Determination of Rainfall Distributions from Microwave Radiation Measured by the Nimbus 6 ESMR

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ABSTRACT

The polarization of 37 GHz microwave radiances emerging from rain clouds above land, rough and calm water surfaces was computed from the equation of radiative transfer. Scattering was assumed to be characterized by a Rayleigh phase matrix. The radiative transfer equation was solved by means of a Neumann solution. It was found that the brightness temperatures of the upward directed radiances emerging from rain clouds were relatively independent of polarization. The weak polarization of radiation emitted by rain clouds can be used to discriminate between cool brightness temperatures emerging from rain clouds and open water. The brightness temperatures of radiances emerging from rain clouds over water can be transformed into a single-valued function of the rainfall rate if polarization effects are considered. A sample of Nimbus 6 data is analyzed in accord with the results of the theoretical analysis.

1. Introduction

The Electrically Scanning Microwave Radiometer (ESMR) on board the Nimbus 6 satellite measures the brightness temperatures of radiances emerging from the earth in two perpendicularly polarized directions. Savage and Weinman (1975) showed that the 37 GHz imagery from this ESMR rendered it feasible to observe precipitation over land. Radiances from rain clouds are characterized by brightness temperatures which are lower than those emerging from surrounding dry land areas. Flooded areas also emit microwave radiances which have lower brightness temperatures than dry land. The radiation emitted by calm water surfaces is strongly polarized. Thus the determination of the polarization of radiances from cool areas may discriminate between rain clouds and land areas covered by water. Such discrimination depends on an understanding of the polarization of rain clouds. This paper is addressed to a theoretical analysis of that problem. The results of that analysis will be applied to a sample of Nimbus 6 ESMR data.

2. Analysis

We will consider a plane parallel cloud composed entirely of water drops. The vertical distribution of hydrometeor density is assumed to be uniform. The size distribution is also assumed independent of altitude. We will neglect the dependence of the refractive index of water on temperature and assume that the refractive index at 273 K is valid at all altitudes. The hydrometeor size distribution is assumed to be that given by Marshall-Palmer (see Battan, 1973). The small effect of absorption by water vapor and oxygen is included. The temperature profile is defined by the 1962 U. S. Standard Atmosphere. This temperature profile is approximated by a linear function of height. The source term for radiation emitted by the cloud is assumed to be proportional to the local temperature in accord with the Rayleigh-Jeans law of radiation. No radiation is assumed incident on the top of the rain cloud, and the radiation incident at the base of the cloud consists of one part which is emitted by the surface and another which is due to radiation propagating downward from the cloud which is reflected upward.

a. *Radiative properties of rain at 37 GHz*

Savage (1976) computed the radiative parameters of ensembles of hydrometeors characterized by Marshall-Palmer size distributions for 37 GHz. Table A1 shown in the Appendix, presents a summary of these parameters as a function of the rainfall rate parameter R (mm h^{-1}) which appears in the Marshall-Palmer function.

The extinction coefficient (km^{-1}) is given by

$$k = AR^{\alpha}. \qquad (1)$$

The albedo for single scattering is given by

$$\tilde{\omega}_0 = 1 - [(BR^{\beta})z^* + \tau_{air}]/[(AR^{\alpha})z^* + \tau_{air}], \qquad (2)$$

where z^* is the cloud thickness and τ_{air} is the optical thickness of the gaseous absorbers in the air. The total optical thickness τ^* is the denominator in this expression.

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TABLE 1. Parameters which define the cases presented in this study.

T_0	258K (cloud-top altitude: 15 000 ft)
T^*	288K (surface temperature)
τ_{air}	0.07 (up to 15 000 ft at 37 GHz)
$\bar{A}_{land}$	0.100 (unpolarized land albedo at 37 GHz)
$\bar{A}_{rough\ water}$	0.538 (unpolarized rough water albedo at 37 GHz)

37 GHz polarized calm water reflection coefficients (Hollinger, 1973)

	$\mu=0.23862$	$\mu=0.66121$	$\mu=0.93247$
$\hat{R}_l(\mu)$	0.150	0.395	0.510
$\hat{R}_r(\mu)$	0.860	0.667	0.563

	R (mm h^{-1})					
	1	2	4	8	16	32
τ^*	0.370	0.710	1.33	2.59	5.11	10.2
$\tilde{\omega}_0$	0.20	0.23	0.27	0.33	0.37	0.40

Savage expanded his phase functions as a series of Legendre polynomials; he then expressed the coefficients of the polynomials of order l as

$$\tilde{\omega}_l = z_l R^{\zeta_l}, \quad 1 \leqslant l \leqslant 4. \tag{3}$$

Savage's parameters are approximated by

$$\tilde{\omega}_1 \approx 0, \quad \tilde{\omega}_2 \approx 0.5 \quad \text{and} \quad \tilde{\omega}_l \approx 0 \quad \text{for} \quad l \geqslant 3 \tag{4}$$

for the purposes of this analysis. We will therefore assume that scattering of 37 GHz radiation by raindrops can be approximately represented by a Rayleigh phase matrix.

b. Definition of the problem regarding the transfer of polarized radiation

We will consider the radiances $I_l(z,\theta)$ and $I_r(z,\theta)$ at depth z measured downward from the top of the cloud and inclined at an angle θ with respect to nadir. The states of polarization in which electric vectors oscillate along the principal meridian and at right angles to it are desginated l and r, respectively. The radiative transfer equations which define this problem are

$$\begin{aligned} \mu\frac{dI_l}{d\tau} &= -I_l + \tilde{\omega}_0[J_{l,l}(\tau,\mu) + J_{r,l}(\tau,\mu)] + (1-\tilde{\omega}_0)\mathcal{B}, \\ \mu\frac{dI_r}{d\tau} &= -I_r + \tilde{\omega}_0[J_{l,r}(\tau,\mu) + J_{r,r}(\tau,\mu)] + (1-\tilde{\omega}_0)\mathcal{B}, \end{aligned} \tag{5}$$

where $d\tau = kdz$ and $\mu = \cos\theta$, and the $J_{p',p}(\tau,\mu)$ are respective scattering source terms which transform radiation polarized with a subscript p' to that characterized by p. Chandrasekhar (1946) showed that if the phase matrix was determined by Rayleigh scattering then

$$\begin{aligned} J_{l,l}(\tau,\mu) &= \frac{3}{8}\int_{-1}^{1} I_l(\tau,\mu')[2(1-\mu'^2)+\mu^2(3\mu'^2-2)]d\mu' \\ J_{r,l}(\tau,\mu) &= \frac{3}{8}\mu^2\int_{-1}^{1} I_r(\tau,\mu')d\mu' \\ J_{l,r}(\tau,\mu) &= \frac{3}{8}\int_{-1}^{1} I_l(\tau,\mu')\mu'^2 d\mu' \\ J_{r,r}(z,\mu) &= \frac{3}{8}\int_{-1}^{1} I_r(\tau,\mu')d\mu'. \end{aligned} \tag{6}$$

The far right-hand side source term in Eq. (5) represents thermal emission by hydrometeors in the cloud, i.e.,

$$\mathcal{B} = K\lambda^{-4}T = \mathcal{B}_0 + \mathcal{B}_1\tau. \tag{7}$$

It is assumed that the Rayleigh-Jeans law describes this emission (K is the appropriate constant and λ the radiation wavelength). All radiances will be given in units of brightness temperature so that $K\lambda^{-4}\equiv 1$. The cloud-top temperature is $\mathcal{B}_0 = T_0$ and $\mathcal{B}_1 = (T^* - T_0)/\tau^*$, where T^* is the cloud base (ground) temperature and τ^* the total optical thickness of the cloud.

The boundary conditions which will be applied to $I_p(\tau,\mu)$ are as follows: At the upper boundary ($\tau=0$)

$$I_p(0,\mu) = 0, \quad \mu \geqslant 0 \text{ and } p = l,r. \tag{8}$$

At the cloud base ($\tau = \tau^*$), two conditions are considered, one for a nonpolarizing surface such as land or rough seas and the other for a polarizing surface such as calm water.

(i) For a nonpolarizing surface

$$I_p(\tau^*,|\mu|) = \bar{A}\int_0^1 [I_l(\tau^*,\mu') + I_r(\tau^*,\mu')]\mu' d\mu' + (1-\bar{A})(\mathcal{B}_0 + \mathcal{B}_1\tau^*), \ \mu' \geqslant 0, \mu < 0 \text{ and } p = l, r, \tag{9a}$$

where

$$\bar{A} = \int_0^1 [\hat{R}_l(\mu') + \hat{R}_r(\mu')]\mu' d\mu' \tag{9b}$$

or

$$\bar{A} = \text{constant}. \tag{9c}$$

The $\hat{R}_l(\mu)$ and $\hat{R}_r(\mu)$ are reflection coefficients determined from Fresnel's equation (see Hollinger, 1973). Values of these coefficients for water surfaces are presented in Table 1. Values for $\bar{A}$ for water and dry land surfaces are also presented in Table 1.

(ii) For a polarizing surface

$$I_p(\tau^*,|\mu|) = \hat{R}_p(\mu)I_p(\tau^*,\mu') + [1-\hat{R}_p(\mu)][\mathcal{B}_0 + \mathcal{B}_1\tau^*], \quad \mu' \geqslant 0, \mu < 0, |\mu'| = |\mu| \text{ and } \quad p = l, r. \tag{10}$$

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c. Solution of the equation of transfer of polarized radiation

The radiances will be determined from Eqs. (5) by means of Neumann's solution (see Busbridge, 1960). The radiances can be expressed in terms of successive orders of scattering j, i.e.,

$$I_p(\tau,\mu)=\sum_{j=0}^{J} I_{p,j}(\tau,\mu), \quad p=l,\, r. \tag{11}$$

The unscattered downwelling radiances are determined by Eq. (5), where the $J_{p',p}=0$ and by Eq. (8). These quantities can be evaluated analytically:

$$I_{p,0}(\tau,\mu)=(1-\tilde{\omega}_0)\left\{\mathcal{B}_0(1-e^{-\tau/\mu}) + \mathcal{B}_1\mu\left[\left(\frac{\tau}{\mu}-1\right)+e^{-\tau/\mu}\right]\right\}, \quad \mu\geqslant 0. \tag{12}$$

The upwelling unscattered radiances can also be derived analytically:

$$\begin{aligned} I_{p,0}(\tau,|\mu|)=(1-\tilde{\omega}_0)\{&\mathcal{B}_0(1-\exp[-(\tau^*-\tau)/|\mu|]) \\ &+\mathcal{B}_1((|\mu|+\tau)\exp[-\tau/|\mu|] \\ &-(|\mu|+\tau^*)\exp[-(\tau^*-\tau)/|\mu|])\} \\ &+I_{p,0}(\tau^*,|\mu|)\exp[-(\tau^*-\tau)/|\mu|], \\ &\mu<0 \quad \text{and} \quad p=l,\, r. \end{aligned} \tag{13}$$

Here $I_{p,0}(\tau^*,|\mu|)$ is defined by Eqs. (9) or (10).

The radiances scattered j times (where $j\geqslant 1$) can then be readily generated from

$$I_{p,j}(\tau,\mu)=\frac{\tilde{\omega}_0 e^{-\tau/\mu}}{\mu}\int_0^\tau \mathcal{J}_{p,j-1}(\tau',\mu)e^{\tau'/\mu}d\tau', \quad \mu>0 \quad \text{and} \quad p=l,\, r, \tag{14}$$

and

$$I_{p,j}(\tau,|\mu|)=I_{p,j}(\tau^*,|\mu|)\exp[-(\tau^*-\tau)/|\mu|] +\tilde{\omega}_0\int_\tau^{\tau^*} \mathcal{J}_{p,j-1}(\tau',\mu)\exp[-(\tau'-\tau)/|\mu|]\frac{d\tau'}{|\mu|}, \quad \mu<0 \quad \text{and} \quad p=l,\, r,$$

where

$$\left.\begin{aligned} \mathcal{J}_{l,j-1}(\tau,\mu) &=\frac{3}{8}\int_{-1}^{1}\{[\mu^2(3\mu'^2-2)+2(1-\mu'^2)]I_{l,j-1}(\tau,\mu') +\mu^2 I_{r,j-1}(\tau,\mu')\}d\mu' \\ \mathcal{J}_{r,j-1}(\tau,\mu) &=\frac{3}{8}\int_{-1}^{1}\{\mu'^2 I_{l,j-1}(\tau,\mu') +I_{r,j-1}(\tau,\mu')\}d\mu' \end{aligned}\right\}. \tag{15}$$

The $I_{p,j}(\tau^*,|\mu|)$ are defined by Eqs. (9) or (10).

TABLE 2. Brightness temperatures of 37 GHz radiances emerging from a rain cloud with a top at 4.57 km located over land. [$\bar{I}$ = scalar radiances computed by Savage (1976).]

μ		R (mm h^{-1}) 1	2	4	8	16	32
−0.23862	I_l	254.4	247.9	240.3	231.4	225.6	223.6
	I_r	253.1	246.0	238.0	228.6	222.5	220.2
	$\bar{I}$	255.5	247.2	239.4	230.4	224.2	—
−0.66121	I_l	263.4	260.8	254.7	245.4	238.5	235.4
	I_r	262.9	259.8	253.1	243.3	236.1	232.4
	$\bar{I}$	265.4	260.9	254.6	245.5	238.7	—
−0.93247	I_l	265.3	264.0	259.0	250.0	242.3	238.6
	I_r	265.2	263.8	258.7	249.5	241.8	237.9
	$\bar{I}$	267.5	264.6	259.7	251.0	244.0	—

3. Computational results and analysis of Nimbus 6 data

The integrals which appear in Eq. (15) can be solved with sufficient accuracy by using six-point Gaussian quadrature. Adaptive numerical integration was used to evaluate the integrals in Eq. (14).

The cases summarized in Table 1 were evaluated numerically to derive the orthogonally polarized radiances.

Results of computations for three situations are presented in Tables 2–4. In spite of the simplifying assumption that the phase matrix is characterized by a Rayleigh phase function, the polarized radiances agree well with the scalar radiances found by Savage (1976) which are designated as $\bar{I}$. (Savage used a scalar phase function which was represented by the sum of five Legendre polynomials of the cosine of the scattering angle.)

Table 2 presents the brightness temperature of radiances emerging from rain clouds over land [see Eqs. (9a) and (9c)]. The most self-evident conclusion that may be drawn from this table is that 37 GHz radiances emerging from rain clouds are weakly polarized. Be-

TABLE 3. As in Table 2 except for a cloud located over rough water.

μ		R (mm h^{-1}) 1	2	4	8	16	32
−0.23862	I_l	235.6	243.7	239.8	231.4	225.6	223.6
	I_r	233.8	241.2	237.2	228.5	222.5	220.0
	$\bar{I}$	240.6	242.7	238.8	230.4	224.2	220.0
−0.66121	I_l	219.6	242.8	250.8	245.1	238.5	235.4
	I_r	218.9	241.5	249.1	243.1	236.1	232.4
	$\bar{I}$	226.2	242.6	250.5	245.2	238.7	234.5
−0.93247	I_l	214.4	240.1	252.5	249.4	242.3	238.6
	I_r	214.3	239.8	252.1	248.9	241.8	237.9
	$\bar{I}$	221.3	240.3	253.1	250.5	244.0	239.5

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TABLE 4. As in Table 2 except for a cloud located over calm water.

μ		R (mm h^{-1}) 1	2	4	8	16	32
−0.23862	I_l	253.8	247.2	240.0	231.3	225.6	223.6
	I_r	235.8	241.9	237.0	228.5	222.5	220.2
−0.66121	I_l	230.7	247.2	251.7	245.1	238.5	235.4
	I_r	200.6	233.7	247.3	242.9	236.1	232.4
−0.93247	I_l	203.7	231.0	248.4	248.7	242.3	238.6
	I_r	196.0	226.7	246.8	248.1	241.8	237.9

cause radiances emerging at a zenith angle of 50° ($\mu = -0.6428$) are negligibly polarized, it will be shown that it is possible to discriminate between radiances emerging from calm water and from rain clouds. The radiances are monotonically decreasing functions of the rainfall rate.

Table 3 presents the brightness temperatures of radiances emerging from rain clouds over rough water; Eqs. (9a) and (9b) are assumed to apply to the lower boundary. Comparing Tables 2 and 3 shows that the underlying surface has little effect on the emerging radiances when $R > 4$ mm h^{-1}. It is also noteworthy that the brightness temperatures of radiances emerging from clouds over water are double-valued functions of the rainfall rate. This effect was noted earlier by Savage and Weinman (1975). The brightness temperature of radiances polarized in both directions are comparable.

Table 4 presents the brightness temperatures of radiances emerging from rain clouds over calm water. The Fresnel reflection coefficients are used in the boundary conditions defined by Eq. (10) in this case. The main differences between results shown in Tables 3 and 4 occur for $R < 4$ mm h^{-1} where the dependence of the water's reflection on polarization can still be observed through the rain cloud.[1]

Data presented in Tables 2 and 4 can be empirically related by

$$I_l^{\mathrm{land}} = \overset{\mathrm{calm}}{I_l^{\mathrm{water}}} - 1.2(\overset{\mathrm{calm}}{I_r^{\mathrm{water}}} - \overset{\mathrm{calm}}{I_l^{\mathrm{water}}} + 2.5°). \quad (16)$$

Nimbus 6 data obtained at approximately 1735 GMT 31 July 1975 were analyzed numerically. The unprocessed picture which summarizes these data is presented in Fig. 2 of Savage and Weinman (1975). A 3×3 bionomial spatial filter was applied to the pixels with $\tilde{I}_l < 280$ K. A 7×7 binomial spatial filter was applied to the 285 K$>\tilde{I}_l>$280 K measurements. Scan line offsets were eliminated by a statistical analysis of the radiances which appears in each scan line; this reduced the streaks which are evident in Fig. 2 of Savage and Weinman. It was found that contrast between water and land surfaces could be eliminated by the transformation

$$\tilde{I}_l = I_l - 1.5(I_r - I_l + 2.5°). \quad (17)$$

subject to the constraint that $I_l > I_r$. This empirical transformation is similar to Eq. (16) except that the polarization correction is enhanced because the reflection coefficient of open water depends less on polarization than it did our calm water model. This is consistent with the findings of Webster *et al.* (1976) who showed that the reflection coefficients of water surfaces are less dependent on polarization than those presented in Table 1 and that they are a function of surface wind stress.

It should be further noted that Eq. (17) provides a method to remove the double-valued dependence of the brightness temperature on rainfall rates for precipitation at sea which was cited in the discussion of the data in Table 4, i.e., $\tilde{I}_l$ is a single-valued function of the rainfall rate over land *and* water.

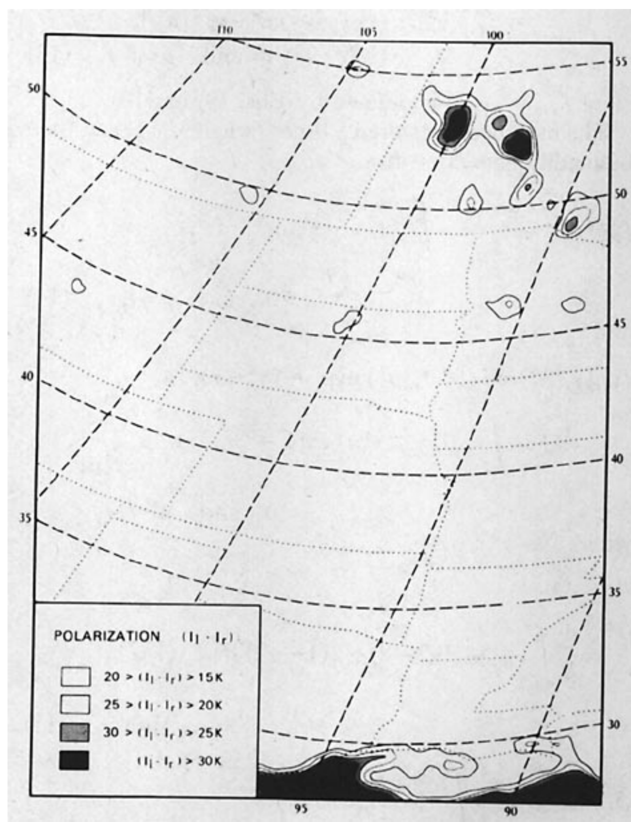

FIG. 1. ESMR 6 37.0 GHz polarization, $I_l - I_r > 15$ K. Data were obtained on 31 July 1975, interrogation orbit 663 at approximately 1735 GMT. Various lakes in the northern United States and in Canada are evident as is the Gulf of Mexico.

[1] The present theoretical model was also applied to clouds of ice hydrometeors. The phase matrix of 37 GHz radiation scattered from ice hydrometeors is not as well approximated by a Rayleigh matrix as is scattering from liquid hydrometeors. The agreement with Savage's scalar solution of the transfer equation is thus poorer. However, it was found that the emerging radiances are also insensitive to polarization.

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Fig. 1 shows $I_l - I_r > 15$ K. Lakes such as Winnipeg, Winnipegosis, Manitoba, Cedar, Red, Mille Lacs and Lake of the Woods are evident as is the coast of the Gulf of Mexico. The precipitation along the Louisiana and east Texas Coasts has gaps which allow the Gulf to be observed. There are also a few spurious occurrences in Saskatchewan and Wyoming.

Fig. 2 shows $\tilde{I}_l \leqslant 285$ K, where $\tilde{I}_l$ is computed by means of Eq. (17) and the various filters cited above. Fig. 3 shows the National Weather Service Radar Summary Chart for the same period observed by Nimbus 6. The agreement between $\tilde{I}_l$ in Fig. 2 and Fig. 3 is gratifying. The low values of $\tilde{I}_l$ in Fig. 2, shown along the borders of Minnesota-Iowa and Montana-Saskatchewan, do not correspond to precipitation in Fig. 3; however, it should be noted that equipment was out for maintenance in these regions.

It is especially revealing to compare the distribution of precipitation in Louisiana and eastern Texas with 11.5 μm imagery for the same region which is shown in Fig. 2 of the article by Savage and Weinman. Clouds in that region appear to be relatively warm in the infrared; they might therefore be ignored as probable sources of precipitation. This study demonstrates the necessity of incorporating microwave imagery in conjunction with imagery obtained at shorter wavelengths to monitor precipitation from satellites.

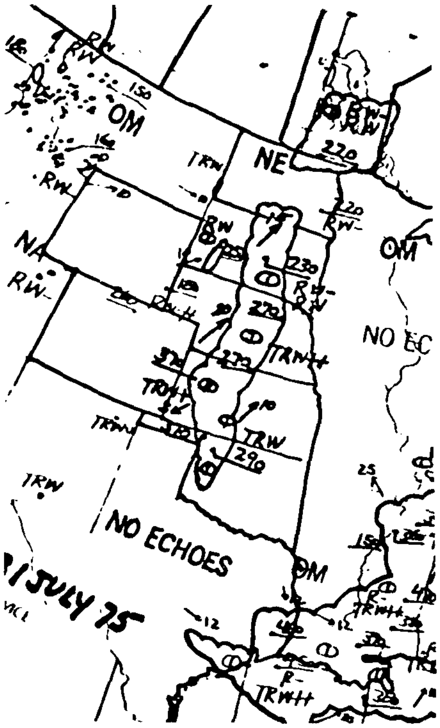

Fig. 3. Part of National Weather Service Summary Chart for 1735 GMT 31 July 1975 covering the area shown in Fig. 2.

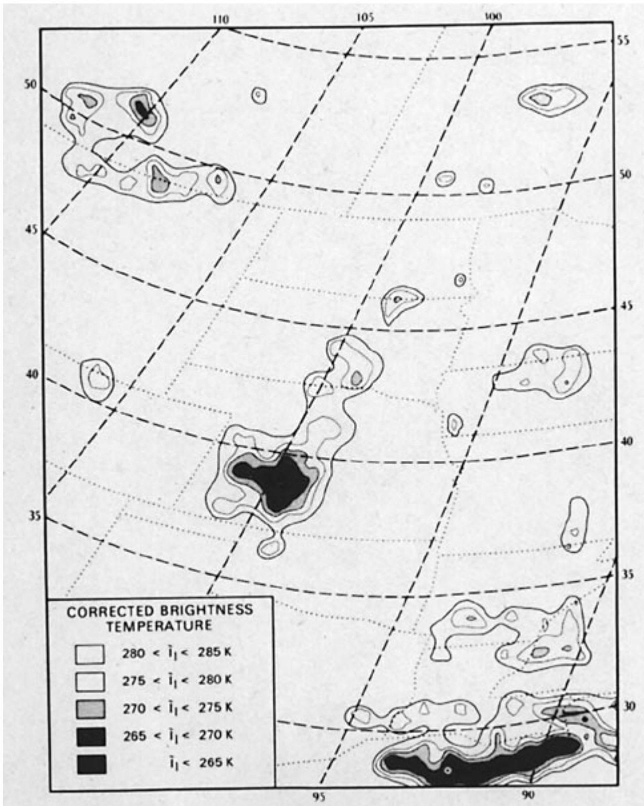

Fig. 2. Brightness temperatures of radiances $\tilde{I}_l$ as defined in Eq. (17) for $\tilde{I}_l < 285$ K (see legend).

4. Conclusions

This study has shown that brightness temperatures of 37 GHz radiances emerging from rain clouds and from calm water are considerably less than those emerging from land surfaces. However, the radiances emerging from calm water depend markedly on polarization whereas those emerging from rain clouds do not. This result can be used to discriminate between precipitation and bodies of water; rain cloud distributions can thus be determined over land by means of 37 GHz ESMR measurements. The double-valued dependence of the brightness temperature of radiances emerging from clouds over water can also be removed when polarization effects are considered.

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APPENDIX

TABLE A1. Summary of the radiative parameters of rain at 37 GHz which appear in Eqs. (1)–(3) (Savage, 1976).

A	0.070
α	1.01
B	0.054
β	0.92
z_1	0.062
ζ_1	0.35
z_2	0.50
ζ_2	-8.4×10^{-3}
z_3	0.037
ζ_3	0.32
z_4	2.5×10^{-3}
ζ_4	0.76

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