Iterative Correction for Attenuation of 5 cm Radar in Rain[1]

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ABSTRACT

The attenuation of 5 cm radar in rain is investigated theoretically for stratiform and thunderstorm drop size distributions. An iterative attenuation estimation scheme is presented. The effects of attenuation on radar precipitation measurements and the capabilities of the attenuation estimation technique are considered for a variety of hypothetical storm sizes and errors in radar calibration, assumed temperature and assumed drop size distribution.

This study indicates that 5 cm radar is an adequate precipitation measuring radar for storms under about 50 dBZ, and that if calibrated correctly and used with the iterative attenuation correction scheme, the 5 cm radar can function moderately well up to about 60 dBZ. Radar calibration accuracy is seen to be a limiting criterion for attenuation correction. The results of this study point out the need for raingages in most situations requiring accurate rainfall measurement.

1. Introduction

Current use of 5 cm weather radars for meteorological research raises the question of attenuation of the 5 cm radar signal by hydrometeors. Here the subject is reviewed, and a method is presented which gives an estimate of the original unattenuated signal. The results are interpreted in terms of the ability of 5 cm radar to estimate rainfall amounts.

2. Attenuation of radar signals

For wavelengths longer than 3 cm the major attenuation of a radar signal is due to hydrometeors, with only minor effects from atmospheric gases (Hitschfeld and Bordan, 1954). If the hydrometeors are cloud or precipitation particles, then the attenuation is [Atlas, 1964, Eq. (33)]

$$K=0.4343\sum Q_t \quad [\mathrm{dB\ km^{-1}}], \tag{1}$$

where the summation is the total attenuation cross section per unit volume, and Q_t is in cm^2. Values of Q_t for water can be obtained from Herman *et al.* (1961) and for ice spheres from Herman and Battan (1961). In this study, we shall assume the hydrometeors are water drops. Since the measured values of Q_t for water are functions of temperature, both the temperature and particle size distribution must be known to calculate the attenuation of the radar signal.

Perhaps the most commonly used raindrop size distribution is that of Marshall and Palmer (1948), who for stratiform rain situations approximated the raindrop size distribution by

$$N_D=N_0e^{-\Lambda D}\ [\mathrm{cm^{-4}}], \tag{2}$$

where

$$N_0=0.08\ [\mathrm{cm^{-4}}],\ \Lambda=41R^{-0.21}\ [\mathrm{cm^{-1}}], \tag{3}$$

and R is the rainfall rate (mm h^{-1}). This drop spectrum is frequently accurate for stratiform precipitation drops >1 cm.

For thunderstorm rain, Sekhon and Srivastava (1971) used vertically pointing Doppler radar data to derive a similar exponential drop size distribution with

$$N_0=0.07R^{0.37}\ [\mathrm{cm^{-4}}],\ \Lambda=38R^{-0.14}\ [\mathrm{cm^{-1}}]. \tag{4}$$

These dropsize distributions are based on data in the range $1\leqslant R\leqslant 60$ mm h^{-1}; hence, their use should be limited to roughly those rainfall rates. Jones (1956), Mueller and Jones (1960) and others have reported thunderstorm drop size distributions which differ from the above in that they show a strong decrease in drop concentration for diameters <1 mm.

When the attenuation of a radar signal is calculated using Eq. (1) and the tables of Herman *et al.* (1961), curves such as those shown in Fig. 1 can be derived for the Marshall-Palmer (MP) dropsize distribution (right-hand set) and the Sekhon-Srivastava (SS) distribution (left-hand set). (These curves are off-set from each other for clarity in plotting.) These curves assume drop sizes ranging up from 0.35 mm, and include variations

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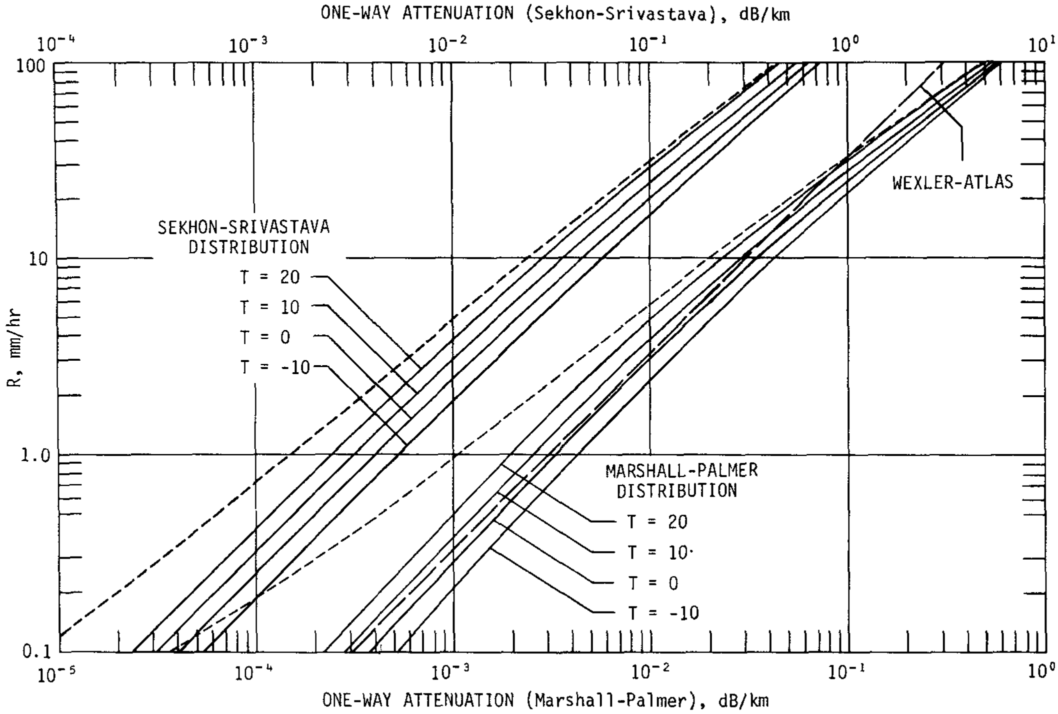

Fig. 1. One-way 5 cm radar attenuation curves for MP and SS drop size distributions, for temperatures between −10 and 20°C. The MP and SS curves are displaced 10 dB horizontally from each other for clarity in plotting. The short dashed lines represent the MP and SS curves at 20°C with drop size distributions limited to sizes larger than 1 mm. The long dashed line is the Wexler-Atlas (1963) attenuation relation for 5 cm radar at 0°C.

in temperature from −10 to +20°C. To test the effects of the observed lowering in drop concentration below 1 mm, additional curves (short dashed lines) are plotted for the MP and SS distributions for a temperature of 20°C, with drop sizes limited to larger than 1 mm. The long dashed line plotted within the MP curves is the attenuation relation $K = 0.0031\,R$ proposed by Wexler and Atlas (1963) for a Marshall-Palmer distribution at a temperature of 0°C. The Wexler-Atlas curve is fairly accurate for low rainfall rates, but produces large underestimates of attenuation for high rainfall rates. This can lead to sizeable errors if attenuation estimates are needed for precipitation measurement.

These curves show that for a 5 cm radar, attenuation is small for rainfall rates $\lesssim 10$ mm h^{-1} ($\sim$40 dBZ). It can be seen that errors in assumptions about the temperature of the attenuating drops may possibly be important in attempts to correct for attenuation. Errors in the assumed drop size distribution are likely to produce errors in attenuation estimate of about the same magnitude as a 10°C error in temperature. [This conclusion is not in agreement with Waldteufel's (1973) conclusion that temperature effects are small in comparison with differences between the MP and SS drop-size distributions.] Finally, for rainfall rates above 5–10 mm h^{-1} the uncertainty of the drop size distribution at sizes less than 1 mm is not important in comparison with likely errors in assumed drop size distribution and temperature.

The curvature of the K-R lines of Fig. 1 and their variation with temperature suggests that a simple power law K-R relation (e.g., Wexler and Atlas, 1963) is not likely to produce accurate results for a range of rainfall rates. Therefore, empirical K-R relations were derived (Table 1) which account for temperature variations in the range −10 to +20°C. Due to the limitations

Table 1. Empirical relations between one-way attenuation and rainfall rate using Marshall-Palmer (stratiform) and Sekhon-Srivastava (thunderstorm) raindrop size distributions, for temperatures between −10 and 20°C. One-way attenuation is in dB km^{-1} and rainfall rate is in mm h^{-1}.

Rainfall rate	Marshall-Palmer*	Sekhon-Srivastava*
<10	$K = (0.0045 - 0.00085T')\,R^{(0.98+0.02T')}$	$K = (0.0054 - 0.0010T')\,R^{(1.02+0.02T')}$
>10	$K = (0.0030 - 0.0007T')\,R^{(1.155+0.065T')}$	$K = (0.0043 - 0.0008T')\,R^{(1.115+0.025T')}$

* $T' = (T/10.) + 1.0$.

on the drop size distributions, use of these relations should be limited to reflectivities less than 60 dBZ.

3. Correction for attenuation

a. Past work

Fig. 1 shows that for 5 cm radar, attenuation can become substantial for high precipitation rates, particularly for widespread storms. This has been demonstrated by Geotis (1975) and Weible and Sirmans (1976) in their comparisons of 10 cm radar data with real or calculated 5 cm data. The continuing interest in deriving accurate precipitation estimates from radar has prompted attempts to correct the received radar signal for attenuation.

Hitschfeld and Bordan (1954) developed the radar equation in terms of the familiar *Z-R* and *K-R* relations, then derived a solution for rainfall rate. They showed that, for most reasonable cases, direct use of their rainfall equation (that is, correction for attenuation) is ill-advised for it produces errors which will dominate the determined rainfall amounts, particularly in the case of erroneous overestimates of received power (*viz.*, in the case of radar calibration errors). They therefore recommend against attempting to correct for attenuation, noting the absurd possibility of an infinite value of radar rainfall in cases of a large overestimation of the received power.

Sims *et al.* (1964), in an investigation of precipitation estimation using 3 cm radar, suggest a scheme of attenuation underestimation as an alternative to previous schemes. In this technique, *Z-R* and *K-R* (or *Z-K*) equations are used. In order to correctly solve for attenuation, the correct (unattenuated) radar reflectivity must be used in the *Z-K* equation. Sims *et al.* use the attenuated, measured value of radar reflectivity in the *Z-K* equation, thus resulting in an underestimate of the attenuation. They show several examples which demonstrate this technique to be an improvement over not making any correction for attenuation.

Geotis (1975) used 5 cm radar measurements and a *Z-K* relation based on drop size measurements to estimate attenuation corrections for the 5 cm radar. When the corrected 5 cm radar measurements were compared with concurrently collected 10 cm radar measurements, large residual attenuation errors remained. He therefore concluded that an acceptable correction for attenuation had not been reached and suggested that even with an improved *Z-K* relation, automatic attenuation corrections can easily become unreasonable.

b. Iterative attenuation estimation

The attenuation estimation scheme presented here is an extension of the method of Sims *et al.* (1964). As in that method, the attenuated radar reflectivity factor measurements $Z_a(r)$ at range r are converted to attenuated rainfall rates and to attenuation estimates $K_a(r)$. Appropriate assumptions concerning drop size distribution and temperature are made and the radar is assumed to be correctly calibrated. With these data, a new estimate of reflectivity factor can be produced with the relation

$$\log Z'(r)=\log Z_a(r)+2\sum_{x=1}^{r-1} K_a(x). \tag{5}$$

If these new reflectivity factor estimates $Z'(r)$ are used to derive revised attenuation estimates $K'(r)$, another set of reflectivity factor estimates can be obtained from

$$\log Z''(r)=\log Z_a(r)+2\sum_{x=1}^{r-1} K'(x). \tag{6}$$

In cases where the temperature and drop size distribution are known and the radar is correctly calibrated, this iterative procedure can be continued until successive iterations do not produce a significant change in the estimated total attenuation along the radar beam. For the purposes of these calculations, iterations were terminated when

$$2\sum_{x=1}^{r\,\max} K'(x)$$

changed by less than 1 dB from the previous iteration.

In cases when the attenuation correction produces an overestimate of attenuation, the attenuation estimation calculation will diverge after several iterations. This can result from radar calibration errors, an overestimate of temperature, or the assumption of a MP drop size distribution when the SS would have been more appropriate. In these cases the calculations were reinitiated, then stopped after a few iterations. The number of iterations on this second pass was empirically related to the number of iterations on the first pass.

4. Sample calculations

The attenuation correction scheme was tested for various hypothetical storm sizes, errors in radar calibration, random errors, and errors in assumed temperature and drop size distribution. Fig. 2 illustrates the effects of the attenuation correction scheme for four storm sizes, assuming no sources of error, an MP drop size distribution and a temperature of 0°C. The actual reflectivity profiles are indicated by the solid lines, the attenuated (measured) profiles by the short dashed lines, and the corrected profiles by the long dashed lines. For the curves in each test, the total range-integrated rainfall

$$R=\sum_{x=1}^{r\,\max} R(x)$$

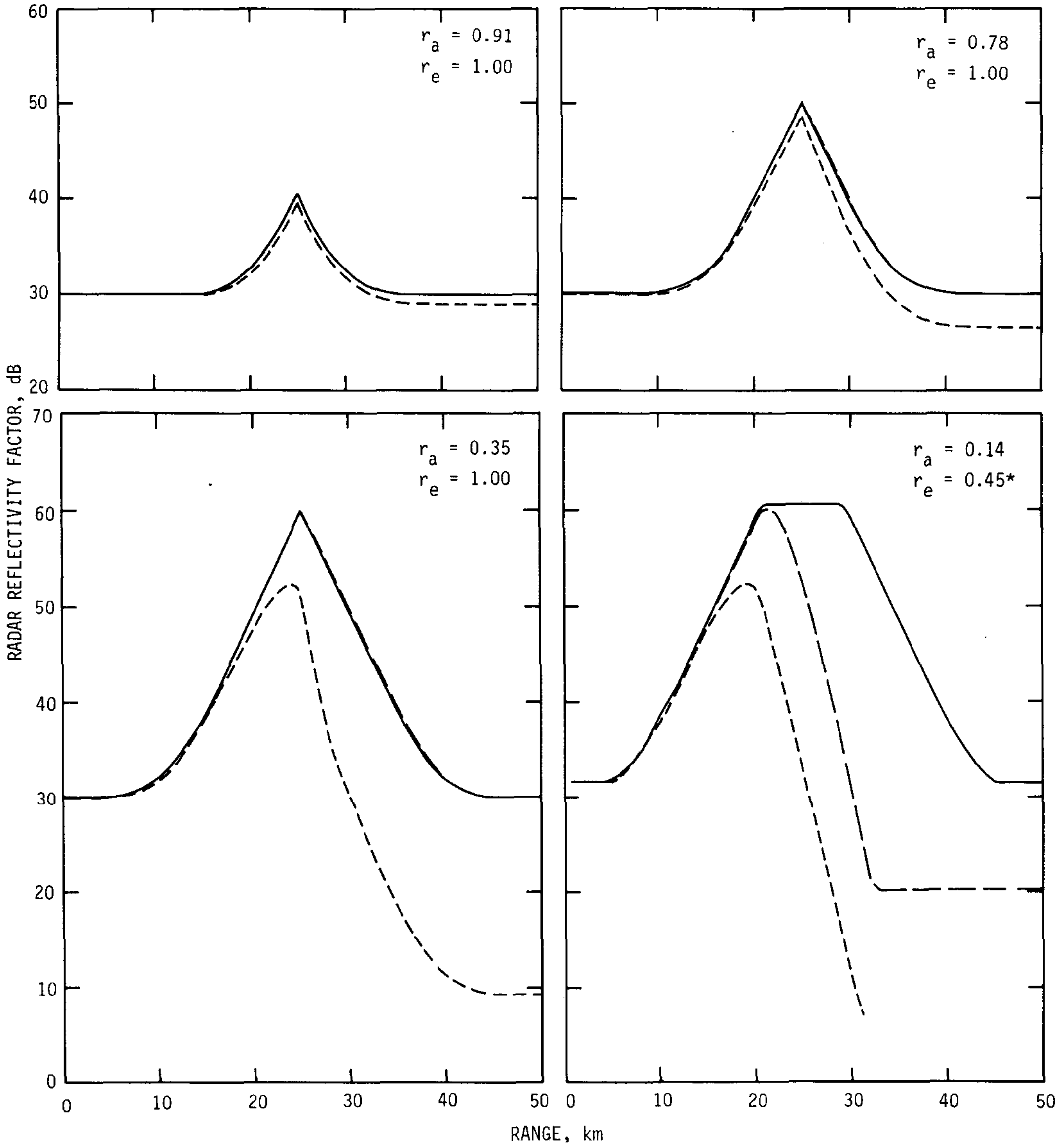

Fig. 2. Effects of storm size on 5 cm radar estimates of rainfall and attenuation. The solid curve represents the actual radar reflectivity factor values, the short dashed curve indicates the attenuated reflectivities measured by the radar and the long dashed curve indicates the results of the attenuation estimator.

was calculated for the true rainfall (R_t), the attenuated rainfall (R_a) and the corrected rainfall estimate (R_e). The values of the ratios $r_a=R_a/R_t$ and $r_e=R_e/R_t$ are presented in Fig. 2 and Table 2 as a convenient means of evaluating the capabilities of the attenuation correction scheme in the context of rainfall estimation. This figure shows that for 5 cm radar, attenuation is not a significant problem for storms with radar reflectivity maxima $\lesssim 40$ dBZ. In addition, it appears that the iterative attenuation estimation scheme works well, even for storms with 60 dBZ maximum reflectivities. For the extended 60 dBZ maximum storm the iterative solution diverged and had to be terminated prematurely. This is due to inaccuracies in the K-R relation (Table 2) for reflectivities $\geqslant 60$ dBZ. A similar run with an extended 57 dBZ maximum storm was handled correctly. As noted in Section 2, the drop size distributions are not necessarily applicable for reflectivities >60 dBZ, and in addition, for many climates hail is likely for high Z. Cases of divergence of the solution are indicated in Table 2 by asterisks located next to the r_e values. The attenuation estimation scheme was terminated in range at the point where the attenuated reflectivity dropped below 10 dBZ.

a. Effect of radar calibration error

Hitschfeld and Bordan (1954) and Fedi (1974) point out that errors in radar calibration can inhibit attempts

TABLE 2. Ratios of attenuated rainfall amount to true amount (r_a) and attenuation corrected rainfall amount to true amount (r_e) for various conditions of drop size distribution, temperature, radar calibration error and maximum storm reflectivity. The asterisk next to some r_e values indicates the iterative attenuation estimation calculation diverged and had to be stopped prematurely. In the drop size distribution and temperature column, (1) indicates values used for attenuation calculations and (2) values used for correction calculations.

Dropsize distribution		Temperature (°C)		Radar calibration error (dB)	$Z_{\max}$ (dBZ)	r_a	r_e
1	2	1	2				
MP	MP	0	0	0	40	0.91	1.00
MP	MP	0	0	0	50	0.78	1.00
MP	MP	0	0	0	60	0.35	1.00
MP	MP	0	0	0	60 (8 km)	0.14	1.45*
MP	MP	10	10	−4	60	0.19	0.24
MP	MP	10	10	−2	60	0.27	0.39
MP	MP	10	10	2	60	0.51	0.98*
MP	MP	10	10	4	60	0.69	1.97*
MP	MP	10	0	0	60	0.37	0.60*
MP	MP	0	10	0	60	0.35	0.74
MP	SS	10	10	0	60	0.37	0.59*
SS	MP	10	10	0	60	0.32	0.71
MP	SS	10	0	−2	60	0.27	0.15
MP	SS	10	0	0	60	0.37	0.65*
MP	SS	10	0	+2	60	0.51	1.22*
SS	MP	0	10	−2	60	0.20	0.28
SS	MP	0	10	0	60	0.28	0.51
SS	MP	0	10	+2	60	0.40	0.71*

to correct received radar reflectivities for attenuation. The second section of Table 2 illustrates the effects of radar calibration errors ranging from −4 to +4 dB in 2 dB steps. The test storm used here is the 60 dBZ storm illustrated in the lower-left corner of Fig. 2 and in line 3 of Table 2. Here we see that the effects of radar miscalibration are serious, producing errors in the range-integrated radar rainfall as large as $r_a=0.69$ and 0.19, and $r_e = 1.97$ and 0.24 for +4 and −4 dB calibration errors, respectively. The value of $r_e=1.97$ for the +4 dB calibration error illustrates that when the calibration error is larger than the maximum attenuation (dB km^{-1}), attempts to correct for attenuation degrade the rainfall estimate in comparison with the attenuated rainfall estimate. Thus, for 5 cm radar calibrated within ±2 dB, no attempts to correct for attenuation should be made for storms with reflectivity maxima of about 45 dBZ or less.

In the illustrated case of the 60 dBZ maximum storm with a ±2 dB calibration error, the attenuation estimation scheme improves the values of $r_a=0.51$ and 0.27 to values of $r_e=0.98$ and 0.39, respectively. Although these r_e values are closer to 1.0 than the r_a values, the $r_e=0.98$ is a fortuitous result of early termination of the calculation, and $r_e=0.39$ still does not represent an accurate precipitation estimate.

b. *Effect of random errors*

For the small sample sizes used in many experiments, the measured radar reflectivity factor may have a sampling error on the order of 1–2 dBZ. To simulate the effects of such errors on the attenuation estimation scheme, a set of random errors ranging from −2 to +2 dBZ was added to the attenuated power measurements. For all cases tried, the effects of these random errors were observed to be small in comparison to the other likely sources of error.

c. *Effect of temperature*

As mentioned in Section 2, the attenuation of the radar signal in precipitation is temperature dependent. The effects of an error in the assumed temperature are illustrated in the third section of Table 2, using the 60 dBZ test storm of Section 4a, no radar calibration error and a MP dropsize distribution. In all cases, the attenuation correction scheme improves rainfall estimates; however, it can be seen that assuming too high a temperature produces an underestimate of attenuation and assuming too low a temperature produces an overestimate of attenuation and consequent divergence of the iterative scheme. Since, in many cases, the temperature is known within better than 10°C (the temperature error illustrated here), temperature errors are not a serious source of error in rainfall estimation. This conclusion is in partial agreement with the findings of Waldteufel (1973) who suggested that temperature had a negligible effect on attenuation. Although the temperature effect is small, incorrect temperature assumptions will degrade attempts to correct for attenuation.

d. *Effect of drop size distribution*

The effects of the assumed drop size distribution are illustrated in the fourth section of Table 2, for a temperature of 10°C, no radar calibration error, and using the 60 dBZ tests storm of Section 4a. Here likely errors due to an incorrectly assumed drop size distribution are seen to be equivalent to likely temperature-induced errors and are small in comparison with the effects of attenuation. As before, the attenuation estimation scheme improves the quality of the rainfall estimate.

e. *Effects of multiple errors*

In the preceding paragraphs, it was shown that the effects of typical temperature and drop size distribution errors are small in comparison to the effects of attenuation. Radar calibration errors, however, present a serious problem, with the expected 1–2 dB calibration errors frequently producing significantly degraded attenuation estimates.

The effects of combinations of temperature, drop size distribution and calibration errors are shown in the last section of Table 2 for the 60 dBZ test storm used in the preceding sections. The combination of temperature and drop size distribution errors were chosen so that they both produced underestimates or overestimates of attenuation; for example, an underestimate of temperature was coupled with assumption of the SS distribution instead of MP, since both errors produced overestimates of attenuation in the attenuation estimation scheme. The values of r_a and r_e for 0 dBZ calibration error show that combined temperature and drop size distribution errors are equivalent to either error taken alone.

If a radar calibration error is added to the other errors, the attenuated rainfall estimates are significantly modified, and the attenuation estimation scheme results are degraded. Additional test calculations for other reflectivity distributions and assumed temperature and dropsize distribution errors confirm these results.

5. Results and conclusions

This study indicates that 5 cm radar measurements of storms with reflectivity maxima $\gtrsim 50$ dBZ will be seriously attenuated. These results agree with the findings of Geotis (1975) and Weible and Sirmans (1976), and suggest that some sort of attenuation estimation scheme must be implemented if 5 cm radars are used to measure rainfall from large storms.

An iterative attenuation estimation scheme is presented which removes a large portion of the effects of attenuation, thereby producing a significant improvement in rainfall estimates. The effects of radar calibration error and errors in assumed drop size distribution and temperature are considered, and it is shown that drop size distribution and temperature errors can significantly degrade the attenuation estimates, but that these degradations are small in comparison with the effects of radar calibration errors. It is shown that use of the iterative attenuation estimation scheme must be limited to cases in which the radar calibration error is smaller than the maximum attenuation (dB km^{-1}). For a 5 cm radar calibrated to within 2 dB, this limits use of the scheme to storms with reflectivity maxima $\geqslant 45$ dBZ. Due to limitations mentioned in Section 4, the scheme does not work well for storms with reflectivity maxima >60 dBZ. For many climates this does not represent a problem, because high reflectivity radar rainfall estimates are frequently inaccurate due to the presence of hail.

The condition that the iterative attenuation estimation scheme cannot be applied in cases when the maximum attenuation is less than the radar calibration error is similar to the infinite rainfall rate of Hitschfeld and Bordan (1954), and suggests that inaccuracies in radar calibration are likely to set the basic limit on any corrections for attenuation using single-wavelength radar.

This study of 5 cm radar attenuation thus suggests that the most viable method of obtaining precipitation estimates is with a fairly dense raingage network, used in conjunction with the radar in a manner similar to that suggested by Brandes (1975). The gages can "calibrate" the radar precipitation estimates, correcting for radar calibration errors, errors in assumed drop size distribution (*Z-R* relation) and the effects of attenuation and evaporation. The strong time-space variability of precipitation (Joss *et al.*, 1974) underscores the need for the gages and suggests the possibility that raingage spacing, for use in conjunction with radar, should be derived on the basis of the typical storm size—a measurable feature of the climate and storm type. Such a criterion would usually insure a gage within areas of significantly differing precipitation characteristics. As Hitschfeld and Bordan (1954) noted, accurate radar precipitation measurement requires a high enough gage density to assure a gage located in areas of significant attenuation. Due to the large time and space variability of precipitation, no sort of daily *Z-R* or *Z-K* relation or average value of gage-radar ratio will suffice. This conclusion suggests that an empirical *Z-K* approach is not likely to be promising. It should be noted, however, that in the case of large-area, long-term precipitation measurement (a case not considered here) many of the radar rainfall estimation problems may disappear in the average if carefully selected *Z-R* and *Z-K* relations are used. However, for most applications, the errors noted here and in other works underscore the ultimate reliance of accurate precipitation measurements on raingage data.

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