Maximum Rise of Cooling Tower Plumes

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ABSTRACT

The application of the maximum plume rise formula for bent-over industrial plumes to visible cooling tower plumes depends on the validity of two assumptions: the neglect of finite source size and the neglect of condensation effects. These produce errors which, in many cases, tend to compensate. The effect of finite source size is examined analytically and an alternative formula for maximum plume rise is derived.

The entrainment theory for bent-over plume rise in a stable environment leads to an elegant approximate formula for maximum height of rise

$$Z_m=\left(\frac{6F_0}{U\alpha^2N^2}\right)^{\frac{1}{3}}, \qquad (1)$$

where U is the horizontal wind speed, α an entrainment parameter, N the Väisälä frequency and F the flux of buoyancy (the subscript zero denotes initial conditions). This result has been derived by Slawson and Csanady (1971) and others and shown to match data from dry industrial plumes by a number of investigators (see, e.g., Briggs, 1969).

Eq. (1) is derived by neglecting initial plume radius (except in calculating F_0) and the possible effect of condensation on plume rise. For condensed (visible) cooling tower plumes, which usually have a large initial radius, these two approximations are less acceptable. Hanna (1972), Wigley (1975) and others have shown that condensation and the attendant release of latent heat can increase cooling tower plume rise by more than 30% so that Eq. (1) should underestimate cooling tower plume rise. Frick and Winiarski (1975) have analyzed the effect of finite source radius on plume rise. Their numerical solutions of the plume equations for a finite source of 30 m radius and an equivalent point source shows that the finite source rise is some 20% below the rise from a point source. The effect of finite source size thus tends to compensate for the effect of condensation and Eq. (1) will give a better estimate of maximum plume rise for cooling tower plumes than might otherwise be expected.

The amount of increased plume rise due to condensation depends on atmospheric stability, being greater in less stable environments. For a plume with a long condensed section, the increase in maximum plume rise will exceed 30% for lapse rates $\gtrsim -0.002°\text{C m}^{-1}$ (see Wigley, 1975, Fig. 2). Such stability conditions are common, but they need not be associated with *long* condensed plumes, and a 30% or more boost in rise due to condensation may be relatively infrequent. The effect of finite source radius is, however, always present.

The effect of either condensation or finite source size alone would be difficult to isolate if maximum plume rise observations were compared with Eq. (1). It is, however, possible to derive analytically an alternative plume rise formula which includes the effect of finite source size. In deriving Eq. (1), Slawson and Csanady (1971) use the boundary condition $R^2w=0$ at $x=0$ (R is plume radius and w plume vertical velocity) and the approximation $R=\alpha z$. If, instead, one uses $R^2w=R_0^2w_0$ at $x=0$ and $R=R_0+\alpha z$, then one obtains

$$Z_m^*=\frac{1}{\alpha}\left\{\left[\frac{3\alpha F_0}{UN^2}(1+\sqrt{1+\beta^2})+R_0^3\right]^{\frac{1}{3}}-R_0\right\} \qquad (2)$$

as the maximum rise [occurring at $x=(\pi U/N)-\tan^{-1}\beta$]. In Eq. (2)

$$\beta=\frac{w_0UNR_0^2}{F_0}=\frac{UNT_{a0}}{g\Delta T_0},$$

where T_{a0} is the environment temperature and ΔT_0 the plume excess temperature at the release point; g is the acceleration due to gravity.

Two approximations must be made to obtain Eq. (1) from Eq. (2); $\beta\ll 1$ and $R_0\ll\alpha Z_m$ [where Z_m is given by Eq. (1)]. A better result can be achieved by retaining first-order terms in $R_0/\alpha Z_m$. For cooling towers the parameter β is invariably small and even in extreme circumstances can only affect Z_m^* by a few percent. If β is ignored, Eq. (2) becomes

$$Z_m^*=Z_m\left\{\left[1+\left(\frac{R_0}{\alpha Z_m}\right)^3\right]^{\frac{1}{3}}-\frac{R_0}{\alpha Z_m}\right\}. \qquad (3)$$

For $R_0\ll\alpha Z_m$ this reduces to $Z_m^*=Z_m$. Retaining first-order terms in $R_0/\alpha Z_m$ gives

$$Z_m^*\approx Z_m-\frac{R_0}{\alpha}. \qquad (4)$$

The effect of finite source radius is immediately apparent from this result. For a typical natural-draft cooling tower plume ($Z_m\approx 400$ m, $R_0=30$ m, $\alpha=0.3$) the difference between Z_m and Z_m^* is approximately 20%, in accord with the numerical result of Frick and Winiarski (1975). The correction R_0/α is only significant for large sources. For a typical industrial plume ($R_0/\alpha\sim 10$ m) the correction is not significant and lies within the observational uncertainty in estimating maximum plume rise.

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