On the Response of Superpressure Balloons to Displacements from Equilibrium Density Level

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ABSTRACT

An approximate analytical solution of the vertical equation of motion for constant density superpressure balloons is obtained. It is shown how this solution can be used to filter out neutrally buoyant oscillations in balloon records.

1. The balloon response

The response of a superpressure balloon to an initial displacement from its constant-density floating level is discussed. The response to sinusoidal variations of vertical wind was treated earlier by Hanna and Hoecker (1971), and the frequency of the balloon's neutral buoyancy oscillations (NBO) was discussed by Levanon *et al.* (1974).

The vertical equation of motion for the balloon (Hanna and Hoecker, 1971) has the form

$$M_b\frac{\partial\omega_b}{\partial t}=M_a\frac{\partial\omega_a}{\partial t}+\frac{1}{2}M_a\left(\frac{\partial\omega_a}{\partial t}-\frac{\partial\omega_b}{\partial t}\right)-M_b g\left(\frac{\rho_b-\rho_a}{\rho_b}\right)-\rho_a A\frac{C_D}{2}(\omega_b-\omega_a)|\omega_b-\omega_a|, \quad (1)$$

where the symbols have the following meaning:

- M_b mass of the balloon system (the balloon, the gas inside it, string and other payload)
- M_a mass of the air displaced by the balloon system
- ω_b vertical speed of the balloon
- ω_a vertical speed of the air
- g acceleration due to gravity
- ρ_b density of balloon system
- ρ_a density of air
- A cross-sectional area presented to the vertical air stream by the balloon
- C_D drag coefficient.

The term on the left-hand side of (1) is the net force on the balloon. The terms on the right-hand side are the dynamic buoyancy force due to the acceleration of the air, the acceleration drag due to the difference between the air's and the balloon's acceleration, the static buoyancy force, and the drag force. Inserting

$$M=\rho V, \quad (2)$$

$$V=\tfrac{4}{3}Ar, \quad (3)$$

where V is the volume and r the radius of the balloon, in (1) and dividing by V, we get

$$\rho_b\frac{\partial\omega_b}{\partial t}=\rho_a\frac{\partial\omega_a}{\partial t}+\frac{1}{2}\rho_a\left(\frac{\partial\omega_a}{\partial t}-\frac{\partial\omega_b}{\partial t}\right)-\rho_b g\left(\frac{\rho_b-\rho_a}{\rho_b}\right)-\rho_a\frac{3C_D}{8r}(\omega_b-\omega_a)|\omega_b-\omega_a|. \quad (4)$$

Assuming $\rho_a=\rho_b$, except in the term where they occur in conjunction with g (the Boussinesq assumption), will reduce (4) to

$$\frac{3}{2}\frac{\partial}{\partial t}(\omega_b-\omega_a)=-g\left(\frac{\rho_b-\rho_a}{\rho_b}\right)-\frac{3C_D}{8r}(\omega_b-\omega_a)|\omega_b-\omega_a|. \quad (5)$$

Since at the equilibrium level $\rho_a=\rho_b$, we make the approximation that

$$\frac{\rho_b-\rho_a}{\rho_b}=-z\frac{\partial}{\partial z}\left(\frac{\rho_a}{\rho_b}\right), \quad (6)$$

where z is the displacement above the equilibrium level.

The hydrostatic stability s is defined by

$$s=-g\frac{\partial}{\partial z}\left(\frac{\rho_a}{\rho_b}\right). \quad (7)$$

With ρ_b assumed constant,

$$s=\frac{g}{T}\left(\frac{g}{R}-\gamma\right), \quad (8)$$

where T is ambient temperature, R the gas constant, and γ the lapse rate.

Substitution of (6) and (7) in (5) reduces the balloon equation to

$$\frac{\partial}{\partial t}(\omega_b-\omega_a)=-\tfrac{2}{3}sz-\frac{C_D}{4r}(\omega_b-\omega_a)|\omega_b-\omega_a|. \quad (9)$$

If we ignore the vertical velocity of the atmosphere, then (9) reduces to

$$\frac{\partial}{\partial t}\omega_b+\tfrac{2}{3}sz+\frac{C_D}{4r}(\omega_b)|\omega_b|=0, \tag{10}$$

which could also be written as

$$\frac{\partial^2 z}{\partial t^2}+\tfrac{2}{3}sz+\frac{C_D}{4r}\left(\frac{\partial z}{\partial t}\right)\left|\frac{\partial z}{\partial t}\right|=0. \tag{11}$$

Such a differential equation has the approximate solution (Minorsky, 1962)

$$z=\frac{z_0}{1+C_Dz_0\omega t(3\pi r)^{-1}}\cos\left[\omega t-\frac{0.122C_D}{4\pi r}\times z_0\left(1-\frac{1}{C_Dz_0\omega t(3\pi r)^{-1}}\right)+\psi_0\right], \tag{12}$$

where ω (without a subscript) is a frequency given by

$$\omega=(\tfrac{2}{3}s)^{\frac{1}{2}}=\left[\frac{2}{3}\frac{g}{T}\left(\frac{g}{R}-\gamma\right)\right]^{\frac{1}{2}} \tag{13}$$

and z_0 and ψ_0 are initial conditions.

A typical Tropical Wind Energy Conversion and Reference Level Experiment (TWERLE) balloon has the following parameters (Levanon *et al.*, 1975):

$r=1.8$ m	$\gamma=0$
$c_D=0.4$	$z_0=10$ m
$\rho_b=0.25$ kg m^{-3}	$\psi_0=0$
$T=210$ K	

In Fig. 1, the approximate solution based on (12) with the above listed parameters is compared to a numerical calculation of the differential equation (10) for the same parameters, to show the good agreement.

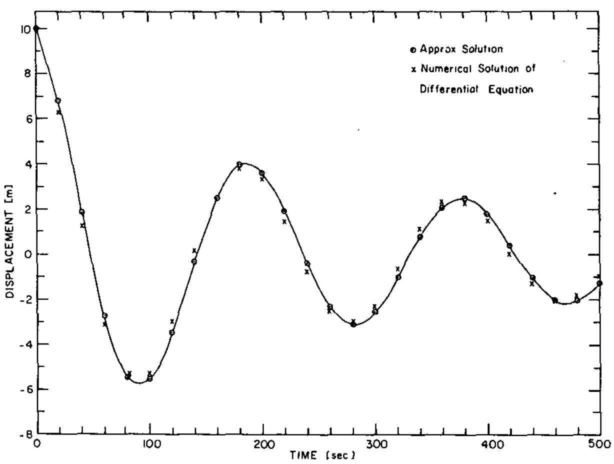

FIG. 1. A superpressure balloon response to an initial displacement (typical TWERLE balloon).

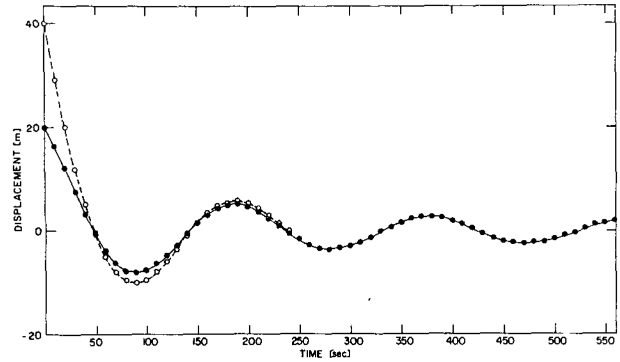

FIG. 2. Balloon responses to two different initial displacements.

In Fig. 2, (12) is plotted for $z_0=20$ m and 40 m, while the other parameters remain as listed above. Fig. 2 clearly shows how the nonlinear damping reduces the effect of the initial displacement on the magnitude of the first overshoot.

Eq. (12) implies that, for a large initial displacement (z_0), the amplitude of the first overshoot (near $\omega t=\pi$) is approximately $3r/C_D$, which, for a typical TWERLE balloon, would be equal to 13.5 m. An example of a large initial displacement would be the arrival at ceiling.

2. Eliminating the NBO

The NBO near the equilibrium level are sometimes large enough in amplitude to mask longer term variations of constant-density level. A filtering technique is desired that will eliminate the NBO from the vertical trajectory of the balloon.

The first signal processing methods that come to mind are digital band-reject or low-pass filters. However, those filters do not take into consideration the damping law of the NBO. Had the balloon response been linear, the optimum filtering method would have been a deconvolution of the measured altitude (or pressure) with the balloon response (Robinson, 1967).

We will attempt to apply this technique despite the nonlinear behavior of the balloon. For a typical initial displacement of a TWERLE balloon we choose 10 m. Below this value the law of decay changes rather slowly, until the amplitudes are too small to be of any interest. Above that value, and in particular for strong displacements, we showed above that the first overshoot is approximately 10 m; hence the filtering will be effective from that overshoot on. The response to a 10 m displacement was given in Fig. 1. The first 22 samples (20 s apart) from the plot in Fig. 1 are normalized and listed in Table 1 as the "balloon wavelet."

The balloon wavelet in Table 1 has an inverse wavelet, and the deconvolution with the balloon wavelet can be easily implemented by convolution with the inverse wavelet (Robinson, 1967).

TABLE 1. TWERLE balloon and inverse wavelets.

Balloon wavelet	Inverse wavelet
1.000	1.000
0.681	−0.681
0.185	0.280
−0.276	0.212
−0.542	0.158
−0.552	0.112
−0.344	0.072
−0.031	0.040
0.250	0.018
0.395	0.005
0.364	0.001
0.181	0.002
−0.040	0.005
−0.232	0.008
−0.310	0.008
−0.255	0.006
−0.102	0.003
0.081	−0.001
0.215	−0.003
0.251	−0.004
0.181	−0.003
0.042	−0.000

The inverse wavelet $b_0, b_1, b_2, \ldots, b_n$ is calculated from the balloon wavelet $a_0, a_1, a_2, \ldots, a_m$, using the two equations

$$a_0 b_0 = 1, \tag{14}$$

$$\sum_{i=0}^{m} a_i b_{n-i} = 0 \quad \text{for} \quad n = 1, 2, 3, \ldots. \tag{15}$$

The inverse wavelet of the balloon wavelet is also listed in Table 1. A test of the deconvolution with the theoretical balloon response was performed on the raw pressure data from TWERLE balloon 1406, launched from Ascension Island on 18 September 1974.

The raw pressure data of flight 1406 are given in Fig. 3. The horizontal scale is time/20 s (20 s is the row data sampling period). The vertical scale is in pressure sensor reading units. The plot is of the pressure sensor reading subtracted from the average reading of 222.55. Each unit of the pressure sensor reading is equal to 0.107 mb. (The average pressure is 149.7 mb and the average ambient temperature is −68°C.)

Fig. 4 is the output of a convolution between the inverse wavelet listed in Table 1 and the raw pressure data which appear in Fig. 3. Comparing Figs. 4 and 3, one sees clearly the strong filtering-out of the NBO.

Several other band-reject and low-pass filters could be used. For comparison, Fig. 5 is the balloon trajectory when passed through a low-pass filter with a cutoff period of 5 min and a 48 dB per octave slope.

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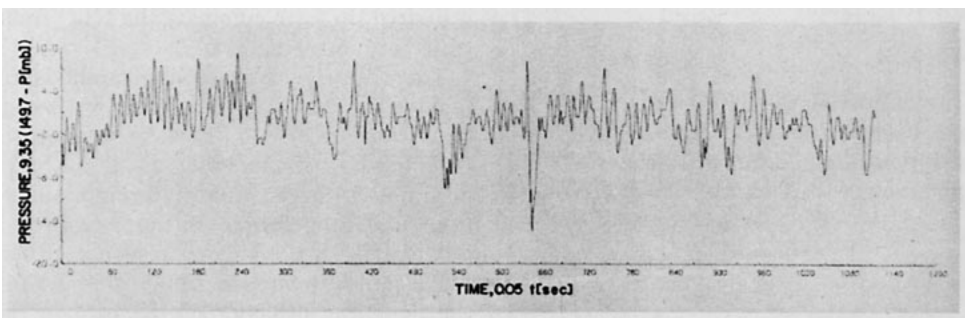

FIG. 3. Raw pressure data of TWERLE balloon flight 1406 (Ascension Island, 18 September 1974).

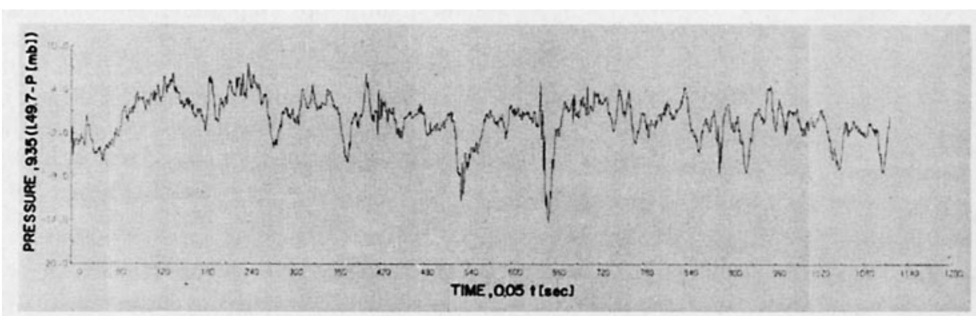

FIG. 4. The pressure data after deconvolution between the raw data (Fig. 3) and the normalized balloon wavelet (Fig. 1).

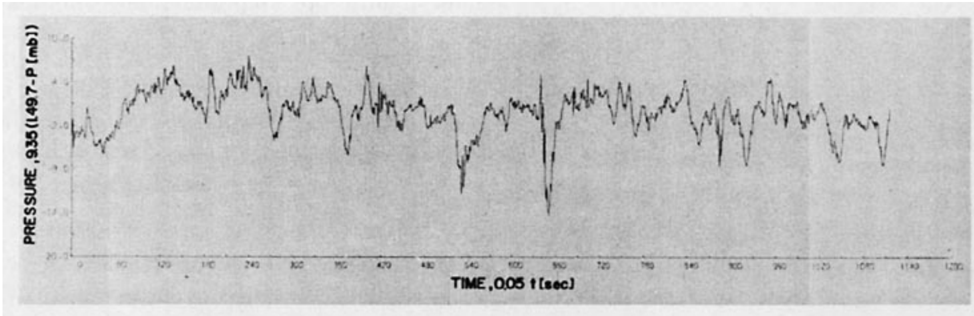

FIG. 5. The pressure data aftr low-pass filtering with a 5 min cutoff period.

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