A Direct Method for Obtaining Ballistic Densities from Satellite Radiance Observations

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ABSTRACT

A nonstatistical method for obtaining ballistic densities directly from satellite radiance observations is derived. The method takes advantage of the fact that both the ballistic density and the satellite radiances depend upon weighted vertical integrals of the atmospheric temperature. Tests of the method on realistically simulated satellite radiances indicate root-mean-square retrieval errors of $\frac{1}{4}$–$\frac{1}{3}$ of the standard deviation of ballistic density for individual months. The method thus appears to be suitable for application to areas of the globe with a paucity of conventional radiosonde observations.

1. Introduction

Satellite radiance observations have been used for some years now for remote sounding of atmospheric temperature profiles. Because of the physical nature of the atmospheric radiative transfer process, the satellite observations are basically sensitive to the temperatures of broad atmospheric layers rather than to temperatures at individual levels. Quantitative estimates of the deterioration in the accuracy of temperature retrievals as the vertical resolution is improved have been presented by Conrath (1972). In view of this basic theoretical limitation on vertical resolution, we have looked for atmospheric parameters of interest that depend upon vertically integrated temperatures. Such parameters would appear to be ideally suited for satellite sounding applications. One such parameter is the thickness of atmospheric layers of prescribed pressure interval; Fleming (1972) has explored the possibility of obtaining this parameter directly from satellite radiance observations. As pointed out by Ruggles (Elsberry and Martin, 1971), another such parameter is the ballistic density.

The ballistic density is essentially a vertically integrated weighted density of the atmosphere and is a quantity that is used, for example, in computing trajectories of vehicles reentering the atmosphere. Elsberry and Martin (1971) and Elsberry *et al.* (1972) have already shown that this quantity can be retrieved from satellite radiance observations with the use of regression techniques. Such techniques require simultaneous sets of satellite radiance observations and conventional radiosonde observations for their development. For certain regions of the globe it may not be possible to obtain the required sets of data. In the present paper, we derive a technique for direct retrieval of ballistic density from satellite radiance observations and test it on a set of realistically simulated satellite radiances.

2. Development of method

The ballistic density may be written as

$$D=\int_0^{x_0} F\rho dx, \tag{1}$$

where D is the ballistic density, F the ballistic density weighting factor and, ρ the density; x, the vertical coordinate, is given by

$$x=-\ln(p/p_g), \tag{2}$$

where p is pressure and p_g the surface pressure, and x_0 represents a pressure level high in the atmosphere where F is effectively zero. The shape of the weighting function is such that it has a maximum in the middle of the atmosphere and decreases to very small values at the surface and at high altitudes. An example of a ballistic density weighting function is shown in Table 1 (after Elsberry and Martin, 1971).

Eq. (1) shows that the ballistic density is a vertical integral of the weighted density through the depth of the atmosphere. The satellite radiance at each observing wavelength can be expressed as a vertical integral of the weighted temperature through the depth of the atmosphere. The weighting functions for the temperature are narrower that that for the density and are displaced in height depending on the observing wavelength. Since atmospheric densities are related to temperature through the equation of state, it seems reasonable to attempt to obtain the ballistic density directly from radiance observations. Furthermore, since ballistic

TABLE 1. *Ballistic density weighting function F.*

Pressure (mb)	F	Pressure (mb)	F
1000		20	
	0.1680		0.0323
850		10	
	0.2421		0.0252
700		7	
	0.4066		0.0193
500		5	
	0.4853		0.0141
400		3	
	0.4637		0.0099
300		2	
	0.4448		0.0061
250		1	
	0.3469		0.0039
200		0.7	
	0.2805		0.0027
150		0.5	
	0.2114		0.0018
100		0.3	
	0.1553		0.0010
70		0.2	
	0.1171		0.0007
50		0.1	
	0.0836		0.0003
30		0.07	
	0.0673		
20			

density is an integral quantity, as is radiance, errors in the determination of ballistic density should be less than the errors in the determination of point values.

Let us define a ballistic density for a standard atmosphere or climatological mean atmosphere as

$$D_s=\int_0^{x_0} F\rho_s dx, \tag{3}$$

where ρ_s is the standard or climatological density profile.

The deviation of the actual ballistic density at a particular time from the climatological value can be written as

$$D'=D-D_s=\int_0^{x_0} F(\rho-\rho_s)dx. \tag{4}$$

With the use of the equation of state $(\rho-\rho_s)$ may be written as

$$(\rho-\rho_s)=\frac{p}{R}\left(\frac{T_s-T}{TT_s}\right), \tag{5}$$

where R is the gas constant and T temperature.

We assume that T in the denominator is equal to T_s. Since the temperature does not vary by more than a few percent about the climatological mean, this assumption introduces, at most, an error of a few percent in the *deviation* $(\rho-\rho_s)$. And since the ballistic density deviation is a vertical integral of $(\rho-\rho_s)$, errors at one level might cancel errors at another level thus further reducing the final error introduced into the determination of the ballistic density deviation by this assumption.

With this assumption Eq. (4) becomes

$$D'=\int_0^{x_0} W(T_s-T)dx, \tag{6}$$

where

$$W=Fp/RT_s^2. \tag{7}$$

We turn now to the satellite observed radiance, which may be written as

$$I_i=B_i(T_g)\tau_{ig}+\int_0^{x_0} B_i(T)(d\tau_i/dx)dx, \tag{8}$$

where i is an index representing wavenumber, B the Planck function and τ the transmittance.

The radiance of a standard or climatological atmosphere is

$$I_{is}=B_i(T_{gs})\tau_{ig}+\int_0^{x_0} B_i(T_s)(d\tau_i/dx)dx. \tag{9}$$

Using the definition (see, e.g., Fleming, 1972)

$$r_i=[I_i-B_i(T_g)\tau_{ig}]-[I_{is}-B_i(T_{gs})\tau_{ig}], \tag{10}$$

we obtain

$$r_i=\int_0^{x_0} [B_i(T)-B_i(T_s)](d\tau_i/dx)dx, \tag{11}$$

which, with the assumption

$$\Delta B_i=\frac{dB_i(T_s)}{dT}\Delta T, \tag{12}$$

becomes

$$r_i=\int_0^{x_0} K_i(x)(T-T_s)dx, \tag{13}$$

where

$$K_i(x)=\frac{dB_i(T_s)}{dT}\frac{d\tau_i}{dx}. \tag{14}$$

The radiances I_i are observed by the satellite. We assume that the surface temperature T_g, which is needed to evaluate the boundary term in (10), can be obtained from either satellite window or conventional observations. Then, since the quantities in (10) depending on the standard or climatological atmosphere can be computed from (9), r_i can be evaluated. Comparison of Eqs. (6) and (13) shows that they both represent weighted vertical integrals of temperature deviation. Multiplication of Eq. (13) by a set of coefficients c_i and summation over the N observing wavenumbers yields

$$\sum_{i=1}^{N} c_i r_i=\int_0^{x_0}\left(\sum_{i=1}^{N} c_i K_i\right)(T-T_s)dx. \tag{15}$$

Comparison of the right-hand sides of (6) and (15) indicates that if it were possible to find a set of coefficients c_i such that $\sum_{i=1}^{N} c_iK_i$ was exactly equal to W at all values of x then we could obtain $-D'$, the negative of the ballistic density deviation, directly and exactly from the left-hand side of Eq. (15). In practice such a set of coefficients cannot be found but we can find the best approximation to such a set of coefficients by minimizing the form

$$J=\int_0^{x_0}\left(\sum_{i=1}^{N} c_iK_i-W\right)^2 dx. \qquad (16)$$

The solution to this minimization problem is

$$\mathbf{c}=\mathbf{S}^{-1}\mathbf{u}, \qquad (17)$$

where

$$S_{ij}=\int_0^{x_0} K_i(x)K_j(x)dx, \qquad (18)$$

$$u_i=\int_0^{x_0} W(x)K_i(x)dx. \qquad (19)$$

Thus, once the c_i are determined, the estimated deviation of the ballistic density from the climatological value can be obtained directly from the satellite radiance observations by means of the equation

$$\hat{D}'=-\sum_{i=1}^{N} c_ir_i. \qquad (20)$$

The estimate of the actual ballistic density D is then

$$\hat{D}=D_s+\hat{D}'. \qquad (21)$$

3. Simulation of radiances

We have tested the technique on realistically simulated radiances. The radiances can be simulated with the use of Eq. (8), using numerical integration for evaluating the integral, if the vertical temperature profile and the transmittances are available. We have simulated radiances that would be observed by the six CO_2 band wavenumbers of the NOAA-2 VTPR radiometer (see Table 2), using transmittances that have been given by McMillin *et al.* (1973). Radiances were simulated daily for the 1200 GMT temperature profile obtained from the Bet Dagan radiosonde station (32.0°N, 34.5°E, elevation 30 m) of the Israel Meteorological Service for the months of January, April, July and October of 1973. These temperature profiles included data from both standard and significant levels. To take advantage of these complete temperature profiles in the numerical integration of Eq. (8), they were combined with the transmittance profiles in the following way. Temperatures were interpolated linearly in ln$p$ to the 50 levels at which transmittances were available. The transmittances were then interpolated linearly in ln$p$ to all levels at which temperature observations were available. The combined levels were used in the numerical integration. The level x_0 was taken at 0.02 mb and temperatures were extrapolated to this level from the highest level for which a radiosonde temperature was available. The extrapolation was based upon the 30°N standard atmosphere temperature model for the particular month (*U. S. Standard Atmosphere Supplements*, 1966), the April and October standard temperatures being obtained from the average of the winter and summer values. A similar computation of radiances was performed for the mean temperature profile for each of the four months, based upon the preceding 5-year record of radiosonde observations for Bet Dagan. These sets of radiances represented the standard or climatological radiances.

TABLE 2. Wavenumbers of NOAA-2 VTPR radiometer and assumed standard deviation of noise σ_e(mW cm^{-2} s^{-1} sr^{-1} cm^{-1}) at each wavenumber.

Wavenumber (cm^{-1})	σ_e
747.6	0.45
725.9	0.38
709.0	0.33
695.2	0.28
678.0	0.20
668.2	0.20

To simulate the effect of errors due to instrumental effects, clear radiance extraction from cloud-contaminated radiances, and other sources, random noise [distributed normally with standard deviations shown in Table 1 (after Fleming, 1972)] was added to the computed radiances.

An additional source of error is the uncertainty in the surface temperature T_g that is used in evaluating the boundary term in Eq. (10). To simulate this error, random noise, again normally distributed, with a standard deviation of 1°C was added to the surface temperatures when computing the boundary term in (10).

4. Results

Ballistic densities obtained directly from the simulated radiances with the use of (20) and (21) were compared to the actual ballistic densities computed from the observed temperature profiles with the use of (1). Table 3 summarizes the results obtained. The standard values of ballistic density D_s are computed from the standard temperature profiles, which are based upon the average temperature profile for the month during the 5-year period 1968–72.

The results for error free radiances indicate that the ballistic density can be retrieved with an rms error between 2 and 6, while the standard deviation of ballistic density (σ_D) has values ranging from 14 to 39 (units: 10^{-7} g cm^{-3}).

TABLE 3. Results of tests of direct method for ballistic density determination from satellite observations (units: 10^{-7} g cm^{-3}).

Month	D_s*	σ_D**	Root-mean-square error Error free radiances	Root-mean-square error Radiances with errors
January	4436	39.1	5.9	10.8
April	4391	32.4	6.0	10.8
July	4236	14.5	1.9	4.6
October	4357	28.9	4.1	7.4

* D_s = standard ballistic density for the month.
** σ_D = standard deviation of ballistic density.

When realistic errors are included in the simulations, the accuracy is degraded but the results are still extremely encouraging. For example, for the month of January $\sigma_D = 39$, while the rms error in retrieved ballistic density is only 11. The other months also show that the rms error is only about $\frac{1}{4}$–$\frac{1}{3}$ of the standard deviation of the ballistic density.

These results using a direct retrieval method may be compared to the results obtained by Elsberry *et al.* (1972), who used a regression technique. They developed their regression relationships from sets of Nimbus 3 SIRS-A clear-column radiances and ballistic densities computed from radiosonde data over Eurasia. These regression relationships were then applied to an independent set of satellite observations over Eurasia. The ballistic densities retrieved from the satellite observations were then compared to those computed from radiosonde observations for the same area. In one set of 24 comparisons the rms error in ballistic density was about one-third of the standard deviation of ballistic density for the set; in another set of 16 comparisons, the rms error was about one-fourth of the standard deviation of ballistic density. These errors are very similar to those that we obtain using the direct method on realistically simulated radiances.

5. Conclusions

Based on simulated observations, it appears that the direct technique for ballistic density determination from satellite radiance observations that has been developed here is a viable alternative to the regression technique used by Elsberry *et al.* (1972). The advantage of a direct method over a regression technique is that there is no requirement for large sets of simultaneous and colocated satellite radiance and conventional radiosonde observations in order to develop the specification equations. On the other hand, the direct technique requires a knowledge of the atmospheric transmittances; to the extent that the true values of the transmittances are uncertain, additional errors will be introduced into the direct technique. It would be of interest to compare both methods using a large set of simultaneous and colocated radiance and radiosonde observations.

This work also suggests that similar procedures can be used for the direct inference from satellite radiance observations of other meteorological parameters that are related to the vertical distribution of atmospheric temperature.

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