1) to be valid, the scattering length must be within inertial subrange scales where isotropic turbulence exists. We will next derive expressions for m in the case of acoustic waves and use this in (1) to determine an acoustic C_n; electromagnetic radiation will be considered in Subsection b. Vapor pressure was chosen as the moisture variable instead of either specific humidity or mixing ratio because vapor pressure is least cumbersome to employ in the equations for the electromagnetic refractive indices. For the sake of consistency, vapor pressure is used in the expressions of acoustic refractive index also.

a. Acoustic waves

It is necessary to see how variations in e affect n. On a local scale, water vapor added to air displaces some of the air gases, but the total static pressure p is not changed (unless by wind-induced dynamic effects). By examination of the equation of state, it can be shown that the effect of changing water vapor pressure on air density, and thus on acoustic refractive index, is similar to a temperature change. Indeed, the use of virtual temperature T_v, instead of ambient temperature T, in the equation of state takes into account changes in density induced by changes in moisture content. From use of this modified equation of state in the formulations given in Table 112 of the *Smithsonian Meteorological Tables* (List, 1949), it follows that the speed of sound in air is $s=(\zeta R_a T_v)^{\frac{1}{2}}$, where ζ is the ratio, at temperature T, of the specific heat at constant pressure to the specific heat at constant volume, and R_a is the gas constant for dry air. Since $n\equiv s_0/s$ and the reference speed $s_0=(\zeta_0 R_a T_0)^{\frac{1}{2}}$, it is easily shown that

$$n=[(\zeta_0/\zeta)(T_0/T_v)]^{\frac{1}{2}}.$$

Another useful relationship in Table 112 of *Smithsonian Meteorological Tables* is that

$$\zeta/\zeta_0\approx 1-e/(10p\zeta_0),$$

which implies that the temperature dependency of ζ can be neglected. As a result, the acoustic refractive index can be expressed as

$$n=\{(T_0/T_v)[1-e/(10p\zeta_0)]^{-1}\}^{\frac{1}{2}},$$

where T_0 is a constant absolute temperature at which arbitrarily $n\equiv 1$, and later will be assumed to be the mean temperature. Substituting the definition for T_v,

$$T_v=T/(1-0.378e/p),$$

$$n=\{(T_0/T)(1-0.378e/p)/[1-e/(10p\zeta_0)]\}^{\frac{1}{2}}. \quad (2)$$

It will be recognized that $0.378=1-\epsilon$, where ϵ is the ratio of the molecular weights of dry air and water vapor.[2]

[2] It has been pointed out by W. L. Clink (private communication) that an *acoustic* virtual temperature T_{va} is sometimes used. It follows from (2) that $T_{va}=T(1+0.307e/p)$.

We will now derive an expression for C_n^2. To simplify (2), we note that $e\ll p$ so that

$$[1-e/(10p\zeta_0)]^{-\frac{1}{2}}\approx 1+e/(20p\zeta),$$

and

$$(1-0.378e/p)^{\frac{1}{2}}\approx 1-0.189e/p.$$

We shall adopt the notation that $\eta=e-\bar{e}$, $\psi=p-\bar{p}$, and $\theta=T-\bar{T}$. Since $\theta\ll\bar{T}$, another useful approximation is that

$$T^{-\frac{1}{2}}\approx\bar{T}^{-\frac{1}{2}}[1-\theta/(2\bar{T})].$$

An expression for m can be derived by substitutions of the above three binomial expansions into (2). The resulting expression can be greatly simplified using the assumption that $\psi/\bar{p}\ll\eta/\bar{e}$ and with the omission of terms that include products of more than one fluctuation component, in anticipation that all triple or higher order correlation terms that result from the multiplications to follow can be neglected when normalized by the constituent mean values. When about 50 terms that contain products of fluctuation components are ignored, the refractive index component can be expressed as

$$m=(-\theta-\eta D\bar{T}/\bar{p})/(2\bar{T}), \quad (3)$$

where $D=1-\epsilon-0.1/\zeta_0\approx 0.307$. The effective scattering distance of acoustic energy, which for most applications is one-half the wavelength, is assumed to be within the scales of existing inertial subranges so that (1) applies to acoustic scattering. Substituting (3) into (1), we have

$$C_n^2=[C_T^2/(4\bar{T}^2)]\alpha_a^2, \quad (4a)$$

$$\alpha_a^2=1+r_{eT}\left(\frac{2DC_e\bar{T}}{C_T\bar{p}}\right)+\left(\frac{DC_e\bar{T}}{C_T\bar{p}}\right)^2, \quad (4b)$$

where $r_{eT}=C_{eT}^2/(C_eC_T)$ is the structural correlation coefficient and C_{eT}^2 is the crossed structure function coefficient. In the derivation of (4) from (2), terms such as $\overline{\theta^2\eta}/(\bar{T}^2\bar{e})$ are neglected because third moments are likely to have values near zero for near-Gaussian fluctuations at various frequencies within the inertial subrange. Fourth-moment terms such as $\overline{\theta^2\eta^2}/(\bar{T}^2\bar{e}^2)$ are nonzero when $\overline{\theta\eta}$ is nonzero. Since it is likely that $\overline{\theta^2\eta^2}^{\frac{1}{2}}$ is only two or three times the absolute value of $\overline{\theta\eta}$, the former term can be neglected because it appears as $\overline{\theta^2\eta^2}/(\bar{T}^2\bar{e}^2)$, which is much less than the comparable term, $\overline{\theta\eta}/(\overline{Te})$, that is found. Justification for neglecting the fifth- and sixth-order moment terms are similar to those for the third- and fourth-moment terms, respectively.

The behavior of α_a is shown in Fig. 1. When the scattering power of the turbulent atmosphere is calculated, α_a^2 can be considered a correction factor to be applied to C_T^2. Conversely, the amplitude of the return signal must be corrected by multiplication with

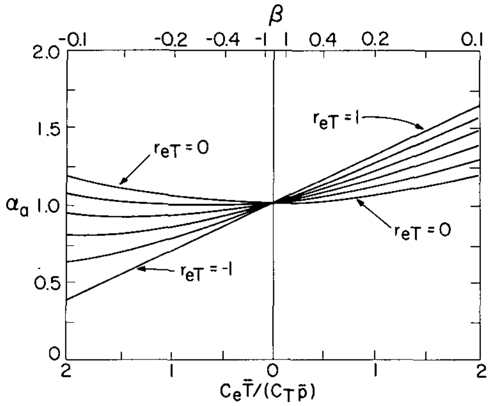

FIG. 1. Correction factors to be applied to C_T in computing acoustic C_n, to take into account humidity fluctuations and their correlation (in increments of 0.2 for r_{eT}) with temperature variations. For $r_{eT}=\pm 1$, $\alpha_a = 1+0.06/\beta$ and for $r_{eT}=0$, $\alpha_a=[1+(0.06/\beta)^2]^{\frac{1}{2}}$.

α_a^{-1} to obtain an estimate of C_T from backscattered sound.

In the surface and free convection layers, the ratio $C_e\bar{T}/(C_T\bar{p})$ can be readily interpreted. By methods described by Wesely and Alcaraz (1973), it can be shown that in the atmospheric surface layer,

$$C_e/C_T = |\overline{w\eta}/\overline{w\theta}|,$$

where w is the vertical velocity, $\overline{w\eta}$ the mean turbulent vertical flux of water vapor pressure, and $\overline{w\theta}$ the corresponding temperature flux. Using free convection formulas [e.g., an extension of equations given by Wyngaard *et al.* (1971)], it can be shown that the above equation is likely to be valid for free convection, which is nearly achieved in convective plumes above the surface layer. The sensible heat flux H and the latent heat flux L_wE are given as

$$\left.\begin{aligned} H &= \rho c_p \overline{w\theta} \\ L_wE &= \rho L_w \epsilon \overline{w\eta}/\bar{p} \end{aligned}\right\},$$

where c_p is the specific heat of air at constant pressure and L_w the latent heat of vaporization of water. It follows that

$$C_e\bar{T}/(C_T\bar{p}) = c_p\bar{T}/(\epsilon L_w|\beta|),$$

where $\beta = H/L_wE$ is the Bowen ratio. When $T=300$ K and $p=1000$ mb,

$$C_e\bar{T}/(C_T\bar{p}) = (5.03|\beta|)^{-1}. \qquad (5)$$

Therefore, the values for the lower horizontal axis in Fig. 1 are multiplied by 5.03 to obtain β^{-1} on the right and $-\beta^{-1}$ on the left, giving the values at the upper horizontal axis. Of course, in the free convection layer, a local value of β is assumed rather than the surface layer Bowen ratio. Inspection of Fig. 1 reveals that $\alpha_a \approx 1$ above dry land surfaces where typically $|\beta|>0.05$, but α_a can be significantly different from unity over wet surfaces.

Direct measurements, rather than remote, can be obtained with the sonic anemometer–thermometer, which responds directly to the fluctuations in the speed of sound s along a short path. Variations in temperature are detected as changes in s, or equivalently as changes in $1/n$ where n is given by (2). Using the same procedure that allows (3) to be derived from (2), we can show that the variations in $1/n$ are approximately

$$(s-\bar{s})/s_0 = (\theta + \eta D\bar{T}/\bar{p})/(2\bar{T}). \qquad (6)$$

If the covariance of the vertical wind component and the sonic anemometer estimate of $\theta = 2\bar{T}(s-\bar{s})/\bar{s}$ is used for computing the sensible heat flux, a correction factor that takes into account humidity fluctuations can be derived for the computed flux. This factor is α_a^{-1} for $r_{eT}=\pm 1$ and can be obtained from Fig. 1 if the Bowen ratio is known. Without this correction, the computed sensible heat flux evidently can be in error by considerably more than 10% for $|\beta|<0.6$.

b. *Electromagnetic waves*

We shall first consider the combined effects of T and e on visible radiation, and then on microwave radiation. The optical refractive index for visible radiation is given by (e.g., Fleagle, 1950)

$$n-1 = A_1(p-e)/T + A_2e/T, \qquad (7)$$

where A_1 and A_2 can be calculated as described by Wesely and Alcaraz (1973) from available data sources. Thus, $A_1 = 78.7\times10^{-6}$ K mb^{-1} and $A_2 = 66.3\times10^{-6}$ K mb^{-1}. [Note that values corresponding to A_1 and (A_1-A_2) are usually listed instead of A_1 and A_2 as assumed by Wesely and Alcaraz.] With the same technique and assumptions that resulted in the derivation of (3) from (2), we obtain

$$m = (A_1/\bar{T})(\psi - \eta - \bar{p}\theta/\bar{T}) + (A_2/\bar{T})(\eta - \bar{e}\theta/\bar{T}). \qquad (8)$$

The scale length associated with scintillation of laser beams or quivering and blurring of images is about $(\lambda l)^{\frac{1}{2}}$, where λ is the wavelength and l the total propagation distance, and is within inertial subranges for most practical lines of sight near the earth's surface (Tatarski, 1971; Hufnagel and Stanley, 1964). As a result, (1) is valid in the case of optical radiation, and with the same assumptions used in deriving (4), we can substitute (8) into (1) to obtain

$$C_n^2 = (C_T^2A_1^2\bar{p}^2/\bar{T}^4)[C_p^2\bar{T}^2/(C_T^2\bar{p}^2) + \alpha_v^2], \qquad (9a)$$

$$\alpha_v^2 = 1 + r_{eT}\left[\frac{2(1-A_2/A_1)C_e\bar{T}}{C_T\bar{p}}\right] + \left[\frac{(1-A_2/A_1)C_e\bar{T}}{C_T\bar{p}}\right]^2. \qquad (9b)$$

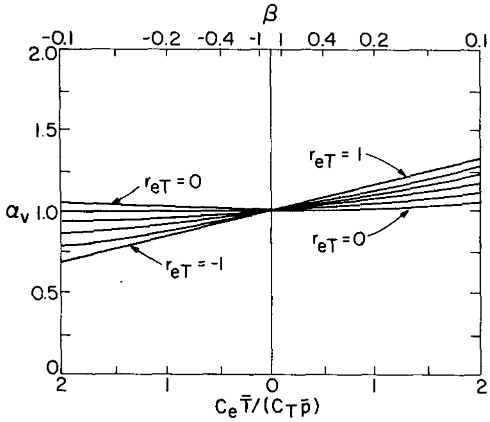

FIG. 2. Correction factors to be applied to C_T in computing optical C_n, to take into account humidity fluctuations and their correlation with temperature variations. For $r_{eT}=\pm1$, $\alpha_v=1+0.03/\beta$, and for $r_{eT}=0$, $\alpha_v=[1+(0.03/\beta)^2]^{\frac{1}{2}}$.

Since $C_p^2\approx\rho^2C_u^4$, where ρ is the density of air and C_u^2 the longitudinal wind structure function coefficient (Hinze, 1959), it can be shown that C_p^2 can be neglected unless C_T^2 and C_e^2 are very small and the mean wind is at least 5 m s^{-1}. Thus, α_v can be considered a correction factor that is sometimes necessary to determine C_n from C_T. In Fig. 2, α_v is graphed as a function of $C_e\bar{T}/(C_T\bar{p})$ for various values of r_{eT}. This correction factor is small above land where typically $|\beta|>0.5$.

It is noteworthy that when the term involving C_p^2 is neglected and $r_{eT}=\pm1$, Eq. (9) reduces to

$$C_n=C_TA_1\bar{p}/\bar{T}^2(1+0.03\beta^{-1}). \qquad (10a)$$

Also, when $r_{eT}=0$,

$$C_n=C_TA_1\bar{p}/\bar{T}^2[1+(0.03\beta^{-1})^2]^{\frac{1}{2}}. \qquad (10b)$$

The above two equations are the "linear" and "vector" methods of calculating C_n from C_T and β, as were used with some success by Wesely and Derzko (1975).

According to Bean and Dutton (1966), the refractive index for radio waves is given by

$$n-1=A(p-e)/T+(e/T)(B+C/T), \qquad (11)$$

where $A=77.6\times10^{-6}$ K mb^{-1}, $B=72\times10^{-6}$ K mb^{-1}, and $C=0.375$ K^2 mb^{-1}. With the same technique and assumptions used to derive (3) from (2), it can be shown that

$$m=(A/\bar{T})(\psi-\eta-\bar{p}\theta/\bar{T}) +(B/\bar{T}+C/\bar{T}^2)(\eta-\bar{e}\theta/\bar{T}). \qquad (12)$$

The effective scattering distance of electromagnetic energy in the radio frequencies is one-half the wavelength for most applications and is assumed here to be within the scales of existing inertial subranges.

An equation equivalent to (9) can be derived with $A_1=A$ and $A_2=B+C/\bar{T}$, but since temperature effects are much less important for radio wave propagation than are water vapor effects, we should consider a relationship that differs from (9) in that C_T^2 appears only in a correction term. This may be accomplished by introducing a factor γ_r^2 that includes the effects of temperature fluctuations as a correction to the effects of fluctuations in water vapor content. Hence, it follows that

$$C_n^2=C_e^2A_1^2(1-A_2/A_1)^2\gamma_r^2/\bar{T}^2 \qquad (13a)$$

and

$$\gamma_r^2=1+r_{eT}\left[\frac{2C_T\bar{p}}{C_e\bar{T}(1-A_2/A_1)}\right] +\left[\frac{C_T\bar{p}}{C_e\bar{T}(1-A_2/A_1)}\right]^2, \qquad (13b)$$

where pressure fluctuations are neglected. Values of γ_r are plotted in Fig. 3. It is evident that when $|\beta|>0.32$ (i.e., $C_T\bar{p}/C_e\bar{T}>1.61$) the relationship between C_n and C_e with $\gamma_r=1$ in (13a) is not accurate within 10%, if T and e are highly correlated. If T and e are uncorrelated, the 10% accuracy level can be extended to $|\beta|>1.45$ ($C_T\bar{p}/C_e\bar{T}>7.3$).

If the covariance of the vertical wind component and a microwave refractometer estimate of $\eta=m\bar{T}/(-A+B+C/\bar{T})$ are used for computing latent heat flux, the correction factor for the computed flux is γ_r^{-1} for $r_{eT}=1$ or -1, corresponding to Bowen ratio values that are positive and negative, respectively. For this case, $\overline{w\psi}$ can be neglected [typical values measured by Dobson (1971) even over sea waves lead to errors in values of latent heat fluxes of only about 0.5 W m^{-2}]. Thus, this method for computing latent heat flux can lead to errors considerably greater than 5% for $|\beta|>0.1$.

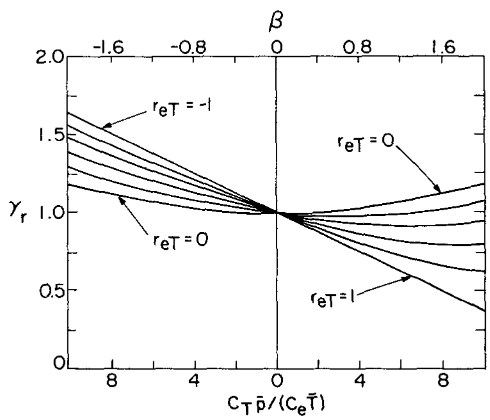

FIG. 3. Correction factors to be applied to C_e in computing radio C_n, to take into account temperature fluctuations and their correlation with humidity variations. For $r_{eT}=\pm1$, $\gamma_r=1-0.31\beta$ and for $r_{eT}=0$, $\gamma_r=[1+(0.31\beta)^2]^{\frac{1}{2}}$.

4. Effects of e and T correlation on spectra

The spectra of refractive index fluctuations should be mathematically dependent on the spectra of temperature and humidity in the same manner that the variance of n is dependent on the variances of T and e. That is, using (3), (8) and (12) we can compute $\overline{m^2}$ in terms of $\overline{\theta^2}$ and $\overline{\eta^2}$, and apply the results to the variance spectral density S_n. For acoustic waves, this results in

$$S_n=[S_T/(4\bar{T}^2)]\alpha_a^2, \tag{14}$$

where the same assumptions and notation used in deriving (4) have been used. Of course, the spectral densities are functions of frequency, and accordingly, r_{eT} is now assumed to be a spectral correlation coefficient. The term $S_e\bar{T}^2/(S_T\bar{p}^2)$ replaces $C_e^2\bar{T}^2/(C_T^2\bar{p}^2)$ in (4b).

In a similar manner, it can be shown that for optical propagation studies,

$$S_n=(S_TA_1^2\bar{p}^2/\bar{T}^4)\alpha_v^2, \tag{15}$$

where pressure fluctuations have been neglected. For radio waves,

$$S_n=S_eA^2[1-(C+B/\bar{T})^2/A^2]\bar{T}^2\gamma_r^2. \tag{16}$$

At this point it is evident that Figs. 1–3 can be applied to spectral densities as well as to structure function coefficients.

5. Conclusions

Since acoustic sounder echo signals are proportional to $C_T\alpha_a$, a substantial portion of the returning acoustic energy can be generated by water vapor pressure (or specific humidity) fluctuations, especially in the lower atmosphere where r_{eT} is large and above wet surfaces where β is small. Furthermore, when $\beta<0$, as can occur when warm, dry air is advected over an evaporating and relatively cool surface, water vapor fluctuations can decrease the scattering power of the atmosphere close to the surface. This situation may occur frequently over lakes and other large bodies of water.

Quantitative measurements of backscattered acoustic energy, which require use of an appropriately calibrated acoustic sounder system, are dependent upon the extent of temperature and water vapor pressure correlation, especially for the following applications:

1) Estimation of inversion "strength." If the reflected energy associated with mean gradients in the inversion is small, most of the signal derived from backscattered acoustic energy is approximately proportional to fluctuations in virtual temperature, or density fluctuations where locally the static pressure is constant. When $0<r_{eT}<1(0>r_{eT}>-1)$, temperature and water vapor in the scattering volume do not always simultaneously add to (subtract from) the density fluctuation, and as a result the local density fluctuations would be less (more) than the linear sum (difference) of the contributions from the temperature and water vapor. On the other hand, mean temperature and water vapor pressure usually increase or decrease monotonically with height through an inversion, resulting in the local density gradient being equal to the linear sum or difference of the contribution from temperature and water vapor. Hence, the lack of information regarding r_{eT} can add to the difficulty of determining density gradients from acoustical returns.

2) Determination of vertical latent and sensible heat transfers from optical measurements of C_T and C_e. The value of r_{eT} is needed in the calculation of C_T and C_e.

The relative importance of the magnitude of C_e^2 as compared to C_T^2 on the value of optical C_n^2, and on laser beam scintillation and image blurring, is slightly smaller than for acoustical backscattering power. Sensible heat flux in the atmospheric surface layer can be determined from optical C_n^2 by the reverse of the process described by Wesely and Alcaraz (1973), but if the effect of water vapor fluctuations on C_n^2 is not directly measured, the sensible heat flux should be corrected by multiplication with α_v^{-1}. [This correction factor when $r_{eT}=\pm1$ is smaller than the one given by Wesely and Alcaraz, because they used an erroneous value for A_2 in (5).] For electromagnetic radiation at radio frequencies, both temperature and water vapor fluctuations can contribute significantly to the scattering power of the atmosphere. A microwave refractometer's measurement of m in the surface layer can be used to estimate spectra of humidity fluctuations, but a closely positioned thermometer should be used to obtain θ, so that it's effect can be removed immediately via (10) (e.g., Martin, 1972). An alternative method, if β and r_{eT} are known, is to use γ_r^{-2} from Fig. 3 as a correction factor for the spectra. Finally, a refractometer can also be used to estimate latent heat flux directly with eddy correlation techniques provided the temperature effects are removed from the refractometer signals, or alternatively, the correction factor γ_r^{-1} for $r_{eT}=\pm1$ in Fig. 3 is applied.

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