A Multi-Level Model of the Planetary Boundary Layer Suitable for Use with Mesoscale Dynamic Models

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ABSTRACT

In this paper a simple model of the planetary boundary layer (PBL) is proposed. The surface layer is modeled according to established similarity theory. Above the surface layer a prognostic equation for the mixing length is introduced. The time-dependent mixing length is a function of the PBL characteristics, including the height of the capping inversion, the local friction velocity and the surface heat flux. In a preliminary experiment, the behavior of the PBL is compared with observations from the Great Plains Experiment.

1. Introduction

The major sink of momentum and sources of heat and moisture are at the earth-atmosphere interface. The vertical fluxes of momentum, heat and moisture are therefore usually largest in the planetary boundary layer (PBL), and considerable effort has been directed toward the understanding of this relatively thin layer next to the earth's surface. Many PBL models of varying complexity have been developed with the purpose of studying the details of the physical processes in this layer alone. However, the treatment of the PBL is also an important part of numerical models of mesoscale or large-scale atmospheric phenomena. In such models, the behavior of the PBL itself is often of secondary interest; what is required is an adequate representation of the coupling between the PBL and the atmospheric flow in the rest of the model.

The degree of accuracy and the detail of the PBL to be resolved in a dynamic model depend mainly on the characteristics of the phenomenon to be modeled. For example, the short-range behavior of the quasi-geostrophic waves in the westerlies may be modeled without any representation of the PBL at all. In long-term integrations of the general circulation, or if the behavior of individual cyclones and smaller scale phenomena is being investigated, the PBL becomes more important. In some of these latter models, the detailed structure of the PBL is unimportant and only its gross effect needs to be considered. In others, a more detailed representation of the PBL is necessary.

2. Approaches to the boundary layer modeling problem

In most models, the atmospheric boundary layer is divided into a surface boundary layer next to the ground or sea surface, in which the fluxes of heat, momentum and water vapor are considered approximately constant with height, and a deeper layer in which these fluxes decrease with height. In general, two approaches are possible toward the incorporation of the effects of the boundary layer into a large-scale model, as discussed by Clarke (1970) and Deardorff (1972). One is to resolve the structure of the boundary layer explicitly by including several computational levels within the boundary layer. The second method is to relate the gross effects of the PBL to a number of parameters derived from the dynamic large-scale model. As an example of the second approach, Deardorff (1972) proposes a relatively sophisticated bulk parameterization of the PBL in which the surface stress and the surface fluxes of heat and moisture vary as functions of a bulk Richardson number which is determined from the large-scale model. In this scheme, an important quantity is a time-dependent height h of the PBL, which is predicted explicitly.

If the detailed structure of the PBL is important in the dynamical model, the turbulent fluxes of momentum, heat and moisture must be resolved at several levels within the PBL. The difficulty lies in relating these fluxes to the large-scale mean variables that are predicted by the model. Two types of closure have been attempted. Most investigators have utilized first-order closure schemes which relate the turbulent fluxes to gradients of the mean quantities through eddy coefficients (e.g., Estoque, 1973). These schemes are economical but suffer from the absence of an accepted formulation of the diffusivities that applies under general conditions.

Recently attempts have been made at the development of higher order closure schemes (Donaldson, 1973; Lumley and Khajeh-Nouri, 1974; Wyngaard *et al.*, 1974). The closure problem in these schemes is the modeling of the third-order moments. Although the second-order closure schemes are intellectually appealing, they do not yet appear to be ready for use in large-scale dynamical models. At present, the second-order closure schemes are controversial, and a large number of constants remain to be determined. A practical limitation at the present is the amount of computer time and storage necessary for these schemes. For example, the boundary layer model of Wyngaard *et al.* consists of 14 equations, twice as many as normally required in a dynamical model. Thus, in spite of the long-range benefits which may be derived from higher order schemes, there remains a need in numerical models for simple schemes that produce results consistent with the known behavior of the PBL.

After establishing a need for relatively detailed representation of the boundary layers in hurricane models, we present a simple, time-dependent model of the PBL suitable for use in dynamical models. Preliminary results from this model are encouraging, and suggest that it be tested against real data and higher order closure schemes as they become available.

3. The need for simple, high-resolution PBL models for hurricane studies

One phenomenon which is strongly dependent not only on the gross effects of the PBL but probably also on the details of the boundary layer structure is the hurricane. It is in the hurricane boundary layer that friction destroys the gradient wind balance and produces the inflow branch of the circulation. This inflow produces the tangential circulation through conservation of the angular momentum and provides the convergence of water vapor that supports the cumulus convection. Thus, in contrast to many other atmospheric phenomena in which the boundary layer effects are secondary in importance, the boundary layer is critical to the development and maintenance of the hurricane.

Tropical cyclones have been realistically simulated by numerical models (e.g., Ooyama, 1969; Rosenthal, 1970, Anthes, 1972). The structure of these model storms agree remarkably well with the structure of typical hurricanes, in spite of the crude parameterizations of the PBL, including the vital sea-air interactions. However, the intensities of these model hurricanes show an unrealistically strong dependence on the temperature of the underlying ocean, which supplies crucial heat and water vapor to the storm. This may be due to the greatly simplified treatment of the PBL employed in the models. Both the surface and the boundary layers are contained in a single, approximately 1 km thick layer. The surface stresses are computed from the quadratic stress law, and the surface fluxes of heat and moisture are calculated from bulk aerodynamic formulas. Under the assumptions that the fluxes of momentum, heat and moisture vanish at the top of the layer, the frictional force and the rate of addition of heat and moisture to the model's lowest layer are obtained.

Although this simple approach apparently accounts for the effects of the PBL to a sufficient degree to obtain fairly realistic model hurricanes, a number of defects in the scheme make improved treatment of the PBL desirable in order to more realistically model the interactions between the hurricane and the ocean. One difficulty is the assumption of a boundary layer of constant height, which is determined by the choice of the lowest computational level in the model.

In reality, the depth of the boundary layer varies both in time and space during the lifetime of the hurricane, although the degree of variability is not well known. A second important defect in the one-layer parameterization scheme is the crude dependence of the turbulent surface fluxes of heat, moisture and momentum on static stability, wind shear and state of the sea surface. In reality, these fluxes will be significantly dependent on the details of the PBL such as the stability of the surface boundary layer (Deardorff, 1972). This dependence may be very significant for the modification of hurricanes which move over water of varying temperature, or as the ocean temperatures change in response to the hurricane.

Finally, even if the above formulation correctly predicts the vertically averaged properties of the hurricane boundary layer, the details of the structure are missing entirely. A better resolution of the strong vertical gradients of temperature, moisture and wind is becoming more important as cumulus parameterization schemes are improved.

The importance of vertical resolution may be illustrated by the height variation of mixing ratio r. In the mature hurricane, r varies from about 18.7 gm kg^{-1} at 1000 mb to 14.0 gm kg^{-1} at 850 mb (Sheets, 1969). In addition, the vertical shear of the radial (inflow) component of the wind is a maximum in the PBL. Thus an accurate representation of the horizontal water vapor convergence, which is vital to the cumulus convection, requires improvement in the vertical resolution.

In the next section, a simple model of the PBL is proposed which appears suitable for a more realistic simulation of the detailed structure of the hurricane boundary layer. Although designed with some of the hurricane problems in mind, the scheme is general and should be suitable for use in a wide variety of atmospheric models.

4. A time-dependent model of the planetary boundary layer

For simplicity, only one dimension is considered in this model; the extension to two or three dimensions is straightforward by the inclusion of horizontal advective effects, as long as the small-scale turbulent structure of the PBL can be considered horizontally quasi-homogeneous at each grid point. The equations of motion and the thermodynamic equation then are

$$\frac{\partial u}{\partial t}=f(v-v_g)-\frac{1}{\rho}\frac{\partial}{\partial z}\rho\overline{u'w'}, \tag{1}$$

$$\frac{\partial v}{\partial t}=f(u_g-u)-\frac{1}{\rho}\frac{\partial}{\partial z}\rho\overline{v'w'}, \tag{2}$$

$$\frac{\partial T}{\partial t}=-\frac{1}{\rho}\frac{\partial\rho\overline{w'T'}}{\partial z}, \tag{3}$$

where the symbols have their usual meteorological meaning and fluctuations in density have been neglected. We now consider the eddy flux terms in the surface layer and in the PBL between the top of the surface layer and the base of a capping inversion.

a. The surface layer

The structure of the surface layer is relatively well known. Here the lowest 10 m of the PBL are assumed to be within the surface layer, and the nondimensional gradients of wind and potential temperature are solved from similarity theory:

$$\frac{u}{u_{*0}}=\frac{1}{k}[\ln(z/z_0)-\psi_m(z/L)], \tag{4}$$

$$\frac{\theta-\theta_0}{\theta_{*0}}=0.74[\ln(z/z_0)-\psi_h(z/L)], \tag{5}$$

where L is the Monin-Obukhov length, defined by

$$L=-\frac{c_p\rho T_0 u_{*0}^3}{kgH_0},$$

with H_0 and ρu_{*0}^2 being the surface fluxes of heat and momentum, respectively. The functions ψ_m and ψ_h are integrals of universal functions Φ_m and Φ_h as given by Businger (1973). Then the fluxes of momentum and heat in the surface layer can be derived from

$$-\overline{u'w'}=u_{*0}^2\left|\frac{\partial u}{\partial z}\right|\Big/\left|\frac{\partial \mathbf{v}}{\partial z}\right|, \tag{7}$$

$$-\overline{v'w'}=u_{*0}^2\left|\frac{\partial v}{\partial z}\right|\Big/\left|\frac{\partial \mathbf{v}}{\partial z}\right|, \tag{8}$$

$$\overline{w'T'}=-u_*\theta_{*0}=H_0/(c_p\rho). \tag{9}$$

The calculation marches forward in the following way: First, an initial guess of L is estimated from the values of u_{*0} and θ_{*0} at time $t_0-\Delta t$. This value of L is used in (4) and (5) to estimate u_{*0} and θ_{*0} at time t_0, which are then used to calculate an updated value of L. This iterative procedure continues until the solution converges.

b. The PBL above the surface layer

Some preliminary efforts were directed toward quasi-stationary modeling of the PBL above the surface layer by use of mixing lengths which approach constant values with increasing heights above the surface (e.g., Panofsky, 1973, p. 153). The efforts included attempts at modification of the mixing lengths for adiabatic cases, so that in the surface boundary layer the behavior of the wind and temperature profiles would be in accordance with Businger *et al.* (1971).

A conceptual difficulty in this approach is that the turbulent fluxes at great heights above the surface layer depend directly and instantaneously on the surface layer characteristics, mainly the surface heat flux. The approach failed to treat realistically the effect of a capping inversion and the behavior of the atmosphere across and above such an inversion. Furthermore, the model predicted unlimited growth of the PBL. These results suggest that the mixing length above the surface layer should explicitly depend on the local conditions and on the height of the capping inversion.

We first define a mixing length λ, which is the scale of the energy-containing eddies, by

$$u_*^2\equiv\lambda^2\left|\frac{\partial \mathbf{v}}{\partial z}\right|^2\equiv K_m\left|\frac{\partial \mathbf{v}}{\partial z}\right|, \tag{10}$$

where u_* is the *local* friction velocity. We then propose a prognostic equation for λ based on simple intuitive ideas. In this model, the rate of change of the mixing length is assumed to be proportional to $(\lambda_s-\lambda)$, where λ is the actual value of the mixing length and λ_s the value λ would attain if the boundary conditions (including the geostrophic wind) were kept constant at any given time. Furthermore, the time scale for the change of the size of the energy-containing eddies is assumed to be inversely proportional to the strength of the mixing characterized by the local friction velocity u_* and the convective velocity w_*, where

$$w_*=\left(\frac{g}{T_0}\overline{w'T'}h\right)^{\frac{1}{3}} \tag{11}$$

and h is the height of the PBL.

The prognostic equation for λ is then

$$\frac{\partial\lambda}{\partial t}=\frac{\lambda_s-\lambda}{a\lambda/(u_*^2+w_*^2)^{\frac{1}{2}}}. \tag{12}$$

In the work presented here, the constant a is arbitrarily set equal to unity.

In general, λ_s will be a slowly varying function of time and will depend on the past history of the flow field. Here we shall assume that

$$\lambda_s=\begin{cases}\dfrac{kz}{\phi_m}\left(1-\dfrac{z}{h}\right), & \text{for} \quad z\leqslant h' \\ \lambda_B, & \text{for} \quad z>h'\end{cases} \tag{13}$$

where h' ($\sim h$) is the height at which λ_s equals λ_B, a residual mixing length which accounts for the slight mixing above the PBL. When u_* or w_* is relatively large, 0.25 m s^{-1} say, the time scale $a\lambda/(u_*^2+w_*^2)^{\frac{1}{2}}$ for eddies of a typical size of 500 m is 2000 s (for $a=1$). Since this time scale is short compared to the characteristic time scale of most dynamical models, it might be appropriate to simply utilize λ_s rather than λ under these conditions. During the transition from an unstable to stable PBL, however, when u_* and w_* may decrease by an order of magnitude, the time scale becomes on the order of several hours. In order to model realistically this transition period, a time-dependent λ may be necessary. Because of the minor increase in computational complexity associated with the prognostic equation for λ, we tentatively retain it pending further sensitivity tests.

The definition (10) of the mixing length is of course somewhat arbitrary and may (as pointed out by J. W. Deardorff in a personal communication) lead to overestimation of the wind shear, especially in weak wind and strong heating conditions. Under these conditions, it would be more satisfactory to define the mixing length in terms of the turbulent energy e, i.e.,

$$u_*^2\equiv b\lambda e^{\frac{1}{2}}\left|\frac{\partial \mathbf{v}}{\partial z}\right|\equiv K_m\left|\frac{\partial \mathbf{v}}{\partial z}\right|, \tag{14}$$

where b is a constant and

$$e=\tfrac{1}{2}\overline{u_i'u_i'}.$$

The expression for K_m would then be obtained from the turbulent energy equation, which for horizontal homogeneity may be written in the form

$$\frac{\partial e}{\partial t}=b\lambda e^{\frac{1}{2}}\left[\left|\frac{\partial \mathbf{v}}{\partial z}\right|^2-\frac{g}{T_0}\frac{K_h}{K_m}\frac{\partial\theta}{\partial z}-\frac{Ae}{b\lambda^2}\right], \tag{15}$$

where we have assumed that the dissipation rate ϵ may be approximated by

$$\epsilon=A\frac{e^{\frac{3}{2}}}{\lambda} \tag{16}$$

and that the divergence of the vertical flux of turbulent energy is negligible compared to the other terms. Neglecting the time-dependent term $\partial e/\partial t$ and using (14), we obtain

$$K_m=A^{-\frac{1}{2}}b^{\frac{3}{2}}\lambda^2\left[\left|\frac{\partial \mathbf{v}}{\partial z}\right|^2-\frac{g}{T_0}\frac{K_h}{K_m}\frac{\partial\theta}{\partial z}\right]^{\frac{1}{2}}. \tag{17}$$

Under neutral conditions, K_m should be given by (10), which implies that $b=A^{\frac{1}{4}}$. For moderate wind conditions and weak heating, the shear term in (17) dominates the convective term and the expression for K_m becomes similar to that given by (10). In the experiment reported here, which involves moderate wind speeds (geostrophic wind 10 m s^{-1}), K_m is calculated from (10). A comparison experiment utilizing (17) instead of (10) to compute K_m showed similar momentum and potential temperature profiles.

Under the assumption that the stress is in the direction of the shear, the vertical flux terms are related to the mixing length by

$$-\overline{u'w'}=K_m\frac{\partial u}{\partial z}, \tag{18}$$

$$-\overline{v'w'}=K_m\frac{\partial v}{\partial z}, \tag{19}$$

$$-\overline{w'T'}=K_h\frac{\partial\theta_c}{\partial z}, \tag{20}$$

where $\partial\theta_c/\partial z=(\partial\theta/\partial z)-\gamma_c$. The inclusion of γ_c in (20) allows an upward heat flux in weakly stable conditions, and represents the countergradient heat flux (Deardorff, 1966). Deardorff (1966) suggests from theoretical considerations that $\gamma_c\approx 0.7\times10^{-3}$ K m^{-1}. The expression for K_h in (20) is derived from the Kansas experiment (Businger *et al.*, 1971), i.e.,

$$\frac{K_h}{K_m}=\left(\frac{\phi_m}{\phi_h}\right)=\begin{cases}\dfrac{1.0+4.7\ z/L}{0.74+4.7\ z/L}, & \text{for} \quad z/L>0 \\ \dfrac{1.35\ (1-9\ z/L)^{\frac{1}{2}}}{(1-15\ z/L)^{\frac{1}{4}}}, & \text{for} \quad z/L<0.\end{cases} \tag{21}$$

The height h of the PBL might be chosen in a number of ways, for example, a fraction of u_*/f under stable conditions or as the height of the capping inversion under unstable conditions. In the present study we use the first height above the ground at which the local Richardson number

$$\mathrm{Ri}=\frac{g}{\theta}\frac{\partial\theta}{\partial z}\Big/\left|\frac{\partial \mathbf{v}}{\partial z}\right|^2 \tag{22}$$

exceeds a critical value of $\frac{1}{4}$.

c. Initial and boundary conditions

The vertical temperature profile observed at 0835 CST 25 August 1953 at O'Neill, Nebr., during the Great Plains Experiment (Stull, 1973) is used to specify the initial conditions for potential temperature. A nearly adiabatic layer is capped with a strong inversion above 400 m (Fig. 6a). The orientation of the model is chosen such that its x axis points toward the east. The geostrophic wind components are assumed to be $u_g = 10$ m s^{-1} and $v_g = 0$. The initial vertical wind profiles of u and v are obtained by integrating the model for 4 h keeping the height of the inversion fixed and without any surface heat flux until a quasi-steady state wind profile is reached. The grid consists of 20 points in the vertical. The first level is located at 0 m, the second at 10 m. Above 10 m, the levels are located every 100 m (at 110, 210, 310 m, etc.). Since the formulation of the heat and momentum fluxes in the PBL and in the surface layer approach each other asymptotically, the choice of the height of the surface layer (10 m in the present model) affects the solution only slightly. The fluxes, mixing length, wind shear and temperature gradients are defined at levels halfway between the levels at which the velocity components and the temperature are computed. At the topmost level, the wind is geostrophic and the temperature is constant. The surface roughness parameter z_0 is 1 cm and the mixing length λ equals λ_s initially, with λ_s given by (13) and $\lambda_B = 3$ m.

The heat flux at the ground [K m s^{-1}] is specified by the function

$$\overline{w'T'} = \mathrm{Max}\begin{cases} 0.25 \sin[2\pi(t-t_0)/24h] \\ -0.06 \end{cases} \tag{23}$$

The sinusoidal representation of the surface heat flux cycle is limited by -0.06 K m s^{-1} after about 12.9 h to simulate crudely the nearly constant heat flux during the night. Sunrise ($t = t_0$) is set at 0635 CST. The maximum heat flux associated with this function is about 18% of the solar constant. Amplitudes of 0.20 and 0.30 K m s^{-1} were also tested, but the value of 0.25 K m s^{-1} gave the closest agreement between the predicted and the observed heights of the inversion. The value for the negative heat flux is also arbitrary, and was chosen to test the model under stable conditions.

The model was integrated for 24 h beginning at 0835 CST using a forward-in-time, centered-in-space finite difference scheme. The time step Δt was varied from 10 to 60 s, so that the computational stability criterion ($K\Delta t/\Delta z^2 < 0.25$) was maintained at every level during the entire integration. Here K is the maximum of K_m or K_h. The 24 h integration requires about 45 s on an IBM 370/168.

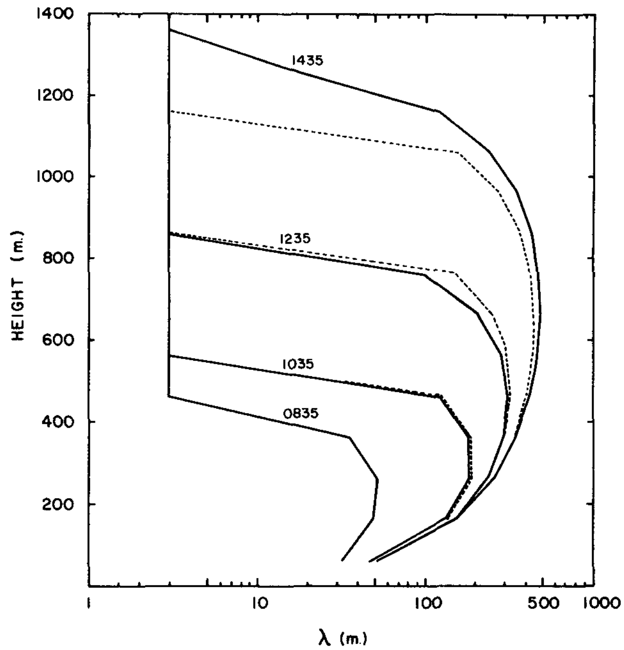

FIG. 1. The time variation of mixing length λ under unstable conditions. The dashed lines denote values of λ_s.

5. Results

a. Time-height variation of mixing lengths

The time-height variation of the mixing length λ is shown in Fig. 1. After 2 h of integration (1035 CST) the maximum value of λ has grown from 50 to 200 m beneath the inversion. The dotted lines show the "stationary" values. As more heat is added, the inversion rises and λ continues to increase below the inversion. In the vertical, λ is proportional to kz near the surface, reaches a maximum at a level which is slightly higher than half the inversion height, and decreases to nearly zero at the inversion height. This vertical variation is similar to Blackadar's (1962) and Shir's (1973) results for the neutral PBL. However, because λ in this model depends on the upward heat flux through (12), the magnitudes here are much larger than Blackadar's and Shir's values, reaching about one-third of the height of the inversion.

The time scale of λ in (12) is inversely proportional to the local stress. Near the ground, where the stress is large, λ approaches its stationary value rapidly. In the upper part of the PBL, where the local stress is less, λ responds more slowly. Also, the stationary value of the mixing length λ_s is changing at this level in response to the rising inversion height [Eq. (13)].

After 1835 CST the surface heat flux becomes negative and the turbulence in the PBL begins to decay, as shown by the decrease of λ in Fig. 2. Because of the reduced values of stress, the mixing length changes at a slower rate during the night than during the day. Near the surface, λ adjusts to the stationary value in

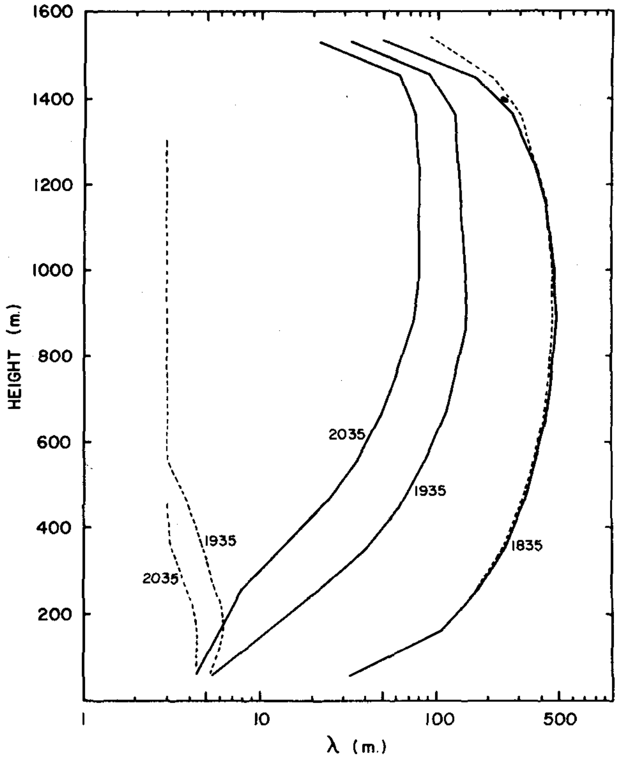

FIG. 2. The time variation of mixing length λ under stable conditions. The dashed lines denote values of λ_s.

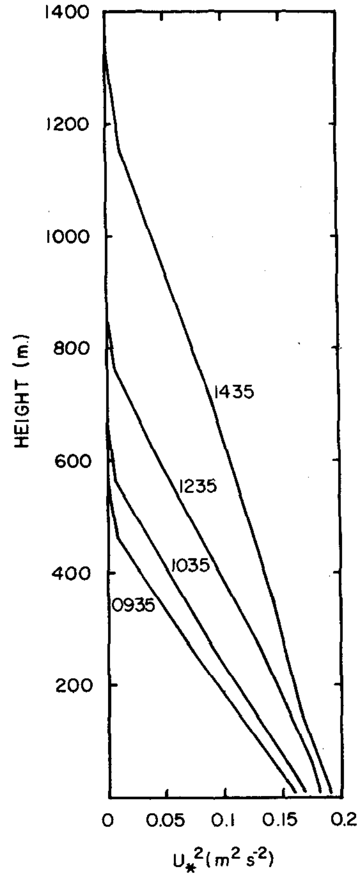

FIG. 4. The time variation of the vertical profiles of u_*^2 under unstable conditions.

1–2 h, while in the upper portion of the PBL the adjustment requires 3–4 h. As shown in Fig. 1, the time-dependent mixing length is very close to the stationary value under unstable conditions, which indicates that the use of the prognostic equation for λ during these conditions is unnecessary. However, under stable conditions (Fig. 2), the actual mixing length differs by an order of magnitude from the stationary value, indicating that a prognostic equation for the mixing length [Eq. (12)] is important in this parameterization of the stable PBL.

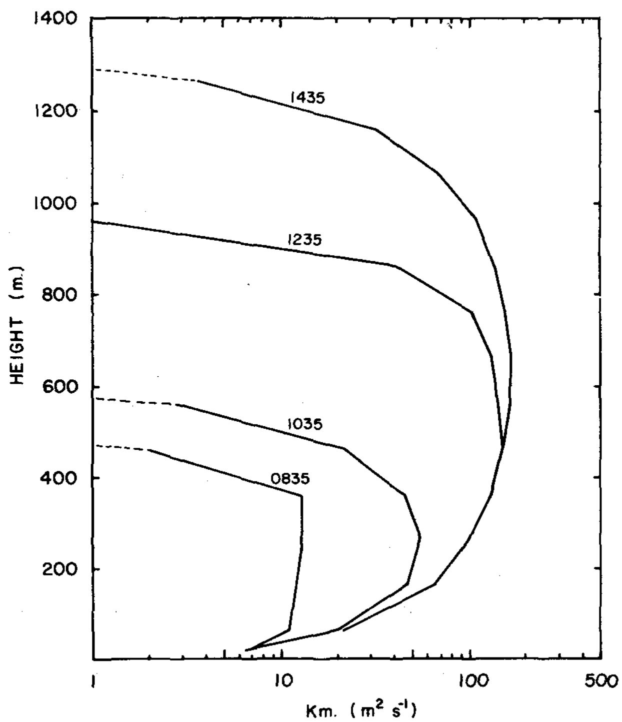

FIG. 3. The time variation of the kinematic coefficient of eddy viscosity for momentum.

The vertical diffusivity for momentum, K_m, is plotted in Fig. 3. The vertical profiles of K_m resemble those proposed by O'Brien (1970) and Agee *et al.* (1973). Here K_m not only varies with the height of the inversion but also changes with the wind shear and stability in the PBL. The profiles show a nearly linear increase of K_m near the ground, a maximum value near one-third of the height of PBL, and a rapid decrease near the inversion. The maximum value of K_m varies by an order of magnitude as the PBL changes from neutral to unstable.

b. *Vertical profiles of stresses and heat flux*

The profile of total stress is shown in Fig. 4. Both Deardorff's (1974) three-dimensional simulations and Wyngaard and Coté's (1974) results from a second-order closure scheme indicate that the stresses in the boundary layer vary linearly with height. The stress shown in Fig. 4 is nearly linear, but concave slightly upward in agreement with Shir's (1973) results for a neutral boundary layer.

The heat flux profiles shown in Fig. 5 are very close to the idealized profile given by Stull (1973, p. 1093). Below the inversion the heat flux is a linear function of height. A downward heat flux occurs at the inversion. This heat flux profile produces a rising inversion with time (Fig. 6b). The magnitude of the downward heat flux is about 10% of the surface heat flux, which is the correct order of magnitude (Lilly, 1968; Wyngaard and Coté, 1974). The behavior of the downward heat flux in this model represents an improvement over simpler K theory models which produce a larger downward heat flux above the inversion and an unrealistically large cooling above the inversion.

c. The rise of the inversion

The observed time variation of the inversion during the Great Plains Experiment (Stull, 1973) is shown in Fig. 6a and the variation predicted by the model is shown in Fig. 6b. The model reproduces the rising inversion, a slightly stable lapse rate below the inversion and a slightly superadiabatic lapse rate near the ground. Minor differences in the predicted and observed profiles may be attributed to the arbitrarily specified surface heat flux, the arbitrary initial conditions for λ, and the neglect of other physical processes such as radiation, horizontal advection and synoptic-scale vertical motions.

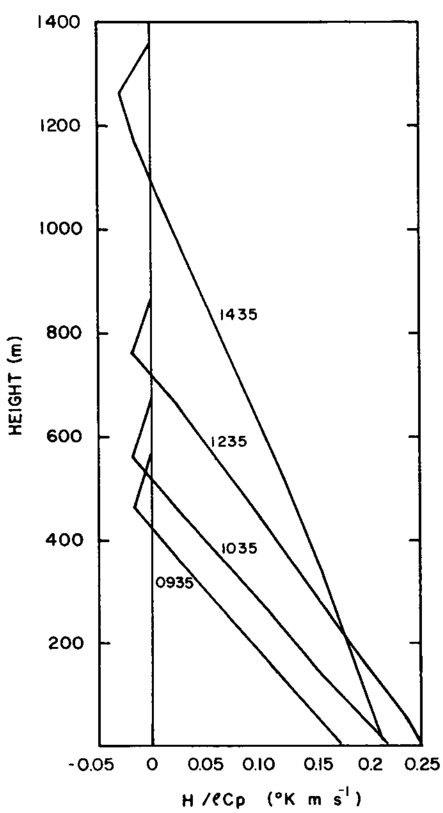

FIG. 5. The time variation of the vertical profiles of heat flux under unstable conditions.

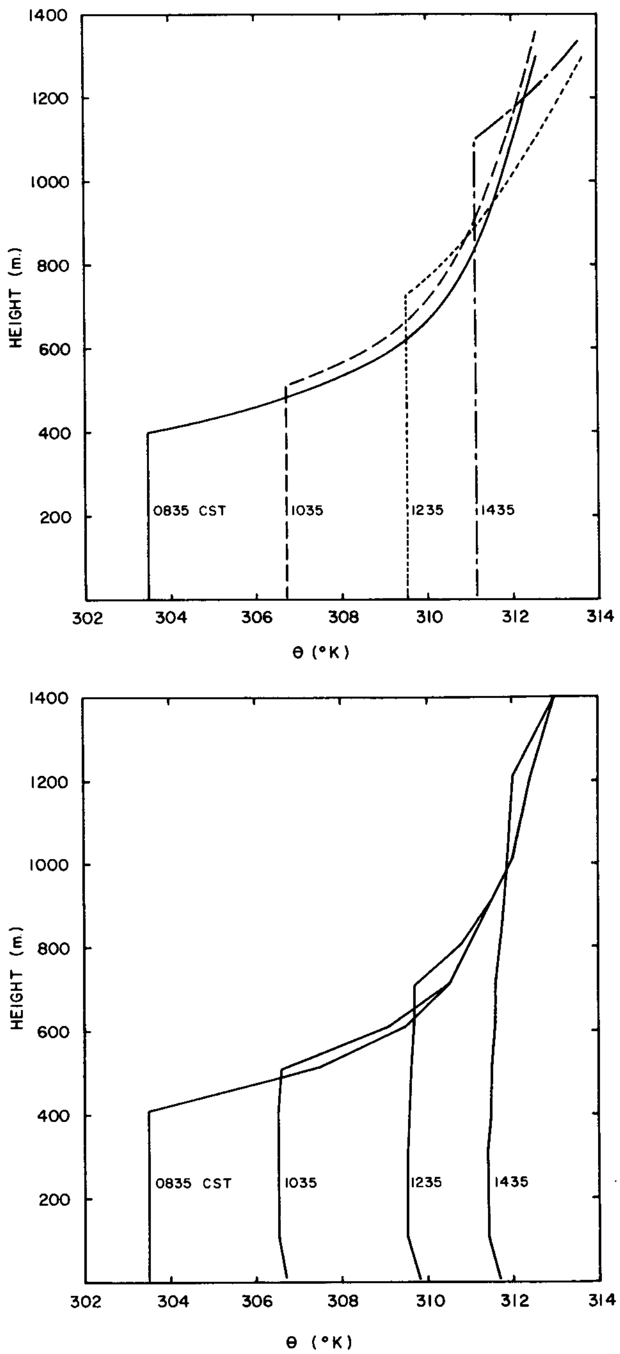

FIG. 6. Observed vertical profiles of potential temperature (a) and predicted values (b) under unstable conditions. Observed values are from the Great Plains Experiment (Stull, 1973).

d. Behavior of the stable PBL

After 1835 CST the specified heating function becomes negative. The magnitude of the downward heat

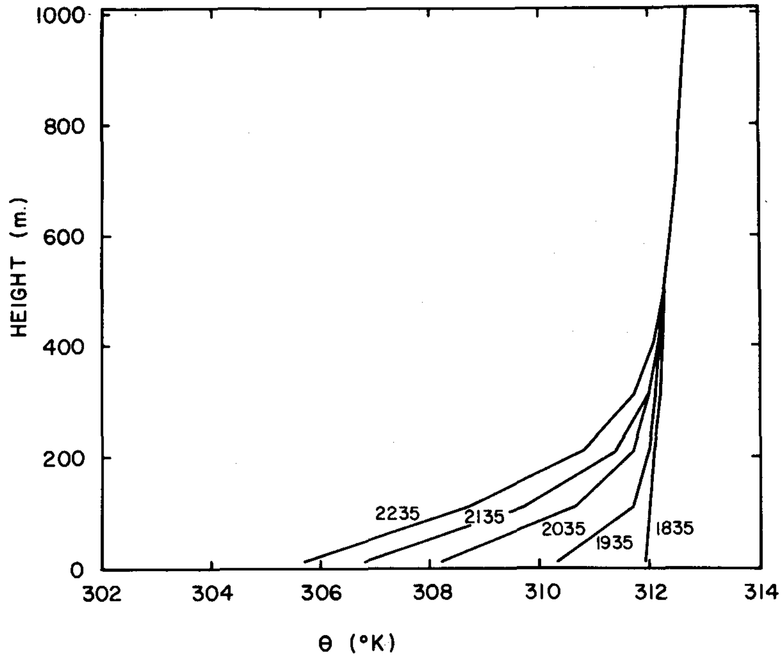

FIG. 7. Predicted vertical profile of potential temperature field under stable conditions.

flux and the associated low-level cooling may not be realistic because radiation is neglected. It is mainly the longwave radiation, not the sensible heat flux divergence, that is responsible for the nocturnal cooling. However, the arbitrary specification of cooling should be sufficient to illustrate the time-dependent behavior of the PBL under stable conditions.

Fig. 7 shows that potential temperature profile at 1835, 2035 and 2235 CST. An inversion forms near the ground as the height of the upper inversion decreases slowly. The heat flux is downward below and weakly positive above the lower inversion.

During the day when the vertical mixing is strong, the directional wind shear is small, averaging only 2°–3° over 400 m (see Fig. 8). As the PBL becomes stable, the winds adjust to the pressure gradient force and the decreased frictional force. In Fig. 8, the wind above 410 m veers toward the geostrophic wind direction when the downward momentum transfer is reduced. The wind below 410 m, deprived of downward momentum transfer from higher levels, backs toward a larger cross-isobar flow angle. Fig. 8 also shows a rapid change of wind direction at lower levels and a more delayed response at higher levels.

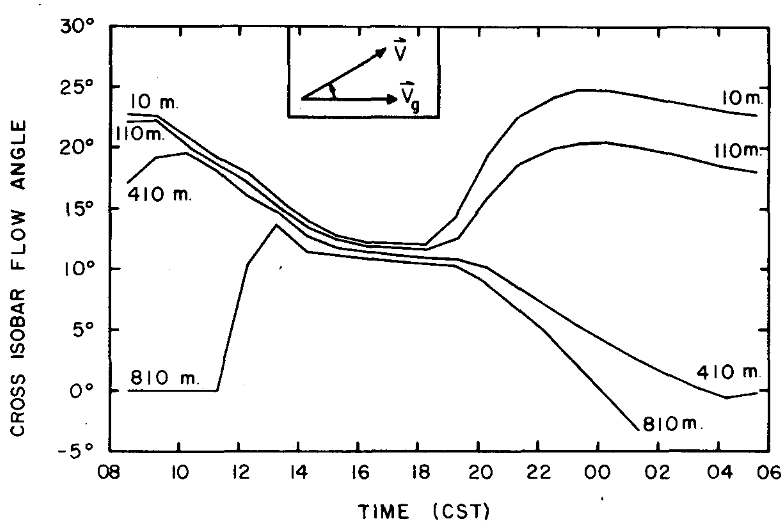

FIG. 8. The time variation of the cross-isobaric flow angle at 10, 110, 410 and 810 m.

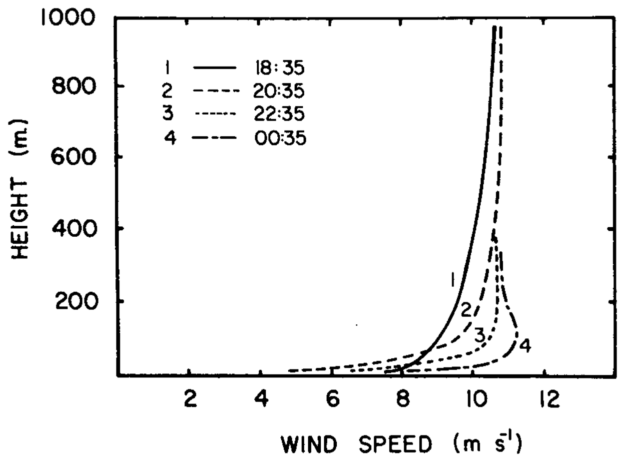

FIG. 9. Vertical profiles of the wind speed showing the formation of a nocturnal low-level jet.

Fig. 9 shows the formation of a nocturnal low-level jet in the stable PBL (Blackadar, 1957). The jet is mainly confined below 400 m where the wind direction first backs due to reduced mixing then veers (after 2300) due to Coriolis forcing. However, the wind in the surface layer *decreases* considerably during the formation of the nocturnal jet. Fig. 10 illustrates the wind speeds at 10, 110 and 410 m. The wind at anemometer level decreases by 4 m s^{-1} in the stable PBL, while the wind at 110 m increases by 2–3 m s^{-1}. Recently Goff and Duchon (1974) presented results from over half a year's observation from a 444 m tower. Although their results contain synoptic-scale variations, the diurnal variation in wind predicted by this model is similar to the observed diurnal variations of mean wind speed.

e. Vertical diffusion of a passive substance

In this section the model is utilized to predict the vertical diffusion of a passive substance q, which is governed by the equation

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho}\frac{\partial \rho \overline{w'q'}}{\partial z}, \quad (24)$$

where

$$\overline{w'q'} = -(\phi_m/\phi_h)\lambda^2\left|\frac{\partial \mathbf{v}}{\partial z}\right|\frac{\partial q}{\partial z}. \quad (25)$$

Fig. 11 shows the concentration of a passive substance q when the flux of q is constant at the ground. This experiment simulates the diffusion of a pollutant from a site with a constant rate of emission. The concentration during the "day" stays nearly constant below the inversion as the substance diffuses upward following the rising of the inversion. The upward flux is limited by the height of the inversion and a sharp vertical gradient of q is maintained at the inversion level at all times. After "sunset," when the surface heat flux becomes negative, the low-level concentration increases rapidly under the newly formed inversion, while the concentration in the upper parts of the PBL remains unchanged. It is often observed in the early morning that the polluted layer over the urban area is much thicker than the height of the lowest inversion. This pollution had been mixed upward during the previous day.

Of course, in order to predict urban pollution, horizontal inhomogeneities are important and advection must be included in the model. However, this simple vertical diffusion model appears to calculate the vertical turbulent fluxes in a realistic way, and hence to have the potential for incorporation into mesoscale dynamic models.

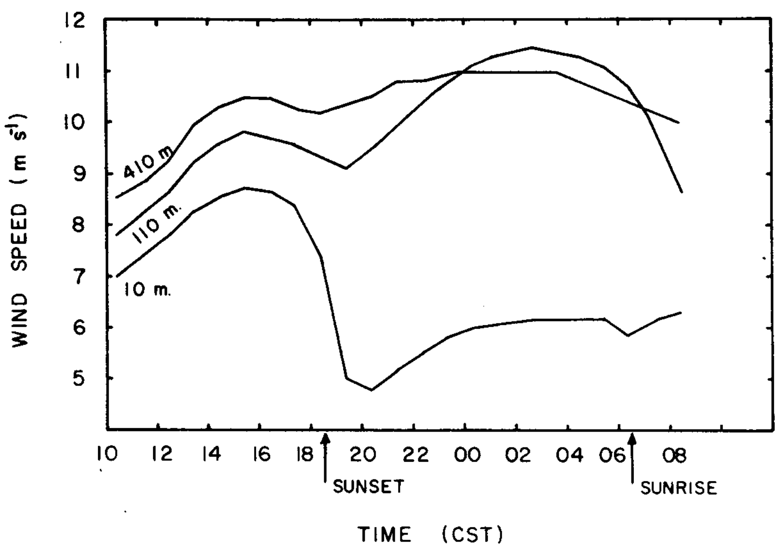

FIG. 10. The time variation of wind speed at heights of 10, 110 and 410 m.

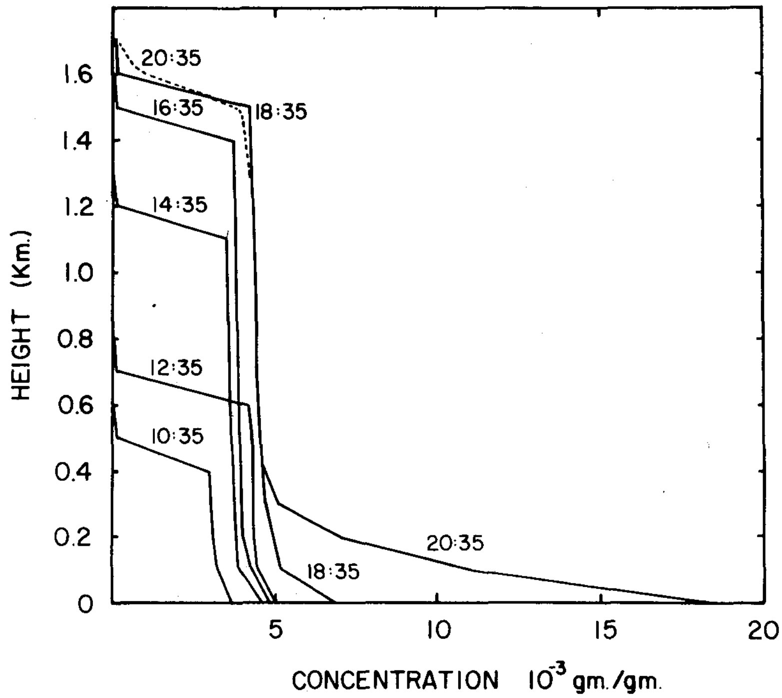

FIG. 11. The time variation of concentration of q in the PBL when the upward flux of q is kept constant at the surface.

6. Summary

Although second-order closure schemes have considerable theoretical appeal, they are time-consuming. Therefore, when it is necessary to describe the PBL in some detail for the purposes of resolving important interactions of the PBL with a coupled dynamic model, simpler first-order closure schemes (K theories) still have an important role to play. In this paper a simple model of the PBL is proposed. The surface layer is modeled according to established similarity theory. Above the surface layer, a time-dependent mixing length is utilized which is a function of the PBL characteristics, including the height of the capping inversion as well as the surface heat flux. In a preliminary experiment, the behavior of the PBL agrees well with the observations of the Great Plains Experiment.

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