A Method for Determining Probabilities of Surface Temperature Extremes

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(Manuscript received 2 May 1977, in revised form 16 August 1977)

ABSTRACT

An objective method for determining probabilities of surface temperature extremes is described herein. Least-squares linear regression equations have been developed to estimate temperatures that would be equaled or surpassed 1, 5 and 10% of the hours at any given location during the warmest and coldest months of the year. These equations are based on an index calculated from three generally available parameters: 1) mean monthly temperature (usually July and January), 2) mean daily maximum temperature for the month and 3) mean daily minimum temperature for the month.

1. Introduction

Knowledge of surface temperature extremes is important for the design and use of equipment which must operate in the natural environment. Although climatic temperature records are readily available for most of the world's population centers, there are many areas for which little or no temperature information (or other climatic data) exists. What is available is usually in the form of averages and/or absolute extremes. The period of record, however, has a profound influence on absolute extremes (the maximum and minimum recorded temperatures), so that, even if available, the data may be of limited value for use in establishing design criteria. Furthermore, it is often economically impractical to design for or expect to operate during periods of absolute extremes. It is more realistic to design equipment to function with an acceptable small risk of inoperability, that is, during extremes which occur or are exceeded during only a small percentage of time.

The ideal method for determining frequency distributions of temperature would be to obtain actual distributions of hourly observations taken over a long period. Unfortunately, these data are either unavailable or incomplete for most regions outside of North America. Consequently, to establish environmental criteria for the design of military materiel, an objective method has been developed, relating the frequency of warm and cold temperature extremes to an index calculated from more readily available climatic data. This concept is used herein to calculate extremes occurring 1, 5 and 10% of the time for any given location during the warmest and coldest months of the year.

2. Derivation

a. Warm temperatures

In a study on the frequency of warm temperature extremes (Tattelman *et al.*, 1969), it was determined that the warmest temperatures in the world are found at locations where the monthly mean temperatures are high and the mean daily range is large. A simple index using these values is expressed by

$$I_w=\bar{T}+(\bar{T}_x-\bar{T}_n), \tag{1}$$

where I_w is the warm temperature index, $\bar{T}$ the monthly mean, $\bar{T}_x$ the mean daily maximum and $\bar{T}_n$ the mean daily minimum temperature for the month.

Since good climatic records are usually available for these three parameters (Meteorological Office, 1966; ETAC, 1971), Eq. (1) was tested for application on a more general basis (for locations where at least 10% of the hourly warmest month temperatures were $\geqslant 20°C$) than at only the very hot locations for which it was originally designed. The index was correlated with 1, 5 and 10% warm temperatures determined from observations taken during the warmest month (June, July or August) at 30 National Weather Service stations throughout the contiguous United States, plus 10 stations in Europe. The stations and pertinent data are listed in Table 1.

The following regression lines for the 1, 5 and 10% warm temperatures were found by the method of linear least squares:

$$\hat{T}_{1\%}=0.676\,I_w+10.657 \tag{2}$$

$$\hat{T}_{5\%}=0.733\,I_w+\ 5.682 \tag{3}$$

$$\hat{T}_{10\%}=0.762\,I_w+\ 2.902. \tag{4}$$

Linear correlations are 0.97 for $T_{1\%}$, 0.98 for $T_{5\%}$ and 0.98 for $T_{10\%}$. The standard errors of estimate (SEE) are 1.34, 1.22 and 1.24°C, respectively. Scatter diagrams and least squares regression lines are shown in Fig. 1. The addition of the 10 European stations to the 30 United States stations originally used in the 1969 study (Tattelman *et al.*, 1969) improved the correlations and changed the SEE only slightly as shown in Table 2.

TABLE 1. List of stations showing 1, 5 and 10% warm temperatures during the warmest month, mean ($\bar{T}$) and mean daily range ($\bar{T}_x - \bar{T}_n$).

Station	Number of years	Altitude (m MSL)	Latitude (°N)	Longitude	$\bar{T}$ (°C)	$\bar{T}_x - \bar{T}_n$ (°C)	Observed $T_{1\%}$ (°C)	Observed $T_{5\%}$ (°C)	Observed $T_{10\%}$ (°C)
United States									
Amarillo	9	1098	35	102°W	27.2	15.0	37.2	35.0	33.3
Atlanta	10	297	34	84°W	26.1	10.6	36.1	33.3	31.7
Bakersfield	10	145	35	119°W	28.9	15.5	40.6	38.9	37.2
Birmingham	9	192	34	87°W	26.7	11.7	37.2	34.4	32.8
Boston	10	9	42	71°W	23.3	9.4	34.4	31.1	29.4
Brownsville	9	6	26	97°W	28.9	9.4	35.0	33.9	33.3
Caribou	8	191	47	68°W	18.3	11.7	31.1	27.2	25.6
Death Valley	34	−59	36	117°W	38.9	16.1	50.6	48.9	47.2
Des Moines	10	294	42	94°W	25.0	12.2	35.6	32.8	31.1
El Paso	9	1194	32	106°W	27.8	14.4	38.3	36.1	34.4
Fargo	9	274	47	97°W	21.7	13.9	34.4	30.6	28.9
Fresno	9	100	37	120°W	27.2	20.6	40.0	37.2	36.1
Goodland	9	1124	39	102°W	25.0	17.2	38.3	35.6	33.3
Houston	9	19	30	95°W	28.3	10.0	36.7	34.4	33.3
Jacksonville	9	9	30	82°W	28.3	10.6	36.7	34.4	32.8
Laredo	10	150	28	99°W	31.1	12.8	40.6	38.9	37.2
Little Rock	9	81	35	92°W	27.8	12.2	37.8	35.0	33.9
Miami	9	4	26	80°W	27.8	7.8	34.4	31.7	31.1
Minneapolis	11	255	45	93°W	22.8	13.3	35.0	31.7	30.0
New Orleans	9	9	30	90°W	27.8	10.0	34.4	32.8	31.7
Oklahoma City	9	397	35	98°W	27.2	12.2	38.3	35.6	33.9
Phoenix	9	337	33	112°W	32.8	15.0	43.9	41.7	40.0
Rapid City	8	966	44	103°W	23.3	15.6	36.7	33.3	31.7
Sacramento	10	8	39	122°W	25.0	20.0	39.4	36.1	33.9
Salt Lake City	9	1288	41	112°W	24.4	17.2	37.2	34.4	32.8
Sault Ste Marie	9	221	46	84°W	18.3	12.2	30.6	27.2	25.0
Tucson	10	788	32	111°W	30.0	14.4	41.7	38.9	37.2
Washington	10	20	39	77°W	25.6	10.0	36.1	33.3	31.7
Wichita	9	408	38	97°W	27.2	12.8	40.6	36.7	35.0
Yuma	8	63	33	115°W	34.4	15.6	44.4	42.2	41.1
Germany									
Bitburg AB	21	377	50	7°E	16.7	9.2	30.0	25.6	22.8
Erding AS	11	461	48	12°E	18.4	11.1	31.7	28.3	26.1
Zweibrucken AB	16	345	49	7°E	17.6	9.6	30.0	26.7	24.4
Berlin	25	50	52	13°E	19.0	8.7	31.1	27.2	25.0
France									
Chateauroux AS	13	157	47	2°E	18.9	11.1	32.8	28.3	25.6
England									
Bentwater RAF	21	29	52	1°E	16.2	8.1	25.0	22.2	20.6
Burtonwood AB	10	27	53	3°W	15.9	7.9	25.6	22.2	20.0
London	10	25	51	0	17.7	9.5	27.8	25.0	22.8
Mildenhall RAF	22	10	52	0	16.6	9.1	26.7	23.9	21.7
Italy									
Aviano AB	17	131	46	13°E	22.3	10.2	32.8	30.0	28.3

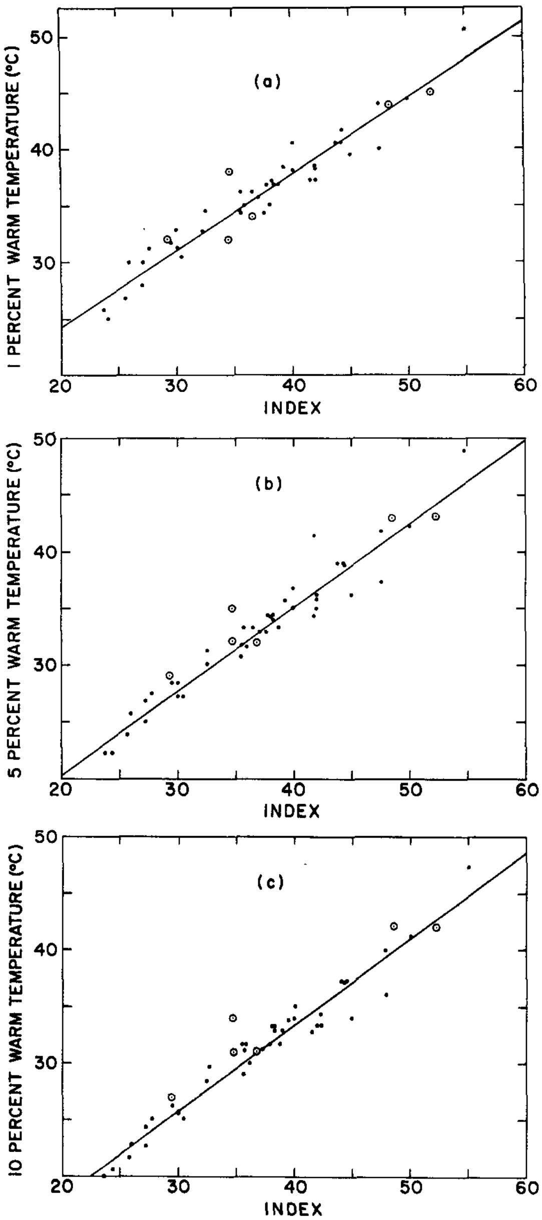

FIG. 1. Percent warm temperature vs index: (a) 1%, (b) 5%, (c) 10%. Circled dots are independent values from Table 3.

As an independent check of (2), (3) and (4), the observed 1, 5 and 10% temperatures at six locations which were not used in this analysis were compared with the estimated percentile temperatures. These comparisons are shown in Table 3. The independent data appear as circled dots in Fig. 1. There is good agreement between observed and estimated values.

TABLE 2. Comparison of correlations and SEE for regression equation at specified warm temperature extremes.

Warm temperature extremes (%)	Correlations		SEE (°C)	
	North America	North America & Europe	North America	North America & Europe
1	0.94	0.97	1.30	1.34
5	0.95	0.98	1.30	1.22
10	0.95	0.98	1.37	1.24

TABLE 3. Comparison of observed warm temperatures and temperatures from Eqs. (2), (3) and (4).

Station	Percentile	Warm temperature (°C) Observed	Estimated
Fort Flatters, Algeria (28°N, 7°E)	1	45	46
	5	43	44
	10	42	43
Dahran, Arabia (26°N, 50°E)	1	44	44
	5	43	41
	10	42	40
Vienna, Austria (48°N, 16°E)	1	32	30
	5	29	27
	10	27	25
Belize, Br. Honduras (18°N, 88°W)	1	32	34
	5	32	31
	10	31	29
Nanking, China (32°N, 119°E)	1	38	34
	5	35	31
	10	34	29
Osaka, Japan (35°N, 136°E)	1	34	35
	5	32	33
	10	31	31

b. Cold temperatures

Since (1) proved successful for describing warm temperature extremes, the same principle was used to estimate cold temperature extremes. A cold temperature index I_c is expressed by

$$I_c = \bar{T} - (\bar{T}_x - \bar{T}_n), \tag{5}$$

where $\bar{T}$ is the monthly mean, $\bar{T}_x$ the mean daily maximum and $\bar{T}_n$ the mean daily minimum temperature for the coldest month. The index was correlated with the 1, 5 and 10% cold temperatures during the coldest month at 30 National Weather Service stations in North America, and 13 stations in Europe and the North Atlantic where at least 10% of the hourly coldest month temperatures were $\leqslant 0$°C. The stations and pertinent data are listed in Table 4.

The following regression lines for the 1, 5 and 10% cold temperatures were found by the method of linear least squares:

$$\hat{T}_{1\%} = 1.069\, I_c - 7.013 \tag{6}$$

$$\hat{T}_{5\%} = 1.084\, I_c - 3.050 \tag{7}$$

$$\hat{T}_{10\%} = 1.082\, I_c - 0.704. \tag{8}$$

TABLE 4. List of stations showing 1, 5 and 10% cold temperatures during the coldest month, mean ($\bar{T}$) and mean daily range ($\bar{T}_x-\bar{T}_n$).

Station	Number of years	Altitude (m MSL)	Latitude (°N)	Longitude	$\bar{T}$ (°C)	$\bar{T}_x-\bar{T}_n$ (°C)	Observed $T_{1\%}$ (°C)	Observed $T_{5\%}$ (°C)	Observed $T_{10\%}$ (°C)
North America									
Alert	10	66	82	62°W	−32.4	5.7	−45.0	−40.6	−38.9
Baker Lake	10	9	64	96°W	−33.5	6.2	−45.6	−43.9	−42.2
Boston	10	5	42	71°W	− 1.6	7.4	−16.1	−11.7	− 9.4
Cambridge Bay	10	14	69	105°W	−35.0	5.2	−46.7	−43.9	−42.8
Caribou	8	190	47	68°W	−11.8	10.2	−30.0	−25.6	−23.1
Coral Harbor	10	62	64	83°W	−29.7	6.4	−45.0	−41.7	−39.4
Des Moines	10	286	42	94°W	− 7.0	9.0	−23.9	−20.3	−17.5
Edmonton	10	676	54	114°W	−13.7	7.4	−33.9	−29.4	−26.7
Ennadai Lake	10	325	61	101°W	−30.9	6.8	−46.7	−42.2	−40.6
Eureka	10	10	80	86°W	−36.3	5.8	−48.3	−46.1	−44.4
Fairbanks	23	167	65	147°W	−23.9	10.0	−47.2	−43.9	−40.0
Fargo	9	273	47	97°W	−14.5	10.6	−33.9	−30.0	−27.8
Ft. Chimo	10	36	58	68°W	−22.6	7.5	−41.7	−37.2	−34.4
Ft. Nelson	10	379	59	123°W	−23.2	7.2	−41.7	−37.8	−35.0
Ft. Smith	10	203	60	112°W	−26.1	8.8	−44.9	−39.9	−37.2
Frobisher	10	21	64	69°W	−25.2	6.5	−41.3	−39.1	−37.4
Isachsen	10	30	79	104°W	−34.8	5.4	−47.8	−43.9	−42.2
Medford	10	121	42	123°W	2.6	8.3	−10.6	− 5.3	− 3.3
Minneapolis	11	254	45	93°W	−11.0	10.0	−29.7	−25.6	−22.8
Moose Jaw	10	566	50	106°W	−14.6	8.8	−33.9	−30.0	−27.8
Mould Bay	10	58	76	119°W	−34.7	5.5	−46.7	−43.9	−41.7
Norman Wells	10	88	65	127°W	−28.3	7.0	−46.7	−44.4	−41.7
Rapid City	8	964	44	103°W	− 5.6	13.7	−28.6	−24.4	−21.7
Resolute	10	40	75	95°W	−32.4	5.6	−45.6	−42.8	−40.6
Sachs Harbor	10	84	72	125°W	−31.1	5.0	−44.4	−40.6	−38.9
Salt Lake City	9	1286	41	112°W	− 2.2	10.5	−20.0	−20.3	−12.2
Watson Lake	10	685	60	129°W	−25.1	9.3	−47.8	−44.4	−40.6
Whitecourt	10	741	54	116°W	−14.8	9.7	−37.8	−32.2	−28.9
Whitehorse	10	675	61	135°W	−18.6	6.7	−43.3	−39.4	−35.6
Yellow Knife	10	208	62	114°W	−28.3	7.2	−44.4	−41.7	−39.4
Germany									
Bitburg AB	20	377	50	7°E	− 0.2	4.3	−12.2	− 8.3	− 6.1
Erding AS	11	461	48	12°E	− 1.4	5.9	−16.1	−11.1	− 8.3
Zweibrucken AB	18	345	49	7°E	0.3	3.3	−12.8	− 8.3	− 6.1
Berlin	24	50	52	13°E	− 0.4	4.2	−15.0	−10.0	− 7.8
France									
Chateauroux AS	13	157	47	2°E	2.9	5.8	−10.0	− 5.6	− 3.9
England									
Bentwater RAF	21	29	52	1°E	3.6	4.5	− 5.0	− 2.2	− 1.1
Burtonwood AB	10	27	53	3°W	3.6	4.9	− 5.0	− 2.8	− 1.1
London	10	25	51	0	4.2	5.6	− 4.4	− 1.7	− 0.6
Mildenhall RAF	20	10	52	0	3.6	5.0	− 7.2	− 3.3	− 1.7
Italy									
Aviano AB	17	131	46	13°E	2.7	8.4	− 8.9	− 5.6	− 3.9
Iceland									
Keflavik	24	52	64	23°W	0.2	4.2	−11.7	− 7.8	− 6.1
Greenland									
Sondrestrom AB	25	50	67	51°W	−17.8	8.9	−40.0	−34.4	−31.1
Thule AB	19	80	77	69°W	−23.8	8.3	−37.2	−35.0	−33.3

Linear correlations are 0.98 for $T_{1\%}$, 0.99 for $T_{5\%}$ and 0.99 for $T_{10\%}$; the respective SEE are 2.91, 2.60 and 2.09°C. Scatter diagrams and least-squares regression lines are shown in Fig. 2. Table 5 shows a comparison of observed and estimated temperatures at six stations not used in the regression analysis. They appear in Fig. 2 as circled dots. Except for Kazachye and Yakutsk, observed and estimated temperatures compare favorably. Since observations of Kazachye were limited to 4 or 5 per day rather than 24 per day, its frequency distribution of observed temperatures does not reflect its full range. Similarly, observations at Yakutsk were limited to 4 to 8 per day over a 10-year period so that agreement between observed and estimated values is better than at Kazachye but not as good as at the other locations shown in Table 5.

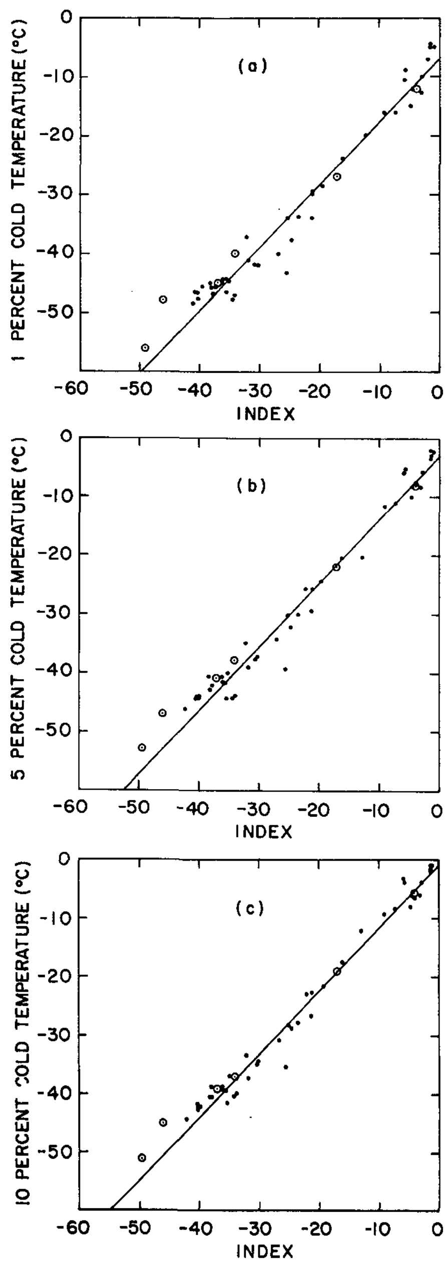

FIG. 2. Percent cold temperature vs index: (a) 1%, (b) 5%, (c) 10%. Circled dots are independent values from Table 5.

TABLE 5. Comparison of observed cold temperatures and temperatures from Eqs. (6), (7) and (8).

Station	Percentile	Cold temperature (°C) Observed	Estimated
Churchill, Canada (59°N, 94°W)	1	−40	−43
	5	−38	−40
	10	−37	−37
Montreal, Canada (45°N, 74°W)	1	−27	−25
	5	−22	−22
	10	−19	−19
Frankfurt, Germany (50°N, 09°E)	1	−12	−11
	5	− 8	− 7
	10	− 6	− 5
Nord, Greenland (82°N, 17°W)	1	−45	−46
	5	−41	−43
	10	−39	−41
Kazachye, USSR (71°N, 136°E)	1	−48*	−56
	5	−47*	−53
	10	−45*	−50
Yakutsk, USSR (62°N, 130°E)	1	−56*	−60
	5	−53*	−57
	10	−51*	−54

* Based on fewer than 24 observations per day (see text).

Although good results were obtained with both (1) and (5), we attempted to improve these equations by using a coefficient other than 1 for the temperature range. We tested the equation

$$I=\bar{T}-b(\bar{T}_x-\bar{T}_n) \tag{9}$$

for different values of b at each percentile temperature. We found that optimum values of b are 1.3 for $T_{1\%}$, 1.3 for $T_{5\%}$ and 1.0 for $T_{10\%}$ with SEE for the 1 and 5 percentile regression lines of 2.82 and 2.53°C. The resulting reductions in the SEE, 3% for $T_{1\%}$ and <3% for $T_{5\%}$, are not sufficiently important to justify using several different values of b.

3. Utility

This method for determining probabilities of surface temperature extremes represents a unique tool for estimating warm and cold temperature extremes. The authors have utilized this technique to map global temperature extremes which enable designers, for the first time, to easily determine percentiles of surface temperatures for any geographic area of concern

(Tattelman and Kantor, 1976a,b). Examples of maps of 10% warm and cold temperature extremes for the Northern Hemisphere are shown in Figs. 3 and 4.

4. Summary

Least-squares linear regression equations have been developed and can be used to calculate temperatures that would be equaled or exceeded 1, 5 and 10% of the hours at a given location during the warmest month of the year. These equations are based on an index calculated from three readily available parameters: 1) the mean monthly temperature, 2) the mean daily maximum temperature for the month and 3) the mean daily minimum temperature for the month. Analogous regression equations have been developed and used to calculate temperatures equaled or less than 1, 5 and 10% of the hours at a given location during the coldest month of the year. These equations, based on a "cold" index, are also derived from the mean monthly temperature, mean daily maximum temperature for the month and mean daily minimum temperature for the month.

This method has been used successfully to produce Northern and Southern Hemisphere maps depicting 1, 5 and 10% warm and cold surface temperatures for the warmest and coldest months.

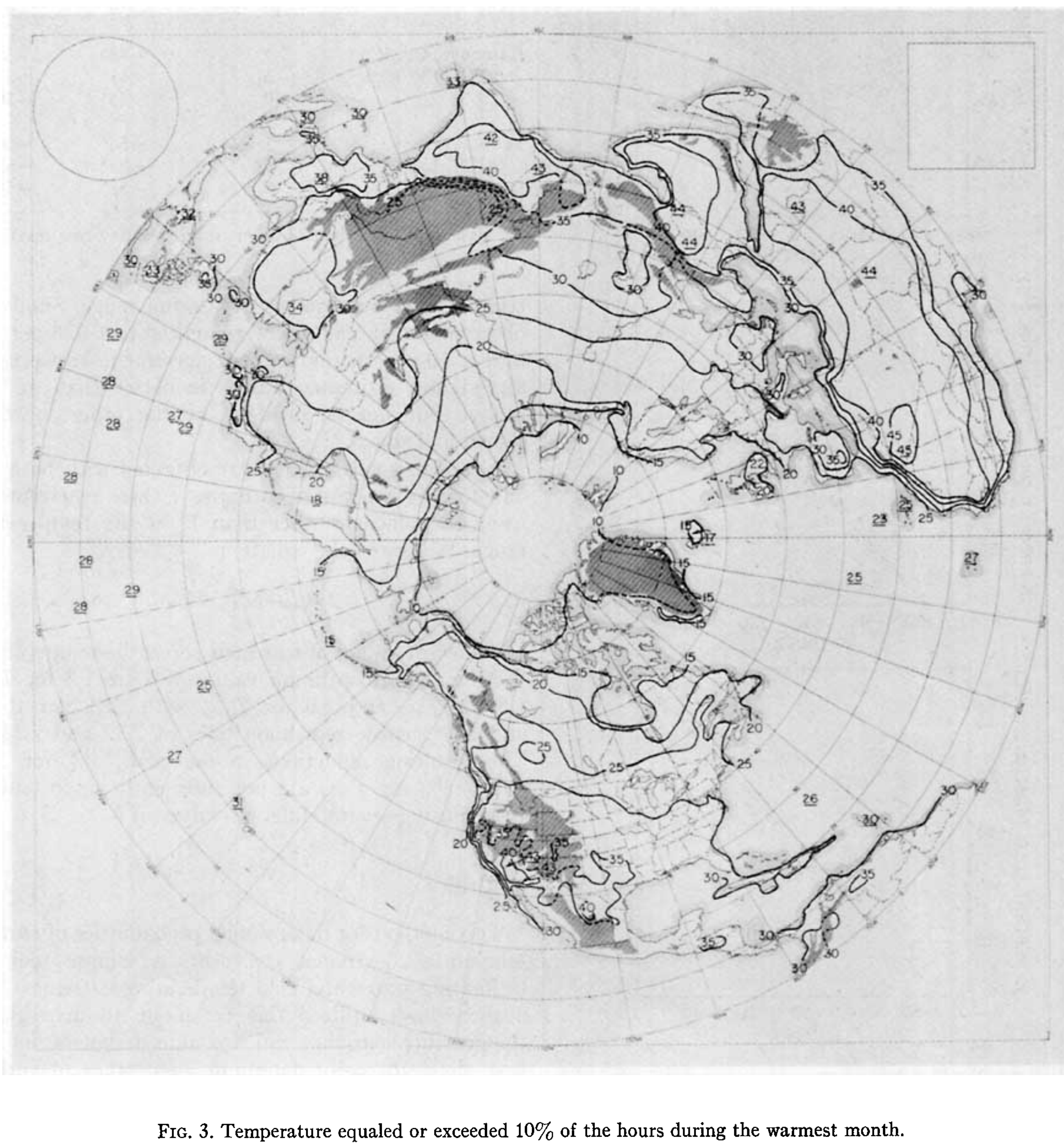

FIG. 3. Temperature equaled or exceeded 10% of the hours during the warmest month.

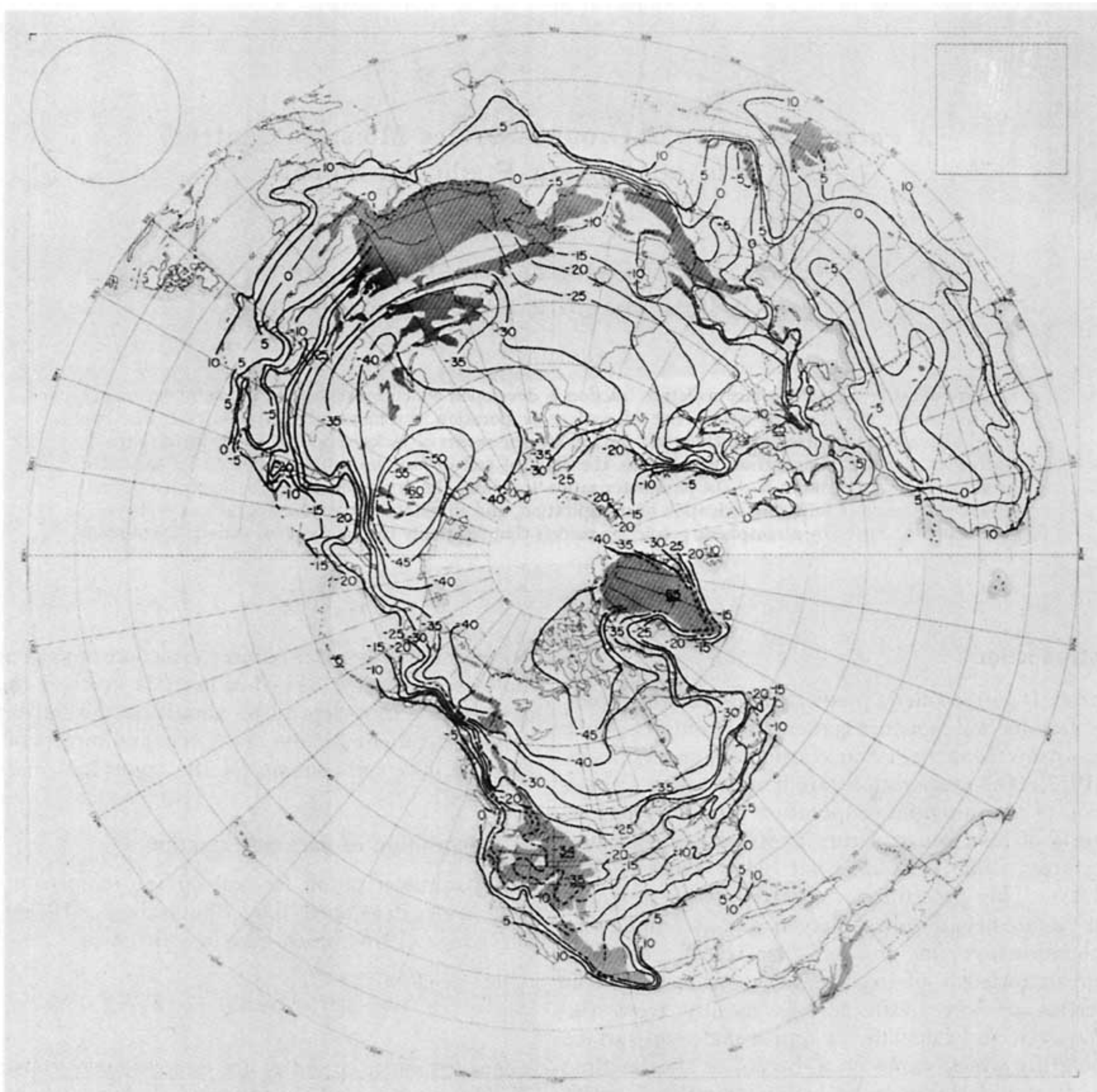

FIG. 4. Temperature equaled or colder 10% of the hours during the coldest month.

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