# An Improved Method for Determining Measurement System Noise Spectra

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## ABSTRACT

A simultaneous equations solution technique is developed to estimate the noise variance density spectra of three or more independent measurement systems observing the same input. Using dew-point data collected during a National Hail Research Experiment aircraft intercomparison flight, this technique is compared to a previously developed method and is found to be superior.

## 1. Introduction

A cross-spectral analysis method was developed by Duchon and Goerss (1976) to estimate the noise variance spectra of three or more independent measurement systems observing the same input. The heart of that method was the modeling of the noise as a first-order autoregressive process. This was accomplished by using a cumbersome technique which minimized the differences between the observed average noise spectra and the average of the individual model noise spectra. Furthermore, the method depended upon the user arbitrarily choosing values for each model noise spectrum at the zero frequency.

In this note the previously developed cross-spectral method is simplified so that an estimate of each system's noise spectrum is directly obtained by solving a set of linear equations. The time-consuming modeling process is avoided and no arbitrary choices of parameters must be made by the user. The simplified technique is then applied to one of the cases treated in the aforementioned paper, and the results are compared with those obtained using the modeling process.

## 2. Theoretical development

The basis for this simplified analysis scheme is the following equation set developed by Duchon and Goerss for an intercomparison among three independent measurement systems $(X, Y, Z)$ with identical response functions each sampling the same input signal:

$$\left.\begin{aligned} \bar{C}_{N_xN_x}(f)/2+\bar{C}_{N_yN_y}(f)/2 &\approx \bar{A}_{xy}(f)\\ \bar{C}_{N_xN_x}(f)/2+\bar{C}_{N_zN_z}(f)/2 &\approx \bar{A}_{xz}(f)\\ \bar{C}_{N_yN_y}(f)/2+\bar{C}_{N_zN_z}(f)/2 &\approx \bar{A}_{yz}(f) \end{aligned}\right\}. \qquad (1)$$

The estimated average noise spectrum between systems $X$ and $Y$ at frequency $f$ is denoted by $\bar{A}_{xy}(f)$ and is determined by subtracting the estimated co-spectrum of the outputs of systems $X$ and $Y$ from the average of the estimated output autospectra of those systems. In practice the actual input signal to each measurement system will be different due to spatial separation. These differences along with those caused by the fact that the actual response functions of the measurement systems will not be identical will be embedded in the average noise spectra. The estimated noise spectra at frequency $f$ for systems $X$, $Y$ and $Z$ are denoted by $\bar{C}_{N_xN_x}(f)$, $\bar{C}_{N_yN_y}(f)$ and $\bar{C}_{N_zN_z}(f)$, respectively. These equations are approximations because the co-spectra of the noises between the systems are zero only in the expected sense. Because of this Duchon and Goerss used the solution of (1) merely as guidance in their modeling process. When dealing with a finite sample, noise co-spectra deviate significantly from zero at some frequencies, and noise spectra computed by solving (1) display wild fluctuations like those graphically displayed in Figs. 12 and 13 of Duchon and Goerss (1976). If one is to use (1) to estimate the population noise spectra such behavior must be eliminated.

While the spectra of meteorological variables often show rapid and large changes of variance with frequency, the noise spectra of the systems measuring the variables often display a more uniform distribution of variance. One must use a spectral estimator with a narrow bandwidth in order to accurately estimate the peaks in the former spectra, but one should be able to use an estimator with a much wider bandwidth in order to estimate the noise spectra. Furthermore, the use of such an estimator will improve the approximation made

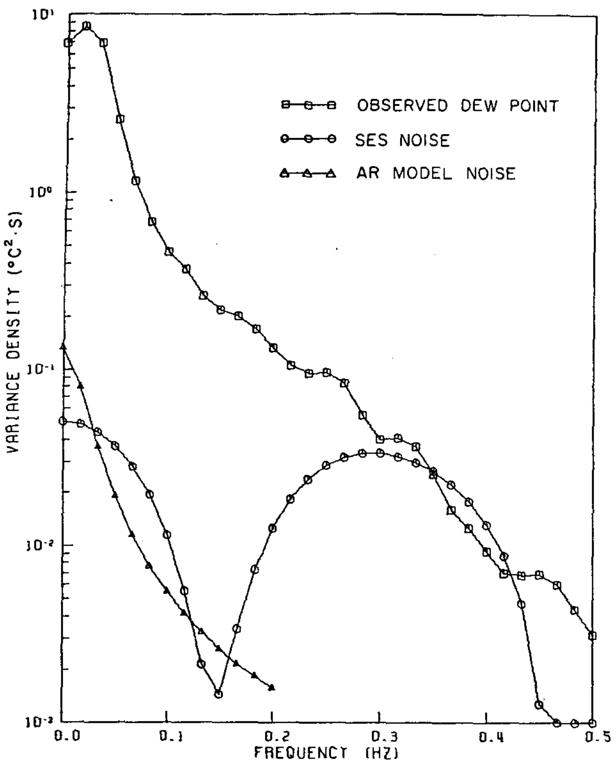

FIG. 1. NCAR Buffalo dew-point spectra for 1203:16–1208:15 MDT 1 June 1972.

in (1), since by averaging over a wider bandwidth one would expect the sample estimates of the co-spectra of the noises to approach more closely their expected value of zero. Therefore, the new scheme for estimating the system noise spectra consists simply of solving (1) after estimates of the average noise spectra are computed using a spectral window with a suitably wide bandwidth. Hereafter, this technique will be referred to as the SES (Simultaneous Equation Solution) method.

## 3. Application

The SES method was applied to dew-point data collected during one of the National Hail Research Experiment intercomparison flights analyzed by Duchon and Goerss (1976). The flight investigated was flown at 7000 ft MSL on 1 June 1972. The participating aircraft were the NCAR Buffalo (326D), the NCAR Queen Air (304D), and the University of Wyoming Queen Air (10UW). The data analyzed were collected at 1 s intervals between 1203:16 MDT and 1208:15 MDT.

Before any spectra were computed the data sets were aligned in time to the Buffalo (the lead aircraft in the flight) by measuring the time displacement from the zero of the peak in the cross-covariance functions. Trend removal was not performed.

The appropriate autospectra and co-spectra were computed using a Tukey spectral window with a band-

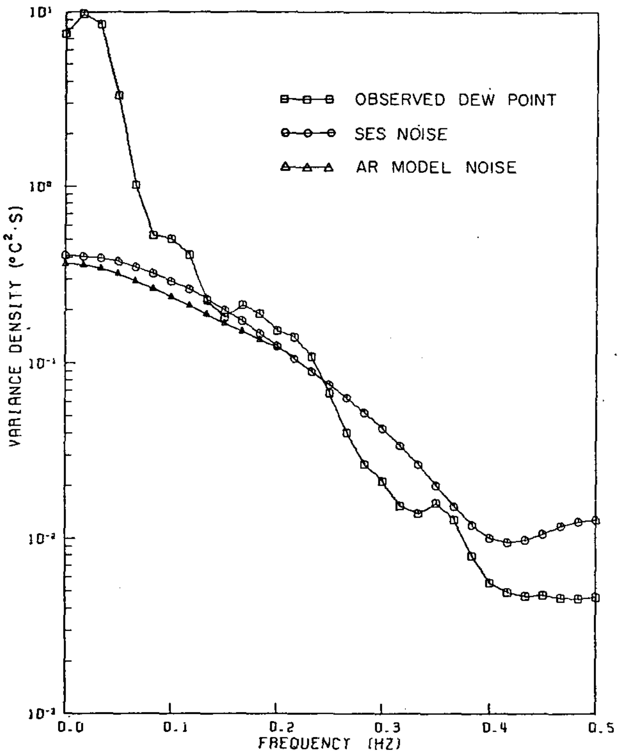

FIG. 2. As in Fig. 1 except for the NCAR Queen Air (304D).

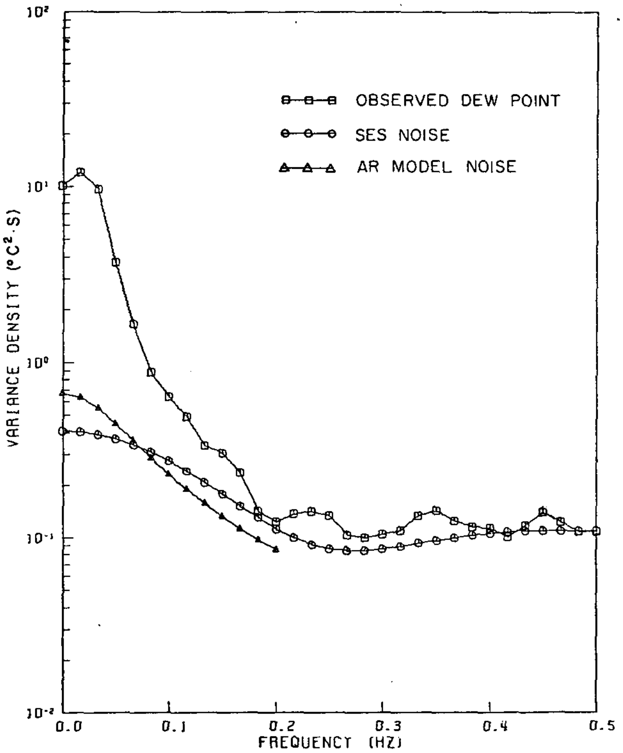

FIG. 3. As in Fig. 1 except for the University of Wyoming Queen Air.

width of 0.222 Hz (Jenkins and Watts, 1968, p. 256), and (1) was solved giving the estimated noise spectra plotted in Figs. 1, 2 and 3 for the Buffalo, NCAR Queen Air and Wyoming Queen Air, respectively. Autospectra for each system were also computed using a Tukey spectral window with a 0.044 Hz bandwidth and plotted in Figs. 1–3 along with the estimated noise spectra obtained by the autoregressive modeling technique. The slight differences between the autospectra displayed in Figs. 1–3 and those shown in Figs. 2–4 of Duchon and Goerss (1976) are due to the fact that biased and unbiased estimates of the autocovariance function (Jenkins and Watts, 1968, p. 174) were used in the respective cases. The fluctuations displayed by the Buffalo noise spectrum in Fig. 1 are due to the approximate nature of (1). The Buffalo possesses a total noise variance nearly an order of magnitude less than that of the Queen Airs; its noise spectrum, therefore, is much more sensitive to the effects of the non-zero values of the sample noise cospectra. In a more extensive comparison of the autoregressive modeling technique and the SES method, Goerss (1975) found from simulation studies in which one system possessed a small noise variance compared to that in the other two systems that neither method can effectively determine the shape of the former system's noise spectrum. However, the SES method was found to more accurately estimate the total noise variance for that system as well as for the other two systems. The average error in the noise variance estimate for the former system was 7.5% for the SES method and 12.6% for the autoregressive modeling technique while those found for all three systems were 4.4% and 12.8%, respectively. As one can see in Figs. 2 and 3 the absolute differences between the autospectra and the noise spectra are quite small above 0.2 Hz for both Queen Airs. Between 0.35 and 0.43 Hz in Fig. 1 and above 0.25 Hz in Fig. 2, the values of the estimated noise spectra are actually greater than those of the estimated autospectra. In Fig. 3 it can be seen that at frequencies >0.25 Hz the values of the noise spectrum for the University of Wyoming Queen Air are almost an order of magnitude greater than those for the other two aircraft. Thus at those frequencies the effect of non-zero sample noise cospectra is much more pronounced on the estimated noise spectra of those two systems and the observed paradox is produced Following the procedure outlined by Duchon and Goerss, the noise spectra were used to compute 95% confidence limits for each system's noise. The resulting confidence limits were almost identical to those found by Duchon and Goerss using the autoregressive modeling technique.

Thus, the SES method is a refinement of the cross-spectral analysis method developed by Duchon and Goerss (1976) to estimate the noise variance spectra of measurement systems. Since it requires very little computation time and no arbitrary decisions to be made by the user, it is a better noise estimation technique from time, understanding and economic points of view.

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## REFERENCES

Duchon, C. E., and J. S. Goerss, 1976: Variance spectrum analysis of aircraft dew-point measurement system noise. *J. Appl. Meteor.*, **15**, 77–93.

Goerss, J. S., 1975: Evaluation of a simultaneous equations technique for the determination of aircraft measurement system noise. M.S. thesis, Dept. of Meteorology, University of Oklahoma.

Jenkins, G. M., and D. G. Watts, 1968: *Spectral Analysis*. Holden-Day, 525 pp.