# A New Criterion for Locating the Subtropical High in West Africa[1]

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ABSTRACT

After reanalyzing the potential temperature data of Peixoto (1958, 1960), a criterion is derived for locating the mean latitude of the subtropical high over West Africa. Using this criterion, it is found that the subtropical high tends to move to higher latitudes, and therefore the amount of rainfall in the Sahel tends to increase, as the concentration of carbon dioxide and particulate matter in the atmosphere increases. We conclude that increasing amounts of atmospheric pollutants are probably not the cause of the severe drought experienced in the Sahel in the early 1970's. This conclusion contradicts that of Bryson (1973) who used the so called $Z$ criterion to predict an opposite response of the subtropical high to changes in the concentration of atmospheric pollutants.

## 1. Introduction

There has been interest in recent years in predicting the long-term trends in the variation of the latitude of the subtropical high (STH) over West Africa based on changes in other measurable parameters. This interest has arisen mainly as a result of the severe drought experienced several years ago in the Sahel zone of West Africa. The northward movement of the Intertropical Discontinuity (ITD) brings monsoon rains over West Africa south of the ITD (Ilesanmi, 1971) and a correlation has been indicated by Bryson (1973) and Beer *et al.* (1977) between the location of the STH and the ITD. Thus, predictions of the location of the STH indicate the northward movement of the monsoon rains and, in particular, help to predict the amount of rainfall to be expected in the Sahel which is the region of the farthest northward advance of these rains.

A measure of the location of the STH, the so-called $Z$ criterion, has been developed by Bryson (1973, 1974) based on earlier work by Smagorinsky (1963) and Flohn (1964, 1965). This criterion relates the location of the STH to the vertical lapse rate and the horizontal meridional temperature gradient. Using this criterion, one is able to show, due to increasing amounts of carbon dioxide and particulate matter in the lower atmosphere, that the lapse rate and meridional temperature gradient have been changing in such a way as to decrease the average latitude of the STH and therefore decrease the amount of expected rainfall in the Sahel. Thus, one finds a correlation between increasing industrial, agricultural and volcanic activity on the one hand, and the presence and continued threat of Sahelian drought on the other.

In this paper, we reexamine the work of Smagorinsky (1963). The relationship derived there, forming the basis of the $Z$ criterion, is known to fit the potential temperature data of Peixoto (1958, 1960) only at mid-latitudes. In this paper, we analyze the data of Peixoto at all latitudes and show that the variation of the data with latitude can be described by empirical relationships containing a few parameters. The parameters are first fit to the data. The Smagorinsky result is made consistent with the empirical relationship for the potential temperature and an equation giving the latitude of the STH is then obtained based on these parameters. We then conjecture that it is the parameters of the potential temperature empirical relationship that change when there is a change in the concentration of $CO_2$ and particulate matter in the atmosphere. The change in these parameters, in turn, produces a change in the location of the STH. This new criterion for locating the subtropical high predicts a response of the STH to changes in atmospheric pollutants that is opposite to the response predicted by the $Z$ criterion.

In Section 2, we review Smagorinsky's derivation of the relationship which forms the basis of the $Z$ criterion and discuss the criterion itself. In Section 3, the potential temperature data are reexamined and the values of the parameters in the empirical relationship are obtained. When Smagorinsky's results are incorporated, an equation is obtained which is satisfied

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for two values of latitude. The more southerly value is interpreted as the average location of the STH, i.e., the approximate boundary between the Hadley and Rossby circulation regimes. The more northerly value of latitude is interpreted as the average location of the boundary between the Rossby and polar circulation cells.

In Section 4, we discuss the results. In particular, we show that the new criterion predicts a change in location of the STH when there are changes in the lapse rate and in the meridional temperature gradient, but that the response is opposite to that predicted by the $Z$ criterion. We summarize our results in Section 5 and in the Appendix give the mathematical details necessary for the discussion in Sections 2 and 4.

## 2. The $Z$ criterion

The relationship derived by Smagorinsky (1963) and subsequently used by Bryson as the basis of his $Z$ criterion is

$$\tan\phi = \frac{h}{a}\frac{\xi}{|\eta|}, \qquad (1)$$

where $\phi$ is the latitude, $h$ the scale height of the atmosphere which we take to be 9 km, $a$ the radius of the earth (6371 km) and

$$\xi = \frac{\partial(\ln\theta_{PE})}{\partial z}, \quad \eta = \frac{\partial(\ln\theta_{PE})}{\partial y}. \qquad (2)$$

In Eq. (2), $\theta_{PE}$ is the partial equivalent potential temperature and $y$ and $z$ are the northward and upward components, respectively. The derivation of (1) relies on the work of Phillips (1954) on a two-level model for meridional circulation. A stability criterion for the baroclinic waves is derived and can be written as

$$W\begin{cases} <0, & \text{stable} \\ =0, & \text{neutral} \\ >0, & \text{unstable} \end{cases}$$

where

$$W = \left(\frac{n}{a}\right)^4\left(\frac{gh}{2\Omega}\right)^2\eta^2 - \left(\frac{2\Omega}{a}\right)^2\cos^2\phi\sin^2\phi - \left(\frac{n}{a}\right)^8\left(\frac{g^2h^3}{16\Omega^3}\right)^2\eta^2\xi^2\sin^{-4}\phi. \qquad (3)$$

Here $n/a$ is the magnitude of the two-dimensional wavenumber, $g$ the acceleration due to gravity ($9.8\times10^{-3}$ km s$^{-2}$) and $\Omega$ the angular velocity of the earth's rotation ($7.3\times10^{-5}$ rad s$^{-1}$). Minimizing the stability criterion with respect to $n$, i.e.,

$$\frac{\partial W}{\partial n} = 0,$$

gives

$$n^4 = \frac{32a^4\Omega^4}{g^2h^4\xi^2}\sin^4\phi. \qquad (4)$$

When this value of $n$ is put into (3) and $W$ set equal to zero, Eq. (1) is obtained.

Smagorinsky uses the potential temperature data of Peixoto (1958, 1960) to calculate average values of $\xi$ and $\eta$ at different latitudes. When these values are substituted into (1), fairly good agreement is obtained with the equation for latitudes between 30° and 60°, where $\phi$ is interpreted as the latitude at which the logarithmic gradients $\xi$ and $\eta$ are measured. Outside the interval between 30° and 60°, the agreement with (1) is poor.

In obtaining the $Z$ criterion, Bryson interprets Eq. (1) in a different way. The approach is to examine the global values of the temperature gradients $\xi$ and $\eta$ and deduce typical values for each of these. When these values are substituted into (1), a single value of $\phi$ is obtained which is then interpreted as an average latitude for the location of the subtropical high, i.e., the transition between the Rossby and Hadley regimes.

The increasing amounts of atmospheric pollution over the years has no doubt had an effect on the globally averaged values of $\xi$ and $\eta$. Bryson (1973, 1974) considers two major pollutants, $CO_2$ and particulate matter. Increases in $CO_2$ will increase the strength of the greenhouse effect, which, in turn, will increase the average temperature at the surface, leaving temperatures at high altitudes unchanged. Thus, there will be an increase in the average lapse rate. As an example, suppose that the average surface temperature increases by 0.03°C but remains unchanged at 3 km. The lapse rate is then increased by 0.01°C km$^{-1}$ and, according to Eq. (A8) in the Appendix, $\xi$ will decrease by $-8\times10^{-5}$ km$^{-1}$. In the summer, the STH is located at a latitude of $\sim$35°N. In the next section we shall find that for this latitude in summer, a typical value of $\xi$ is $8\times10^{-3}$ km$^{-1}$ (see Table 1). Thus, an increase in lapse rate of 0.01°C km$^{-1}$ causes a decrease in $\xi$ of $\sim-1\%$. If this decrease in $\xi$ is used in (1), we find a decrease in $\phi$ of $\sim-0.27°$.

It has been found, in both Nigeria (Ilesanmi, 1971) and Ghana (Beer *et al.*, 1977), that changes in the location of the STH give rise to changes in the latitude of the ITD which are 2–3 times as large. Thus, in our example, we would expect the average latitude of the ITD to decrease by about $-0.7°$. Furthermore, the rainfall gradient south of the ITD is known to be very steep, $\sim$180 mm per degree of latitude in northern Nigeria (Ilesanmi, 1971). A decrease in the latitude of the ITD of $-0.7°$ would cause a decrease in the average annual rainfall of $\sim$130 mm. This is a drastic reduction in the Sahel, where the average annual rainfall is between 200 and 600 mm (Glantz, 1977).

TABLE 1. The values of $\xi$ and $\eta$, defined in Eqs. (5) and (6), as a function of latitude $\phi$, for annual, winter and summer average values of $\theta_{PE}$. The values of $\theta_{PE}$ used in the calculation of $\xi$ and $\phi$ are from Smagorinsky (1963).

| Annual | | | Winter | | | Summer | | |
|---|---|---|---|---|---|---|---|---|
| $\phi$ (deg) | $\xi\times10^3$ (km$^{-1}$) | $\eta\times10^5$ (km$^{-1}$) | $\phi$ (deg) | $\xi\times10^3$ (km$^{-1}$) | $\eta\times10^5$ (km$^{-1}$) | $\phi$ (deg) | $\xi\times10^3$ (km$^{-1}$) | $\eta\times10^5$ (km$^{-1}$) |
| 0 | 2.3 | | 0 | 1.7 | | 0 | −1.3 | |
| 10 | 1.0 | −1.80 | 10 | 2.7 | −1.51 | 10 | −0.3 | −0.90 |
| 20 | 3.0 | −1.37 | 20 | 6.5 | −1.69 | 20 | 1.0 | −0.91 |
| 30 | 7.5 | −1.85 | 30 | 10.7 | −2.34 | 30 | 5.1 | −1.22 |
| 40 | 14.4 | −2.20 | 40 | 16.8 | −2.73 | 40 | 11.1 | −1.85 |
| 50 | 20.8 | −2.10 | 50 | 24.2 | −2.48 | 50 | 17.2 | −1.90 |
| 60 | 26.3 | −1.81 | 60 | 31.6 | −2.02 | 60 | 23.2 | −1.93 |
| 70 | 32.4 | | 70 | 36.3 | | 70 | 28.8 | |

The increase in particulate matter in the atmosphere and the surface cooling that has resulted from this has occurred mainly in northern latitudes, with very little surface cooling in the tropics. This, in turn, leads to an increase in the meridional temperature gradient. As an example, suppose that the meridional temperature gradient increases by 0.01°C (1000 km)$^{-1}$. According to Eq. (A13) $|\eta|$ will increase by $\sim 4\times10^{-8}$ km$^{-1}$. In the next section we find that in the summer, for $\phi=35°$, a typical value of $|\eta|$ is $1.5\times10^{-5}$ km$^{-1}$. Thus, the resulting 0.3% increase in $|\eta|$ produces a decrease in $\phi$ of about $-0.1°$ from Eq. (1). As discussed previously, this, in turn, will cause a decrease in the average ITD latitude of $\sim -0.25°$ and consequently a decrease in the amount of expected rainfall in the Sahel of $\sim$45 mm.

Based on these arguments, Bryson concludes that increasing amounts of carbon dioxide and particulate matter in the atmosphere has to some extent been the cause of the drought conditions experienced in the Sahel. In the next section we shall show that Eq. (1), which is the basis of the $Z$ criterion, must be modified to fit potential temperature data at all latitudes. Once this modification is made, an equation is obtained which we interpret as a new criterion for locating the STH.

## 3. A new criterion for locating the subtropical high

We begin the derivation of the new criterion for the STH by obtaining values of $\xi$ and $\eta$ using the data of Peixoto (1958, 1960) as analyzed by Smagorinsky (1963). These are presented as annual, winter and summer averages of $\theta_{PE}$ for the year 1950. We obtain values of $\xi$ from the equation

$$\xi(\phi)=\frac{\partial(\ln\theta_{PE})}{\partial z}=\frac{1}{\theta_{PE}(500)}\left[\frac{\theta_{PE}(250)-\theta_{PE}(1000)}{h}\right], \quad (5)$$

where $\theta_{PE}(N)$ is the partial equivalent potential temperature at a height $N$ in millibars. The values of $\eta$ are obtained from

$$\eta(\phi)=\frac{\partial(\ln\theta_{PE})}{\partial y}=\frac{1}{\theta_{PE}(\phi)}\left[\frac{\theta_{PE}(\phi+10°)-\theta_{PE}(\phi-10°)}{2000\text{ km}}\right]. \quad (6)$$

In this equation, all values of the partial equivalent potential temperature are taken at 500 mb and $\phi$ is the latitude at which $\eta$ is calculated. The results of using Eqs. (5) and (6) are given in Table 1 for annual, winter and summer averages.

We find that the variation of $\xi$ with $\phi$ has a power law dependence for $\sin\phi$ of the form

$$\xi=\xi_{90}\sin^{\nu}\phi, \quad (7)$$

where $\xi_{90}$ is the value of $\xi$ at $\phi=90°$. In Fig. 1, log-log plots are made of $\xi$ vs $\sin\phi$ for the three sets of values in Table 1. The data are seen to lie on nearly straight lines in all cases. We have omitted the data for $\phi=0°$ in all cases since this value cannot be included in a log plot. The negative value of $\xi$ for $\phi=10°$ (summer) is also omitted for the same reason. Least-squares fits to the data are shown in each case. The resulting values of the parameters in Eq. (7) are as follows:

Annual: $\nu=2.11\pm0.08$, $\xi_{90}=(36\pm3)\times10^{-3}$ km$^{-1}$

Winter: $\nu=1.54\pm0.08$, $\xi_{90}=(36\pm3)\times10^{-3}$ km$^{-1}$

Summer: $\nu=3.27\pm0.20$, $\xi_{90}=(40\pm4)\times10^{-3}$ km$^{-1}$.

Since the rains in the Sahel occur when the ITD, and therefore the STH, makes its furthest northward advance, and since this occurs during the summer months, we shall restrict our attention to the summer variation of $\xi$ with $\phi$. From Fig. 1c, to a good approximation, we have

$$\xi=0.04\sin^3\phi\ [\text{km}^{-1}]. \quad (8)$$

We use this expression for $\xi$ in our subsequent discussion.

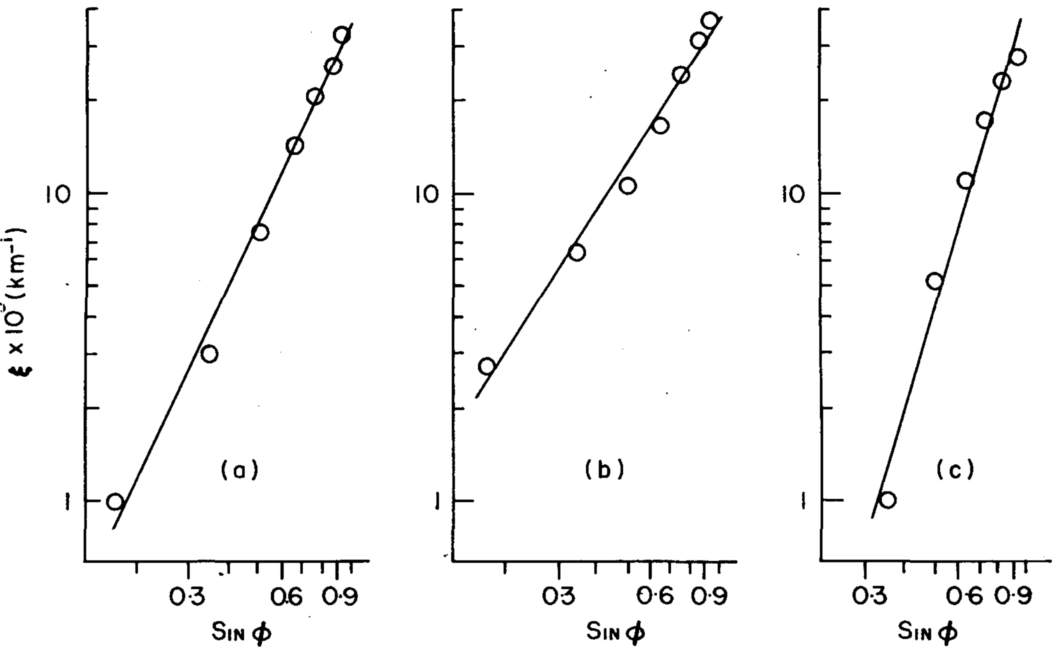


FIG. 1. Log-log plot of $\xi=\partial(\ln\theta_{PE})/\partial z$ as a function of $\sin\phi$. The data are for (a) annual, (b) winter and (c) summer and are given in Table 1. The curves are least-squares straight line fits to the data.

In order to obtain the latitude dependence of $\eta$ for the summer months, we note that for the mid-latitudes, Eq. (1) fits the data fairly well. For $\xi$ varying as $\sin^3\phi$, Eq. (1) implies that $\eta$ should vary as $\sin^3\phi\,(\tan\phi)^{-1}=\sin^2\phi\cos\phi$. We therefore plot $|\eta|$ for the summer months, given in Table 1, against $\sin^2\phi\cos\phi$ in Fig. 2. The points fall approximately on a straight line and a least-squares fit gives

$$\left.\begin{aligned}\eta&=\eta_0+\eta_1\sin^2\phi\cos\phi\\ \eta_0&=-(0.660\pm0.116)\times10^{-5}\ \mathrm{km}^{-1}\\ \eta_1&=-(3.33\pm0.43)\times10^{-5}\ \mathrm{km}^{-1}\end{aligned}\right\}. \tag{9}$$

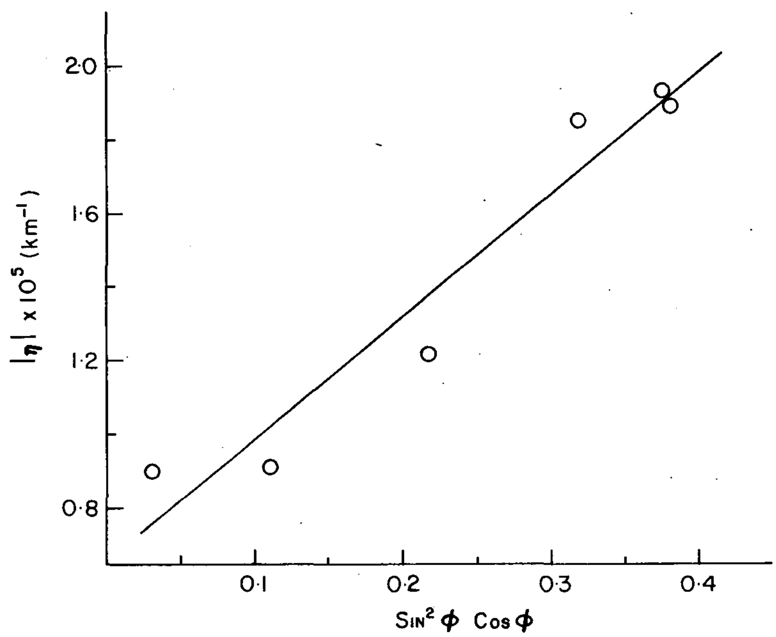


FIG. 2. Absolute value of $\eta=\partial(\ln\theta_{PE})/\partial y$ for the summer plotted against $\sin^2\phi\cos\phi$. The data are from Table 1. The straight line is a least-squares fit to the data.

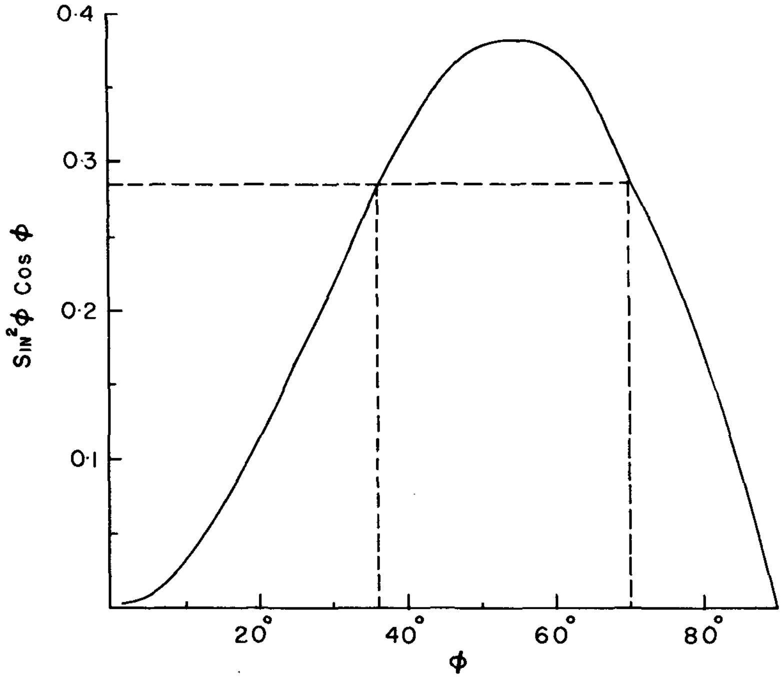


FIG. 3. The function $\sin^2\phi \cos\phi$. The value of 0.28 is obtained from the parameters of Eqs. (8) and (9) and gives an average location of the subtropical high in the summer months at a latitude of 36°. The high-latitude solution at ~70° is interpreted as the average boundary between the polar cell and the Rossby regime.

This result contains a term which varies as $\sin^2\phi \cos\phi$ as expected, and in addition contains the constant $\eta_0$. If this constant were zero, then the $\sin^2\phi \cos\phi$ dependence of $\eta$ and the $\sin^3\phi$ dependence of $\xi$ would simply make Eq. (1) an identity satisfied for all values of $\phi$. This is the point of view taken by Smagorinsky (1963).

Here we have shown that the constant $\eta_0$ is substantially different from zero. When Eqs. (8) and (9) are substituted into (1), we obtain

$$\sin^2\phi \cos\phi = \frac{a|\eta_0|}{h\xi_{90} - a|\eta_1|}. \tag{10}$$

This equation is the new criterion for locating the STH. We justify this by noting that our derivation is based on the stability criterion of Phillips which, in turn, leads to Eq. (1) from which Eq. (10) is derived. The specific values of $\phi$ which can be obtained from (10) therefore give latitude boundaries between baroclinic wave stability and instability. The subtropical high is one such boundary.

Using the values of $\xi_{90}$, $\eta_0$ and $\eta_1$ obtained previously, the right-hand side of Eq. (10) has the value 0.28. In Fig. 3 we plot the function $\sin^2\phi \cos\phi$. The value 0.28 corresponds to a latitude $\phi \approx 36°$ which is in good agreement with the average location of the STH in the summer months (Beer *et al.*, 1977). In addition there is a second solution at $\phi \approx 70°$. This latitude is interpreted as the average boundary between the polar cell and the mid-latitude Rossby regime.

## 4. Discussion: Behavior of the subtropical high under the new criterion

We now show that the behavior of the average STH latitude based on the criterion of Eq. (10) is opposite to that predicted by the $Z$ criterion discussed in Section 2.

We first consider the effect of increasing $CO_2$ in the atmosphere. As discussed in Section 2, the increased greenhouse effect will tend to increase the average lapse rate and, by Eq. (A8), this will decrease the value of $\xi$. If we assume that this decrease in $\xi$ is proportionally the same at all latitudes, then the change will be observed as a change in the scale factor $\xi_{90}$. As an example, we assume, as in Section 2, that the lapse rate increases by 0.01°C km$^{-1}$. By Eq. (A8), this will decrease $\xi_{90}$ by $-8\times10^{-5}$ km$^{-1}$ and this, in turn, will increase the right-hand side of (10) by ~0.4%. From Fig. 3, we see that an increase in the right-hand side of (10) will increase the average location of the STH.[3] In our example the average latitude increase is ~0.08°. Since the

[3] A detailed analysis shows that $\Delta\phi \approx 14\Delta l_1$, where $\Delta l_1$ is the change in the lapse rate (°C km$^{-1}$) and $\Delta\phi$ the corresponding latitude change in the STH location.

motion of the ITD is obtained by multiplying by a factor of about 2.5, the latitude of the ITD is increased by ~0.2°. This is smaller by a factor of 3 and opposite in direction to the −0.6° change in latitude predicted by the $Z$ criterion in Section 2. The increase in ITD latitude predicted here corresponds to an increase of ~36 mm in rainfall in the Sahel when one takes into account the 180 mm per degree of latitude rainfall gradient south of the ITD.

As discussed in Section 2, an increase in particulate matter in the atmosphere at high latitudes will tend to increase the meridional temperature gradient and lead to an increase in the average value of $|\eta|$ according to Eq. (A13). Since our analysis indicates that $\eta$ contains two parameters, $\eta_0$ and $\eta_1$, we consider the effect of changing each of these separately.

We assume that the meridional temperature gradient increases by 0.01°C $(1000 \text{ km})^{-1}$ as in Section 2 for the $Z$ criterion. If the change affects $|\eta_0|$, then from (A13) this parameter would increase by $4\times10^{-8}$ km$^{-1}$ and this, in turn, will increase the right-hand side of (10) by ~0.6%. This increase produces an increase in the average location of the STH[4] by ~0.12°. The ITD latitude then increases by ~0.3°.

The same increase in the meridional temperature gradient may instead cause an increase in $|\eta_1|$, which from (A13) would amount to $4\times10^{-8}$ km$^{-1}$. An increase in $|\eta_1|$ will again produce an increase in the right-hand side of (10), this time by 0.16% and therefore an increase in the location of the STH[5] by 0.04°. The increase in the ITD latitude would then be 0.1°.

In both cases, for either a change in $|\eta_0|$ or $|\eta_1|$, an increase in meridional temperature gradient causes an increase in the STH and ITD latitudes. The amount of increase is approximately the same in magnitude, but opposite in direction, to the changes in latitude obtained in Section 2 using the $Z$ criterion.

## 5. Summary

Smagorinsky (1963) has given an equation for the boundary between regions of baroclinic wave stability and instability [Eq. (1)] which relates the vertical and horizontal derivatives of the potential temperature to the latitude at which they are measured. Using the data of Peixoto (1960), he has shown that the relationship holds fairly well for latitudes between 30° and 60°.

Bryson (1973, 1974) extended this interpretation of Eq. (1) and has assumed that it gives the average latitude of the subtropical high when one uses typical values of the potential temperature derivatives. This he called the $Z$ criterion. He shows that increases in carbon dioxide and particulate matter in the atmosphere will change the potential temperature derivatives in (1) in such a way as to decrease the latitude of the STH. This in turn causes a decrease in the latitude of the Intertropical Discontinuity and therefore a decrease in the expected amount of rainfall in the Sahel.

In this paper, we have re-examined the potential temperature data and have shown that smooth functions can fit the data for the derivatives $\xi$ and $\eta$ at all latitudes. When these functions are put into Eq. (1), a new criterion [Eq. (10)] results for locating the latitude of the STH. In general, Eq. (10) yields two values of latitude, each of which is a boundary between baroclinic wave stability and instability. The lower value is the average STH latitude (~36° in the summer) and the higher value is interpreted to be the boundary between the polar and Rossby regimes (~70°).

Increasing amounts of atmospheric pollutants will cause changes in the potential temperature derivative parameters $\xi_{90}$, $\eta_0$ and $\eta_1$ in Eq. (10). These changes, in turn, will cause changes in the average latitude of the STH. Using Eq. (10), we find that increasing amounts of $CO_2$ and particulate matter in the atmosphere cause the average location of the STH to move to higher latitudes. Consequently, the ITD also moves to more northern latitudes in the summer which, in turn, increases the amount of expected rainfall in the Sahel. These results are opposite those obtained using the $Z$ criterion and do not support the conclusion reached by Bryson (1973) that increasing amounts of industrial, agricultural and volcanic activity is a cause of the recent severe drought in the Sahel of West Africa.

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APPENDIX

## Mathematical Details

Here we give the mathematical details necessary for the discussions in Sections 2 and 4. We first note that the partial equivalent potential temperature is given by

$$\theta_{PE}=K\theta, \qquad \text{(A1)}$$

where $\theta$ is the potential temperature and (Haltiner and Martin, 1957)

$$K=\exp\left\{\frac{18}{29}e_0\frac{L_v}{C_p}\frac{\rho}{pT}\exp\left[\frac{L_v}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)\right]\right\}. \qquad \text{(A2)}$$

The function $K$ introduces modifications in the potential temperature due to the presence of moisture in

[4] For changes in $|\eta_0|$, $\Delta\phi\approx18\Delta l_2$, where $\Delta l_2$ is the change in meridional temperature gradient [°C $(1000 \text{ km})^{-1}$] and $\Delta\phi$ is the corresponding latitude change in the STH location.

[5] For changes in $|\eta_1|$, $\Delta\phi\approx5\Delta l_2$.

the air. The various symbols in (A2) are defined as follows:

- $p$ pressure
- $T$ absolute temperature
- $e_0$ saturation vapor pressure at temperature $T_0$
- $L_v$ latent heat of vaporization
- $\rho$ relative humidity
- $R$ 0.461 J kg$^{-1}$ K$^{-1}$

In practice we are interested in the logarithmic derivative of $\theta_{PE}$, and from (A1) this is given by

$$\xi=\frac{\partial(\ln\theta_{PE})}{\partial z}=\frac{\partial(\ln K)}{\partial z}+\frac{\partial(\ln\theta)}{\partial z}. \tag{A3}$$

From data given by Smagorinsky (1963) for both $\theta_{PE}$ and $\theta$ the logarithmic derivative of $\theta$ can be determined, i.e.,

$$\frac{\partial(\ln\theta)}{\partial z}=\frac{1}{\theta(500)}\left[\frac{\theta(250)-\theta(1000)}{h}\right] \tag{A4}$$

which is analogous to Eq. (5). Using the average values of $\theta$ for the summer months in Eq. (A4), we obtain the results shown in the first column of Table A1. From (A3) we see that the logarithmic derivative of $K$ can be obtained empirically by subtracting the logarithmic derivative of $\theta$ from $\xi$. Subtracting the values in the first column of Table A1 from the summer values of $\xi$ given in Table 1, we obtain the results for the logarithmic derivative of $K$ shown in the second column of Table A1. By examining the two columns of Table A1, we find that they satisfy the approximate relationship

$$\frac{\partial(\ln K)}{\partial z}\approx\frac{\partial(\ln\theta)}{\partial z}-C_1,$$

where $C_1$ is a constant approximately equal to 0.030 km$^{-1}$. Eq. (A3) then becomes

$$\xi\approx 2\frac{\partial(\ln\theta)}{\partial z}-C_1. \tag{A5}$$

The potential temperature is given by (Beer, 1974)

$$\theta=T\left(\frac{p}{p_g}\right)^{(1-\gamma)/\gamma}, \tag{A6}$$

where $p_g$ is some reference pressure and $\gamma=1.4$. The logarithmic derivative of the potential temperature is then

$$\frac{\partial(\ln\theta)}{\partial z}=\left(\frac{1-\gamma}{\gamma}\right)\frac{1}{p}\frac{\partial p}{\partial z}-\frac{1}{T}l_1, \tag{A7}$$

where $l_1=-\partial T/\partial z$ is the vertical lapse rate. Using (A5) and (A7), it can be seen that a small change $\Delta l_1$ in the lapse rate gives rise to a change in $\xi$ given by

$$\Delta\xi\approx-\frac{2}{T}\Delta l_1, \tag{A8}$$

where $T$ is taken to be 250 K, the approximate temperature at a height of 500 mb.

A similar analysis can be performed for changes in the meridional temperature gradient. From (A1) we have

$$\eta=\frac{\partial(\ln\theta_{PE})}{\partial y}=\frac{\partial(\ln K)}{\partial y}+\frac{\partial(\ln\theta)}{\partial y}. \tag{A9}$$

We again use the potential temperature data for the summer months given in Smagorinsky (1963), to derive an equation analogous to Eq. (6), i.e.,

$$\frac{\partial(\ln\theta)}{\partial y}=\frac{1}{\theta(\phi)}\frac{\theta(\phi+10°)-\theta(\phi-10°)}{2000\text{ km}}. \tag{A10}$$

Here $\phi$ is the latitude at which the logarithmic derivative of $\theta$ is being calculated and all the values of $\theta$ are taken at 500 mb. The results are shown in the first column of Table A2. From (A9), if we sub-

TABLE A1. The vertical logarithmic derivatives of the potential temperature and of the factor $K$ in Eq. (A1), as a function of latitude $\phi$, for the average over the summer months. The method used for calculating the logarithmic derivations is given in the Appendix. The values of $\theta$ used in the calculation are from Smagorinsky (1963).

| $\phi$ (deg) | $\frac{\partial(\ln\theta)}{\partial z}\times 10^3$ (km$^{-1}$) | $\frac{\partial(\ln K)}{\partial z}\times 10^3$ (km$^{-1}$) |
|---|---|---|
| 0 | 14.6 | −15.9 |
| 10 | 16.3 | −16.6 |
| 20 | 16.4 | −15.4 |
| 30 | 17.9 | −12.8 |
| 40 | 20.2 | − 9.1 |
| 50 | 24.2 | − 7.0 |
| 60 | 27.9 | − 4.1 |
| 70 | 32.9 | − 3.9 |

TABLE A2. The horizontal logarithmic derivatives of the potential temperature and of the factor $K$ in Eq. (A1), as a function of latitude $\phi$, for the average over the summer months. The values of $\theta$ used in the calculation of the logarithmic derivatives are from Smagorinsky (1963).

| $\phi$ (deg) | $\frac{\partial(\ln\theta)}{\partial y}\times 10^5$ (km$^{-1}$) | $\frac{\partial(\ln K)}{\partial y}\times 10^5$ (km$^{-1}$) |
|---|---|---|
| 10 | −0.46 | −0.44 |
| 20 | −0.62 | −0.29 |
| 30 | −0.93 | −0.29 |
| 40 | −1.56 | −0.29 |
| 50 | −1.76 | −0.14 |
| 60 | −1.62 | −0.31 |

tract these results from the values of $\eta$ given in Table 1 for the summer, we obtain empirical values for the logarithmic derivative of $K$. These are shown in the second column of Table A2. Since the logarithmic derivative of $K$ is seen to be approximately constant as a function of latitude, from (A9) we can write

$$\eta \approx \frac{\partial(\ln\theta)}{\partial y} + C_2. \tag{A11}$$

Taking the logarithmic derivative of the potential temperature in (A6) we obtain

$$\left|\frac{\partial(\ln\theta)}{\partial y}\right| = \left(\frac{1-\gamma}{\gamma}\right)\frac{1}{p}\frac{\partial p}{\partial y} + \frac{1}{T}l_2, \tag{A12}$$

where $l_2$ is the absolute value of the meridional temperature gradient. Using Eq. (A12) in (A11), we find that for a small change $\Delta l_2$ in the absolute value of the meridional temperature gradient there is a change in $|\eta|$ given by

$$\Delta|\eta| \approx \frac{1}{T}\Delta l_2, \tag{A13}$$

where $T \approx 250$ K, the approximate temperature at a height of 500 mb.

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